Formats of Representation in Scientific Theorising

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1. Introduction

This paper is intended to sketch the definition of a methodological tool – the notion of a format of representation – for the study of scientific theorising. "Theorising", as I understand it, comprises the various reasoning tasks scientists perform in their practice of science: hypotheses forging, developing, comparing, and testing, as well as teaching, communicating, giving arguments in controversies, etc. All these activities imply the construction and use of models, which are at the same time expressions of the theoretical structure from which they might be drawn, representations of the phenomena, and tools for predicting and explaining them.

The various functions of models are generally conceived of as depending on their representational function: a model is a device standing for a target system in virtue of some relationship between its features and features of the system. This representational relationship enables scientists to draw inferences concerning the target system by reasoning with – which sometimes implies literally manipulating – the model.

At least two kinds of questions arise concerning the representational function of scientific models. The first one concerns the semantic relationship between the model and the target; it has been coined the "constitution question" by Callender and Cohen (2006): What does the representational relationship between models and their target consist in? And how does it enable scientists to infer successfully from properties of the model to properties of the target? This kind of question has been much addressed in the recent literature (see, e.g., Suárez, 1999, 2003, 2004, Frigg, 2002, 2006, 2009, French, 2003) and is often referred to as "THE problem of scientific representation". Answers to these questions have to handle the problem of the truth and accuracy of the descriptions provided by models, given that various abstractions, idealisations, and approximations are made in modelling physical systems. This paper will not tackle such issues.

The second kind of questions concerns the relationship between the model and its user. To be clear: in reasoning with models, scientists gain knowledge concerning the target by inferring from properties of the model to properties of the target. But, beforehand, they have to gain knowledge about the content of the model itself: the information they need is not always displayed in an immediately readable form in the model, and they have to "make it speak", so to speak. The question is: How do models enable cognitive agents to obtain information *within* them, independently of the true or approximately true statements that can be inferred about their target? In terms of Hughes' (1997) "DDI" (for Denotation-Demonstration-Interpretation) account, the question concerns the second step, namely "demonstration": once the model is designed, and before the sentences concerning it are interpreted as sentences about the target, the scientist has to use the "internal dynamics" of the model, as Hughes calls it, in order to "read off" the information within the model. This paper is centred on this second kind of questions. More precisely, I aim at providing a tool to analyse the interaction between what Hughes metaphorically calls the "internal dynamics" of models, and agents' reasoning abilities.

Tackling this issue requires that one first makes more precise what is to be understood by "model". Indeed, this term is highly polysemic, ranging from abstract mathematical structures or idealised systems to particular graphs, equations, and even 3D material models. But the fundamental assumption of this paper is that the reasoning processes undertaken by agents in order to gain knowledge within a model are always led on *formatted* representations. In other words, I assume that agents do not reason with models *in abstracto*: even when they use abstract or imaginary models such as, for instance, the simple pendulum, they always reason with a particular equation or graph. More generally, I will consider all the concrete devices[1] (both linguistic and non-linguistic) scientists construct, use, and manipulate in their practice of science: equations, diagrams, linguistic descriptions, graphs, pictorial images, 3D material models[2]. These will be my units of analysis for the study of the user-model interaction. From now on, in this paper, "model" has to be understood in this restricted sense of concrete representing device.

[1] In some cases, one can certainly reason with a model by merely imagining it, without a graph or an equation being there in black and white, written down on a paper. Acknowledging that one nevertheless always reasons on formatted representations means that, even in this case, one does not draw inferences on, say, a pendulum, without couching it – even mentally – in some particular graph or equation. For the sake of simplicity, I will exclusively focus on external representing devices.

[2] I will not consider simulations here, although it would be a valuable extension of my account.

Acknowledging that reasoning is always led on formatted representations goes hand in hand with another initial assumption of this paper: the particular form of a representation matters to its user's reasoning processes. Independently of what is being modelled, different kinds of representational media do not facilitate the same kind of inferences. In consequence, following Paul Humphreys (2004), I claim that attention has to be paid to the computational aspects and to the tractability of our theoretical tools, in parallel to their representational relationship to the phenomena. Therefore, as he suggests, one should focus on the "concrete pieces of syntax" that are constructed and used in prediction and explanation.

My perspective, though, is substantially different from Humphreys'. His proposal is meant to do justice to the increasing importance of the use of computers in scientific practice, and his notion of "template" is not centred on the individual agent with limited cognitive abilities, but rather on what it is possible to calculate, in practice, once we have "extended" our computing abilities by means of extremely powerful computers. However, I am interested in the user-model interaction, from the perspective of the individual agent, taking into account his cognitive limitations, personal skills, background knowledge, prior beliefs, interests, etc. Therefore, the individual differences between agents become central to my picture of theorising. The inferences an agent typically does to gain knowledge within a model depend *both* on the particular form (or "syntax") of the representation *and* on his cognitive abilities. And I claim that paying attention to this double dependence is a fruitful method to analyse scientific theorising.

In section 2, I define the notion of the format of a representation in terms of the inferences this representation enables a particular agent to do, taking into account both the "syntax" of the representation and the agent's cognitive abilities and interests. In section 3, I give a broad outline of two case studies – classical mechanics and Feynman's diagrams – to show how focusing on concrete representing devices and analysing them in terms of formats, as defined in section 2, might well shed some light on various important aspects of theorising.

2. Formats

The definition I give of the notion of format in this section relies on "toy" examples, which are chosen to make the definition clear, without raising specific issues concerning the use of models in theorising. How formats, so defined, enable us tackle such issues is studied in section 3.

2.1 Representations and their informational content

Let me start with the following picture of representation, or, better said, of a representation-situation (*R*-situation): an agent or a group of agents is/are using a device (the *representation*) to represent some feature of the world (the *target*), in some context and for some purpose[3]. The representation can be any external device or artefact[4], marks on a paper or on a computer screen, a 3D material object, etc., whose perceptual properties are accessible to the user[5]. The target can be a system, properties of a system, the evolution of some value, the relation between various values, an event, a pattern, etc. A representation can be used for various purposes (e.g. mnemonic, aesthetic, epistemic). I will only consider epistemic, or knowledge-seeking uses of representations; this refers to all the situations in which a representation is used to draw inferences concerning its target[6].

I restrict my analysis to representations that function in virtue of a *representational scheme*, as defined by Haugeland (1991, 172), namely such that "(i) a variety of possible contents can be represented by a corresponding variety of possible representations; (ii) what any given representation […] represents is determined in some consistent or systematic way by the scheme […]; and (iii) there are proper (and improper) ways of producing maintaining, modifying, and/or using the various representations under various environmental and other conditions." Focusing on external representations, one can describe how they function within a scheme by referring, following the Goodmanian tradition[7], to their syntax and semantics[8]. Rules for producing and interpreting representations can be of very different kinds, according to the scheme in which they function. Some representations are produced by using natural

[3] This is roughly the conception advocated by Giere (2006, 60).

[4] In this paper, I focus on external representations. Integrating some results of works on mental representations, and on their interactions with external ones, in particular from the perspective of distributed cognition (see, e.g. Clark, 1997), would certainly be a valuable extension of my account.

[5] This does not imply any particular theory of perception, and is just meant to exclude abstract objects such as mathematical structures, propositions, or fictions, as well as unobservable things.

[6] Knowledge-seeking uses of representations are more comprehensive than genuine *scientific* representations: using a map to know how far Washington is from New York is a knowledge-seeking use. Without getting into this issue, I would be inclined to adopt Callender and Cohen's (2006) claim that a demarcation criterion for scientific representations cannot be found in any special semantics, but rather in pragmatic aspects of their uses.

[7] This tradition stems from Goodman (1968/1976).

[8] Kulvicki (2003) gives a detailed account of representational systems in such terms.

causal processes (e.g. photographs, electron micrographs, X-ray images); some rely on much more arbitrary relationships between their properties and their content (e.g. languages).

What is there to be understood by the "content" of a representation? Since my analysis concentrates on the knowledge-seeking uses of representation, I only consider their *informational content* (as opposed, for instance, to what Goodman calls their "expressive content"). Before giving a more precise definition of what I mean by "informational content", let me state that a piece of information is any propositional content that it can be the object of belief[9].

The perceptual properties of a representation standing for a target in virtue of some representational scheme are signals carrying *information* about this target for agents mastering this scheme. Such agents can in principle extract this information from the representation and thus gain knowledge about the target. This implies at least two steps[10].

For one's knowledge-seeking enterprise to be successful, one has to know which elements of the model refer, what kind of approximations and idealisations were made in producing the representation, and many other things concerning the status and the accuracy of the information one extracts from the model. Consider a graph representing the variations of temperature in Paris during 2008. According to how precise one wants one's knowledge about the temperature in Paris in 2008 to be, one might want to know how and at what time of the day measurements were taken, how accurate the thermometer was, etc. In other words, one has to be able to interpret the information carried by the graph *as information about* the temperature in Paris. This is where misrepresentations can happen. For instance, if I take the graph to be about Paris, whereas it displays data obtained by measurements that were taken in Madrid, my knowledge-seeking enterprise fails; likewise if I believe the numerals shown in the graph to represent the exact value of the temperature during the whole day, whereas they were taken at 8am every morning.

But, before inferring – soundly or not – from the features of the graph to the features of its target, one has to *know how to read the graph itself*, in order to extract information from it. The graph functions under a particular representational scheme. This scheme determines which of its features are syntactically relevant properties, and how they are to be interpreted. Let me insist: reading off the information from the graph – even before interpreting it *as about* Paris – requires the knowledge of the scheme's semantic rules (although not necessarily

[9] I don't exclude the possibility of non-linguistic propositional contents.

[10] They are often simultaneous, but need to be distinguished by the analysis. As will appear, they correspond to the two kinds of questions about the representational function of models mentioned in the introduction.

of its actual referent), and not only of its syntax. In other words, the representation itself has content, independently of whether it is about some target or not. Consider again a graph of temperature in function of time: whether it is taken to be about Paris, about Madrid, or about any other place, even fictional or not specified, there is a sense in which it "says" to its user that the temperature was of 0°C on January 10th. It does contain such information, whether it is true of any real place or not. And, if no numerals feature on the graph, I can still state that the temperatures increase or fall over time. Whether one can genuinely speak of representation and informational content when there is no actual referent is a difficult issue, which I shall not tackle here[11]. But, since I will not consider issues concerning successful representations or misrepresentations, it is worth stating clearly that I take the liberty not to use the term "information" as a success term; rather, I use "informational content" to refer to the set of all statements that can in principle be made within a model by an agent who masters the scheme under which it functions, whether or not this agent is mistaken concerning the target, and whether or not there is any such target.

Thus, *the informational content of a representation consists in the set of all the pieces of information, at any level of abstraction, which an agent mastering its scheme could, in principle, extract from it and put in explicit[12] form*. As for the levels of abstraction[13], consider again a graph representing the variations of the temperature in Paris in 2008. One can read off information at various levels of abstraction: one can infer that temperature was of 0°C on January 10th, that it was colder in January 10th than in June 20th, that temperature globally increased from February to June, etc. Note that the informational content, so construed, is infinite: one can also infer that temperature on January 10th was less than 1°C, and less than 2°C, etc. "In principle" means that the definition of the informational content does not take into account whether the cognitive abilities and personal interests of agents, which direct their inferences, will practically lead them to extract such or such information. Neither does it take into account how difficult it is to extract different pieces of information. Therefore, the

[11] This raises the difficult problem of fictions. One more problem is: when exactly can we say that a principle such as Newton's Second Law, which, as such, does not have any representational content, becomes a representation: once the forces are specified? Or once precise values are given to the variables? It makes sense, here, to speak of levels of interpretation. And, even at the most abstract level, speaking of pure "syntax" (as Humphreys, 2004, about his "templates") seems to be a non-standard use, since there is always at least a minimal mathematical interpretation.

[12] I understand "explicit" in an intuitive sense. For a detailed analysis, see e.g. (Kirsh, 1991)

[13] For a recent analysis of this notion, see (Floridi, 2008).

informational content of a representation depends on its perceptual features and on the scheme under which it functions, but it is an "objective" notion, in the sense that it is not agent-relative[14].

2.2 Informational content *versus* cognitively accessible content

In practice, though, all the pieces of information contained in a representation are not equally easy to access, and some are practically inaccessible without the aid of external devices (such as computers). Unlike its informational content, the cognitively accessible content of a representation is agent-relative. When no more precision is given, I assume the agent to be an average human adult with normal cognitive abilities (this excludes, for instance, blind subjects, babies, or subjects with exceptional computational capacities). According to the kind of representation one is dealing with and to the kind of information one is seeking, obtaining this information might require various cognitive operations, which might be mentally processed, and which sometimes consist in drawing other external representations from the original one, with the help, for instance, of pencil and paper. This process has a certain cognitive cost. The cognitive cost required to obtain a particular piece of information depends both on the representation and on the agent.

One can say that a piece of information has actually been obtained when it is explicitly displayed, either mentally or in the external representation which might result from the process of drawing another representation from the original one (think, for example, of the process of solving an equation with pencil and paper, which results in the writing down of its solutions). The cognitive cost that an agent will typically have to pay to obtain a piece of information from a representation is inversely proportional to the relative immediacy of the availability of this piece of information, as defined by John Kulvicki (2009). A piece of information is immediately available for an agent if it is (i) displayed in extractable form[15];

[14] It is certainly agent-dependent, in the sense that nothing is a representation unless someone uses it as such; but it does not depend on the actual inferences agents typically draw from the representation. See page 7 for more precision on this point.

[15] A piece of information is extractable if there is a "non-semantic feature of the representation in virtue of possessing which it carries the piece of information in question and *no other*, *more specific piece of information*" (Kulvicki, 2009). In other words, it is extractable if no inference, even very quick, is required to put it in an explicit form.

(ii) syntactically salient[16]; and (iii) semantically salient[17]. Syntactic and semantic salience depend, as Kulvicki shows, on the user's cognitive abilities, and they are a matter of degree. Indeed, a map where shades of red and green stand for ranges of temperature would display some information in an immediately available form for a subject with normal perceptual abilities, but not for a colour-blind subject. According to the colour contrast, this information is more or less syntactically salient (for the normal subject). As for semantic salience: a diagram representing the succession of historical events on a time's arrow starting from the right would display information in a less immediately available form than if it started from the left, though one could get used to such scheme.

Therefore, the so-called "immediacy" of availability of a piece of information (and, accordingly, the cognitive cost required to obtain it) also admits degrees, depending (i) on the relative syntactic and semantic salience of the perceptual features of the representation that either display the desired piece of information in extractable form, or facilitate the inferential process the agent will typically perform in order to obtain this information (either mentally or by drawing a representation where this piece of information lies in an – absolutely – immediately available form), (ii) on the relative cognitive cost of this latter process.

Hence, different pieces of the informational content of a representation are not equally cognitively costly to access. Consider again the graph representing the variation of the temperature in Paris. Reading off the value of the temperature at time *t*, as well as assessing its global evolution are easy (not costly) tasks for an average user; in Kulvicki's terms, these pieces of information are immediately available. Now, for instance, giving the exact value of the difference between temperature on June 20th and on January 10th, however it is not very costly, requires some cognitive operations, and even in some cases the use of pencil and paper to write down the values and make the subtraction (e.g. for children, or if the values are very precise).

Likewise, some pieces of information within a representation are not equally easy to obtain for different users. For example, the solutions of an equation are more immediately available, or less costly to obtain, for a trained mathematician than for a beginner.

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[16] A piece of information displayed in an extractable form is syntactically salient if the properties of the representation responsible for this information are perceptually salient.

[17] Semantic salience depends on the relationship between perceptual properties of the representation and the data to which they correspond. For example, a map of temperature were shades of red would stand for cold areas and shades of blue for warm areas would be less semantically salient, given our habits, than the reverse.

Finally, two representations can contain (partially or totally) the same information, though displaying it in such different ways that they do not make the same pieces of information equally accessible. To borrow one of Kulvicki's (2009) examples, the results of a temperature survey can be presented as a list of triples of numerals, the first two standing for the coordinates of the different places where measurements were taken and the third one for the corresponding temperature values; the very same data can be presented in a two-dimensional diagram, on which the locations of the triples of numerals keep the relative distance between the places where measurements were taken. Colours corresponding to ranges of temperatures could also be added. The information contained in these three representations is the same. But the diagrams, and particularly the coloured one, make some information much more easily available: for instance, one can quickly assess the relative temperature of different areas. In order to extract such information from the corresponding list of numerals, one would need to achieve various inferential steps. But, if one is seeking the value of the temperature for given coordinates, one will rather use the list.

To sum up, the relative accessibility of a piece of information depends on the perceptual properties of the representation, on the scheme under which it functions, and on the agent's cognitive abilities (which include his skills, training, perceptual abilities, habits, background knowledge, prior beliefs, and particular interests). Within a representation, different pieces are not equally easy to access for a given agent, and the same piece of information might not be equally easy to access for different agents; and finally, two representations can have the same content without making the same pieces of information easily accessible.

2.3 Formats: a definition

According to our common understanding of the word "format", it makes perfectly sense to say that the list of numerals and the diagrams mentioned above are in different formats. My aim is to define the very notion of format in terms of the type of inferences[18] particular representations facilitate for individual agents with particular cognitive abilities.

[18] To dispel any ambiguity: though my account is compatible with and sympathetic to Suárez's (2004) inferential conception, it does not have the same status. Suárez claims that the representational relationship between models and the world consists in their enabling us to draw inferences about the phenomena (which corresponds to the first question stated in introduction); my point is to study the inferential processes agents perform within the model (second question).

Therefore, the format of a representation depends on its perceptual features, but also on the agent who is using it, and on the particular situation in which it is used (which includes the kind of information the agent is seeking); it is a highly context-dependent notion.

Consider a particular *R*-situation, where an agent *a* is using a representation *r* standing for some target in virtue of some representational scheme in order to draw a certain type of information. Let me call *I* the full informational content of *r*[19]. *I define the format of* r *in this* R-*situation as the type of* r*'s inferential enabling for* a. *The inferential enabling of* r *for* a *is a function of the different cognitive costs, which* a *will typically have to pay in order to draw different pieces of information from* r. *In other words, the relative length of the inferential processes that* a *would typically require to obtain the various pieces of information she might be seeking.* One can, in addition, define the *distance in format* between two representations r_1 and r_2 for agent *a* as the typical inferential length – or cognitive cost – required for *a* to draw r_2 from r_1.

Therefore, the format of a representation is agent- and context-relative, in addition to depending on the representational scheme and on the perceptual features of the representation. This is an important difference with apparently close notions, such as Goodman's (1968/1976) "symbol systems" or Haugeland's "representational schemes". Certainly, there is a sense in which a representational scheme depends on the users of the representations: reading Chinese characters, for instance, requires some abilities that only some agents possess; moreover, to assess whether such "black wiggly line on white backgrounds" is a "momentary electrocardiogram" representing heartbeats or "a drawing of Mt. Fujiyama"[20] depends on the user's knowing the scheme in which it is intended to be read. But formats, as I define them, are agents-relative in stronger a sense: although it seems counterintuitive, the "same" equation, that is, the *same* marks on a paper, intended to represent the *same* values, in virtue of the *same* representational scheme, is not in the same format for a trained mathematician who can immediately see the typical form of its solutions as for the beginner

[19] Within *I*, certainly, some pieces of information are irrelevant to *a*'s inquiry. The particular kind of information *a* is seeking is part of the *R*-situation and of the determination of the format: it directs her inferences in such a way that some pieces of *I* will absolutely not be considered. I could hence exclude from my definition the cognitive cost needed to access irrelevant pieces of information; nevertheless, since these inferential processes will typically not happen, keeping them aside would be useless. In addition, this accommodates with the case where unexpected – though relevant – information is found in a reasoning process.

[20] Goodman, 1976, 229-230.

who needs pencil, paper, and time, to achieve the reasoning that might lead him to the same results.

Conversely, any difference in the perceptual properties of two representations does not necessarily count as a change in format, since it is not necessarily a change in the inferential enabling of the representation for the agent. For instance, in a graph representing the motion of a pendulum, the relevant properties are the coordinates of the points representing its position in function of time. The colour of the line does not matter: whether the line is blue or red, the inferential enabling is the same. Now, if one wants to represent the motions of two different pendulums with different frequency of oscillations, it can be useful to draw the two lines in different colours. A bicolour graph with two colours reduces the cost of some inferences. Therefore, the bicolour and the unicolour graph are not in the same format – though their formats are quite close. However, for a colour-blind person, they are in the same formats. Finally, if one draws a graph representing the motion of a pendulum with various colours, corresponding for instance to the varying temperature of the room, the format again changes, since more information is available; and if one is only interested in the position of the pendulum, the colours create a useless noise, which can render inferences concerning the position less easy than in the case of a unicolour graph.

A precision is needed concerning the definition of formats as *types* of inferential enabling. Till now, I have been comparing between particular representations having the same target and at least partially the same informational content, in terms of their inferential enabling for individual agents. However, one would like to be able to say that two representations, which do not have the same target, hence having different informational content and incomparable inferential enabling[21], can nevertheless be in the same format. For instance, there must be a way to state that a map of Paris and a map of London at the same scale, representing the same type of aspects of both cities (roads, touristic highlights, etc.), are in the same format. Resorting to counterfactuals, one could argue that for a representation *a* of target *A* to be in the same format as a representation *b* of target *B* means that, if the process by which *A* has been constructed had been exactly the same, except that *B* would stand instead of *A* (wherever and however it features in this process), the inferential enabling of *a* and *b* would have been the same. This might not be very satisfactory, but let me just assume, for now, that

[21] Since inferential enabling is defined as a function of the relative accessibility of particular pieces of information, if no piece of information is common to two representations, their inferential enabling is incomparable.

there can be a notion of format as a type of inferential enabling, corresponding to types of inferences and types of information.

The notion of format, so defined, is a comparative tool rather than a tool to characterise isolated representations in given *R*-situations. Indeed, it is certainly practically impossible to measure the inferential enabling of some representation for an agent. But it is possible to compare the format of a representation for different agents, as well as the respective formats of two representations for the same agent, and to assess their distance. As to the second case, it enables us characterising differences between equations and graphs, linguistic descriptions and images, but also between representations belonging to the same type of representational schemes (linguistic, iconic, diagrammatic, etc.): two equations or two graphs in different coordinates, for instance. As such, it has to be distinguished from the philosophical analyses of the differences between types of symbol systems (Goodman, 1968/1976) or what Haugeland (1991) calls "representational genera". Of course, there might be typical inferential enablings associated to different types of systems – and many analyses, in the field of Artificial Intelligence and cognitive science have shown how diagrammatic representations can dramatically enhance agents' problem-solving capacities[22]. Such studies are of course relevant to my analysis. But my point is to define the very notion of format – of which we have a common intuitive understanding – in terms of inferential enabling for particular agents. As a result, it helps us gather various phenomena, which are not usually captured under the same heading.

Why insist so much on agents' relativity of formats? Our common understanding of the word "format" might make us feel reticent about admitting that the same equation standing for the same values in virtue of the same scheme is in two different formats for two different agents. Indeed, in many cases, assuming a human agent with average cognitive abilities – or, in the context of a study of theorising, assuming a scientist with the same skills and background knowledge as other members of his scientific community – seems natural. To qualify a change in coordinate as a change in format, or the difference between a list of numerals and a map as a difference in format, one does need to appeal to psychological peculiarities of individual agents. Nevertheless, as soon as one takes into consideration inferential aspects, that is, epistemic differences, as opposed to logical differences, it is quite

[22] See, e.g., Larkin and Simon, 1987. For a study from the perspective of distributed cognition, see Zhang, 1997. Sometimes, such considerations come along with cognitive hypotheses concerning the format of mental representations (see Johnson-Laird, 1983). But such hypotheses are not required for an external descriptive analysis such as mine.

difficult to draw a clear-cut frontier between such epistemic differences and psychological ones. Once representations are approached from the perspective of the inferences one can draw with them, agents are an important parameter. And once agents are in the picture, differences between agents have to be considered to give a precise definition of formats.

In addition, and more importantly for my purpose, there are issues concerning theorising in which individual differences matter much. For example, for a study of scientific learning or popularising, the question of the individual skills of agents is crucial. Normative issues concerning the best way to teach theories could be stated in terms of formats. Beside, learning a theory could be described as a process of changing – in fact, reducing – the distance in formats between the different representations that are used in this theory. More simply, when we learn mental calculation, we get skilled in such a way that the distance between a problem and its solution gets smaller. This corresponds to a very intuitive idea of what becoming an expert means, as the case of Classical Mechanics will suggest below.

Agents' relativity of formats does not prove useful only in expert-novice cases. As the case of Feynman's diagrams will show, there are examples where, even between experts, different skills, individual preferences, and theoretical commitments, make a difference in the use of theoretical representations, which can fruitfully be treated in terms of formats.

3. Two cases

3.1 Classical Mechanics

Classical Mechanics can be expressed under various formulations (Newtonian, Lagrangian, Hamiltonian, Hamilton-Jacobi theory), which are generally considered as equivalent, since a chain of mathematical deduction lead from any of them to any other. The differences between them can be assessed in various ways: by comparing their fundamental principles (Newton's Law *versus* Principle of Least Action under its various forms), their mathematical language (differential equations *versus* variational principles, scalar *versus* vectorial quantities), their basic concepts (force *versus* energy), the system of coordinates they use to describe the motion of mechanical systems (Cartesian coordinates versus generalised coordinates, the latter being different in the Lagrangian and the Hamiltonian formulations), or the form of the equations of motion they result in (second order *versus* first order).

Since the formulations are logically and empirically equivalent[23], there is a sense in which these differences are "purely formal". Nevertheless, although it is surely not a case of theory reduction or of incommensurability, one might intuitively feel like describing these differences as *conceptual*, or as differences in the *way they represent* the world. However, a difference in representation should mean that the various formulations do not have the same truth conditions, which is not the case.

In parallel to studies of the formulations of mechanics in terms of their underlying mathematical structure, or to approaches – more historical in character – focusing on the evolving concepts, principles, and laws of mechanics[24], I suggest that one could adopt a local perspective. By "local", I mean that, instead of considering the whole edifice of mechanics, one could look at the "concrete pieces of syntax", to borrow Humphreys' expression, that are used in theorising, and assess their differences in terms of format[25]. This might shed some light on the issues raised above.

Consider the description of the motion of a physical system – e.g., the bob of a grandfather clock – by means of the laws of Mechanics. As mentioned above, I deliberately ignore, in this paper, the difficult issues of idealisation and abstraction, and I adopt a naïve glance on the relationship between the grandfather clock and its idealised representation *as* a simple pendulum. Since I am interested in how inferences are processed *within* the model, understood as the particular equation or graph used by the agent, let me assume, for the sake of the argument, that there exists such object as the simple pendulum[26] (that the grandfather clock *is* a simple pendulum) and that the equations of mechanics are used to gain knowledge about its behaviour. "Gain knowledge", here, can refer to various situations: when an expert inquires into the mathematical consequences of an equation in order to develop his theory, as well as when a student learns how to describe and predict the typical behaviour of mechanical

[23] This equivalence would require further analysis, which is not within the scope of this paper. See, e.g., (North, 2009). Some studies of mechanics in the structuralist view of theories, such as (Balzer et al., 1987) show that this equivalence is far from trivial. However, that does not matter for my argument.

[24] The most famous of such analyses is probably the one given by Mach, 1883. To be clear: my point is not to show that such analyses are fruitless, but to propose another way to approach these issues.

[25] In a different vein, see (Wimsatt & Griesemer, 1987), for a detailed assessment of the advantages of using "isolable" representations as units of analysis for a study of conceptual evolution.

[26] We are facing again the problem of representation of fictional entities. For reasons already invoked (that I am interested in the inferences *within* the representation, and not as being *about* their target), I will ignore this problem here. To be clear: what I call the "representation", here, is the equation, not the pendulum. For a detailed account of how fictions themselves work as representations, see (Frigg, 2009).

systems, thus learning the meaning of the laws of mechanics, by writing and solving the equations for what Kuhn (1974) calls an "exemplar"[27]. All these practices, where particular representations are constructed and manipulated in order to deepen one's understanding of a theory are precisely what I mean by "theorising".

Back to the pendulum. One can represent its motion by a differential equation of the form of Newton's Second Law ($\vec{F} = m\vec{a}$), where the force $\vec{F}$ is specified: $m\vec{a} = -\left(\frac{mg}{l}\right)x\cos(\alpha)$, with $\vec{a}$ standing for the acceleration, m for the mass of the bob, x for the position of the bob, l for the length of the thread, g for the gravitational force, and α for the angle of oscillation. Knowing the initial conditions at time t_0, one can write down the corresponding differential equations. The following step it to solve them. The solutions of the equations enable one to give the precise value of the position of the bob at time t. According to the information one is seeking (general evolution, precise values…), one might also draw the graph of these solutions within two instants t_1 and t_2.

However, some mechanical problems cannot, in practice, be solved that way (for all agents). This is, for instance, the case for constrained systems[28]. In such cases, the very task of writing down the Newtonian equations of motion is practically impossible, because the forces maintaining the constraints are unknown. For this kind of problem, the analytical formulation is more tractable, since one does not need to specify the local equations of the forces acting on the system at any instant. Knowing the constraints of the system enables one to describe it in terms of generalised coordinates[29], from which one can draw the Lagrangian L of the system, which is, in simple cases, the difference of its kinetic and potential energies. Hamilton's principle ($\delta A = 0$) prescribes that the integral A of L between t_1 and t_2 be stationary. From this principle, one can draw the Lagrangian equations of motion, which have the following form: $\frac{d}{dt}\left(\frac{\partial L}{\partial q_i}\right) - \frac{\partial L}{\partial q_i} - 0$.

Like the Newtonian equations, to which they can be proven to be equivalent, the Lagrangian equations are second order differential equations. In some cases, although one can write them down by the procedure described above, they do not have analytical solutions, and

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[27] Exemplars are typical problems given one finds in handbooks as exercises for the students.

[28] A system is constrained when its different points cannot move independently because of internal forces relating them.

[29] Generalised coordinates take into account the constraints of the system, and correspond to its degrees of freedom.

are therefore intractable. The Hamiltonian formulation offers a solution to such cases: by using a different kind of generalised coordinates[30], one obtains first order differential equations. The so-called Legendre transformations enable one to change an intractable Lagrangian equation into two corresponding Hamiltonian equations, which are tractable.

The equivalence of the equations of motion in their Newtonian, Lagrangian and Hamiltonian form, and of Hamilton's Principle, can be easily shown at an abstract level (when they do not have any representational content). Therefore, the particular representations one can construct with them are, in principle, equivalent, in the sense that they have the same informational content. Nevertheless, their inferential enabling, in practice, is different. Writing down the equations by specifying the vectorial quantities representing the forces acting on the system at instant t_0, or obtaining them by determining scalar quantities (energies) and by relying on a variational principle (Hamilton's principle), which concerns the trajectory taken globally, does certainly not consist in the same inferential processes. Once the equations are written, solving them does not, again, require the same cognitive cost. According to the case at hand, one will rather use one or the other formulation.

These differences in inferential order and in cognitive cost can be assessed without a reference to the cognitive abilities of particular agents. Now, describing the various equations and principles of mechanics as being equivalent representations in different formats enable us shed light on at least two more aspects of theorising, where individual agents get into the picture.

Throughout their history, and in particular during the XIXth century[31], the principles of mechanics have been the object of various philosophical or meta-scientific reflections among physicists, who worked towards the most intelligible and "rational" presentation of these principles. According to their reasoning style[32], theoretical commitments, and assessments of what it is for a theory to be intelligible and explanatory[33], different physicists do not present the principles of mechanics in the same order: some consider that the whole theory is "implicitly" contained in the Newtonian equations[34], others claim that D'Alembert's

[30] Lagrangian generalised coordinates have the dimension of positions and of velocities, whereas Hamiltonian ones have the dimension of positions and momenta.

[31] See, e.g., Hertz and 1894, Mach, 1883.

[32] Think of Maxwell's (1890) reflections on the use of visual analogies.

[33] See e.g. Mach's (1883) notion of "economy of thought".

[34] This view is advocated by Mach (1883), whereas Duhem (1903) argues for a genuine difference between force-based and energy-based explanations.

Principle "says" something more than Newton's Law[35]; some handbooks deduce the Lagrangian equations from D'Alembert's Principle, others chose to deduce them from Hamilton's Principle, etc. According to which principle is taken as fundamental, and in which order other principles are drawn from it, the whole edifice of mechanics, and each of its principles, look slightly different. Therefore, in addition to saying that Newtonian and Lagrangian equations are not in the same format, one can claim that Newton's Second Law, for instance, is not in the same format for someone who takes it as fundamental, and for someone who prefers to start from a variational principle[36]: although the logical relations between these principles are independent of such order, the inferences one will typically draw are different. And this gives a meaning to the claim that there is a conceptual[37] difference – despite logical and empirical equivalence[38] – between Newtonian and Lagrangian representations of motion[39]. Moreover, the historical development of mechanics, which relied in mathematical developments rather than on empirical novelties, can be described as a process of changing the formats of the various principles, by inquiring into their mathematical consequences, thus showing unnoticed relations and equivalence between them, and modifying their inferential enabling[40].

Finally, as I suggested at the end of section 2, formats give us a tool to describe what becoming an expert and understanding a theory consist in. Learning mechanics is a process of

[35] See Lanczos' (1970) reflections on what he calls the "*A*-postulate", and Butterfield's (2004) comments on them.

[36] This echoes Kuhn's (1970) view that Newton's Law, as it is given in Newton's theory, does not have the same meaning as when it is deduced from Einstein's theory. However, here, there is an *intra-theoretical* "conceptual" difference, without any logical difference.

[37] So-called "conceptual (or inferential) role semantics", as advocated by Gilbert Harman (1982, 1987), defines the content of mental representations and linguistic expressions (note that Greenberg & Harman also consider non linguistic representations, such as maps), as its role in the cognitive life of agents. The psychological version of Harman results in divorcing semantics from considerations of truth-conditions. As clearly appears, my account is in agreement with such approach.

[38] Feynman (1965) speaks of a "psychological difference" within "scientifically equivalent" theories.

[39] As Kuhn (1974) claims, understanding a theory is a matter of know-how as much as of know-that. Learning *how to use* a "symbolic generalisation", such as $\vec{F} = m\vec{a}$, and acquiring a certain world-view (a picture of the world *as* Newtonian), is one and the same thing. Therefore, there is a sense in which one does not "see" the world the same way when using Newtonian equations or Hamilton's principle.

[40] See Anouk Barberousse (2008) description of Hamilton's work as a process of "making explicit", in Brandom's (2000) sense, the content of Newtonian mechanics. It shows in which sense one can speak of "conceptual change" without there being changes in truth-conditions.

getting trained to use its various equations according to the problem at hand, and might result in being able to "see" easily the typical form of the solutions of an equation by reading it[41], as well as to transform it into another, more tractable equation, and thus to see in a more immediate way the equivalence of various formulations of the same content. In other words, learning, like developing a theory, consists in changing the formats of its different principles, by reducing the distance between them[42]. Note that experts are much more prone to state the equivalence of the various principles of mechanics, than novices who are generally struck by their difference and who have the feeling that representations of a pendulum in Newtonian and Hamiltonian formulations do not give the same explanation of its motion. Showing their equivalence is cognitively more costly for novices than for experts. Their formats are, for the novice, more distant to each other than for the expert.

3.2 Feynman's Diagrams

As a second example of the fruitfulness of the notion of format, I will briefly consider some issues raised by the case of Feynman's Diagrams. For historical and technical details, I refer the reader to the works of David Kaiser (2000, 2005), from which I drew all my material.

Feynman first introduced his famous diagrams in 1948, as a mean to help physicists get rid off the infinities of quantum electrodynamics (QED), which prevented them to give predictions about complex interactions of atomic particles. At that time, he presented them as "mnemonic devices" (Kaiser, 2000, 52) to complete complex high order calculations[43] without confusing or omitting terms; "they were a form of bookkeeping" (*ibid.*). A year later, Dyson (1949a,b) demonstrated the equivalence of Feynman's diagrams with the mathematical derivations given at the same time by Schwinger (1948a,b) as a workable calculational

[41] See De Regt & Dieks' (2005, 151): "A scientific theory *T* is intelligible for scientists (in context C) if they can recognise qualitatively characteristic consequences of *T* without performing exact calculations." They report that Heisenberg (1927, 172) and Feynman (et al., 1965, vol. 2, 2-1) express similar views.

[42] See Andrea Woody's (2004, 792) notion of "articulate awareness" within a representational scheme as a "hallmark of nontrivial knowledge".

[43] The process called "renormalisation" consisted in approaching the solutions for complex interactions by adding higher-order corrections to solutions of less complex situations.

scheme for QED. Moreover, Dyson made Feynman's methods "available to the public"[44], by codifying the rules for constructing the diagrams, on the basis of a one-to-one correspondence of features of the diagrams to particular mathematical expressions. As is well known, this was the beginning of an amazingly successful career for these diagrams, which, as Kaiser (2005) tells in details, were eventually used in almost every field of theoretical physics.

In some obvious sense, Feynman's diagrams and their corresponding mathematical formulae – along the lines of the rules of derivation given by Dyson – have the same informational content, though different inferential enablings. Such differences are handled by various deep analyses about visual or diagrammatic reasoning, and its role for agents' problem-solving abilities, as mentioned in section 2. As in the case of mechanics, I suggest that attention to such differences in formats, which are obvious when one looks at concrete problem-solving tasks, is also fruitful for a study of theorising: one does not have the same understanding of the general theoretical framework one is exploring, developing, or learning when one uses a mathematical formula or a diagram. This is a way to give a precise sense to Kaiser's (2005, 75) suggestive remark, that Dyson demonstrated "the mathematical – though by no means conceptual – equivalence between Schwinger's and Feynman's formalisms."

Now, the interest of the case of Feynman's diagrams for a study of the importance of formats in theorising is not exhausted by such remarks. Indeed, the diagrams themselves happen to be in different formats[45] for different users, as Kaiser's (2005) analysis of the "Feynman-Dyson split" shows. Relying on explicit remarks they wrote in letters and personal papers, he indeed shows that Feynman's and Dyson's use and assessment of the status of the diagrams within QED was different to many respects.

Dyson's application to showing the rules of derivations of the diagrams was a sign of his conception of them as secondary, psychological aid to perform mathematical calculations. If they had not been proven rigorously derivable from mathematical *formulae*, their use would not have been legitimate[46]. He conceived of them as means to "visuali[se] the formulae which we derive rigorously from field theory" (Dyson, 1951, 129-130, quoted in Kaiser, 2005, 190). Hence, to him, they had a meaning only *within* QED, to which they added nothing except cognitive tractability.

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[44] Dyson, Letter to his parents, 4 Dec. 1948, quoted by Kaiser (2005, 77).

[45] Kaiser (2005) gives an impressive analysis of the "plasticity" of diagrams throughout their "spreading" in theoretical practices in modern physics, and of their varying uses and interpretations in different context.

[46] Dyson (1979, 62) writes that "until the rules were codified and made mathematically precise, [he] could not call [Feynman's method] a theory."

On the other hand, Kaiser reports that Feynman never felt the need to show how to derive diagrams from mathematical expressions, and expressed clearly in various occasions his theoretical preference for diagrams over mathematical formulae: “All the mathematical proofs were later discoveries that I don’t thoroughly understand but the physical ideas I think are very simple.” (Feynman, Letter to Ted Welton, 16 Nov. 1949, quoted in Kaiser, 2005, 178) [47]. Hence, contrary to Dyson, he thought of them as primary and more important than any mathematical derivation they might be given. In addition to being mnemonic devices, they provided an intuitive dimension to the theory, and Feynman took them as “intuitive pictures” (Kaiser, 2005, 176): as Dyson notes, Feynman “regard[ed] the graph as a picture of an actual process which is occurring physically in space-time”[48]. Rather than visualisations of the formulae, they were primary visualisations of the physical processes themselves.

As Kaiser suggests, this difference in use by the two physicists was based on different theoretical commitments and preferences. Contrary to Dyson, who demonstrated how to cast both Feynman’s diagrams and Schwinger’s equations within a consistent field-theoretic framework[49], Feynman’s renormalisation approach, from which the diagrammatic method arose, was based on particles, rather than on fields, as the primary ingredients of his theory (Kaiser, 2005). And, in 1949, he explicitly divorced the diagrams from QED. More generally, as Kaiser notes, Feynman had a preference for a semi-classical approach, and worked almost entirely in terms of particles, trying to remove fields from theoretical descriptions altogether.

This is an interesting case of the same representation being in two formats for two different experts. The diagrams have different inferential enablings for Dyson and Feynman: they stand in a reverse inferential order for the two physicists, who do not even draw them the same way, since Dyson deduces them from mathematical formulae, whereas Feynman draws them intuitively. Nevertheless, it makes sense to claim that they have the same informational content, at least in some cases. Feynman does not reject the proof of the deducibility of diagrams from formulae, whereas they are not relevant for him to see the meaning of his diagrams, which he takes to be full-blown representations of the phenomena.

I suggest, finally, that this sheds light on what I take to be an important feature of highly theoretical science, in addition to the other points already mentioned. Contrary to the “toy” examples taken in section 2, the nature of the “target” of the representation is far from

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[47] He also spoke of the “physical plausibility” of the diagrammatic approach (quoted in Kaiser, 2005, 177).

[48] Dyson, 1951, 127 (quoted in Kaiser, 190).

[49] Dyson (1965, 23) claims that he contributed to allow “people like Pauli who believed in field theory to draw Feynman diagrams without abandoning their principles”.

being a trivial issue, and has been the object of one of the most difficult problem in philosophy of science, namely the question of the reference of theoretical terms and of scientific realism, which is (fortunately) not in the scope of this paper. Nevertheless, I suggest that attention to the representations that are used in theorising from the perspective of the inferences they enable different agents to do can shed a new light on such issues: different scientists[50], according to their individual theoretical commitments and skills, do not use the same representation the same way, which means that they relate it differently to other representations, and, finally, to the physical world[51].

4. Conclusion

Theories have most of the time been approached as representational wholes, from what Humphreys (2004) calls a "no-ownership perspective". They are nevertheless complex entities, which are always constructed, developed, learned, and hence understood, from the point of view of individual cognitive agents. Approaching theorising by focusing on the interaction between agents and concrete devices displaying information in a particular form, thus gathering different issues which are usually not treated under the same heading (theory development, individual scientists' ontological commitments, expert versus novice understanding of theories), seems a promising path to understanding how computational and representational aspects of theorising relate. When it comes to highly theoretical issues, it is worth paying attention to computation and to forms of reasoning, rather than exclusively focusing on the representational relationship between theories – or models – and the world.

The approach I have advocated, which consists in focusing on particular devices and in analysing them in terms of formats, is, I suggest, a way to enlighten aspects of theorising, which have usually been studied from much more global perspectives. Indeed, contextual

[50] This can of course be extended to the scale of communities. Another interesting case, which I study in another paper, is provided by the debates around linkage mapping in the 1920's, where maps are alternatively taken to be spatial visualisations of numerical data and "pictorial" – in some broad sense – visualisations of the chromosomes. I suggest that such an ambiguity in the meaning of representations, far from being pathological, is characteristic of highly theoretical science. Many historical debates happen to be controversies about the best format to use.

[51] What this example suggests is something along the lines of what Humphreys (2004, 82, 83) calls "selective realism". Kuhn (1974) also suggests that different scientists using the same symbolic generalisations can have different epistemic attitudes towards them, which are related to their own "ontology".

relativity of theory understanding has usually been approached by philosophers of science through global units of analysis such as Quinean “conceptual schemes” and Kuhn’s (1970) “paradigms”. Hacking’s (1982, 1992) “styles of reasoning” and Kitcher’s (1989) “argument patterns” put more emphasis on reasoning schemes, but are still quite general and abstract[52]. The notion of format is intended to enable us capturing these important aspects of theorising, by studying concrete local units, in interaction with individual agents.

[52] Note, however, Andrea Woody’s (2004) analysis of diagrammatic representational schemes in crystal field theory as explanatory standards along the line of Kitcher’s argument patterns.

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