Models and Formats of Representation

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1 Introduction

Models are generally used by scientists to obtain predictions and to provide explanations about phenomena. Their predictive and explanatory power is usually thought of as depending on their representative power: a model is a device standing for a target system in virtue of some relationship between its features and features of the system. This representational relationship enables scientists to draw inferences and to obtain information concerning the target system by reasoning with - and in some cases literally manipulating - the model. It is still not clear, though, in virtue of which features models allow such inferences to be drawn.

Some philosophers[1] have argued that appealing to isomorphism[2] between models and target systems is not sufficient to explain how models are used by cognitive agents, in practice, to represent the systems they study. The fact that the structure of theories seems to adequately capture the "structure" of the world is a genuine philosophical problem - roughly, the problem of the use of mathematics in science[3] - but it does not solve the problem of how scientists use representing devices to draw inferences about the systems they study. Examining this problem does not commit anyone to any particular conception of the structure of scientific theories. When it comes to representation, one has to pay attention to the kind of cognitive operations and sometimes to the concrete manipulations scientists perform when they learn, develop and

[1][Frigg, 2002], [Frigg, 2008], [Suárez, 2003].

[2]According to the semantic conception of theories, whose main defenders are Patrick Suppes, Frederick Suppe, Bas Van Fraassen, and Ronald Giere, theories are sets of mathematical models which stand in an isomorphic relationship with the phenomena. Ronald Giere, though, stands slightly apart, by speaking of similarity between models and phenomena, and by insisting on the pragmatic and cognitive aspects of representing.

[3][Frigg, 2008], section 6 argues for this distinction between the problem of representation and the problem of the application of mathematics to reality.

apply theories. They do not use theories as wholes, but rather representing devices of various kinds - which are most often called "models": diagrams, graphs, 3D scale models, imaginary models such as the simple pendulum, etc.

In this paper, I will focus on a particular kind of models, namely imaginary models (from now, I-models), like the simple pendulum. The importance of such models has been forcefully underlined by recent contributions[4], and they appear to be omnipresent in scientific learning, theorizing and practice. In addition, they pose a special problem to an analysis of representation, since they are not particular concrete devices like equations or diagrams. On the other hand, they are not reducible to the mathematical structures they might instantiate[5]. Roman Frigg[6] distinguishes between two sets of questions concerning I-models (which he calls "model systems"), namely ontological questions about what kind of entities I-models are, and semantic questions about what kind of relationships I-models have with the phenomena they stand for. Frigg addresses the first problem as a preliminary to solve the second one. I take a different route, and I shall address the semantic problem, by turning it in the following way: how do scientists *use* I-models to draw inferences and gain knowledge about the systems they stand for?

First, I propose a clarification of the very notion of representation (2.1) by emphasizing the importance of what I call the format of a representation (2.2) to the inferences cognitive agents can draw from it. Then, I turn to the core question and I analyze the various representational relationships that are in play in the use of I-models (3.1 and 3.2). I finally conclude that there is no special semantics to be applied to I-models, and that the study of the representational power of models in general should instead focus on the variety of the formats that are used in scientific practice.

2 Formats of representation

In this section, I shall present a conception of representation which is not to be restricted to the scientific domain, though focusing only on the knowledge-seeking aspects of representing, and I will introduce a new notion, namely the notion of a format, which is intended to help us capture some important phenomena at play when cognitive agents use representations in order to gain

[4][Frigg, 2002], [Frigg, 2008], [Giere, 2006].

[5]As Frigg states, they "do not exist spatio-temporally but are nevertheless not purely mathematical or structural in that they would be physical things if they were real." [Frigg, 2008], section 2.

[6][Frigg, 2008].

knowledge on something. That is a necessary preliminary to a study of the use of I-models in representing target systems, which I will start in section 3.

2.1 Representation

I propose to define representation - or representing[7] - as a cognitive activity which consists in using some device (the representing device, or *representans*) in order to gain knowledge concerning something it stands for (the *representatum*). In the most straightforward and less problematic cases, the *representans* is a particular concrete object, which can be marks on a paper, vibrations of the air, or anything else to which one can have a direct perceptual access[8]. The *representatum* can be a physical object, some properties of an object, the temporal evolution of a value, a causal process, etc. I will neutrally speak of the "scene" represented.

The perceptual properties of the *representans* are signals carrying information for us concerning the *representatum*. "Information", here, is to be understood as referring to any propositional content that can be object of belief. Thus, "informational content" is synonymous with "propositional content". Since I am exclusively interested in the knowledge or information-seeking function of representation, I restrict my analysis to the propositional content of representing devices[9].

In order to gain knowledge about the *representatum* by means of the *representans*, a cognitive agent has to master the rules of interpretation of these signals. Indeed, a device can be used in representing a scene in virtue of various - at a pinch, an infinity of - kinds of relationships between its perceptual properties and the properties of the scene. Any device can be decreed to represent any scene by *fiat*. The representational relationship can also be grounded in non conventional relationships between the *representatum* and the *representans*: for instance, a photograph of a dog represents that dog - under some conditions of luminosity and framing, and provided the photograph is taken from the adequate distance and with the correct focus - in virtue of the causal relationship between the light reflected by the dog

[7]One has to be careful with the term "representation", which can be used to refer to 1. a type of actions (representing), in which case it comes without any pronoun, 2. a particular representing device, and 3. an equivalence class of representing devices. Most of the time, the context dispels any ambiguity.

[8]By speaking of "direct perceptual access", I am not committing myself to any particular theory of perception; I think there is an uncontroversial way of assessing that I have a direct perceptual access to the concrete physical objects around me, whereas abstract objects such as mathematical structures or propositions are not accessed the same way.

[9]For instance, I am not interested in the *expressive* content of a representation, in the Goodmanian sense of "expression" ([Goodman, 1976], ch. 2).

and the exposure of the film. Of course, even that kind of representational relationship relies on various conventional parameters, but the causal relationship plays a fundamental role in the establishment of the representational relationship between the picture and the dog.

Nelson Goodman[10] conceives the relationships between any *representans* and its *representatum* as a kind of denotation, governed by some specific symbol system. A symbol system consists in a syntax and a semantics: the syntax is the set of the perceptual properties (the "marks") that count as signals carrying information, together with their combination rules, and the semantics establishes the relationship between these properties and what they denote. According to what symbol system is used in interpreting it, the same set of marks can denote very different scenes. For instance, a "black wiggly line on white backgrounds" can be a "momentary electrocardiogram" representing heartbeats in a diagrammatic system, as well as a "drawing of Mt. Fujiyama". The relevant perceptual properties of both systems are different: for instance, the "thickness of the line, its color and intensity, the absolute size of the diagram, etc., do not matter", whereas any of these properties - and any change in them - is significant in the case of the sketch[11].

In this paper, I adopt such a perspective on representation, though I shall propose a new notion, namely the notion of a format, in order to capture cognitive and pragmatic features of the use of representation by human agents, which the notion of a symbol system ignores.

2.2 Formats

By mastering the symbol system under which a representing device has to be interpreted, one can draw information about its *representatum*. Now, it is worth noting that two representing devices can contain the very same information (concerning the same thing) though conveying it in different way to us[12]. Consider for example a digital picture and the linguistic coding for each of its pixel: they are strictly equivalent as to the information they contain. But only the two-dimensional one enables unaided human subjects to draw information concerning the scene that has been pictured. The informational content of the linguistic coding, without any external computing device which would transform it into a two-dimensional picture, is strictly inaccessible to us.

[10] [Goodman, 1976].

[11] [Goodman, 1976], pp. 229-230.

[12] In Herbert Simon and Jill Larkin's terms, such representations are "informationally equivalent" but "computationally different" [Simon and Larkin, 1987].

There are other examples of informationally equivalent representations which are different as to their cognitively accessible content in less dramatic a way. Consider the following one[13]: the results of a temperature survey can be presented as a list of triples of numerals, the first two standing for the coordinates of the different places where measurements were taken and the third for the corresponding temperature values; the very same data can also be presented in a two-dimensional map, on which the locations of the triples of numerals keep the relative distance between the places where measurements were taken. Colors corresponding to ranges of temperatures could also be added on the corresponding areas. The information contained in both representations is exactly the same, and human agents - unlike in the picture/bits case - are able to draw it from any of them. But the map makes some information much more easily available: for instance, if warm shades stand for high temperatures and cold shades for low temperatures, one can quickly conclude that the southern part of the represented area is warmer than its northern part. In order to extract such information from the corresponding list of numerals, one would need to achieve various inferential steps. Moreover, the inferential operations leading to the same conclusion from the consideration of both representations are not of the same kind: reading numerical values and comparing them by taking into account the corresponding coordinates does not consist in the same cognitive operation as comparing the relative locations of different colors on the map.

More generally, one can state that in any representation, some pieces of information are immediately[14] available, though others are extractible *via* an inferential process that can consist in a more or less large number of steps, and which can be of various kinds[15].

[13]This example is freely inspired by a manuscript by John Kulvicki [Kulvicki, 2008].

[14]Whether or not there is a kind of perceptual knowledge which is not the product of any inferential process - which is genuinely immediate - is not relevant to my point; by speaking of "immediately available information" as opposed to information which is obtained through inferences, I am not claiming that there is a difference in nature rather than in degree between both. Let's imagine there is water on the pavement while I am walking on the street; I would say that the proposition "there is water" is an immediately accessible piece of information, whereas "it has rained" is obtained through an inferential process. I cannot get into more details here, but the intuitive notion of immediacy will do the job. [Kulvicki, 2008] proposes an elaboration of this notion.

[15]In the area of Artificial Intelligence, studies in the problem-solving abilities of human agents (for a philosophical treatment, see for instance [Simon and Larkin, 1987]), have shown how the presentation of data in a two-dimensional image can dramatically enhance the inferential capacities of agents. Sometimes, such considerations come along with cognitive hypotheses concerning the format of mental representations and reasonings (see [Johnson-Laird, 1983]). But such hypotheses are not required to state that the mode of presentation matters to human reasoning capacities.

The notion of a format is intended to capture these cognitive or inferential[16] differences such changes in representation make. Certainly, in Goodmanian terms, one can say that these various representations are to be interpreted in various symbol systems: their syntactic and semantic rules are thoroughly different. But my point is to characterize and measure the differences between these various representations from the pragmatic and cognitive perspective of the agent who is supposed to draw inferences from the *representans* in order to gain knowledge about the *representatum*. Therefore, the notion of a format is context and agent-relative, whereas the notion of a symbol system can be fully defined in objective terms of syntax and semantics.

An informal definition of a format would be the following: the format of a representation is the particular way a representation conveys the information it contains to the user. The digital picture and its linguistic coding are in two different formats; the list of numerals and its corresponding map also. More precisely, the format F_{RS} of a particular device R representing some scene S can be isolated according to the *inferential power* of the representation, that consists in

- I: the kind and quantity of information about S a particular agent A in a particular context C can draw from R;
- CC: the relative length of the inferential process P - or the number of inferential steps, if they can be counted - necessary for A in C to draw I from R (the cognitive cost);
- CK: the kind of cognitive operations involved in P.

Certainly, all that depends on the syntax and semantics of the system under which R is interpreted, but this is not sufficient. F_{RS} also depends on A and C: the cognitive abilities, skills and background knowledge of the agent, her cognitive goals (the information she is seeking), the information which is previously available, etc. Indeed, according to the agent's cognitive abilities and goals, any change in the perceptual properties does not necessarily count as a change in format, since it is not necessarily a change in the inferential power of the representation: for instance, in a graph representing the temporal evolution of the position of a pendulum, the relevant properties are the coordinates of the points representing its position. The

[16] I speak equally of cognitive or inferential equivalence and difference. Although there are cognitive processes that cannot be characterized as inferential, all the cognitive processes with which I am concerned are inferential in kind, since I am exclusively interested in the propositional content of representations.

color of the line does not matter: whether the line is blue or red, I, CC, and CK are strictly the same, thus the format is the same. Now, if one wants to represent the temporal evolution of two different pendulums with different frequency of oscillations, it can be useful to draw the two lines in different colors. A graph with two colors facilitates the inferences, as compared with a graph representing the two pendulums in the same color. Therefore, the bicolor graph and the unicolor graph are not in the same format - though their formats are quite close. Finally, if one draws a graph representing the temporal evolution of the position of a pendulum with various colors, corresponding for instance to the varying temperature of the room, the format again changes, since more information is available; but if one is only interested in the position of the pendulum, the colors create a useless noise, which can render inferences concerning the temporal evolution of the position less easy than in the case of a unicolor graph[17].

Now, certainly, I, CC, and CK are practically impossible to measure in a precise way. Nevertheless, one can compare two representations a and b by saying that, for instance, drawing such piece of information from a demands a greater cognitive cost than from b, or that some piece of information which is available in a is practically inaccessible in b. Likewise, despite the fact that we still do not have any precise description of the format of mental representations and processes, one can intuitively acknowledge that drawing information from a graph and from an equation does not consist in the same kind of cognitive process. The notion of a format is mainly intended to be a comparison tool between different representations of the same scene, which are (at least partially) informationally equivalent.

Finally, I define a *representation-type* as the class of all possible and actual representing devices of the same scene in the same format. In other words, a representation-type is identified on the basis of its inferential power[18]: it is the class of all representing devices that enable agents with equal cognitive

[17]Insisting on the agent and context-relativity of the notion of a format might seem useless, since in my analysis of the use of I-models, I will not mention individual agents' differences, implicitly assuming an average scientist with normal cognitive abilities. Nevertheless, these precisions are worth giving, at least for two different reasons: first, the notion of a format can be used in other kinds of analysis, where individual differences matter; and second, since the notion of a format is intended to account for the cognitive aspects of the interpretation of representations, it is necessarily human-relative, contrary to the notion of a symbol system. Therefore, a formal definition of it cannot ignore the agent's importance.

[18]Such a conception could be provided useful tools from the so-called "conceptual role semantics"(see, for instance, [Greenberg and Harman, 2005]), according to which the meaning or content of a representation consists in its inferential role in the cognitive life of the agents.

abilities in equivalent contexts to draw the exact same information through the same inferential processes.

3 How do we use I-models to represent target systems?

Let me now turn to my main question: how do we use I-models to represent physical systems[19]? I-models are widely used by scientists when they develop hypotheses as well as when they predict and explain phenomena; they also play a central role in scientific learning. In textbooks, laws are most often introduced by reference to an I-model which enables students understand what the law means. When it comes to predicting and explaining the behavior of some physical system, one appeals most of the time to an I-model, claiming that it "applies", with various approximations and idealizations, to the target system: for instance, the motion of the bob of a grandfather clock can be represented by means of the simple pendulum. There are many other examples of I-models in various scientific fields; one could cite, for instance, the billiard ball model for the study of molecular motion in the kinetic theory of gases, isolated populations in population genetics, and perfectly rational agents in economics.

I propose to characterize I-models as the imaginary referents of some scientific laws: they do not exist in the external world, but would they exist, their behavior would be the perfect instantiation of those laws. Some philosophers, such as Roman Frigg[20], claim that I-models ("model systems") are fictions[21]. Though I roughly agree in calling them fictions, my main concern is with how we use I-models in representing, and, as will appear later, such a question can be addressed without adopting any particular position about the nature of fiction. I will therefore remain agnostic on this topic.

Keeping in mind that the main goal of a model is to enable one to draw inferences concerning the system it stands for, in virtue of which features can one say that I-models are used in representing? Is there some special semantics to be applied to them?

My strategy in answering such questions will consist in analyzing and clarifying the different representational relationships that are in play in the use of an I-model - the model of the simple pendulum -, by relying on the

[19]I use "physical system", "target system" and sometimes "system" to refer to the concrete piece of the world whose behavior is being studied.

[20][Frigg, 2008].

[21]Whereas Ronald Giere [Giere, 2006] denies that they are fictions, and defines them as "abstract objects".

account of representation proposed in section 2. Indeed, there are at least two representational relationships involved in what was exposed above. First (3.1), the simple pendulum has to be given by a representation of some sort, since one cannot have a direct perceptual access to it. In that case, I will speak of "characterization" rather than of "representation", for reasons I shall give in the next section. Second (3.2), the physical system under study is represented by means of the simple pendulum. I will show that the key of the representational power of I-models lies in the relation between these two representational processes.

3.1 Characterizations of the I-model

In this paragraph, my aim is to highlight the fact that an I-model - e.g., the simple pendulum - cannot be accessed unless given in some particular format, and that it can be characterized in various formats, according to what information one wants to obtain. Since it does not exist in the spatio-temporal world, it has to be accessed by means of our representational capacities such as language[22]. Following Nelson Goodman, and for the sake of clarity, I shall not speak of "representation" of the simple pendulum, keeping "representation" for the cases in which the *representatum* does exist. I will rather speak of "characterizations"[23].

The simple pendulum is introduced in most mechanics textbooks as a referent of the law of oscillatory motion. It is defined as a mass point m fixed in the inferior extremity of an inextensible thread with no mass and of length l, that can move freely around its fixed superior extremity. Fig. 1 is a schematic drawing of the simple pendulum.

In the theoretical framework of Classical Mechanics, one can describe the motion of the pendulum bob by the following equation, where x stands for the position, $\mathbf{a}$[24] for the acceleration, and g for the gravitational force.

$$m\mathbf{a} = -(\frac{mg}{l})xcos(\alpha) \quad (1)$$

For oscillations of small amplitudes, the angle of swing α is small enough

[22] Again, since I do not want to argue for any particular conception of fiction, I do not claim that the pendulum is *constructed* by our representational capacities; nevertheless, whatever its ontological status is, it cannot be accessed unless *via* concrete characterizations.

[23] In Goodmanian terms, the characterizations of the pendulum are *pendulum-representations*.

[24] Bold characters stand for vectorial quantities.

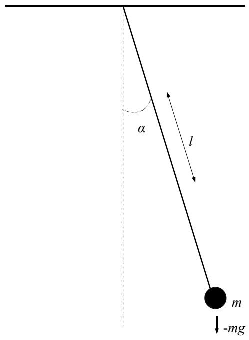

Figure 1: The simple pendulum

to allow one to consider $cos(\alpha)$ as equal to one. Thus, one obtains:

$$m\mathbf{a} = -(\frac{mg}{l})x \tag{2}$$

This is a linear differential equation that describes a linear oscillatory motion. Without such an approximation, the motion described would not be linear.

The solutions of this equation for both position $x(t)$, and velocity $v(t)$, as functions of time have an harmonic form represented by the function

$$f(t) = Acos[(\sqrt{\frac{g}{l}})t] + Bsin[(\sqrt{\frac{g}{l}})t] \tag{3}$$

where A and B are constants to be determined by the initial conditions of the system represented.

By adding the initial conditions into function (3), one can draw a graph both for the position and velocity of m as functions of time. Both graphs will have a sinusoidal form. For initial conditions $x_{(t=0)} = A$ and $v_{(t=0)} = 0$, one obtains the graphs given in Figure 2[25].

In some cases, it becomes advantageous to describe the motion of the pendulum in terms of energy, instead of force. The equations of motion for the system can be written with the Hamiltonian H which is the sum of the kinetic and potential energies of the system:

$$\frac{dx}{dt} = \frac{DH}{Dp} \tag{4}$$

[25] I took this figure from [Giere, 1988], p. 69. In his chapter, these graphs represent position and velocity of a mass and spring system. But, once the approximation of the angle of swing of the pendulum α to zero has been done, the equation for both models is the same.

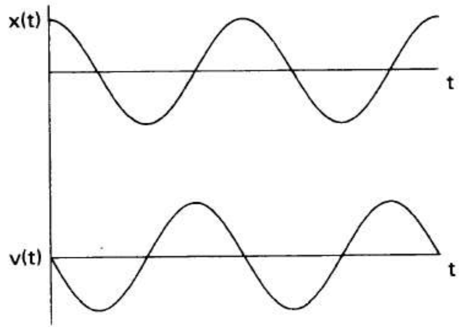

Figure 2: Position x and velocity v as functions of time for the mass of the pendulum. (Taken from [Giere, 1988], p. 69)

and

$$\frac{dp}{dt} = -\frac{DH}{Dx} \tag{5}$$

The solutions of equations (4) and (5) are in terms of position $x(t)$ and momentum $p(t)$. Taking x and p as axis of a two-dimensional euclidian state-space, the state of the system at any instant is represented by a unique point, and its evolution is represented by an ellipse (Figure 3).

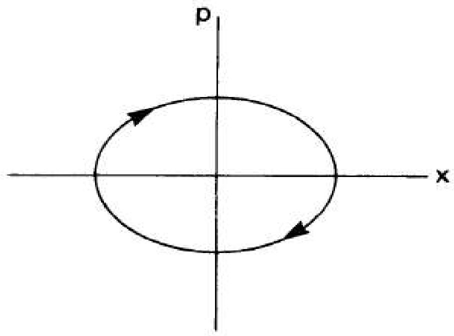

Figure 3: The state of a simple pendulum in a position-momentum space. (Taken from [Giere, 1988], p. 72)

The equations and graphs I have presented are mechanical characterizations of the temporal evolution of the simple pendulum in various formats. Indeed, the notion of a format enables us to distinguish between linguistic and spatial representations, but also between two linguistic representations and between two spatial representations. The differences between formats are of various degrees and occur along various dimensions[26].

The graph in Fig. 2 is drawn from function (3): it represents its values in some specific conditions; displaying those solutions in such a graphical format, though it adds nothing to their informational content, nevertheless makes some information immediately accessible to the agent, who would have to make various inferences to draw it from the consideration of the solutions displayed in linguistic format.

The difference in format between Newtonian and Hamiltonian equations is of another kind: both are linguistic formats, and they are mathematically equivalent, since it can be shown that they are interdeducible. Nevertheless, they are inferentially different. They are not used in the same cases and they do not facilitate the same inferences. Using one format and the other does not imply the same kind of cognitive operation: solving a problem in the framework of the Newtonian format involves dealing with second-order differential equations whose solutions are in terms of position and velocity - and which are in some cases intractable - , while Hamiltonian equations are first-order equations and their solutions are in terms of position and momentum. Moreover, Hamiltonian equations reveal the deep relations between Classical Mechanics and other fields of physics - such as statistical mechanics, quantum mechanics and relativistic quantum mechanics - whose core equations are in close formats.

Consideration of the graphs respectively drawn from the Newtonian and the Hamiltonian linguistic formats makes clearly appear the differences between them. It makes also appear their equivalence: one can indeed understand the connection between the state space of Fig. 3 and the solutions displayed in Fig. 2 by imagining a third axis, representing time, perpendicular to the x-p plane. The state of the system as a function of time is represented by an elliptically shaped spiral moving out along the t axis (Fig. 4).

By projecting this path on the x-t and p-t planes, one obtains the sinusoidal functions whose graphs were given in Fig. 2. Similarly, the projection of the state in the x-t-p space onto the x-p plane yields the ellipse pictured in Fig. 3.[27] That shows the various dimensions along which changes in format

[26]Distance between formats could be thought of as a function of the cognitive cost that is needed to draw a representation in one format from a representation in another format; and the dimension along which a change in format occurs can be thought of as depending on the kind of cognitive operation that is needed: for instance, drawing a graph from an equation is not the same kind of operation as mapping a graph in some reference frame onto another reference frame. Both are changes in formats, occurring along different dimensions.

[27]All this exposition is drawn from [Giere, 1988].

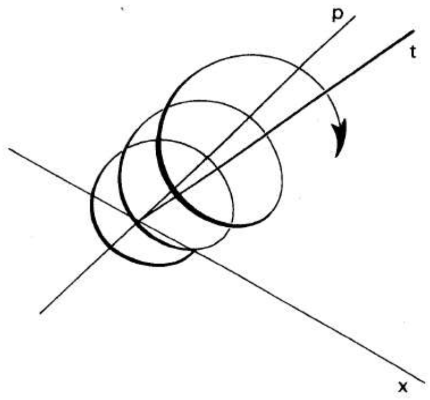

Figure 4: The state of the simple pendulum in a position-momentum-time space. (Taken from [Giere, 1988], p. 73)

can occur, and the different inferences such changes facilitate.

It is worth noting here the strong analogy between my formats and Paul Humphreys' *templates* ([Humphreys, 2004]). A template is a syntactic computational scheme that is relatively theory-independent, in the sense that there can be different templates within the same theoretical field, and that the same template can be found in different fields. Newtonian and Hamiltonian equations are typically two different templates within the field of mechanics, where sometimes a change in template results in a change in tractability; in turn the Hamiltonian template is also used in other fields. Rather than being attached to some determinate theory or domain of phenomena, they embody forms of computation. Humphreys' goal in proposing such a notion is to draw our attention to practice and computability, in contrast with purely logical approaches, which consider as equivalent formulations that are nevertheless not equally tractable. Such a perspective contributed to motivate my elaboration of the notion of a format. Nevertheless, there is a deep difference between templates and formats, which is grounded in a difference in perspective: templates make a computational difference for non human (and thus non cognitive) computing devices, whereas formats are intended to account for cognitive differences in the use of representations by human agents, and many changes in format would make no difference from the perspective of computer science.

I have shown that the simple pendulum can be characterized in different formats, which are not inferentially equivalent. Here, the pendulum is not the representing device, but rather the thing being represented - or, better said, characterized. Now, it is time to turn to the second branch of the question: how can the pendulum itself be the representing device?

3.2 Representations of the target system

Consider a physical system such as a grandfather clock, whose bob has a mass m and whose cable has a length l. The various characterizations of the simple pendulum do not fit precisely the motion of its bob, since its cable has a mass, its bob has a volume, and there are other forces exerting on it, such as the resistance of the air and various frictional forces. Thus, in order to represent this system by means of the model of the simple pendulum, one has, first, to write down the particular values of m and l, and to specify the initial conditions of the system: the position and velocity of the bob at time t_0. Then, one has to determine the additional forces exerting on the system, due to the volume of the bob (which is not a mass point), the frictions, and the resistance of the air. Provided one makes all the appropriate approximations, distortions and idealizations, one can finally write down an equation that enables one to predict the position and velocity of the bob at time t_i. Such an equation is a specification of the equation of the simple pendulum that fits more or less precisely (according to the goal one aims at) the particular conditions at hand. It amounts to what Nancy Cartwright calls a "prepared description"[28] of the system. Note that, in order to write the equation, it can be very useful to make a schematic drawing of the system, in order to specify the forces that apply to it. Such a schematic drawing itself consists in a specification of the schematic drawing of the simple pendulum that fits - with distortions and approximations - the system under study. Likewise, in order to quickly determine the period of oscillation of the bob, it can be useful to draw a graphical representation of its temporal evolution from the equation thus obtained.

In all these cases, what are the devices that are used to represent the clock? An equation, a drawing, a graph. Again, in order to draw inferences and to gain knowledge concerning the behavior of the system being modeled, one has to couch a representation in some particular format, and each format has a particular cognitive utility, according to the intended goal. But in no way does the pendulum *per se* serve as the representing device. The representing device cannot be the I-model *in abstracto*, but rater a concrete representation in some particular format, that can be reasoned with[29].

[28] [Cartwright, 1983].

[29] One could object that my definition of a *representans* as a particular concrete object is question-begging: since the pendulum is not a concrete object, it cannot be a representing device. Nevertheless, some people are so familiar to the behavior of pendulums that they

In what sense, then, can we say that we use the model of the simple pendulum to represent the behavior of the clock? I claim that such a statement means that *we use representations in the same formats* as those that are used in characterizing it. The equation, drawing, and graph representing the behavior of the clock are in the same format as, respectively, the equation, drawing, and graph of the simple pendulum. Since each format has its own cognitive utility, the simple pendulum can be used in drawing various kinds of inferences; therefore, *one cannot identify the model of the simple pendulum with a unique representation-type* - and it can be accessed *via* various characterization-types.

What, then, warrants the unicity of the I-model? Why wouldn't we say that there are as many models as representation-types? Again, the Goodmanian toolbox is useful here. Let me briefly recall some points made in [Goodman, 1976]. Since representation is a kind of denotation, there cannot be representations of non existing objects, such as unicorns. Therefore, what we usually call a "picture of a unicorn" is a unicorn-picture, that is, a kind of thing that is itself denoted by the label "unicorn-picture", but that denotes nothing. Unicorn-names, unicorn-descriptions, and unicorn-pictures have in common their belonging to the broader class of unicorn-representations. Now, to represent an existing object, e.g. Churchill, one has to use some picture, word, or any other representing device. This representing device in turns belongs to some class of representing devices that are given some label, e.g. "unicorn-picture", "bulldog-picture", "man-name", etc., but, which do not necessarily denote an actual man, bulldog, or, *a fortiori*, unicorn. For instance, a caricature of Churchill that denotes him with the features of a bulldog is a representation *of* Churchill, since it denotes the man Churchill, and it is a bulldog-representation, though it does not denote any actual bulldog. It is a representation *of* Churchill *as* a bulldog. Turning back to the simple pendulum, equation (2) is typically what Goodman would call a pendulum-equation, and more precisely a pendulum-newtonian-equation: it has no actual referent, but it is given under the label "equation of a simple pendulum". Likewise, all the characterizations I have given in section 3.2 belong to the same class, namely, the class of things that are denoted by the label "pendulum-representations"[30]. I thus claim that using the simple

do not even need to use pencil and paper to write down the equation and that they can compute its solutions without drawing an external concrete representation of it. But my claim is that, even if merely imagined, the pendulum does not represent *per se*: the representation which is imagined and reasoned with is, again, in a particular format, either an equation or a graph, in either the Newtonian or the Hamiltonian formulation.

[30] It is worth insisting on the fact that this does not intend to be a claim concerning the ontology of mathematical objects. Appealing to Goodman's tools in order to analyze

pendulum to represent a target system consists in representing this system *as* a simple pendulum, that is, in proposing a pendulum-representation of the system[31].

To sum up, an I-model such as the simple pendulum cannot be identified with one unique representation-type, but rather with a class of representation-types (here, the class of all pendulum-representations) that might be logically but not inferentially equivalent. There is not one and only kind of inferential process that can be achieved by using an I-model, but rather as many as there are different formats used in characterizing it[32].

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how representation works, in practice, does not necessarily commit me to any nominalist position in the philosophy of mathematics. I am not claiming that there is no mathematical structure which is common to all the characterizations of the pendulum, but rather that, in an analysis of the inferential processes involved in using representations, formats play the crucial role.

[31]This account is consistent with how science is taught: students first learn the law of oscillatory motion by being given the example of the simple pendulum, which instantiates it. They learn what the pendulum is by being told that it is a mass-point hanging on a mass-free thread, by reading the equation, the graph, etc., exactly in the same way as a child learns what a unicorn is by being shown a unicorn-picture or being told that a unicorn is a horse with a horn. So doing, students learn the formats that are used in mechanics to represent a system as obeying the law and they get familiar to manipulating them and applying them to actual physical systems, that is, to represent physical systems *as* pendulums.

[32]Here, it might be useful, as a mean to clarify my proper claim, to compare it to Ronald Giere's views on representation as he exposes them in [Giere, 2006]. Indeed, he insists on the importance of the concrete manipulation of external devices by scientists in order to do a reasoning concerning the phenomena they study. In his chapter 5, he raises the exact same problem as the one I have been addressing here: how can abstract objects play a role in representation, since representation consists in reasoning with external devices? He thus suggests that "the expert is using the external representations in order to *reconstruct* [italics original] aspects of the abstract model relevant to the problem at hand." (p. 105) He also underlines the variety of the representational means to access and characterize abstract models: "Watson and Crick's physical model of DNA, for example, also serve the purpose of specifying some features of an abstract model of DNA, such as the pitch of the helix and the allowable base pairs." (p. 106). So, Giere acknowledges that, even in the case of abstract models, scientists need to draw (either concretely or mentally) some representation in some format. But I would not say that, doing so they "reconstruct" aspects of the abstract model, in order to compare its properties with properties of the system at hand. Indeed, if the practice of representing relies on similarities, as Giere argues, the similarity holds between the concrete external DNA physical model and the molecule, rather than between the abstract model and the molecule.

4 Conclusion

The analysis of the representational function of models was initially motivated by the project of shedding light on scientists' ability to give predictions and explanations. I have shown that this ability depends rather on the formats of the particular representing devices than on I-models themselves. Indeed, there is no genuine representational relationship between I-models and target systems. Rather, the representational power of I-models lies in the formats that are used in characterizing them and in the inferences they enable us to do.

Therefore, I am led to conclude that there is no special semantics to be applied to I-models[33]: they do not represent their target in virtue of some properties that are specific to them as imaginary entities and that make them similar in some sense to it. Rather, even in the case of representation by means of an I-model, the actual reasoning or manipulation is led on a particular device, whose format matters to the predictions and explanations one can draw from it.

Such conclusions indicate that we are on the way to get a unified account of scientific representation by models in general, since representation by I-models is not more mysterious than representation by means of diagrams, graphs, equations, etc. This suggests that a study of scientific representation should take as object of analysis the variety of formats that are used in scientific practice, in order to determinate their different virtues, the way they relate to each other and to the principles of the theory, and the kind of reasoning they enable scientists to do.

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References

[Callender and Cohen, 2006] Callender, C. and Cohen, J. (2006). There is no special problem about scientific representation. *Theoria*, 55:67–85.

[33] This is quite close to Craig Callender and Jonathan Cohen's claim that "there is no special problem of scientific representation", [Callender and Cohen, 2006].

[Cartwright, 1983] Cartwright, N. (1983). *How the laws of physics lie.* Clarendon Press.

[Frigg, 2002] Frigg, R. (2002). Models and representation: Why structures are not enough. In *Measurement in Physics and Economics Project Discussion Paper Series.* DP MEAS 25/02, London School of Economics.

[Frigg, 2008] Frigg, R. (forthcoming 2008). Models and fictions. *Synthese.*

[Giere, 1988] Giere, R. N. (1988). *Explaining Science. A Cognitive Approach.* The University of Chicago Press.

[Giere, 2006] Giere, R. N. (2006). *Scientific Perspectivism.* The University of Chicago Press.

[Goodman, 1976] Goodman, N. (1968/1976). *Languages of Art. An Approach to a Theory of Symbols.* Hackett Publishing Company (1976, 2nd edition).

[Greenberg and Harman, 2005] Greenberg, M. and Harman, G. (2005). Conceptual role semantics. *UCLA School of Law: UCLA Public Law Series (September 1 2005)*, pages 5–16.

[Humphreys, 2004] Humphreys, P. (2004). *Extending Ourselves. Computational Science, Empiricism, and Scientific Method.* Oxford University Press.

[Johnson-Laird, 1983] Johnson-Laird, P. N. (1983). *Mental Models. Towards a cognitive science of language, inference, and consciousness.* Cognitive Science Series. Harvard University Press.

[Kulvicki, 2008] Kulvicki, J. (2008). Knowing with images: Medium and message. Manuscript.

[Simon and Larkin, 1987] Simon, H. A. and Larkin, J. H. (1987). Why a diagram is (sometimes) worth ten thousand words. *Cognitive Science*, 11:65–99.

[Suárez, 2003] Suárez, M. (2003). Scientific representation: against similarity and isomorphism. *International Studies in the Philosophy of Science*, 17(3):225–244.