Chapter 8

Deontic Logic

Risto Hilpinen

8.1. Introduction

Deontic logic is an area of logic which investigates normative concepts, systems of norms, and normative reasoning. The word 'deontic' is derived from the Greek expression 'déon', which means 'what is binding' or 'proper'. Thus, Jeremy Bentham (1983) used the word 'deontology' for "the science of morality," and the Austrian philosopher Ernst Mally (1926), who developed in the 1920s a system of the "fundamental principles the logic of ought," called his theory 'Deontik'. Normative concepts include the concepts of obligation (*ought*), permission (*may*), prohibition (*may not*), and related notions, such as the concept of right. Systems of deontic logic contain, in addition to the usual sentential connectives and quantifiers, logical constants which represent some of these normative concepts.

Much of the recent work on deontic logic has been based on the view that deontic logic is a branch of modal logic [see chapter 7], and that the concepts of obligation, permission, and prohibition are related to each other in the same way as the alethic modalities *necessity, possibility* and *impossibility.* This view goes back to medieval philosophy; some fourteenth-century philosophers observed the analogies between deontic and alethic modalities, and studied the deontic (normative) interpretations of various laws of modal logic. In the same way, Leibniz (1930) called the deontic categories of the obligatory, the permitted and the prohibited 'legal modalities' (*Iuris modalia*), and observed that the basic principles of modal logic hold for the legal modalities. In fact, Leibniz suggested that deontic modalities can be defined in terms of the alethic modalities; according to him, the permitted (*licitum*) is

> what is possible for a good man to do

and the obligatory (*debitum*) is

> what is necessary for a good man to do

The contemporary development of deontic logic since the publication of von Wright's (1957 [1951]) pioneering paper "Deontic Logic" has been based on the study of the analogies between normative and alethic modalities.

8.2. The Standard System of Deontic Logic (SDL)

A simple system of deontic logic can be obtained by reading Leibniz's definition of the concept of obligation (ought) as

(O.Leibniz$_1$) p is obligatory for a iff (if and only if) p is necessary for a's being a good person

that is,

(O.Leibniz$_2$) $O_a p$ iff $N(G(a) \supset p)$

where 'N' is the alethic necessity operator and '$G(a)$' means that a is 'good' (in the sense intended by Leibniz). Deleting the explicit reference to an agent gives the following definition of the concept of ought:

(O.Leibniz$_3$) $Op \equiv N(G \supset p)$

The corresponding Leibnizian concept of permission (or the concept of may) is expressed by

(P.Leibniz$_3$) $Pp \equiv M(G \,\&\, p)$

(where 'M' is the operator for alethic possibility). These schemata can be regarded as partial reductions of deontic logic to 'ordinary' (alethic) modal logic. The Leibnizian analysis of the concepts of obligation and permission was rediscovered by the Swedish philosopher Kanger in 1950, who interpreted the constant G as 'what morality prescribes' (Kanger, 1981 [1957]). According to this interpretation, Op (it ought to be the case that p) means that p follows from the requirements of morality. Anderson (1967 [1956]) put forward a reduction schema equivalent to Kanger's,

($O.S$) $Op \equiv N(\neg p \supset S)$

where S may be taken to mean the threat of a sanction or simply the proposition that the requirements of law or morality have been violated.

If the alethic N-operator satisfies the axioms of the modal logic **T** (Chellas, 1980, p. 131) [or see chapter 7], viz.

(K) $N(p \supset q) \supset (Np \supset Nq)$

(T) $Np \supset p$

and the modal 'rule of necessitation'

(RN) If p is provable, Np is provable, or briefly, p/Np

it is easy to see that the ought-operator defined by (O.Leibniz$_3$) satisfies the deontic K-principle

(K_D) $O(p \supset q) \supset (Op \supset Oq)$

and the rule of 'deontic necessitation'

(RN_D) p/Op

The additional assumption that being good is possible,

(D_G) MG

yields the principle of deontic consistency

(D_D) $Op \supset Pp$

where 'P' represents the concept of permission, definable in terms of 'O' by

(P) $Pp \equiv \neg O \neg p$

Similarly, the concept of prohibition, F, is defined by

(F) $Fp \equiv O\neg p$

where a state of affairs p is prohibited iff not-p is obligatory. The system of (propositional) deontic logic obtained by adding to propositional logic the axioms (or axiom schemata) K_D and D_D and the rule RN_D is usually called the 'standard system of deontic logic' (SDL). Among its theorems are:

$O(p \& q) \supset (Op \& Oq)$	(Conjunctive distributivity of O)	(8.1)
$Op \& Oq \supset O(p \& q)$	(Aggregation principle for O)	(8.2)
$Op \supset O(p \vee q)$		(8.3)
$O(p \supset q) \supset (Pp \supset Pq)$		(8.4)
$Pp \supset P(p \vee q)$		(8.5)
$P(p \vee q) \supset (Pp \vee Pq)$	(Disjunctive distributivity of P)	(8.6)
$P(p \& q) \supset Pp$		(8.7)

while the rules of inference

(RM_D) $\quad p \supset q / Op \supset Oq$

(RE_D) $\quad p \equiv q / Op \equiv Oq$

are derivable. On the basis of the axioms K_D and D_D, this system may be called the system **KD**, or simply **D**; it is a member of the family of ***normal*** modal logics, all of which contain (a counterpart of) the rule RN; [see chapter 7] (Chellas, 1980, p. 114).

8.3. The Semantics of the Standard Deontic Logic

The sentences of SDL can be interpreted in terms of possible worlds (or world states) in the same way as other normal modalities. A possible worlds' interpretation of SDL is a triple $M = \langle W, I, R \rangle$, where W is a universe of possible worlds, I is an interpretation function which assigns to each sentence a subset of W, i.e., the worlds $u \in W$ where the sentence is true; the truth of p at u under M is expressed '$M, u \vDash p$,' or briefly '$u \vDash p$.' If p is not true at u, it is false at u. R is a 2-place relation on W, called the relation of deontic alternativeness. The interpretation function assigns each sentence a truth value at each possible world. A sentence is called ***valid*** (logically true) iff it is true at every world $u \in W$ for any interpretation M, and q is a logical consequence of p iff there is no interpretation M and world u such that $M, u \vDash p$ and not $M, u \vDash q$. The interpretation function is subject to the usual Boolean conditions which ensure that the truth-functional compounds of simple sentences receive appropriate truth-values at each possible world. The alternativeness relation R is needed for the interpretation of sentences involving the deontic operators. In the semantics of modal logic, necessary truth at a given world u is understood as truth at all worlds which are possible relative to u or ***alternatives*** to u, and possibility at u means truth at some alternative to u. For the concepts of obligation (or ought) and permission (may), these conditions can be formulated as follows:

(CO) $\quad u \vDash Op$ iff $v \vDash p$ for every $v \in W$ such that $R(u, v)$

(CP) $\quad u \vDash Pp$ iff $v \vDash p$ for some $v \in W$ such that $R(u, v)$

For the axiom D_D to be valid, it is necessary to regard R as a serial relation, in other words,

(CD) $\quad$ For every $u \in W$, there is a $v \in W$ such that $R(u, v)$

Different further assumptions about the structural properties of the R-relation validate different deontic principles, and lead to different systems of deontic logic. For example, it is clear that

$$Op \supset p \tag{8.8}$$

is not a logical truth, and therefore R cannot be assumed to be a reflexive relation, but the principle

$$O(Op \supset p) \tag{8.9}$$

seems a valid principle of deontic logic: It ought to be the case that whatever ought to be the case is the case. The validity of (8.9) follows from the assumption that R is secondarily reflexive, in other words,

(C.OO) If $R(u, v)$ for some u, then $R(v, v)$.

((8.9) is not derivable in SDL, but it can be added as an additional axiom. It is derivable from the Kanger–Anderson reduction in alethic modal logic if that logic contains T.)

The semantics sketched above, due initially to Hintikka (1957, 1981) and Kanger (1981 [1957]), may be termed the 'standard semantics' of deontic logic. It gives an intuitively plausible account of the meanings of simple deontic sentences when the deontic alternatives to a given world u are taken to be worlds (or situations) in which everything that is obligatory at u is the case; they are worlds in which all obligations are fulfilled. Hence, the worlds related to a given world u by R may be termed ***deontically perfect*** or ***ideal*** worlds (relative to u). If possible worlds are regarded as possible courses of events or histories which are partly constituted by an agent's actions, the semantics of SDL simply divides such histories into deontically acceptable and deontically unacceptable histories. An action is permitted iff it is part of some deontically acceptable course of events or if there is some deontically acceptable way of performing the action, and an action is obligatory iff no course of events is acceptable unless it exemplifies the action in question. The set of acceptable courses of action (relative to a given action situation) may be termed the ***field of permissibility*** (Lewis, 1979). According to the deontic consistency principle (CD), the field of permissibility is never empty; some action is permissible in any situation.

8.4. Problems and Paradoxes

SDL, like any logical system designed for certain applications, faces two kinds of problems:

(a) Problems of interpretation and application:
How should the deontic operators O, P, and F and the non-logical (propositional) symbols p, q, r, . . . , of the system be interpreted, and how should the metalogical and semantic concepts ***truth***, ***validity***, and ***logical consequence*** be understood in this context?

(b) Problems about the adequacy of the formalization of normative reasoning provided by the standard system:
Does SDL give an adequate and correct account of the logical relationships among norms or normative propositions?

These questions (or classes of questions) are obviously interrelated; the adequacy of a system of deontic logic depends on its interpretation. Both questions have been discussed extensively in the recent literature.

Deontic logic is usually defined as the logic of the basic normative concepts or, more generally, as the logic of normative or prescriptive discourse. This characterization gives rise to an interesting question about the metalogical concepts of validity and logical consequence in deontic logic. These concepts were defined above in the standard way in terms of the concept of truth, but norms and directives cannot be said to be true or false in the same sense as statements and assertions, and therefore the standard concepts of validity and logical consequence, familiar from the logic of descriptive or assertoric discourse, seem inapplicable to the logic of normative discourse. The Danish philosopher Jørgensen presented this observation in the 1930s as an objection to the very possibility of the logic of imperatives (commands). Since imperatives are not true or false, it does not, strictly speaking, make sense to speak about the logic of imperatives. Norms are, in this respect, analogous to imperatives. On the other hand, as Jørgensen (1937/8) observed, it seems clear that directives or imperatives can be inferred from other directives or that two directives can be logically inconsistent. This difficulty is called *Jørgensen's dilemma*; Makinson (1999) describes it as "the fundamental problem of deontic logic."

Many philosophers have proposed to solve this problem by making a distinction between two uses of norm sentences; they can be used for expressing norms or directives and for making normative statements (statements about norms). The latter are descriptive statements which state that something is obligatory, permitted or prohibited according to a given system of norms (Bulygin, 1982). For example, the deontic sentence

Motor vehicles ought to use the right-hand side of a road.

can be regarded as a directive addressed to drivers, or as a statement which gives information about the traffic code of some (unspecified) country. If it is regarded as a statement about the U.S. traffic regulations, it is a true statement; understood as a statement about the U.K. regulations, it is false. Normative statements, unlike the norms themselves, are true or false, and the logical relationships among normative statements can therefore be understood in the usual way. If deontic logic is regarded as the logic of normative statements, Jørgensen's problem does not arise.

This way out of Jørgensen's difficulty does not mean that the prescriptive or genuinely normative use of deontic sentences is not subject to logical laws, because the distinction between the normative and the descriptive use of deontic sentences can be understood as two ways of using normative *statements* (which are true or false). As Kamp (1973/4, 1979) has pointed out, a normative sentence, like the above

Motor vehicles ought to use the right-hand side of the road.

(which is true or false), can be used or uttered *performatively*, to create or sustain a norm, or *assertorically*, to describe an independently existing norm system. In the former case, the utterance of the statement in the appropriate circumstances (by a proper norm authority) has normative force, and is sufficient to make the statement true; in the latter case, the truth of the statement depends on whether it fits a norm system whose content is independent of the utterance in question. Thus the prescriptive–descriptive distinction coincides with the distinction between two uses of deontic statements, the performative use and the assertoric use; and, in both cases, the statements in question can be regarded as true or false. Consequently, the concepts of validity and logical consequence can be defined in deontic logic in terms of the concept of truth in the same way as in other areas of logic. According to Kamp, the assertoric use of deontic sentences should depend on their performative use. In their performative use, the function of *O*- and *F*-sentences (obligation and prohibition sentences) is to restrict the range of normatively acceptable options available to an agent (the addressee), whereas permission-sentences have the opposite effect; they enlarge the set of deontically acceptable action possibilities. For example, Kamp has put forward the following principle concerning the performative and assertoric uses of permission sentences:

(PP) An assertoric utterance of a permission sentence *Ps* in a context *c* is true iff all those worlds already belong to the options of the agent that a performative use of *Ps* would have added to the set of the agent's options if they had not already belonged to it.

Kamp has also observed that it is not always clear whether a deontic sentence is used performatively or assertorically. However, if the assertoric use of deontic sentences is governed by (PP) (and by analogous principles for ought-sentences and prohibition-sentences), assertoric utterances of deontic sentences can guide and direct the agent's actions in the same way as their performative utterances. For example, in the case of a permission sentence, "either the utterance is a performative and creates a number of new options, or else it is an assertion; but then if it really is appropriate it must be true; and its truth then guarantees that these very same options already exist" (Kamp, 1979, p. 264). The practical consequences of the utterance for the addressee are the same in both cases.

According to SDL, deontic logic is a branch of modal logic, and many principles of deontic logic are special cases of more general modal principles. This approach to deontic logic has sometimes been criticized on the ground that it ignores or misrepresents many significant features of normative discourse which distinguish it from other varieties of modal discourse. It has been argued that some principles of SDL, including some of the principles (8.1)–(8.7) listed above, lead to paradoxes and are therefore unacceptable. For example, some philosophers have felt that there is something paradoxical about the formula (8.3), which says that if it ought to be the case that p, then it ought to be the case that $p \vee q$. (8.3) authorizes, e.g., the inference from the directive (8.10) to (8.11):

Peter ought to mail a letter. (8.10)

Peter ought to mail a letter or burn it. (8.11) which seems to some an unacceptable inference. This is known as Ross's paradox, originally due to the Danish philosopher Alf Ross (1941). A somewhat similar (putative) paradox depends on principle (8.5), according to which the permissibility of p entails the permissibility of $p \vee q$ (for any q); for example, according to (8.5), (8.12) entails (8.13):

Peter may drink water. (8.12)

Peter may drink water or drink whisky. (8.13)

which also seems counter-intuitive. These inferences are of course valid if sentences (8.10)–(8.13) are understood in terms of the possible worlds semantics outlined above. If Peter mails a letter in all deontically perfect situations, then he mails a letter or burns it in all such situations, and if Peter drinks water in some deontically satisfactory situation, then he drinks water or whisky in some such situation. But this may be taken as evidence that the semantics of SDL fails to do justice to significant features of normative discourse.

The inferences in question may seem especially paradoxical if the sentences (8.10)–(8.13) are thought of as being used performatively or normatively, i.e., if they are used for issuing a norm or a permission and not merely for describing the content of a system of norms. It is obvious that the effects of a normative utterance of (8.11) are not the same as the effects of (8.10); unlike (8.10), (8.11) does not suffice to make the action of posting the letter obligatory or required (for Peter). (8.10) excludes more possibilities (restricts the field of permissibility more) than (8.11). In the same way, the normative effects of a performative utterance of (8.13) are not the same as those of (8.12). If (8.12) is used performatively, it opens (makes permitted) some possibilities in which Peter drinks water, but (8.13) opens a more vaguely defined set of possibilities, namely, some possibilities where Peter drinks water or whisky. The claim that the inference of (8.11) and (8.13) from (8.10) and (8.12) is paradoxical or unacceptable seems to be tacitly based on the following principle:

(IntP) If a norm (permission) N_1 entails N_2, then the normative (performative) effects of N_1 entail the effects of N_2.

But this principle is obviously false; logical deduction should not be expected to preserve the effects of a norm on a norm system any more than logical deduction preserves the effects of the acceptance of a declarative statement on a person's belief system (or corpus of knowledge). The effects of the acceptance of a disjunctive proposition on a person's belief system are quite different, and usually less significant, than the effects of the acceptance of one of the disjuncts; a disjunctive belief adds less content to a belief system than either of the disjuncts.

The apparent paradox related to disjunctive permissions seems more interesting than Ross's paradox. According to the SDL, the normative use of (8.13) should make acceptable some worlds (or situations) in which Peter drinks water or whisky. This can be accomplished by allowing some situations in which Peter drinks water. However, a normative utterance of (8.13) is normally taken to permit some situations in which Peter drinks whisky as well as some situations in which Peter drinks water; in other words, (8.13) usually seems to have the same effect as the utterance of the conjunction

Peter may drink water and Peter may drink whisky. (8.14)

A disjunctive permission seems to offer a choice between the two disjuncts and thus entail a conjunction of two permissions. This feature of disjunctive permissions cannot be explained on the basis of SDL alone, but depends on some pragmatic features of disjunctive permissions. However, a disjunctive permission does not necessarily permit both disjuncts, but may leave the determination of the field of permissibility partly open, as in the case of the statement (Kamp, 1979, p. 271)

Yes, you may drink water or whisky, but you have to consult your doctor before you drink whisky. (8.15)

(8.15) may be an instance of a normative (performative) use of a permission sentence; a norm authority makes a disjunctive action permitted, but refers to another authority for the determination of the permissibility of one of the disjuncts. This suggests that the principle

$$P(p \vee q) \supset Pp \;\&\; Pq \tag{8.16}$$

should not be regarded as a general principle for the concept of permission.

Another much discussed paradox is related to rule (RM_D). If q is a logical consequence of p, then, according to (RM_D), Op entails Oq. Since knowing that p entails the truth of p,

$$OK_a p \supset Op \tag{8.17}$$

is a valid formula, where '$K_a p$' means that a knows that p. For example, if Gladys, who is a firefighter, ought to know that there is a fire, then, according to (8.17), there ought to be a fire, which is quite counter-intuitive (Åqvist, 1967). In other words, according to (8.17), the following statements cannot be all true:

$$p \supset OK_a p \tag{8.18}$$

$$p \tag{8.19}$$

$$O\neg p \tag{8.20}$$

But a situation in which there is a fire, (8.18)–(8.20) seem all true; if there is a fire, Gladys ought to know it, but there ought not to be a fire. Some philosophers have regarded this paradox (the ought-to-know paradox or the paradox of epistemic obligation) and other similar paradoxes as evidence that (RM_D) is not a valid principle of deontic logic (Goble, 1991).

8.5. Conditional Norms

In the example above, (8.18) expresses a conditional obligation: Gladys ought to know that there is a fire if there is one, not otherwise. As was observed above, in the semantics of SDL, the interpretation of deontic sentences is based a division of possible worlds or situations into 'deontically perfect' or normatively faultless worlds and normatively unacceptable or imperfect worlds. Systems of conditional norms (conditional obligations) are often semantically more complex, and an attempt to formalize them in SDL is apt to lead to paradoxes. Chisholm (1963) has given an example of such a set: The following sentences seem jointly consistent and pairwise logically independent:

(Ch1) Jones ought to go to help his neighbors.

(Ch2) Jones ought to tell his neighbors he is coming if he is going to help them.

(Ch3) If Jones does not go to help his neighbors, he ought not to tell them he is coming.

(Ch4) Jones does not go to help his neighbors.

In the language of SDL, these sentences might be expressed as follows:

$$Oh \tag{8.21}$$

$$O(h \supset t) \tag{8.22}$$

$$\neg h \supset O\neg t \tag{8.23}$$

$$\neg h \tag{8.24}$$

where h says that Jones goes to help his neighbors, and t says that Jones tells his neighbors that he is coming. According to SDL, (8.21) and (8.22) entail

$$Ot \tag{8.25}$$

and (8.23) and (8.24) entail

$$O\neg t \tag{8.26}$$

by propositional logic, in other words, (8.21)–(8.24) entail

$$Ot \,\&\, O\neg t \tag{8.27}$$

and according to the consistency principle (D_D), (8.27) entails

$$Ot \,\&\, \neg Ot \tag{8.28}$$

Thus the suggested interpretation of (Ch1)–(Ch4) makes them jointly inconsistent. This seems intuitively unsatisfactory; (Ch1)–(Ch3) seem a reasonable and consistent set of requirements, and the fact that Jones does not go to help his neighbors should not make them jointly inconsistent.

It may be suggested that there is an unjustified logical asymmetry between (8.22) and (8.23); in (8.22), the O-operator precedes $\supset$, but, in (8.23), their order is reversed. The corresponding asymmetry between (Ch2) and (Ch3) does not seem to be semantically significant. If (Ch2) is represented by

$$h \supset Ot \tag{8.29}$$

or (Ch3) is formalized as

$$O(\neg h \supset \neg t) \tag{8.30}$$

the contradiction is avoided; (8.21) and (8.29) do not entail (8.25), and (8.24) and (8.30) do not entail (8.26). However, according to SDL, (8.29) is a logical consequence of (8.24), and, on the other hand, (8.30) is a logical consequence of (8.21). Both results are intuitively unacceptable; as was noted above, the sentences (Ch1)–(Ch4) seem to be pairwise logically independent of each other.

Sentence (Ch3) tells what Jones ought to do in a situation where he has failed to fulfill his duty to help his neighbors; it expresses a *contrary-to-duty* (CTD) obligation. For this reason, Chisholm's paradox may also be called the paradox of CTD obligation. Chisholm's example shows that systems of norms which contain both primary obligations and CTD obligations cannot be formalized in SDL in a satisfactory way. Some authors have proposed to avoid the inconsistency of between (8.25) and (8.26) by relativizing the concept of obligation (or the concept of ought) to time since, it has been suggested, e.g., by Åqvist and Hoepelman (1981), (8.25) and (8.26) hold at different points of time. However, this does not seem to be an essential feature of Chisholm's paradox. There are many non-temporal versions of the CTD-paradox, such as the following situation: Assume that dogs are not permitted in a certain village, but if anyone happens to have a dog, there ought to be a warning sign about it in front of the owner's house. Moreover, warning signs ought not to be posted without sufficient reason. Thus the following normative statements seem to be true:

(Ds1) There ought to be no dog.

(Ds2) There ought to be no warning sign if there is no dog.

(Ds3) If there is a dog, there ought to be a warning sign.

(Ds4) There is a dog.

(Ds1)–(Ds4) are formally analogous to Chisholm's example, and an attempt to formalize them in SDL leads to a similar inconsistency (Carmo and Jones, 2001; Prakken and Sergot, 1997).

The deduction of the contradiction (8.27) from (8.25) and (8.26) depends on the principle of normative consistency (D_D), $Op \supset \neg O\neg p$. This principle has been criticized independently of Chisholm's example. (D_D) excludes the possibility of normative conflicts, but such conflicts are not unusual in morality and law, and it may be argued that they do not amount to paradoxes (Chellas, 1974, p. 24; Goble, 1999, p. 332). If the consistency principle is rejected, the aggregation principle (8.2), $Op \,\&\, Oq \supset O(p \,\&\, q)$, should be rejected as well, because the latter principle undermines the distinction between a conflict between obligations and the existence of a self-contradictory obligation; the recognition of the possibility of normative conflicts does not mean that one should also admit the possibility of self-contradictory obligations. Thus logicians have developed systems of deontic logic in which (D_D) and the aggregation principle do not hold (Chellas, 1980, pp. 201–2). Nevertheless, such systems do not help to give a satisfactory solution to the puzzles about the CTD-obligations. They enable one to conclude only that CTD-situations involve conflicting obligations without offering any analysis of CTD-obligations and their relationship to the 'primary' obligations.

It is not difficult to see why Chisholm's example cannot be represented in a satisfactory way in SDL. As observed above, the semantics of SDL is based on a division of worlds or situations into acceptable (deontically perfect) and unacceptable worlds, and the *O*-sentences describe how things are in the deontically perfect worlds. But sentence (Ch3) in Chisholm's example does not tell how things are in a deontically faultless world; it tells what the agent (Jones) ought to do under deontically imperfect conditions, i.e., in situations in which Jones does not act in accordance with his duties. (Ch3) is a ***contrary-to-duty*** obligation. The situation could be described by saying that among the (less than perfect) worlds where Jones does not fulfill his duty to help his neighbors, those in which he does not tell them he is coming are preferable to the circumstances where he makes a false promise Thus the interpretation of Chisholm's example seems to require a distinction between different degrees of deontic perfection.

According to this interpretation, (Ch2) can be taken to mean that, in deontically perfect circumstances, where Jones is going to help his neighbors, he tells them that he is coming, and (Ch3) says that in the best worlds where he is not going to help his neighbors, he does not tell them he is coming (Hansson, 1981, p. 143). Express these conditional obligations of by

$$O(t/h) \tag{8.31}$$

$$O(\neg t/\neg h) \tag{8.32}$$

respectively. Call worlds where p is true simply 'p-worlds'; in other words, u is a p-world iff $u \in I(p)$. Let the p-worlds that are normatively least objectionable relative to a given situation u be called deontically optimal p-worlds relative to u, briefly, $\mathrm{Opt}(p, u)$. The concept of deontically optimal p-world is a generalization of the concept of a deontically perfect world of SDL; the (absolute) deontic perfection of w relative to u, i.e., $R(u, w)$, can be taken to mean that w is $\top$-optimal relative to u, where $\top$ is a logical truth:

(COT) $\quad Op =_{\text{df}} O(p/\top)$

The assumption that for any consistent proposition p, there is a nonempty set of deontically optimal p-worlds, is a generalization of the principle (CD) of SDL, i.e., the principle that any world has a nonempty set of deontic alternatives. The truth of the conditional ought-statement $O(q/p)$ at u can be taken to mean that q is true in all deontically optimal p-worlds (relative to u), i.e.,

(CO.cond) $\quad u \vDash O(q/p)$ iff q is true in every world $w \in \mathrm{Opt}(p, u)$

If p entails r and p is true in some r-optimal world (relative to u), the p-optimal worlds (relative to u) are obviously the r-optimal worlds where p is true; in other words, the concept of optimality (or relative deontic perfection) is subject to the following condition:

(C.Opt) $\quad$ If $I(p) \subseteq I(r)$ and $I(p) \cap \mathrm{Opt}(r, u)$ is non-empty, then $\mathrm{Opt}(p, u) = I(p) \cap \mathrm{Opt}(r, u)$.

Thus, according to (C.Opt), the truth of

$$Op \tag{8.33}$$

means that $\mathrm{Opt}(p, u) = \mathrm{Opt}(\top, u)$, and

$$O(q/p) \tag{8.34}$$

means that all optimal p-worlds are q-worlds; hence, according to (C.Opt), (8.33) and (8.34) entail

$$Oq \tag{8.35}$$

Hence, according to this semantics, the principle of 'deontic detachment'

(DDet) $\quad O(q/p) \supset (Op \supset Oq)$

is a valid principle for conditional obligations. On the other hand, the principle of 'factual detachment'

(FDet) $\quad O(q/p) \supset (p \supset Oq)$

does not hold. If (Ch2) and (Ch3) are interpreted in this way, (Ch1)–(Ch4) do not lead to a contradiction; (Ch1) and (Ch2) entail the obligation Ot (8.25), but (Ch3) and (Ch4) do not entail $O\neg t$ (8.26).

Another possible response to Chisholm's paradox is the replacement of the truth-functional conditional in (8.22) and (8.23) by an intensional or subjunctive conditional (Mott, 1973) or even a relevant conditional (Goble, 1999) without introducing a special concept of conditional obligation. It has been known since the beginning of the twentieth century, indeed, from antiquity, that conditional statements are usually not truth-functional. Philosophers have attempted to represent if-then-sentences as truth-functional (or 'material') conditionals for want of a better theory, but the situation changed in the early 1970s when David Lewis (1973) and others developed intensional theories of conditionals. In the representation of Chisholm's example in SDL, the logical asymmetry between (8.22) and (8.23) is required by the assumption of the logical independence of (Ch1)–(Ch4), and this makes it possible to derive the inconsistency (8.25)–(8.26). If the two conditionals are expressed as intensional conditionals with a Lewis-type semantics, this problem does not arise. An intensional conditional, e.g., a subjunctive conditional, 'q if p' can be regarded as true in a situation u iff q is true in all possible worlds (situations) in which p is true but which resemble u in other respects as much as possible. The truth of such a conditional is not a consequence of the falsity of p (or of the truth of q) at u.

If the conditional 'q if p' is symbolized '$p > q$', and (Ch2) and (Ch3) are represented by

$$h > Ot \tag{8.36}$$

$$\neg h > O\neg t \tag{8.37}$$

respectively, no contradiction will arise. If the counterpart of the *modus ponens* principle holds for the conditional connective, i.e., if

(FDet>) $\quad (p > q) \supset (p \supset q)$

is logically true, (Ch3) and (Ch4) entail (8.26), but (Ch1) and (Ch2) do not entail (8.25). The former analysis of conditional obligations, as $O(q/p)$, leads in Chisholm's example to the result that Jones ought to tell his neighbors that he is coming to help them, but the second analysis, $p > Oq$, gives the result that Jones ought not to tell his neighbors that he is coming. Thus the two analyses seem to involve two different senses of 'ought' (or 'obligation'). The first interpretation of (Ch1)–(Ch4) seems to take the statements as expressions of 'ideal' or *prima facie* obligations; (Ch1)–(Ch2) can be regarded as saying that insofar as Jones ought to help his neighbors, he ought to tell them that he is coming – but if he is in fact not going to help his neighbors, he has an 'actual' or practical obligation not to tell them he is coming. There seems to be no logical or deductive connection between the two kinds of ought, but the existence of an ideal (or prima facie) obligation serves as evidence for the corresponding practical obligation. The inference of actual obligations from ideal obligations is an abduction rather than a deduction.

Both forms of conditional obligation, $O(q/p)$ and $p > Oq$, are defeasible in the sense that they do not satisfy the principle of strengthening the condition of the obligation or strengthening the antecedent of the conditional; in other words, the principles

$$O(q/p) \supset O(q/p \,\&\, r) \tag{8.38}$$

$$(p > Oq) \supset ((p \,\&\, r) > Oq) \tag{8.39}$$

are not valid. However, according to Lewis's (1973) semantics for subjunctive conditionals, the counterpart of *modus ponens* holds for the $>$-connective, making the detachment of actual obligations from factual premises possible; the Lewis-type conditionals are 'strict,' albeit only 'variably strict.' In the recent literature, however, many authors have analyzed conditional obligations (including CTD-obligations) by means of defeasible conditionals for which *modus ponens* does not hold, for example, when the conditional $p > q$ is read 'Normally, q holds in circumstances p' (Alchourrón, 1993, p. 75; Makinson, 1993, pp. 363–5). According to this interpretation, (8.24) and (8.37) do not entail (8.26) in the standard sense of logical consequence, but provide only evidence for it. Different variants of Chisholm's example and the attempts to represent various CTD-obligations and other conditional obligations in formal systems of deontic logic have generated an extensive literature on the subject; see Carmo and Jones (2001) and the articles in Nute (1997).

8.6. On the Representation of Actions in Deontic Logic

Above, the schematic letters p, q, r, etc., are propositional symbols; they represent propositions. However, in informal normative discourse deontic concepts are usually applied to actions rather than propositions. Philosophers have made a distinction between two kinds of ought, the ought-to-be (*Seinsollen*) and the ought-to-do (*Tunsollen*) – see, for example, Castañeda (1972 [1970]) – and it has been suggested that since the deontic operators of SDL are propositional operators, the standard deontic logic and the extensions and revisions discussed above should be regarded as theories of the ought-to-be rather than theories of the ought-to-do. It has been argued that in a satisfactory theory of the ought-to-do, deontic operators should be construed as action modalities rather than propositional modalities. Deontic concepts were understood in this way by Leibniz and by other authors of the seventeenth and the eighteenth centuries (Hilpinen, 1993a, pp. 85–6). Von Wright's first system of deontic logic (1957 [1951]) can be regarded as an attempt to articulate and formalize this view. In this system, the deontic operators O, P and F are prefixed, not to propositional expressions (statements), but to expressions for action-types or, in von Wright's terminology, 'act-qualifying properties.' Castañeda (1972 [1970], 1981) adopted a similar approach; he stressed the importance of distinguishing action-descriptions, or *practitions*, from propositional expressions. According to Castañeda, deontic reasoning is reasoning about practitions (as opposed to propositions which describe the conditions or circumstances of action), and deontic operators can be applied only to practitions, not to propositions.

Von Wright's and Castañeda's distinction between propositions and action terms (or practitions) has been formalized and developed further in dynamic deontic logic; see Czelakowski (1997), Meyer (1988) and Segerberg (1982). In dynamic logic, the interpretation of action terms reflects the common philosophical view that an action can often be described as the bringing about of a change in the world. According to this interpretation, an action transforms a given situation or a world-state into a new state (or keeps it unchanged). For example, in his 'action-state semantics' for imperatives, Hamblin (1987) has analyzed actions or deeds in terms of successive world-states. Thus the distinction between propositions and action terms is interpreted in the semantics of (dynamic) deontic logic as the distinction between sets of world-states (propositions) and relations between world-states. Let A, B, C, . . . be action terms or action descriptions. Action terms can be simple or complex; the latter are formed from simple action terms by act-connectives, some of which are analogous to propositional connectives. For example, if A and B are action terms, the following expressions are also action terms:

(ActT1) $A + B$: doing A or B

(ActT2) $A \bullet B$: doing A and B together

It is also convenient to have an expression for the omission of an act,

(ActT3) OmA: omitting A

OmA is applicable to all actions (world state transitions) which fail to exemplify A. Systems of dynamic logic usually also contain act-connectives which have no counterparts in propositional logic, for example,

(ActT4) $A;B$: A followed by B

(ActT5) A^*: doing A a finite number of times

For the sake of simplicity, this chapter considers only complex actions of types (ActT1–3).

Actions change the world, thus an action in a space W of possible worlds or situations may be interpreted as a binary relation, i.e., as a set of ordered pairs (u, w) such that the action in question can transform the first situation into the second. The ordered pairs assigned to an action-term A may be called the possible *performances* of the action A. A world-state w is said to be possible relative to u or accessible from u iff it is possible for some action or sequence of actions to lead from u to w. Denote the accessibility relation by Poss, and let Poss_u be the set of transitions which originate from u. The semantics of SDL can be applied to action terms in a relatively straightforward way. In SDL, possible worlds are divided into acceptable (deontically correct) and unacceptable worlds; and, in the deontic logic of action, world state transitions can be divided in an analogous way into deontically acceptable (legal) and deontically unacceptable (illegal) transitions. Let Leg_u be the set of legal transitions which originate from u, and let Ill_u be the set of illegal transitions from u. It is assumed that any possible transition from u is either legal or illegal (there is no deontic indeterminacy), and no transition is both legal and illegal; in other words

(Ddet) $\mathrm{Leg}_u \cup \mathrm{Ill}_u = \mathrm{Poss}_u$

(Dcons) $\mathrm{Leg}_u \cap \mathrm{Ill}_u = \varnothing$

The assumption that there is some legal way out of every situation, in other words,

(DactD) For every $u \in W$, Leg_u is nonempty

corresponds to principle (D) of SDL, i.e., the postulate that every world (situation) has some deontic alternative. Let I be an interpretation function which assigns to each action A its possible performances (a subset of $W \times W$), and let $I_u(A)$ be the performances of A which originate from u; thus $I_u(A) \subseteq \mathrm{Poss}_u$.

The basic normative concepts (deontic action modalities) can be defined by these truth-conditions:

(CF.act) $u \models FA$ iff $I_u(A) \subseteq \mathrm{Ill}_u$

(CP.act) $u \models PA$ iff $I_u(A) \cap \mathrm{Leg}_u \neq \varnothing$

(CO.act) $u \models OA$ iff $I_u(OmA) \subseteq \mathrm{Ill}_u$

These definitions are simple generalizations of the truth-conditions of normative propositions in SDL. According to (CF.act), an act A is prohibited in a given situation if every possible performance of A at that situation is illegal, and A is permitted iff it can be performed in a legal way. According to (CO.act), A is obligatory at u iff its omission at u would be illegal.

According to (CP.act), the permissibility of an action A means that some possible performances of A (in a given situation u) are deontically acceptable. For example, A may be permitted in this sense if it can be performed together with some other acts. This is a 'weak' concept of permission which corresponds to that defined in SDL. In the present framework, it is possible to define another concept of permission which may be termed a strong permission. When one says that an act A is permitted in a given situation, one often means that A itself is not illegal, i.e., that no sanction is attached to A, and not only that some (possible) performances of A would be deontically acceptable in the situation. This sense of permission can also be expressed in the form of a conditional; if the agent a were to do A, a would not do anything illegal. The truth-conditions of such a conditional can be formulated by means of a selection function f which selects from $I_u(A)$ the transitions which exemplify A but change the original situation u in other respects in a 'minimal' way. Such transitions may often be described by saying that the agent does *only* A. The concept of strong permission may be defined as

(CP^s.act) $u \models P^sA$ iff $f(I_u(A), u) \subseteq \mathrm{Leg}_u$.

One might say that the f-function selects from $I_u(A)$ the ***minimal*** performances of A. For example, if Oscar's mother gives him permission to take one cookie, it means that the action of taking one cookie is acceptable; in other words, the mother would not punish Oscar if he were to take one cookie and do nothing else. On the other hand, it is permitted for a driver to flash her right turn signal – but only if she is going to make a right turn as well. The latter action is an example of weakly permitted action, whereas the former action (taking a cookie) is strongly permitted.

The formulation (CP^s.act) is analogous to one of the standard ways of expressing the truth-conditions of conditionals by means of a selection function $f(I(p), u)$ which selects, for each proposition $I(p)$ and a situation u, the p-worlds closest (most similar) to u; a conditional $p > q$ is true at u iff the consequent q is true at all selected p-worlds. Thus (CP^s.act) fits the most natural reading of a strong permission to do A; if you were to do A, you would not be doing anything illegal. The selection function f used in (CP^s.act) selects the 'minimal' performances of A from the set of all possible performances of A, just as the truth of a conditional $p > q$ is determined by the selection of the p-worlds minimally different from the actual situation (or the situation where the conditional is being evaluated) (Hilpinen, 1993b, p. 309).

If the disjunctive permission 'You may do A or B' is interpreted as a strong permission in the sense defined by (CP^s.act), the truth of

$$P^s(A+B) \supset P^sA \ \& \ P^sB \tag{8.40}$$

depends on whether

$$f(I_u(A), u) \cup f(I_u(B), u) \subseteq f(I_u(A+B), u) \tag{8.41}$$

In other words, it depends on whether the minimal performances of a disjunctive act include the minimal performances of both disjuncts. The example (8.15) (on page 167) suggests that this need not always be the case; therefore (8.40) is not a logical truth, but it may hold in many cases; and, for pragmatic reasons, it may normally be expected to hold in situations in which permission sentences are used performatively, because otherwise it would not be clear what has been permitted, i.e., which performances of $A+B$ have been made deontically acceptable. In the example (8.15), the disjunctive permission is given together with the information that the permissibility of one of the disjuncts will be determined by another norm authority, and, consequently, there is no reason to assume that (8.40) should hold in the example.

8.7. Deontic Logic and the Logic of Agency

In most recent systems of the logic of the ought-to-do, simple action descriptions are not regarded as primitive terms, as outlined above, but are obtained from propositional expressions by means of an action operator which is usually read 'a sees to it that' or 'a brings it about that.' Thus simple action descriptions have the form $Do(a, p)$, where Do is a modal operator for action or agency, a names an agent, and p is a propositional expression. This analysis of action sentences goes back to the eleventh-century philosopher St. Anselm, who investigated the formal properties of the Latin verb *facere*, 'to do' (Segerberg, 1992).

Kanger (1972) presented an interesting analysis of the concept of seeing to it that p. He regarded a statement of the form 'a sees to it that p', $Do(a, p)$, as a conjunction

$$\text{(CDO)} \qquad Do(a, p) \equiv Ds(a, p) \,\&\, Dn(a, p)$$

where *Ds* may be said to represent the ***sufficient condition aspect*** of agency and *Dn* stands for the ***necessary condition aspect*** of agency. Kanger read $Ds(a, p)$ as p is necessary for something a does, and $Dn(a, p)$ as p is sufficient for something a does. These readings are equivalent to

$Ds(a, p)$: Something a does is sufficient for p (8.42)

$Dn(a, p)$: Something a does is necessary for p (8.43)

Kanger interpreted the agency operators *Ds* and *Dn* in terms of two alternativeness relations on possible worlds:

$$\text{(CDS)} \qquad u \vDash Ds(a, p) \text{ iff } w \vDash p \text{ for every } w \text{ such that } S_{DS}(u, w)$$

$$\text{(CDN)} \qquad u \vDash Dn(a, p) \text{ iff } w \vDash \neg p \text{ for every } w \text{ such that } S_{DN}(u, w)$$

The worlds w such that $S_{DS}(u, w)$ can be regarded as worlds in which the agent a performs the same actions as in u. Kanger (1981 [1957]) took $S_{DN}(v, w)$ to mean that 'the opposite' of everything a does in u is the case in w. One possible interpretation of this expression is that a does not do any of the things she does in u, but (for example) is completely passive (insofar as this is possible), or, for any action B that a performs at u, she does something else (i.e., some alternative to B) at w.

This analysis of the concept of agency has a form which has become widely accepted in the recent work on the logic of action. The first condition, the *Ds*-condition, may be termed the ***positive*** condition, and the second condition, the *Dn*-condition, may be termed the ***negative*** condition of agency. The latter condition is a ***counterfactual*** condition of agency; it states that if the agent had not acted the way she did, p would not have been the case. An analysis of this kind was put forward by von Wright (1963, 1968); other versions of the analysis of agency by means of a positive and a negative condition have been formulated by Åqvist (1974), Åqvist and Mullock (1989), Lindahl (1977), Pörn (1977), and more recently by Belnap, Horty, Perloff, and others; see Horty (2000) and the references given in it.

Philosophers have disagreed about the formulation of the negative condition. Pörn (1977) has argued that, instead of Kanger's *Dn*-condition (CDN), one should accept only a weaker negative requirement: $\neg Dn(a, \neg p)$, abbreviated here $Cn(a, p)$.

$$\text{(ACN)} \qquad u \vDash Cn(a, p) \text{ iff } w \vDash \neg p \text{ for some } w \text{ such that } S_{DN}(u, w)$$

This condition can be read: but for a's action it might not have been the case that p (Pörn, 1977, p. 7); in other words, it was not unavoidable for a that p. Åqvist (1974, p. 81) has accepted a similar weak form of the counterfactual condition. According to Pörn and Åqvist, the negative condition should be formulated as a might-statement or a might-conditional, not as a would-conditional. (For a discussion of different forms of the positive and the negative condition of agency, see Hilpinen (1997).)

The *Do*-operator makes it possible to distinguish four modes of action with respect to a result (state of affairs or event) p:

$Do(a, p)$:	a sees to it that p
$\neg Do(a, p)$:	a does not see to it that p
$Do(a, \neg p)$:	a sees to it that $\neg p$
$\neg Do(a, \neg p)$:	a does not see to it that $\neg p$

The combination of different modes of action with deontic concepts makes it possible to represent several types of obligation and permission and different legal or deontic relations between individuals. For example, consider a state of affairs involving two persons, $F(a, b)$. According to Kanger (1981) and Kanger and Kanger (1966), the *Do*-operator can be combined with deontic operators to distinguish four basic types of right (or different sense of the expression 'right'):

(R1) $ODo(b, F(a, b))$

(R2) $\neg ODo(a, \neg F(a, b)) \equiv P\neg Do(a, \neg F(a, b))$

(R3) $\neg O\neg Do(a, F(a, b)) \equiv PDo(a, F(a, b))$

(R4) $O\neg Do(b, F(a, b))$

(R1)–(R4) define four basic normative relations between a and b which from a's perspective can be regarded as different relational concepts of right. In (R1), b has a duty to see to it that $F(a, b)$; this is equivalent to a's *claim* in relation to b that $F(a, b)$. (R2) can be described as a's freedom (or *privilege*) in relation to b that $F(a, b)$; this means that a has no obligation to see to it that $\neg F(a, b)$. Kanger called (R3) a's *power* in relation to b that $F(a, b)$, and (R4) a's *immunity* in relation to b that $F(a, b)$. The replacement of the state of affairs $F(a, b)$ by its opposite $\neg F(a, b)$ yields four additional concepts of right which Kanger and Kanger (1966) called counter-claim (R1′), counter-freedom (R2′), counter-power (R3′), and counter-immunity (R4′). Kanger and Kanger called the eight relations defined in this way *simple* types of right. The normative relationship between any two individuals with respect to a state of affairs p can be characterized completely by means of the conjunctions of the eight simple types of right or their negations. There are $2^8 = 256$ such conjunctions, but the simple types of right are not logically independent of each other; according to the logic of the deontic O-operator and the agency operator *Do*, only 26 combinations of the simple types of right or their negations are logically consistent. Kanger and Kanger (1966) called these 26 relations the 'atomic types of right.' The atomic types provide a complete characterization of the possible legal relationships between two persons with respect to a single state of affairs. It is perhaps misleading to call these 26 relations 'types of right,' because they include as their constituents duties as well as claims and freedoms. Thus Kanger's theory of normative relations can be regarded as a theory of duties as well as rights (Lindahl, 1994).

Kanger's concepts (R1)–(R4) correspond to the four ways using the word 'right' (or four concepts of a right) distinguished by Hohfeld (1919), from which he adopted the expressions 'privilege', 'power', and 'immunity'. Although Kanger apparently intended (R1)–(R4) as approximate explications of Hohfeld's notions, his concepts of power and immunity differ from Hohfeld's. According to Kanger, both power and freedom are permissions; a power consists in the permissibility of actively seeing to it that something is the case, whereas freedom means that there is no obligation to see to it that the opposite state of affairs should be the case. Lindahl (1977) and others have argued that Hohfeld's concept of power should be analyzed as a legal *ability* rather than a permission (a *can* rather than a *may*); see Bulygin (1992), Lindahl (1994) and Makinson (1986).

An agency operator such as the *Do*-operator considered above can be iterated, and it is possible to form sentences which contain several nested occurrences of deontic operators, agency operators (or action operators), and epistemic operators, relativized to possibly different agents. This feature has facilitated the applications deontic logic and the logic of agency to the analysis of complex social and normative phenomena, for example, the analysis of different kinds of rights relations and other normative relations (H. Kanger, 1984; Lindahl, 1994; Makinson, 1986), governmental structures and the concept of parliamentarism (Kanger and Kanger, 1966), normative positions and normative change (Jones and Sergot, 1993; Lindahl, 1977; Sergot, 1999), the analysis of normative control, influence, and responsibility (Pörn, 1989; Santos and Carmo, 1996), and the analysis of trade procedures and the concept of fraud (Firozabadi et al., 1999).

Suggested further reading

Deontic Logic: Introductory and Systematic Readings (Hilpinen, 1981a [1971]) contains some pioneering contributions to deontic logic, including those by Kanger and Hintikka. *New Studies in Deontic Logic: Norms, Actions and the Foundations of Ethics* (Hilpinen, 1981b) contains papers on the ontology of norms, deontic paradoxes, temporal deontic logic, and the interpretation of quantifiers in deontic logic. Horty's *Agency and Deontic Logic* (2000) analyzes the concepts of action and agency in deontic logic and discusses the relevance of deontic logic to ethical theories, for example, utilitarianism. *Norms, Logics, and Information Systems*, edited by McNamara and Prakken (1999), contains recent papers on the philosophical foundations of deontic logic, norm conflicts, and computer systems applications of deontic logic. The papers in Nute's *Defeasible Deontic Logic* (1997) analyze defeasible reasoning in normative discourse, and Carmo and Jones' essay "Deontic Logic and Contrary-to-Duties" (2001) is a good survey of the problems about CTD obligations.

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