# THE LOGIC OF ACTUAL OBLIGATION
## AN ALTERNATIVE APPROACH TO DEONTIC LOGIC

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## 0. Introduction

In this paper we develop a system of deontic logic (**LAO**, the logic of actual obligation) with a rather limited scope: we are only interested in obligations as far as they are relevant for deciding what actions actually ought to be done in a particular situation, given some normative system N. In fact we are interested how actual obligations are derived from the prima facie ones implied by N. Hence statements expressing that certain *states of affairs* are obligatory, such as "the speed-limit ought to be 140 in stead of 100", fall out of the scope. (Roughly speaking **LAO** is what Castañeda calls a logic of "ought-to-do". (cf. [C]).) Since in **LAO** actions can be obligatory while assertions cannot, actions and assertions have to be strictly separated in the language of **LAO**. On this point we follow [M].

In [E] Job van Eck analyzes the relation between actual- and prima facie obligations in terms of tense. We don't agree with the details of his analysis, but we do believe that the role of time is important in deontic logic in general and in obtaining actual obligations from prima facie ones in particular. In section 1 we give a sketch of van Eck's system of temporally relative deontic logic (**QDTL**), to get some idea of the role of time in deontic logic. In section 2 **QDTL** is criticized, especially the fact that obligations are interpreted in terms of perfect alternatives.

In **LAO** we start with prima facie duties which follow from some normative system N. (A typical example of such an N is a predominant system of morality in some society, which e.g. gives rise to the prima facie obligations not to lie, not to steal, etc.) In general it is possible to have conflicting prima facie obligations and **LAO** is intended to tell what actually ought to be done in such situations. The output is intended to be directive, i.e. action guiding. Hence we don't consider statements like "if it is raining then you ought to have brought your umbrella with you", since such statements cannot give direction to (future) action. (In contrast with "if it is likely that it is going to rain, then you ought not to forget your umbrella".)

It is important to note that we are not interested in the question which action is the *best* in a particular situation but only in the question which action is *obligated.* (These questions may be equivalent for utilitarianists, but in general they are not.) Another point is that it doesn't make much sense to apply **LAO** in a context where the normative system N does not allow obligations to be overruled by stronger ones, i.e. where the notions of actual- and prima facie obligation coincide.

After giving a semantics and an axiomatization of **LAO** in section 3, we finish by showing in section 4 that the well-known paradoxes of deontic logic do not arise in **LAO**.

Acknowledgements:

[E] and [M] formed important sources of inspiration while writing this paper. Albert Visser, Theo van Willigenburg and especially Gerard Renardel de Lavalette ought (prima facie *and* actually) to be thanked for valuable discussions.

## 1. Temporally relative deontic logic

In [E] a system of temporally relative deontic logic **QDTL** is developed which is claimed not to have the deficiencies of traditional deontic systems. In particular, temporal relativization makes it possible to represent the difference between having a duty at time t to do some action at t and having a duty at t to do some action at t'>t (Van Eck considers this to be the difference between an actual- and a prima facie duty). Further the phenomenon of a conditional obligation turning into an unconditional one can be adequately represented.

### 1.1 Prima facie and actual obligation.

Suppose John promised Suzy (p) to have a cup of coffee with her (q).

Consider

(1) John ought to have a cup of coffee with Suzy.

Traditionally (1) is formalized as Oq and interpreted by

$v \Vdash Oq$ :⇔ for all $w>v$ $w \Vdash q$, where $w>v$ means that w is a deontically perfect alternative of v.

In [E] it is argued that since the truthvalue of (1) is dependent on the moment of time to which the "ought" pertains, (1) cannot be interpreted in terms of deontically perfect worlds simpliciter. In **QDTL** (1) is formalized as $O_t q_{t+7}$, where t is the moment of time just after John's promise and t+7 is the time of the date. $O_t q_{t+7}$ means: in all courses of the world which are as good as possible from t on $q_{t+7}$ (q holds at t+7).

In [E] (1) is seen as an elliptic sentence bearing a tacit 'ceteris paribus' proviso meaning: provided that between t and t+7 no stronger obligations conflicting with (1) or situations which render the realization of q impossible arise. Such duties which leave room for other things not being equal are called "prima facie duties". If at t+7 the conditions of the 'ceteris paribus' proviso are still met, then $O_t q_{t+7}$ is considered to be an actual duty.

Further the prima facie obligation $O_t q_{t+7}$ is said to imply the actual duty $O_t \Diamond_t q_{t+7}$, not to make q impossible e.g. by killing Suzy or himself. Hence the statement "other things were not equal" is not a sufficient justification for not doing q at t+7.

If at t+2 Suzy dies without John being involved in the matter, then John is not to be blamed for not being able to fulfil his obligation since "all worldcourses that were perfect from t onwards have become inaccesible at t+2 by an accident" ([E],p.72).

If at t+1 a stronger obligation s arises such that fulfilling s makes q impossible, then he is also not to be blamed for not doing q since "the best world seen from t+1 is different from an ideal world seen from t".

## 1.2 Primary and secondary duty.

Suppose John did not fulfil (1) because he did not feel like going.

Consider

(2) If John does not go he has to apologize the next day ($r_{t+10}$)

In **QDTL** (2) is formalized as $\neg q_{t+7}O_{t'}r_{t+10}$, meaning: in all worldcourses that are possible from t' onwards and are as perfect as possible - given that $\neg q_{t+7}$ is the case in them - $r_{t+10}$ is the case.

At t+8 we have $\neg q_{t+7}O_{t+8}r_{t+10}$ and $\neg q_{t+7}$ and hence the unconditional prima facie $O_{t+8}r_{t+10}$. Such a detachment obtaining an unconditional (primary) duty from a conditional (secondary) one is not possible in traditional dyadic systems. (A set like {p, pOq, O¬q}, which clearly represents a possible situation, is contradictory if one allows the inference of Oq from p and pOq.)

## 2. Problems

Although temporal relativization gives **QDTL** a remarkable flexibility, some problems remain. E.g we believe that the difference between an actual obligation and a prima facie one is not the difference between an action which ought to be done now and an action which ought to be done in the future. Some further problems derive from the fact that in **QDTL** only propositions can be obligatory and actions cannot. But our most serious objection to **QDTL** is that obligations are interpreted in terms of the (rather obscure) notion of 'best possible alternative'.

## 2.1 Terminology.

The use in [E] of the terms "prima facie and actual obligation" is not exactly in accordance with the use of these terms in traditional normative theory. The duties called "actual" in [E] are in this paper referred to by "acute prima facie duties" and we shall call actually obligatory only what actually ought to be done in a particular situation.

E.g. one has a prima facie duty not to lie but in a situation where telling the truth has unacceptable consequences it is possible to have an acute prima facie duty to tell the truth but an actual obligation to lie.

## 2.2 Actions and assertions.

$O_t q_{t+7}$ is said to imply $O_t \Diamond_t q_{t+7}$, not to make q impossible. But John cannot always be held responsible for q becoming impossible. $O_t q_{t+7}$ does imply that John has a prima facie obligation not to do anything that makes q impossible (or even improbable), but such an obligation cannot be expressed in a straightforward way in the language of [E]. This problem can be resolved if one uses a language where actions and assertions are separated. We will develop such a language, more or less along the lines of [M], in section 3.

Further, if at t+8 $\neg q_{t+7}$ is the case, then $O_{t+8} \neg q_{t+7}$ is derivable in **QDTL**. However, $O_t \phi_{t'}$, with t>t', is said not to express a real obligation:

"At t $\phi_{t'}$ is already a part of all possible further courses of the world and a fortiori of all best possible further courses". ([E],p.73)

The same reasoning would show that if at t $\phi_{t'}$ with t'≥t is necessary, then $O\phi_{t'}$ holds. Although this possibility is not mentioned by van Eck, he would probably not consider this $O\phi_{t'}$ to express a real obligation either, since he seems to hold that everything which is obligatory by virtue of its necessity does not count as a real (directive) obligation. We prefer to call obligatory only that which is really obligatory. This means that we cannot interpret obligations in terms of perfect alternatives, but we do not consider this to be a great loss:

## 2.3 (Nearly-)perfect alternatives.

The interpretation of $Oq_{t+7}$ in terms of best possible future world courses makes sense only if there is some independent way of determining these best possible future world courses.

Suppose $v \Vdash Oq$. It seems that in [E] the only criterion for being a best possible world course for v is that q is realized somewhere in the course after v. We see not on what other grounds the following world courses are not considered to be best possible world courses for v:

(i) Suzy dies between t and t+7, John not being involved.

(ii) At t+5 Suzy and John meet by some coincidence, have dinner together and decide that there is no need for fulfilling $q_{t+7}$, since they would have seen each other only recently at t+7.

(iii) At t+6 John decides to go shopping and buy Suzy a diamond ring. This makes it impossible for him to meet Suzy before t+9, but (he knows that) she doesn't mind.

We have seen that "for all perfect alternatives w of v: $w \Vdash q \Rightarrow v \Vdash Oq$" makes all what is necessary obligatory (and all what is impossible forbidden). On the other hand, "$v \Vdash Oq \Rightarrow$ for all perfect alternatives w of v: $w \Vdash q$" is likely to be valid only if Oq expresses an actual duty. Given the fact that it is almost always possible to do something supererogatory, like in (iii), an actual obligation will usually have the following form: "One ought to do the prima facie obligated a or some b which is at least as good as a". In section 3 we sketch an alternative approach to deontic logic which enables us to represent this relation between actual and prima facie obligations.

## 3. An alternative approach: the Logic of Actual Obligation (**LAO**)

In this section we first describe a semantics for actions in which the rise of actual obligations out of prima facie ones is represented and which is intended to be adequate for **LAO**. After obtaining some elementary results concerning this semantics, we propose an axiomatization of **LAO**.

## 3.1 Possible worlds and actions.

u,v,w,... denote possible worlds. These possible worlds are assumed to be partially ordered by $<$, which we assume to be transitive, irreflexive and tree-like. $v<w$ means that w is a possible future world seen from v. (We assume $<$ to be serial, i.e. $\forall v \exists w\ v<w$). a,b,c,... denote actions.

$v<_a w$ :⇔ $v<w$ and in the course from v to w action a is done.
$v<_{/a} w$ :⇔ $v<w$ and in the coursethrough v and w action a is not done.
$v<_{\neg a} w$ :⇔ $v<w$ and in the course from v to w action $\neg a$ is done, which implies that it will not be possible to do a in all courses of the world which go through v and w.
$v<_{a\&b} w$ :⇔ $v<_a w$ and $v<_b w$. Thus a&b denotes the action which consists of doing both a and b.
$v<_{a+b} w$ :⇔ $v<_a w$ or $v<_b w$. Thus a+b denotes the action which consists of doing either a or b or both a and b.
$v<_{ab} w$ :⇔ for some u: $v<_a u$ and $u<_b w$. Hence doing ab means doing first a and then b.

Examples: $v \xrightarrow{/a} u \xrightarrow{a} w$ : $v<_a w$, $v<_{/a} u$, $u<_a w$

$v \xrightarrow{\neg a} u \rightarrow w$ : $v<_{\neg a} w$, $v<_{\neg a} u$, $v<_{/a} w$, $u<_{/a} w$, etc.

$v \xrightarrow{a} u \xrightarrow{b,c} w$ : $v<_a u$, $v<_{a+b} u$, $u<_{b\&c} w$, $v<_{ab} w$, etc.

In general $v<_a w$ and $v<_a u$ does not imply $u=w$, not even if u and w are assumed to be direct successors of v. (E.g. it is possible that in addition $v<_b w$ and $v<_{\neg b} u$.)
If for all w $v<_a w$ implies $v<_b w$, then we say "a implies b" or "b is part of a", notation: $a \supset_v b$. Define $a \equiv_v b$ :⇔ $a \supset_v b$ and $b \supset_v a$.
$\varnothing$ denotes the empty action, defined by: for all v there exists no w such that $v<_{\varnothing} w$, and U denotes the universal action, defined by: for all v and all $w>v$ $v<_U w$.
$A_v$, the set of possible actions of v can be defined as follows: $a \in A_v$ iff $\exists w >_a v$.

Examples of some properties: For all v we have:

$a\&b \supset_v a$, $ab \supset_v a\&b$, $a \supset_v a+b$.

$a+U \equiv_v U$, $a\&U \equiv_v a$, $a+\varnothing \equiv_v a$, $a\&\varnothing \equiv_v \varnothing$.

Definition: An action is called positive if:

(i) $v <_a w$ and $w < z$ implies $v <_a z$

(ii) $u < v$ and $v <_a w$ implies $u <_a w$

Remark on the actions /a and ¬a:
The action /a (a-complement) is purely negative characterized: doing /a means just abstaining from doing a, i.e. not doing a. For doing ¬a (not-a) it is not sufficient to abstain from a. In addition it is necessary to make it impossible that a will ever be done. Thus ¬a is a positive action in the sense defined above, while in general /a is not.

Remark on the action a&b:
As the first of the above mentioned examples of some properties shows, we assume that the action a&b, if possible at all, contains the actions a and b intact. E.g. if a is "painting the table blue" and b is "painting the table yellow", then it is not possible to do a and b simultaneously, hence $a\&b \equiv \varnothing$. (Thus a&b is not e.g. "painting the table green".) Or if a is "John marries Suzy" and b is "John marries Anna", then a&b is not empty only in a society where polygamy is allowed.

We can generalize + and & as follows:
If A is a set of actions, then &A is the action which consists of doing all $a \in A$ and +A is the action which consist of doing at least one $a \in A$.
(If A is empty, then $\&A \equiv U$ and $+A \equiv \varnothing$. If $A=\{a\}$, then $\&A \equiv +A \equiv a$. If $A=\{a,b\}$, then $\&A \equiv a\&b$ and $+A \equiv a+b$.) & and + are interdefinable by means of /: $\&A \equiv /(+A^c)$, where $A^c := \{/a \mid a \in A\}$. +!A denotes the action which consist of doing exactly one action $a \in A$ (a+!b is of course +!{a,b}). &, + and +! will not occur in our formal language, but will be used in informal arguments.

We assume that for every world v there exists a partial pre-ordering $\leq_v$ on the set of actions which has to satisfy some obvious properties such as:

$a \leq_v b$ and $b \leq_v c \Rightarrow a \leq_v c$.

$a \leq_v c$ and $b \leq_v c \Rightarrow a+b \leq_v c$.

(Subscripts in expressions like $a \supset_v b$ and $a \leq_v b$ will be omitted when no confusion is likely to arise.)

The intended meaning of "a≤b" is "action b is at least as good as a". The ordering ≤ depends on the normative system N considered (e.g. it is possible that in general drinking coffee with Suzy(a) is not deontically better than drinking coffee with Anna(b) but that, due to a promise, O(a) and (therefore) a>b), but we do not assume ≤ to be completely determined by N. (In our typical case of N being a predominant system of morality in some society, the precise ordering of actions in a particular situation is in general not determined by N.)

3.2 Syntax of **LAO**.

We use $a_0$, $a_1$, $a_2$,... for elementary actions. We use a,b,c,... as variables (and meta-variables) and a,b,c,... as constant names for actions. Two special constant names are U and ∅.

Assertions and (compound) actions are formed as follows:

Actions (**Act**):
- elementary actions ∈ **Act**
- U and ∅ ∈ **Act**.
- if a,b ∈ **Act**, then
  ¬a, the negated action ("not-a),
  /a, the complementary action ("a-complement"),
  ab, the sequential composition ("ab" or "a followed by b"),
  a&b, the joint or simultaneous action ("a and b"),
  a+b, the alternative composition or choice action ("a or b") ∈ **Act**.

Let A be a set of actions. Then Cl(A), the closure of A, denotes the set of actions which can be obtained from A∪{U,∅} by means of the above mentioned operations on actions.

Assertions (**Ass**):
-propositional atoms ∈ **Ass**.
-if φ,ψ ∈ **Ass**, then ¬φ, φ∧ψ, φ∨ψ, φ→ψ, φ↔ψ, □φ, ◇φ and Pφ ∈ **Ass**.

-if a,b $\in$ **Act**, then $a \supset b$, $a \equiv b$, $a \leq b$, $a < b$, $\Box a$, $\Diamond a$, D(a), O(a), O'(a), O*(a), O*(D(a)), $O^s(a)$ and $P^s(a)$ $\in$ **Ass**.

-if i is a positive integer, a $\in$ **Act** and $\phi \in$ **Ass**, then $\Box^i\phi$, $P^i\phi$ and $D^i(a)$ $\in$ **Ass**.

-if $\phi \in$ **Ass** and a $\in$ **Act**, then $[a]\phi$ and $\langle a \rangle\phi$ $\in$ **Ass**.

-if $\phi(a) \in$ **Ass** (where a $\in$ **Act**), then $\phi(\underline{a})$, $\forall a\phi(a)$ and $\exists a\phi(a)$ $\in$ **Ass**.

Let us list the intended meanings of some of these assertions:

$P\phi$ : $\phi$ has been the case somewhere in the past
$P^1\phi$ : $\phi$ was the case in the possible world immediate before this one
$P^{i+1}\phi$ :$\Leftrightarrow$ $P^1P^i\phi$
D(a) : a has been done
$D^1(a)$ : a has been done between this world and the one immediate before it
$D^{i+1}(a)$ :$\Leftrightarrow$ $P^1D^i(a)$
$\Box^1\phi$ : in all immediate successors of this world $\phi$ is the case
$\Box^{i+1}\phi$ :$\Leftrightarrow$ $\Box^1\Box^i\phi$
$[a]\phi$ : if a will be done, then $\phi$ will be the case

(In **LAO** tense operators make it possible to represent the temporal aspects of deontic reasoning, whereas in **QDTL** temporal parameters are used.)

### 3.3 Semantics of **LAO**.

A **LAO**-model $\underline{K}$ is a tuple $\langle K,<,E,N,\Vdash \rangle$, where

- K is a set of possible worlds (for every possible world v there exists a partial pre-ordering $\leq_v$ on Cl(E))

- < is a discrete transitive irreflexive tree-like serial partial ordering on K

- E is a set of elementary actions, which are interpreted as relations on K×K, and this interpretation satisfies:
  (i) if u<v and $v <_a w$, then $u <_a w$
  (ii) if $v <_a w$ and w<z, then $v <_a z$,
  where $<_a$ denotes the interpretation of a.
  (Notice that we assume the elementary actions to be positive.)

- *N* is a set of formulas of the form $\phi \rightarrow O(a)$, with $a \in Cl(E)$ and $\phi$ not containing $\leq$ or

the operator O. We assume $N$ to satisfy the following rule:
for all a,b∈Cl(E): if ($\phi \rightarrow O(a)) \in N$, then ($\phi \wedge b \supset \neg a \rightarrow O(/b)) \in N$

- $\Vdash$ is a forcing relation with the following characteristic clauses:
  - $v \Vdash a \leq b :\Leftrightarrow a \leq_v b$
  - $v \Vdash a \supset b :\Leftrightarrow a \supset_v b$ ($a \supset_v b :\Leftrightarrow$ For all w: $v <_a w$ implies $v <_b w$)
  - $v \Vdash \Box\phi :\Leftrightarrow$ for all $w > v$: $w \Vdash \phi$
  - $v \Vdash \Box^1\phi :\Leftrightarrow$ for all direct successors w of v: $w \Vdash \phi$
  - $v \Vdash P\phi :\Leftrightarrow$ there exists a $w < v$: $w \Vdash \phi$
  - $v \Vdash P^1\phi :\Leftrightarrow$ there exists a direct predecessor w of v: $w \Vdash \phi$
  - $v \Vdash [a]\phi :\Leftrightarrow$ for all $w >_a v$: $w \Vdash \phi$
  - $v \Vdash D(a) :\Leftrightarrow$ there exists a $w <_a v$
  - $v \Vdash D^1(a) :\Leftrightarrow$ there exists a direct predecessor w of v: $w <_a v$.
  - $v \Vdash \forall a \phi(a) :\Leftrightarrow$ for all a∈Cl(E): $v \Vdash \phi(\underline{a})$, where $\underline{a}$ represents a.
  - $v \Vdash O(\underline{a}) :\Leftrightarrow v \Vdash \neg\Box\neg D(\underline{a})$ and for some $\phi$: $v \Vdash \phi$ and $\phi \rightarrow O(a) \in N$

The interpretation of non-elementary actions is determined by:
$<_{a+b} := <_a \cup <_b$, $<_{a\&b} := <_a \cap <_b$, $<_{ab} := <_a . <_b$, $<_{/a} := (<_a)^c$ and $<_{\neg a} := (<_a \cup <<_a . \cup . <_a <)^c$,
where $u <_a . <_b w$ iff $\exists v$: $u <_a v \wedge v <_b w$, $u <<_a . w$ iff $\exists v$: $u < w \wedge w <_a v$, $u . <<_a w$ is defined similarly and $(<_a)^c$ is the complement of $<_a$.

The truth value of a proposition of the form $O(\underline{a})$, denoting the prima facie duty to do a, is determined by the valuation of the propositional atoms together with the interpretation of the elementary actions and the set $N$. This set $N$ consists of formulas which express the relevant prima facie oblgations implied by some system N of norms. We make two assumptions on this system N:

- we assume that it follows from the system N that one cannot be obligated to do something impossible. (This is implemented by the condition $v \Vdash \neg\Box\neg D(\underline{a})$ in the forcing clause for $v \Vdash O(\underline{a})$.)
- we assume that if one has a prima facie duty to do a, then one has a prima facie duty not to do anything which makes a impossible. Hence we assume that the following is forced: $O(a) \wedge b \supset \neg a \rightarrow O(/b)$. (This is implemented by the closure condition on $N$: $\phi \rightarrow O(a) \in N \Rightarrow \phi \wedge b \supset \neg a \rightarrow O(/b) \in N$.)

Example: Suppose that it follows from N that if if John does not fulfil his promise to Suzy ($\neg p$), then he has the prima facie obligation to apologize to her (a). This may be represented in $N$ by $D(\neg\underline{p}) \rightarrow O(\underline{a})$. Hence if $v \Vdash \neg\Box\neg D(\underline{a}) \wedge D(\neg\underline{p})$, then

$v \Vdash O(\underline{a})$. Further, if $v \bigvee D(\neg \underline{p})$, then $v \Vdash [\neg \underline{p}](\neg \Box \neg D(\underline{a}) \rightarrow O(\underline{a}))$.
An unconditional prima facie obligation O(a) is of course treated as $\phi \rightarrow O(a)$, with $\phi$ a tautology.

In a situation where one has prima facie duties one is in general not obligated to act immediately. However, if O(a) holds and not doing a implies doing ¬a, then immediate action is required. Such actions a (and all b such that $a \supset b$) will be called acute duties. Notation O'(a). Hence we arrive at the folowing definition:

$$O'(a) :\Leftrightarrow \exists b(O(b) \wedge [/b]D(\neg b) \wedge b \supset a)$$

The partial pre-ordering on the set of possible actions associated with each possible world has to satisfy the following requirements:

(≤1) $a \equiv b \Rightarrow a \leq b$
(≤2) $(a \geq b$ and $b \geq c) \Rightarrow a \geq c$.
(≤3) $\varnothing \geq a \Rightarrow a \equiv \varnothing$
(≤4) if O'(a), then for all b $(a \leq b \vee a \geq b)$.
(≤5) $(a \geq b$, $a \geq c$ and $a \geq b\&c) \Rightarrow a \geq b+c$; $(a \leq b$, $a \leq c$ and $a \leq b\&c) \Rightarrow a \leq b+c$.
(≤6) if O'(a), O'(b) and $\neg(a\&b \equiv \varnothing)$, then $(a \geq c$ and $b \geq c) \Rightarrow a\&b \geq c$.
(≤7) if O'(b), O'(c) and $\neg(b\&c \equiv \varnothing)$, then $a \geq b\&c \Rightarrow (a \geq b$ and $a \geq c)$.

Some abbreviations:

$\exists a\phi :\Leftrightarrow \neg \forall a \neg \phi$
$\Box a :\Leftrightarrow \Box D(a)$
$\Diamond \phi :\Leftrightarrow \neg \Box \neg \phi$
$\langle a \rangle \phi :\Leftrightarrow \neg [a] \neg \phi$
$a \equiv b :\Leftrightarrow a \supset b \wedge b \supset a$
$a \sim b :\Leftrightarrow a \leq b \wedge b \leq a$
$a < b :\Leftrightarrow a \leq b \wedge \neg(a \geq b)$

Other abbreviations, like $O^*(a)$ and $O^s(a)$, will be introduced in the following sections.

<u>Def.</u> Let $\underline{K}$ be a **LAO**-model.

$\underline{K} \vDash \phi :\Leftrightarrow$ for all $v \in K$ $v \Vdash \phi$.
$\vDash \phi :\Leftrightarrow$ for all **LAO**-models $\underline{K}$ $\underline{K} \vDash \phi$.

## 3.4 Some elementary results.

The following propositions give a rather extensive list of formulas expressing elementary properties of actions which are valid in all **LAO**-models. The proofs are easy and left to the reader.

Prop. 1

$\models a \supset a$

$\models a \supset U$

$\models \emptyset \supset a$

$\models (a \supset b \wedge b \supset c) \rightarrow a \supset c$

Prop. 2

$\models (a \supset b \wedge a \supset /b) \rightarrow a \equiv \emptyset$

$\models a \supset b \rightarrow /b \supset /a$

$\models \neg a \supset /a$

$\models /(/a) \equiv a$

$\models /U \equiv \emptyset$

Prop. 3

$\models (a \supset c \vee b \supset c) \rightarrow a\&b \supset c$

$\models a\&b \supset a \wedge a\&b \supset b$

$\models a\&a \equiv a$

$\models a \supset b \rightarrow a\&b \equiv a$

$\models a\&\emptyset \equiv \emptyset$

$\models a\&U \equiv a$

$\models (a \supset c \wedge b \supset d) \rightarrow a\&b \supset c\&d$

Prop. 4

$\models (a \supset c \wedge b \supset c) \rightarrow a+b \supset c$

$\models (c \supset a \vee c \supset b) \rightarrow c \supset a+b$

$\models a+b \supset a \vee a+b \supset b$

$\models a+a \equiv a$

$\models a+U \equiv U$

$\models a+\emptyset \equiv a$

Prop. 5

$\models ab \supset a \wedge ab \supset b$

$\models (a \supset c \wedge b \supset d) \rightarrow ab \supset cd$

$\models (a \supset c \vee b \supset c) \rightarrow ab \supset c$

$\models a\emptyset \equiv \emptyset$

$\models \emptyset a \equiv \emptyset$

<u>Prop. 6</u> (Associativity)

$\models (ab)c \equiv a(bc)$

$\models (a+b)+c \equiv a+(b+c)$

$\models (a\&b)\&c \equiv a\&(b\&c)$

<u>Prop. 7</u> (Commutativity)

$\models a+b \equiv b+a$

$\models a\&b \equiv b\&a$

<u>Prop. 8</u> (Distributivity)

$\models a\&(b+c) \equiv a\&b+a\&c$

$\models a+(b\&c) \equiv (a+b)\&(a+c)$

$\models a(b+c) \equiv ab+ac$

$\models (a+b)c \equiv ac+bc$

$\models a(b\&c) \equiv ab\&ac$

$\models (a\&b)c \equiv ac\&bc$

<u>Prop. 9</u>

$\models [ab]\phi \leftrightarrow [a][b]\phi$

$\models [a+b]\phi \leftrightarrow ([a]\phi \wedge [b]\phi)$

$\models ([a]\phi \vee [b]\phi) \rightarrow [a\&b]\phi$

<u>Prop. 10</u>

$\models [a]D(a)$

$\models ([a]\phi \wedge b \supset a) \rightarrow [b]\phi$

$\models [a](\phi \rightarrow \psi) \rightarrow ([a]\phi \rightarrow [a]\psi)$

$\models [U]\phi \leftrightarrow \Box\phi$

$\models \neg\Diamond D(a) \leftrightarrow [a]\bot$

$\models [\varnothing]\bot$

## <u>3.5 Actual and strong obligation.</u>

O'(a) does not imply that one is actually obligated to do a (notation: O*(a)), since there may e.g. be stronger duties conflicting with a. More generally, one is not to be blamed in case O'(a) and a is not done, if one does some b which is at least as good as a. (Remember that the ordering $\leq$ depends on the valuation of formulas of the form O(a), hence if $b \geq a$, then the effect of a being obligatory is already taken into account.)

Hence if one has the acute obligation to do a, then one has the actual obligation to do a or some b which is at least as good as a. On the other hand, it seems that an actual obligation a can only be derived from some acute obligation b such that $b \supset a$ and for all $c \geq b$: $c \supset a$. So we arrive at the following definition of actual obligation:

$$O^*(a) :\Leftrightarrow \exists b(O'(b) \wedge b \leq a \wedge \forall c(c \geq b \rightarrow c \supset a))$$

The following proposition shows that actual obligations are also acute ones:

<u>Prop. 11</u> $\models \forall a(O^*(a) \rightarrow O'(a))$

<u>Proof:</u> Assume $v \Vdash O^*(\underline{a})$.

$\Rightarrow v \Vdash \exists b(O'(b) \wedge b \leq \underline{a} \wedge \forall c(c \geq b \rightarrow c \supset \underline{a})$

$\Rightarrow v \Vdash \exists b(\exists d(O(d) \wedge [/d]D(\neg d) \wedge d \supset b) \wedge b \supset \underline{a})$

$\Rightarrow v \Vdash \exists d(O(d) \wedge [/d]D(\neg d) \wedge d \supset \underline{a})$

$\Rightarrow v \Vdash O'(\underline{a})$ □

If a is an acute obligation, then it is actually obligated to do a or some b which is at least as good as a:

<u>Prop. 12</u> $\models \forall a(O'(a) \rightarrow O^*(a +! +!\{b \mid b \geq a\}))$

<u>Proof:</u> Assume $v \Vdash O'(\underline{a})$

$\Rightarrow v \Vdash O^*(\underline{a} +! +!\{b : b \geq \underline{a}\})$, since we have

$\models a \leq (a +! +!\{b \mid b \geq a\})$ and $\models \forall c(c \geq a \rightarrow c \supset (a +! +!\{b \mid b \geq a\})$ □

Notice that in general actual obligations as defined above are not unique. E.g. if $v \Vdash O'(\underline{a}) \wedge \underline{a} \supset \underline{b}$, then both $v \Vdash O^*(\underline{a} +! +!\{c \mid c \geq \underline{a}\})$ and $v \Vdash O^*(\underline{b} +! +!\{d \mid d \geq \underline{b}\})$. Hence doing some actual obligated action is no guarantee for being blameless. In fact one has to do all actual obligated actions. In other words, one is to be blamed if one has not done some actual obligated action.

We introduce $O^*(D(a))$, as notation for the proposition that a actually ought to have been done, as follows:

$$v \Vdash O^*(D(\underline{a})) :\Leftrightarrow \exists u < v \; u \Vdash O^*(\underline{a}).$$

Then we can say that if $v \Vdash O^*(D(\underline{a})) \wedge \neg D(\underline{a})$, then v is not a perfect possible world, since the actual obligated a is not done in it. One may wonder whether it is always possible to do all actual obligated actions. Fortunately it is:

<u>Lemma 1</u> $\models\forall a(O'(a) \rightarrow \neg(a\equiv\varnothing))$

<u>Proof:</u> Assume $v\Vdash\underline{a}\equiv\varnothing$. Then, since we assumed that one cannot be obligated to do something impossible, $v\Vdash\neg O(\underline{a})$. But then $v\Vdash\neg O'(\underline{a})$, since $v\Vdash\underline{b}\supset\underline{a}$ implies $v\Vdash\underline{b}\equiv\varnothing$ and thus $v\Vdash\neg O(\underline{b})$. □

<u>Lemma 2</u> $\models\forall ab((O^*(a) \wedge a\equiv b) \rightarrow O'(b))$

<u>Proof:</u> Assume $v\Vdash O^*(\underline{a}) \wedge \underline{a}\equiv\underline{b}$.

$\Rightarrow v\Vdash O'(\underline{a}) \wedge \underline{a}\equiv\underline{b}$ (by prop. 11)

$\Rightarrow v\Vdash \exists c(O(c) \wedge [/c]D(\neg c) \wedge c\supset\underline{a}) \wedge \underline{a}\equiv\underline{b}$

$\Rightarrow v\Vdash \exists c(O(c) \wedge [/c]D(\neg c) \wedge c\supset\underline{b})$. Hence $v\Vdash O'(\underline{b})$. □

<u>Prop. 13</u> $\models\forall a,b((O^*(a) \wedge O^*(b)) \rightarrow O'(a\&b))$

<u>Proof:</u> Assume $v\Vdash\underline{a}\geq\underline{b} \wedge O^*(\underline{a}) \wedge O^*(\underline{b})$

$\Rightarrow v\Vdash\underline{a}\geq\underline{b} \wedge \exists c(O'(c) \wedge c\leq\underline{b} \wedge \forall d(d\geq c \rightarrow d\supset\underline{b}))$

$\Rightarrow v\Vdash\underline{a}\supset\underline{b}$

$\Rightarrow v\Vdash\underline{a}\&\underline{b}\equiv\underline{a}$

$\Rightarrow v\Vdash O'(\underline{a}\&\underline{b})$ (by lemma 2)

The case $v\Vdash\underline{b}>\underline{a} \wedge O^*(\underline{a}) \wedge O^*(\underline{b})$ is similar. □

From Prop. 13 it only follows that the joint of a finite number of actual obligations is again acute obligation and hence not empty (by lemma 1). But also the joint of the possible infinite set of all actual obligated actions is not empty, since it is not only an acute obligation, but even an actual one:

<u>Prop. 14</u> $\models\forall a((a\equiv\&\{b\,|\,O^*(b)\} \rightarrow O^*(a))$

<u>Proof:</u> Assume $v\Vdash(\underline{a}\equiv\&\{b\,|\,O^*(b)\}$.

$\Rightarrow v\Vdash/\underline{a}\supset/\underline{b}$, for some b such that $v\Vdash O^*(\underline{b})$

$\Rightarrow v\Vdash[/\underline{a}]D(\neg\underline{a}) \wedge /\underline{a}\supset/\underline{b} \wedge O'(\underline{b})$

$\Rightarrow$ for some c $v\Vdash[/\underline{a}]D(\neg\underline{a}) \wedge /\underline{a}\supset/\underline{c} \wedge O(\underline{c}) \wedge [/\underline{c}]D(\neg\underline{c})$

$\Rightarrow v\Vdash[/\underline{a}]D(\neg\underline{a}) \wedge /\underline{a}\supset\neg\underline{c} \wedge O(\underline{c})$

$\Rightarrow v\Vdash[/\underline{a}]D(\neg\underline{a}) \wedge O(\underline{a})$

$\Rightarrow v\Vdash O'(\underline{a})$

Further $v\Vdash\forall d(d\geq\underline{a} \rightarrow d\supset\underline{a})$:

Assume $v\Vdash\underline{d}\geq\underline{a}$

$\Rightarrow$ for all e: $v\Vdash O^*(\underline{e}) \rightarrow \underline{d}\geq\underline{e}$

$\Rightarrow$ for all e: $v \Vdash O^*(\underline{e}) \rightarrow \underline{d} \supset \underline{e}$

$\Rightarrow v \Vdash \underline{d} \supset \&\{e \mid O^*(e)\}$

$\Rightarrow v \Vdash \underline{d} \supset \underline{a}$

Hence $v \Vdash O'(\underline{a}) \wedge \forall d(d \geq \underline{a} \rightarrow d \supset \underline{a})$ and thus $v \Vdash O^*(\underline{a})$. □

We define the notions of strong obligation and strong permission as follows:

Def. $O^s(a) :\Leftrightarrow O^*(a) \wedge \forall b(O^*(b) \rightarrow a \supset b)$.
(One has the strong obligation to do a if one has the actual obligation to do a and if doing a implies doing every other actual obligation.)

Def. $P^s(a) :\Leftrightarrow \neg O^s(/a)$.
(a is strongly permitted if it is not strongly obligatory not to do a.)

Doing a strong obligation is equivalent with doing all actual obligations:

Prop. 15 $\quad \models \forall a(O^s(a) \leftrightarrow a \equiv \&\{b \mid O^*(b)\})$

Proof: "$\leftarrow$" Assume $v \Vdash \underline{a} \equiv \&\{b \mid O^*(b)\}$. Then, by prop 15, $v \Vdash O^*(\underline{a})$ and for all b such that $v \Vdash O^*(\underline{b})$ we have $v \Vdash \underline{a} \supset \underline{b}$.
"$\rightarrow$" Assume $v \Vdash O^s(\underline{a})$. Then $v \Vdash O^*(\underline{a})$ and hence $v \Vdash \&\{b \mid O^*(b)\} \supset a$. On the other hand $v \Vdash \underline{a} \supset \&\{b \mid O^*(b)\}$, since for all b such that $O^*(b)$: $v \Vdash \underline{a} \supset \underline{b}$.
Hence $v \Vdash \underline{a} \equiv \&\{b \mid O^*(b)\}$. □

## 3.6 Axiomatization of **LAO**.

An axiomatization of **LAO** is obtained by adding the following axioms and rules to those of propositional logic:

- the formulas mentioned in the propositions 1-10 (or some more economical equivalent set of formulas).

- formulas expressing the required properties of $\leq$ (viz. ($\leq$1)-($\leq$7)) and introducing the abbreviations mentioned in section 3.3.

(∀1) $\forall a A(a) \rightarrow A(b)$, if b is free for a in $A(a)$.
(∀2) $\vdash(\phi \rightarrow A(a)) \Rightarrow \vdash(\phi \rightarrow \forall a A(a))$, if a not free in $\phi$.

(□1) $\vdash\phi \Rightarrow \vdash\Box\phi$
(□2) $\Box(\phi \rightarrow \psi) \rightarrow (\Box\phi \rightarrow \Box\psi)$
(□3) $\Box\phi \rightarrow \Box\Box\phi$
(□4) $\Box^1\phi \leftrightarrow (((\neg P\phi \wedge \neg\phi) \rightarrow \Box(\phi \vee P\phi)) \wedge ((\neg P\phi \wedge \phi) \rightarrow \Box(P^1\phi \rightarrow \phi)) \wedge$
$(P^i\phi \rightarrow \Box(P^{i+1}\phi \rightarrow \phi)))$
(□5) $\Box^{i+1}\phi \leftrightarrow \Box^1\Box^i\phi$
(□6) $\Box^i(\phi \rightarrow \psi) \rightarrow (\Box^i\phi \rightarrow \Box^i\psi)$

(D1) $D(a) \rightarrow \Box D(a)$
(D2) $D^1(a) \leftrightarrow (D(a) \wedge \neg PD(a))$
(D3) $D^{i+1}(a) \leftrightarrow P^1 D^i(a)$
(D4) $(D^i(a) \wedge P^i(a \supset b)) \rightarrow D^i(b)$

(P1) $PP\phi \rightarrow P\phi$
(P2) $(\phi \vee P\phi) \rightarrow \Box P\phi$
(P3) $P\Box\phi \rightarrow (\phi \wedge \Box\phi)$
(P4) $(P\phi \wedge P\psi) \rightarrow P(\phi \wedge \psi)$
(P5) $P^1\phi \leftrightarrow (P\phi \wedge \neg PP\phi)$
(P6) $P^{i+1}\phi \leftrightarrow P^1 P^i\phi$
(P7) $\phi \leftrightarrow \Box^i P^i\phi$
(P8) $P^i\Box^i\phi \leftrightarrow \phi$

(O1) $(O(a) \wedge b \supset \neg a) \rightarrow O(/b))$
(O2) $O(a) \rightarrow \Diamond a$
(O3) $O'(a) \leftrightarrow \exists b(O(b) \wedge [/b]D(\neg b) \wedge b \supset a)$
(O4) $O^*(a) \leftrightarrow \exists b(O'(b) \wedge b \leq a \wedge \forall c(c \geq b \rightarrow c \supset a))$
(O5) $O^*(D(a)) \leftrightarrow P(O^*(a))$
(O6) $O^s(a) \leftrightarrow (O^*(a) \wedge \forall b(O^*(b) \rightarrow a \supset b))$
(O7) $P^s(a) \leftrightarrow \neg O(/a)$

Notice that most of the axioms listed above are in fact definitions.

It is easy to see that if **LAO**$\vdash\phi$, then $\models\phi$.
Below we list some theorems of **LAO**.

Thm.1 For all i:

(i) $\vdash \Box\phi \rightarrow \Box^i\phi$

(ii) $\vdash P^i\phi \rightarrow P\phi$

(iii) $\vdash D^i(a) \rightarrow D(a)$

Thm. 2

(i) $\vdash O^*(a) \rightarrow O'(a)$

(ii) $\vdash O'(a) \rightarrow \neg(a \equiv \varnothing)$

(iii) $\vdash (O^s(a) \wedge O^s(b)) \rightarrow a \equiv b$

(iv) $\vdash (O^*(a) \wedge a > b) \rightarrow \neg O^s(b)$

(v) $\vdash (O^*(a) \wedge b \equiv a) \rightarrow O^*(b)$

(vi) $\vdash (O^*(a) \wedge O^*(b)) \rightarrow O^*(a \& b)$

Proofs:

Thm. 1 and 2(i) are trivial.

For 2(i)-(vi) reason in **LAO**:

2 (ii): Assume $O'(a)$ and $a \equiv \varnothing$. Then $\exists b\,(O(b) \wedge b \supset a \wedge [/b]D(\neg b))$.
Hence, by the transitivity of $\supset$, $\exists b(O(b) \wedge b \supset \varnothing)$.
Contradiction, since $b \supset \varnothing$ implies $[b]\bot$ (by $[\varnothing]\bot$) and $[b]\bot$ implies $\neg\Diamond b$, which in turn implies $\neg O(b)$ (by (O2)). □

2 (iii): Assume $O^s(a)$ and $O^s(b)$. Then $O^*(a)$ and $O^*(b)$.
$(O^s(a) \wedge O^*(b)) \Rightarrow a \supset b$ and $(O^s(b) \wedge O^*(a)) \Rightarrow b \supset a$. Hence $a \equiv b$. □

2 (iv): Assume $O^*(a)$ and $a>b$ and $O^s(b)$.
It follows from $O^s(b)$ and $O^*(a)$ that $b \supset a$.
On the other hand $O^*(a) \wedge a > b$ implies $a \supset b$. Hence $a \equiv b$.
But then, by ($\leq$1), $a \leq b$, which contradicts $a>b$. □

2 (v): Assume $O^*(a)$ and $b \equiv a$. Then $O^*(a) \wedge a \supset b \wedge b \geq a$ (by ($\leq$1)).
$\Rightarrow \exists c\,(O'(c) \wedge c \leq a \wedge \forall d(d \geq c \rightarrow d \supset a)) \wedge a \supset b \wedge b \geq a$
$\Rightarrow \exists c\,(O'(c) \wedge c \geq b \wedge \forall d(d \geq c \rightarrow d \supset b)$
$\Rightarrow O^*(b)$ □

2 (vi): Assume $O^*(a)$ and $O^*(b)$. By ($\leq$4), $a \leq b$ or $a \geq b$.
If $a \geq b$, then $a \supset b$ (by $O^*(b)$). Hence $a\&b \equiv b$ and thus $O^*(a\&b)$ (by 2 (v)).
The case $a \leq b$ is similar. □

## 4. Applications

Thanks to the rather limited scope of **LAO** and the strict separation of actions from assertions the well-known paradoxes of deontic logic do not arise in **LAO**. Further the logic of actual obligation gives more insight into the normative point of view which lies behind a normative assertion than the perfect alternatives approach does.

### 4.1 The Ross-paradox.

Traditionally, the following principle is considered to be valid:

(O) $((\phi \rightarrow \psi) \wedge O(\phi)) \rightarrow O(\psi)$.

But this leads to paradoxical results, such as "the Good Samaritan paradox" which will be treated below and "the Ross-paradox": "you ought to post the letter" implies "you ought to post the letter or burn it". The latter sentence is intuitively understood as implying that both mailing the letter and burning it are permitted. But it is of course absurd to infer the permission to burn a letter from an obligation to post it.

A possible answer to the Ross-paradox is that the problem is of a pragmatic nature: one usually doesn't assert a disjunction if one is able to assert one of its disjuncts. The logical connective "$\vee$" does not correspond exactly with "or" in ordinary language. However, this is not the whole story. We believe that there is a sense of "ought" for which the inference from "a ought to be done" to "a or b ought to be done" is not only pragmatically, but even semantically invalid: in some situations "you ought to do a" means "you are doing your duty iff you do a". The notion of strong obligation corresponds with this sense of "ought".

In **LAO** we have $a \supset a+b$, the action-equivalent of $\phi \rightarrow \phi \vee \psi$, and although in general the implication $a \supset c \rightarrow (O(a) \rightarrow O(c))$ is not valid, it cannot be excluded that for some systems N it is. We certainly have the validity of $O'(a) \rightarrow O'(a+b)$. However we don't have $O^s(a) \rightarrow O^s(a+b)$, unless $a \equiv a+b$, i.e. $b \supset a$. It is easy to check that if a+b is strongly obligated, for some non-empty a and b, then both a and b are strongly permitted. Hence the Ross-paradox cannot be derived for the notion of strong obligation.

## 4.2 The Chisholm-paradox.

In [Ch] Chisholm formulated in essence the following paradox:
Consider

1. a ought to be done
2. if a is done, then b ought to be done
3. if ¬a is done, then ¬b ought to be done
4. ¬a is done
5. ¬b ought to be done

Intuitively the set consisting of the sentences 1.-4. is both consistent and non-redundant, i.e. no sentence among 1.-4. is derivable from the other three. Further, 3. and 4. intuitively imply 5. However, all known formalizations in monadic deontic logic of 1.-4. render a set of sentences which is either inconsistent or redundant (cf. [Å]) and those in dyadic logic do not allow the inference to 5.

By taking the role of time into consideration, van Eck is able to overcome these difficulties in [E]. He arrives at something like the following formalization: $\{O_t a_{t+7}, a_{t+7} O_t b_{t+17}, \neg a_{t+7} O_t \neg b_{t+17}, \neg a_{t+7}\}$. However, we don't consider his solution entirely satisfactory. E.g. in **QDTL** it is not excluded for a to be obligatory at some time when a is already impossible: suppose doing $c_{t+1}$ implies the impossibility of doing $a_{t+7}$, then after doing $c_{t+1}$ we still have $O_{t+2}a_{t+7}$, but also $\neg\Diamond_{t+2}a_{t+7}$ and hence $O_{t+2}\neg a_{t+7}$, since in **QDTL** $\Box_t\phi$ implies $O_t\phi$.

In **LAO** we have before a or ¬a is done:

$$O(a), [a]O(b) \text{ and } [\neg a]O(\neg b).$$

If ¬a has been done, then we have

$$\neg O(a), D(\neg a) \text{ and } O(\neg b).$$

(If in addition a was actually obligated, then we can see that something went wrong, since we have $O^*(D(a)) \wedge \neg D(a)$.)

Notice that we have assumed that b or ¬b ought to be done *after* doing a or ¬a. In some versions of the Chisholm-paradox doing a (¬a) is assumed to imply that b (¬b) ought to be done *before* a (¬a). We could formalize this e.g. as [a]P(O(b)), but we believe that these kinds of conditionals fall out of the scope of the logic of actual obligation as outlined in the introduction. (Since to derive $v \Vdash O(b)$ from $v \Vdash [a]P(O(b))$ some future information not available in v, viz. that a will be done, is required.)

## 4.3 The Good Samaritan paradox.

Aside from the Ross-paradox, the principle (O) seems to have another paradoxical result:

Let Suzy be the unique girl which will be killed by John and suppose that John ought to marry Suzy (e.g. because he has impregnated her). Then "John ought to be going to kill a girl", since

(1) "John marries the girl he will kill" (a) implies "John will kill a girl" (b)

and (2) John ought to marry the girl he will kill (viz. Suzy).

It is generally assumed that, even without rejecting (O), scope distinctions avoid the paradox. (In (2) marrying is in the scope of the "ought", but killing not. Cf. [C].) In **LAO** (1) is not valid: we do not have $a \supset b$. Hence this paradox does not seem much of a problem. However, in [F] James Forrester formulates the following version of the Good Samaritan paradox, which is not as easily dismantled:

Suppose that it is settled that John will murder Suzy.
Consider

(3) It ought to be that if John murders Suzy (c), then John murders Suzy gently (d).

Hence John ought to murder Suzy gently, which implies that he ought to murder Suzy.

In [LB2] Barry Loewer and Marvin Belzer accept the conclusion that John ought to murder Suzy gently. Their solution of the paradox consists in making a virtue out of necessity (i.e. everything which is settled is obligatory), thereby making the obligation to murder Suzy a vacuous one. Since they are themselves not satisfied with this, they define a deontic operator $O^+$(in their notation: $O^*$) such that $O^+p$ holds whenever Op holds and p is not settled. Then $O^+d$ but not $O^+c$. We agree that being necessary does not imply being obligatory, but on the other hand being necessary does not imply being not obligatory either.

At first sight the above version of the Good Samaritan paradox also seems to arise in **LAO**, since now we do have $d \supset c$. However, as it stands, i.e. in ought-to-be form, (3) does not fall in the scope of **LAO**, nor does any reasonable approximation in ought-to-do form, such as $[c]O^*(D(d))$. In our opinion this is not due to a weakness of **LAO** since we do not believe that (3) can be used to infer that John has any kind of obligation to murder Suzy gently, even in case it is settled that he will murder her. Aside from the prima facie duty not to murder Suzy, John is also prima facie obligated not to cause unnecessary suffering. Hence a cruel murder is worse than a gentle one, but it remains wrong to murder Suzy gently.

## 4.4 Normative point of view.

In the logic of actual obligation the different areas which together decide which action actually ought to be done are clearly separated:

(1) the general statements expressing prima facie obligations, such as "you ought not to lie". (N)

(2) the question whether one of those general statements is applicable on some particular action. ($\mathcal{N}$)

(3) the question which actions are possible in a particular situation and which worlds are the possible outcomes of those actions.

(4) the ordering of the possible actions. (≤)

Since normative arguments can arise out of disagreement in any one of the areas (1) - (4), it is useful to keep them separated.

We believe that a person's normative point of view is better learnt by letting him specify his decisions in all these areas (e.g. by letting him specify a **LAO**-model) than by asking him which possible future courses of the world he considers to be as perfect as possible (cf. [E]) or how he ranks the possible courses (cf. [LB1]).

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