The Theoretician's Gambits: Scientific Representations, Their Formats and Content

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Abstract. It is quite widely acknowledged, in the field of cognitive science, that the format in which a set of data is displayed (lists, graphs, arrays, etc.) matters to the agents' performances in achieving various cognitive tasks, such as problem-solving or decision-making. This paper intends to show that formats also matter in the case of theoretical representations, namely general representations expressing hypotheses, and not only in the case of data displays. Indeed, scientists have limited cognitive abilities, and representations in different formats have different inferential affordances for them. Moreover, this paper shows that, once agents and their limited cognitive abilities get into the picture, one has to take into account both the way content is formatted and the cognitive abilities and epistemic peculiarities of agents. This paves the way to a dynamic and pragmatic picture of theorizing, as a cognitive activity consisting in creating new inferential pathways between representations.

1 Introduction

Philosophers of science have traditionally approached theoretical representations (i.e. theories, models, concepts) from an abstract point of view, by idealizing away both from the actual means of representation used in scientific practice, and from the actual reasoning of scientists who use these representations. From such a perspective, contents are therefore considered as independent both from the form in which they are expressed and from the cognitive abilities and epistemic peculiarities of the agents. In consequence, two logically equivalent representations (e.g. equations of motion in

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polar and cartesian coordinates) are generally considered as descriptions of the same model (e.g. the harmonic oscillator): philosophers of science usually assume that, despite their differences, two such representations have exactly the same content.

In this paper, I adopt a different perspective, by considering scientific representations as tools for theorizing; by "theorizing", I refer to a certain class of cognitive activities implying the construction, use, and development of theoretical hypotheses. Assuming that scientists do not reason *in abstracto* by contemplating abstract logical or mathematical structures, but rather by manipulating concrete representing devices, I shall focus on the external representations they construct and use in their day-to-day practice. I assume that the main function of such representing devices is to enable scientists to draw inferences concerning the systems they stand for. The overall purpose of this paper is to analyze a crucial – though oft-neglected – feature of the functioning of scientific (external) representations as inferential tools, namely the importance of what I call their *format*.

Some studies in cognitive science and Artificial Intelligence on the use of external representations in problem-solving [28, 35, 36] and decision-making [21] show that the very way in which data are displayed (e.g. a list of numerals as opposed to its corresponding graph) has important consequences on the agents' performances. Indeed, two representations coding the same information can nevertheless convey it in different ways, thus facilitating different cognitive processes and making such or such piece of information more or less easy to access. Such differences I shall call differences in *format*. However, studies emphasizing the importance of formats are almost always concerned with tasks involving *data* manipulation and processing by agents in order to achieve a particular task, and few analyzes (if any) have been given of the importance of such phenomena for the use and manipulation of *theoretical* representations. Theoretical representations, as opposed to mere presentations of data, are representations expressing *hypotheses* about a certain domain of phenomena. If one considers theorizing as a kind of cognitive activity, which consists in reasoning with theoretical hypotheses and exploring their consequences, it becomes legitimate to inquire into the consequences of a change in format for theoretical representations as well.

In this paper, my aim is twofold. My main goal is to show that formats, whose importance is quite widely acknowledged in the case of data display, have notable consequences on theorizing as well. In order to assess such consequences and to evaluate their bearing on a philosophical understanding of the content of scientific representations, further analysis is needed of the fact that representations in different formats have different inferential affordances for agents. Giving such an analysis is the second, subordinate aim of this paper.

Firstly (section 2), I shall give a few examples showing the importance of formats for both data manipulation and theorizing. In order to show that a change in representation sometimes induces a change in the agents' reasoning processes, I shall restrict to examples of representations in different formats, which are nevertheless logically – or informationally – equivalent. In section 3, I propose to clarify the very notion of format, as it is used in describing the cases in section 2. It will appear that the intuitions underlying the use of this notion are not fully captured by an account of the syntactic and semantic rules according to which information is coded within a representation (its "symbol system", in Goodman's sense). Indeed, the most relevant feature of the format of a representation, in my analysis, is that it determines the *inferential affordances* or *potential* of this representation *for agents with limited cognitive abilities*. As we will see, the inferential affordances of a representation depend both on the way information is displayed *and* on the cognitive abilities of this representation's users. I will therefore argue that, as soon as one acknowledges the importance of the format under which a certain informational content is displayed for its users's performances, then one has to take into account these two parameters (information display and cognitive abilities of agents). I will finally draw a few consequences of this analysis for a study of theorizing, conceived as the exploration of the content of theoretical representations (section 4).

2 The "Representational Effect": Data Displays and Theoretical Models

In this section, I wish to show that what Zhang [36] has coined "the representational effect" has important consequences for theorizing as well, and not only for tasks involving data manipulation. "Representational effect" refers to the fact that various representations displaying the same information in different ways do not facilitate the same cognitive behavior. After having briefly recalled what it consists in in the case of data display (subsection 2.1), I shall take two examples (the equations of Classical Mechanics and Feynman's diagrams) highlighting the importance of the representational effect for the use of theoretical representations (subsection 2.2).

2.1 External Representations and the "Representational Effect"

Nobody would deny that external representations – as opposed to internal or mental ones – sometimes prove practically indispensable to perform various cognitive tasks, such as problem-solving or decision-making: laying out a mathematical operation in order to solve it, drawing a graph from a set of data in order to see easily the relation between two variables, or constructing a diagram in order to solve a geometrical problem are common practices. For instance, although it is in principle possible to divide 346 by 7 by mere mental computation, such an operation is quite difficult and costly – and doomed to error – for an average agent. One would rather use paper and pencil to keep track of the various steps of the computational process. Moreover, humans have invented special procedures for displaying numerals[1], which turn the solution of a division into a simple manipulation. All pupils have learned how to lay out a division and how to reach its solution by following simple transformation rules of this kind of device:

$$\begin{array}{r|l} 3\,4\,6 & 7 \\ \cline{2-2} 6\,6 & 4\,9 \\ 3 & \end{array}$$

In virtue of its particular spatial display and of its "internal dynamics", this external device, so to speak, "computes" the solution on behalf of the agent. Manipulating it exempts one from drawing various inferences that would otherwise be indispensable.

Note that the cognitive advantage of this procedure does not rely merely on its being externalized: on the one hand, one could imagine that a trained agent be able to "lay out" the division in his/her mind's eye; on the other hand, using paper and pencil to write down this division problem by following a different procedure could prove much more costly[2]. The very advantage of this procedure rather relies on what I propose to call the "format" of the device shown above, namely the way data are displayed[3], which determines the processes agents have to follow in order to extract information. This

[1] See [24] for a review of the artifacts and procedures that were invented, throughout history, to serve as "cognition amplifiers", and which can be thought of as the ancestors of our modern computers.

[2] My focus on external representations is rather based on expediency (it makes the study of formats easier) than on a commitment to any particular thesis concerning the relation between external and mental representations. One can acknowledge that most cognitive processes involve, indeed, both external and mental representations – and should therefore be studied as distributed processes, as suggested by the advocates of distributed cognition [17, 15, 35] – without committing oneself to the metaphysical version of the extended mind thesis, as advocated by Andy Clark and David Chalmers [3, 2]. Note, however, that an interesting empirical question would be: are the processes actually performed on external representations a mere externalization of internal processes that would be performed mentally on the "same representations", if merely imagined? In other words, are these two kinds of processes similar in some relevant sense, the internal ones being only too complex and requiring too much memory skills to be performed without any external aid, but being in principle describable by the same algorithms? Another, related, question is whether agents using external devices have to construct an internal model of the problem to be solved, the external device serving as an aid in this construction process (see [35]). In this paper, I shall not tackle these issues, which have no bearing on my argument.

[3] Zhang [35] speaks of the "form" of the "graphic display".

becomes clear when one considers several representations displaying the same data in different ways.

Consider, for example, a list of numerical data corresponding to the temperature in Paris over a year (the bracketed numerals represent the months, and the numerals in bold characters represent the values of the corresponding temperature in degrees Celsius): [1]**2.5** ; [2]**3**; [3]**9**; [4]**15**; [5]**18**; [6]**23**; [7]**25**; [8]**23**; [9]**22**; [10]**13**; [11]**6**; [12]**3**. From such a list, one can draw a graph, as in figure 1 below.

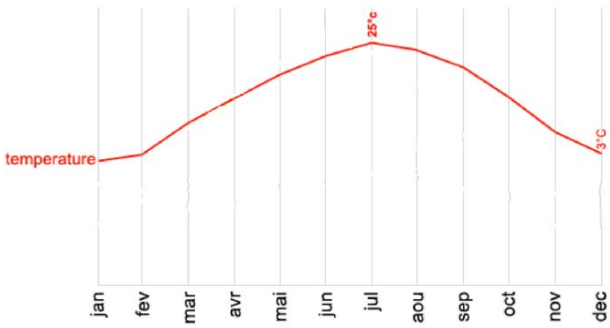

Fig. 1 Annual evolution of temperature in Paris.

The data displayed in these two representations are identical. In other words, the list and the graph contain exactly the same information – since the graph was drawn from the list, and the numerals shown in the list can be retrieved by properly reading the graph. However, as Larkin and Simon [28] would put it, these two representations are *informationally equivalent*, but *computationally different*: though containing the same information, they do not require nor do they facilitate the same cognitive operations. Consequently, the graph and the list do not make the various pieces of information they contain equally accessible to the agents.

Consider, for instance, the task of assessing the global evolution of temperature from January to June. In virtue of the spatial relationships between the points of the graph, one does not need to memorize and then compare the numerals standing for the values of temperature at different times, in order to finally infer the global evolution of temperature over the year; the graph displays in an immediately accessible[4] form the temporal evolution of temperature. The spatial display of the graph, again, computes this information

[4] For a definition of the notion of accessibility, see [32], where I rely on John Kulvicki's notions of "extractability", "syntactic salience" and "semantic salience" [23].

on behalf of the agent. On the other hand, if one wants to know the precise value of the temperature in June, one would rather use the list, since this value is explicitly[5] displayed in the list, whereas it is not so in the graph.

To sum up, differences in the representational format imply differences in the *cost* and in the *type* of the cognitive processes required to access the various pieces of information contained in the two representations. Some pieces of information are easier and quicker – less costly – to access within the graph, while others are so within the list. Entering data into the graph as well as computing information within it do not consist in the same type of processes as entering data and computing them within the list. Certainly, as Larkin and Simon acknowledge [28, p. 67], "ease" and "quickness" are not precise concepts, and it seems therefore difficult to give a measure for the cost of a cognitive task. Similarly, our present knowledge of cognitive processes is too poor to enable us to assess precisely the difference between two *types* of cognitive processes (e.g. those involved in the reading of a list of numerals as opposed to those involved in the reading of a graph). However, without knowing what "happens in the head" of an agent, and without being able to precisely model these processes, we have an intuitive grasp of what a type of cognitive operation is, by analogy with the notion of algorithms; similarly, it seems *prima facie* possible to assess the quickness of a cognitive process by measuring the amount of time involved in performing this task, or by counting the number of (at least conscious) steps involved in it. For an average user with normal cognitive abilities, it seems quite uncontroversial that the process of assessing the general evolution of temperature between June and December (stating whether it increases or decreases) is much less costly by using the graph than the list[6].

Cognitive scientists and AI researchers nowadays pay a growing attention to the role of external representations in tasks involving complex information-processing (see [36] for a review[7]). Some have underlined the importance of what Zhang called the "representational effect" – namely the consequences of the format on the agents' performances in various cognitive tasks, such as

[5] The notion of explicitness needs a further analysis as well. Here, I take it in the intuitive sense – which is also the sense Larkin and Simon [28] seem to rely on – corresponding to the idea that an information is explicitly represented when no inference is needed to access it. For a more refined analysis of the implicit/explicit distinction, see [20].

[6] For a more detailed analysis, see [32].

[7] See also Jiajie Zhang's online bibliography on external representations (thanks to Alex Kirlik for indicating me this link):
`http://acad88.sahs.uth.tmc.edu/resources/ExtRep_Bib.htm`

problem-solving[8] and decision-making[9]. However, although some have suggested that some kinds of representations are particularly well suited to the expression of some kinds of information[10], no clear account of what I have proposed to call "format" has been given[11]. I shall come back to this in section 3. Let me first turn to a few examples revealing the existence of a representational effect in the use of theoretical representations as well.

2.2 The Representational Effect and Theoretical Models

Till now, I have been considering external devices displaying data to be processed by agents in order to achieve simple cognitive tasks. As such, the contents of these representations are sets of data, which were collected by empirical inquiry and entered into the representing device by following rather simple rules. However, theoretical models, such as, for instance, the equation of the simple pendulum, are not mere displays of data. They are rather representations expressing *hypotheses* about a wide range of phenomena and systems' behavior, thus enabling scientists to explain and predict these phenomena. Their *content*, as such, is much richer and more complex to define than the content of the representations considered above.

Indeed, analyzing the content of scientific representations and accounting for their explanatory and predictive power is one of the central problems in the philosophy of science. Philosophers have generally addressed this problem by giving a logical reconstruction of the relation between theoretical representations and the phenomena they stand for. Therefore, logical equivalence has long been taken as a criterion of identity of content for scientific representations: two representations are considered scientifically equivalent if they are inter-deducible, thus having the same set of empirical consequences. On

[8] Zhang [35, 36, 37] shows that different representations of a common abstract structure can generate dramatically different representational efficiencies, task complexities, and behavioral outcomes. He moreover suggests [34] that all graphs could be systematically studied under a representation taxonomy based on the properties of external representations.

[9] Kleinmutz and Schkade [21] showed that different representations (graphs, tables, lists) of the same information can dramatically change decision-making strategies.

[10] Larkin and Simon [28] suggest that different kinds of representations typically display, in an explicit form, different kinds of information: diagrams preserve topological relations, outlines preserve hierarchical relations, and languages are well fitted to display logical or temporal relations. For an analysis of the types of reasoning associated with the use of graphs and diagrams, see the works by Tufte [30, 31]; for diagrammatic logic, see [29, 27].

[11] Note, incidentally, that differences in format can happen between different types of representations (e.g. linguistic *versus* diagrammatic) as well as between representations belonging to the same broad type (e.g. graphs in different coordinate systems, arabic *versus* roman numerals).

such a view, one could feel reluctant to attribute any importance to a mere change in the presentation of this content.

However, if one considers scientific representations as tools for theorizing, namely for drawing inferences enabling agents to explore the content of these representations – and therefore to gain knowledge concerning the systems they stand for, and concerning their link to other representations –, it becomes legitimate to pay attention to the very format of these representations, since it might have consequences for their inferential role for agents, *in practice* – despite their equivalence *in principle*. Let me briefly give two examples showing that formats matter for the use of theoretical representations as well.

2.2.1 The Equations of Classical Mechanics

Consider, first, the equations of Classical Mechanics (CM). Solving a problem in CM, say, predicting and explaining the dynamical evolution of a system, typically consists in finding the functions that describe the temporal evolution of the position and velocity of the system under study. First, one writes down the differential equations governing the dynamics of the system with the help of the information one has about it; and then, one solves these equations.

The equations of CM can be formulated in different ways, according to the kind of coordinate system used to describe the motion of a physical system. One distinguishes generally between the Newtonian and the analytical (Lagrangian and Hamiltonian) formulations. According to the problem at hand, using one or the other formulation can dramatically facilitate both the processes of writing the equations and of solving them. The Newtonian formulation relies on a description of the configuration of systems by means of Cartesian (and sometimes polar) coordinates, which represent the position and velocity of each point of the system at some instant t. Newtonian equations of motion, which govern the dynamical evolution of a system so represented, have the form of Newton's Second Law ($\mathbf{F} = m\mathbf{a}$, where $\mathbf{F}$, the force, and $\mathbf{a}$, the acceleration, are vectorial quantities). The first step in solving a problem in this framework consists in specifying the forces exerted on the various points of the system in order to write down the corresponding equations. In other words, the Newtonian *format* requires that one enters data concerning the forces, since the value of forces is explicitly displayed in the Newtonian equations.

In the case of constrained systems, namely systems with internal forces maintaining constraints between different points (thus preventing them from moving independently from each other) the identification and specification of each force is practically impossible. In such cases, the Lagrangian formulation is more appropriate: it relies on a description of the configuration of systems by means of so-called "generalized" coordinates, which correspond to the degrees of freedom of the system. Transforming the description of the system from Cartesian coordinates into generalized ones (q_i) enables one to write down the Lagrangian equations of the system, without needing to know the forces maintaining the constraints. The Lagrangian equations have the following form:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0.$$

Forces do not appear in them; the dynamical evolution of the system is entirely governed by a scalar quantity, the Lagrangian L, which typically corresponds to the difference between the kinetic and potential energies of the system. These equations implicitly contain the forces maintaining the constraints, since they are expressed in coordinates taking these constraints into account. Therefore, it is possible to retrieve information concerning the forces; nevertheless, the constraints need not appear explicitly. What may be called the Lagrangian *format* consists in presenting, in an explicit form, information concerning the energy of the system, rather than information concerning the forces. Despite their equivalence to the Newtonian equations, the Lagrangian ones are therefore much more appropriate to cases where some forces are unknown.

In some cases, though, Lagrangian equations are only partially integrable; it is thus impossible to achieve the second step of the problem-solving process, namely finding the analytical solutions of the equations. The Hamiltonian formulation, which uses a different kind of generalized coordinates[12], enables one to change an intractable Lagrangian equation into two corresponding first order equations, by means of mathematical transformations called "Legendre transformations". Such first order equations ($\frac{\partial H}{\partial p_i} = \dot{q}_i$ and $\frac{\partial H}{\partial q_i} = -\dot{p}_i$) are integrable. The Hamiltonian H typically equals the total energy of the system.

One can easily show the inter-deducibility of these three kinds of equations. Nevertheless, as we have seen, changing from one to the other can considerably enhances our problem-solving capacities: changing from Cartesian to generalized coordinates sometimes facilitates the process of writing the equations, which is otherwise practically impossible; changing from a Lagrangian to a Hamiltonian representation transforms one intractable equation into two tractable ones. As Paul Humphreys [16] would put it, despite their equivalence *in principle*, these various equations are not equally usable *in practice*. Their formats neither require nor facilitate the same processes.

Moreover, this variety of formulations has consequences on the development of the theory itself. The Lagrangian formalism is suitable for the expression of the theory of relativity, and the Hamiltonian formalism is used in Quantum Mechanics. As Feynman said concerning the various ways of expressing the law of gravitation (*via* Newton's law, field theory, or minimum principles), these formulations are "equivalent scientifically. [...] But psychologically, they are very different" [9, p. 53].

[12] Lagrangian generalized coordinates have the dimension of positions and of velocities, whereas Hamiltonian ones have the dimension of positions and momenta.

2.2.2 Feynman's Diagrams

Consider now the case of Feynman's famous diagrams (see Figure 2). Feynman first introduced them in 1948, as a mean to help physicists get rid of the infinities of quantum electrodynamics (QED) which prevented them from giving predictions about complex interactions of atomic particles[13]. At that time, he presented them as "mnemonic devices" [18, p. 52] to complete complex higher order calculations without confusing or omitting terms, that task being practically impossible by means only of the mathematical formulae.

A year later, Dyson [5, 4] demonstrated the equivalence of Feynman's diagrams with the mathematical derivations given at the same time by Schwinger [25, 26] as a workable calculational scheme for QED. Moreover, Dyson made Feynman's methods "available to the public"[14], by codifying the rules for constructing the diagrams, stating the one-to-one correspondence of features of the diagrams to particular mathematical expressions.

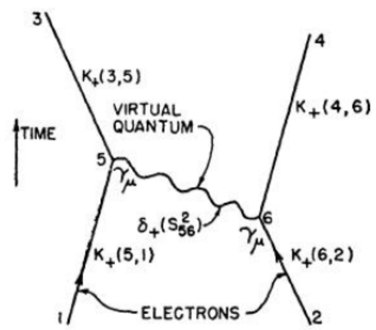

Fig. 2 Diagram corresponding to the following equation: $K^{(1)}(3,4;1,2) = -ie^2 \int\int K_{+a}(3,5)K_{+b}(4,6)\gamma_{a\mu}\gamma_{b\mu} \times \delta_+(s_{56^2})K_{+a}(5,1)K_{+b}(6,2)d_{\tau 5}d_{\tau 6}$ (drawn from [8, p. 772]).

As is well known, this was the beginning of an amazingly successful career for these diagrams, which, as Kaiser [19] describes in detail, were eventually used in almost every field of theoretical physics. The diagrams were initially intended to relieve physicists' memory and help them in performing difficult calculations; they finally became genuine theoretical tools, which went beyond the theoretical frame within which they had first been designed. They indeed played a crucial role in research in high energy post-war physics, and are still taught and used today. Beyond the equivalence of mathematical formulae and diagrams in principle, the latter acquire a genuine independence. As Kaiser [19, p. 75] suggests – thus echoing Feynman's quote about the

[13] For historical and technical details, I refer the reader to the works of David Kaiser [18, 19], from which I drew all my material concerning this case.

[14] Dyson, Letter to his parents, 4 Dec. 1948, quoted by Kaiser [19, p. 77].

law of gravitation –, Dyson demonstrated "the mathematical [...] equivalence between Schwinger's and Feynman's formalisms", but "by no means" their "conceptual equivalence"[15].

In both cases (equations of CM and Feynman's diagrams), representations whose equivalence can be mathematically proven happen to have different consequences in problem-solving and theory-development. According to the intuitive understanding of the notion of "format" suggested by the toy examples given in subsection 2.1, it makes sense to claim that all these are differences in format. Newtonian, Lagrangian, and Hamiltonian equations have different formats; so do Schwinger's formulae and Feynman's diagrams. Although they contain, at least partially, the same information, these representations do not convey this information the same way. Constructing and using them do not consist in the same cognitive operations: identifying and specifying the forces exerted on a system by manipulating vectors does not amount to the same process as identifying the Lagrangian of the system; drawing a diagram enabling one to visualize the different quantities to be remembered in a calculation does not consist in the same operation as performing this calculation by means of a mathematical formula. Since theorizing often consists in drawing inferences by means of theoretical representations in order to explore their content – be it in order to draw predictions concerning particular phenomena or in order to inquire into the logical relationships between various theoretical hypotheses – one can therefore say that formats do matter for theorizing.

In section 4, I shall come back to these two examples, and draw a few consequences for our understanding of both theorizing and the content of theoretical representations. Beforehand, in the next section, I will analyze further the very notion of format; in particular, I shall examine the idea that the format of a representation determines the inferential affordances of this representation for its users.

3 Formats and Inferential Affordances

As we have seen above, representations in different formats can be informationally equivalent though computationally different. This means that such representations can contain the same information – have the same *informational content* – without making it equally accessible to agents with limited cognitive abilities. In other terms, although the informational content of a

[15] So-called "conceptual role semantics" or "inferential role semantics" (see, e.g., [12]), which states that the content of a representation consists in (or depends on) its role in the inferential processes of agents could help us give a precise meaning to Kaiser's suggestive remark: diagrams and equations neither facilitate nor require the same inferential processes. Defending such a view would enable us to state that what Feynman calls a "psychological difference" sometimes counts as a genuine conceptual difference.

representation seems independent from the way this representation is formatted, its format (partially) determines the *inferential procedures* agents have to follow in order to access the various pieces of this content. Representations in different formats do not have the same *inferential affordances* for agents; they neither facilitate, nor require the same inferential procedures.

Before elaborating on such a view, a clarification of the very notion of "informational content" is needed. Although the informational content of a representation depends neither on the way it is coded within it, nor on the inferential procedures agents have to follow in order to access it, it is certainly determined by some features of the representation, namely the syntactic and semantic features in virtue of which this representation has such content. In section 3.1, I propose a definition of the notion of informational content, by appealing to the Goodmanian analysis of symbol systems.

In section 3.2, I shall come back to the notion of format. The intuitive use of this notion obviously relies on an analogy with computer science: a format serves to specify a procedure for both encoding data and retrieving them. *Prima facie*, the format of a representation seems to be one and the same thing as its symbol system: indeed, the symbol system of a representation is the set of syntactic and semantic rules according to which information is encoded within it. However, I shall argue that such rules do not correspond to the actual inferential procedures agents have to follow when they intend to retrieve pieces of information from the representation. Therefore, appealing to the notion of symbol system is insufficient to account for the very idea that representations in different formats have different inferential affordances for agents. I shall conclude that, if one wants to pay attention to the inferential procedures agents have to follow in order to access the content of a representation (rather than concentrate on its mere informational content), then one has to take into account *both* the way information is encoded *and* the cognitive abilities of agents.

3.1 Symbol Systems and the Informational Content of Representations

There are various theories of information[16], but it is unnecessary for my purpose to enter into any detail: let me simply state that a piece of information is a proposition that can be object of belief. One can start by defining the informational content of a representation as *the set of all the pieces of information that an agent mastering its symbol system could in principle extract from it.*

Here, "symbol system" is understood along the lines of Nelson Goodman's definition [11], namely, by analogy with a language, as a set of syntactic and semantic rules. Its syntax defines the set of relevant perceptual properties and their rules of arrangement and transformation. The semantics governs the

[16] See [10] for a recent account.

way these perceptual properties so arranged denote different elements of the domain of reference of the system. According to Goodman, a particular set of marks (visual or auditory) is a representation of some feature of the world (its target)[17] in virtue of a symbol system. Following the initial definition of the informational content given above, one can say that the symbol system under which a certain set of marks has to be interpreted determines its informational content. This definition, though, requires several refinements.

As I said in the introduction, one of the main functions of scientific representations is to enable agents to draw inferences concerning the systems they stand for. This can be generalized to non-scientific representations as well, if one concentrates on their epistemic use (as opposed, for instance, to their aesthetic use), namely their use in a knowledge-seeking enterprise. However, the very phrase "drawing inferences concerning a system" needs to be refined. Indeed, the epistemic enterprise of using a representation to gain knowledge concerning its target consists in *two* inferential steps, which are often simultaneous, but need to be distinguished.

For such an enterprise to be successful, one has to be able to interpret the information carried by the graph as information *about* some particular target. One needs to be aware of the source and the precision of the data one can extract from the representation. For instance, one has to know what approximations and idealizations were made in collecting the data. Consider the graph in figure 1. If one believes that the values shown in it represent the average temperature over one month, whereas measurements were made at 8am every 10th of the month, one's epistemic enterprise fails. Likewise if one takes the graph to be a representation of the variations of temperature in Paris, whereas the measurements were in fact made in Madrid.

But, before inferring – soundly or not – from the features of the graph to the features of its target, one has to *know how to read the graph itself* in order to extract information from it. This is what I call *mastering the system* under which the graph functions. For a given graph, the symbol system that defines it determines which of its perceptual features are syntactically relevant, and how they are to be interpreted, *within* the graph. Let me insist: reading off the information from the graph – even before interpreting it as about some target – requires the knowledge of the system's semantics (although not necessarily of its actual referent), and not just of its syntax. When a teacher draws a graph on a blackboard in order to teach how to read such a representation, he does not intend the graph to represent the evolution of any real quantity. Nevertheless, there is a sense in which the graph "tells" its readers that the intended quantity increases or decreases over time. It does contain such information, whether or not it is true of any real place. In the following, I shall concentrate on this second – in fact, logically first – step of the reading of a representation: the very extraction of the information it contains within

[17] The target can be a material object, properties of an object, the evolution of the value of some quantity, the relation between various quantities, an event, a pattern, etc.

it, independently of the true or approximately true statements its user may formulate about its intended target[18].

Of course, this definition of the informational content of a representation involves some further difficulties. First, some pieces of information are presented in an explicit[19] way, while other pieces require some inferential process to be extracted. Performing such inferences requires the mastering of the rules of the system. But it is far from clear, for a given representation, what rules are to be included in the system's syntax and semantics. For instance, the graph in figure 1 "says" in an explicit way that the temperature in July is 25 degrees Celsius, and that the temperature in December is 3 degrees Celsius. One would also like to say that it contains – though implicitly – the information that the temperature in December is 22 degrees less than in July. Therefore, it seems reasonable to consider that the basic rules of arithmetic, which enable one to make the subtraction, are part of the system's rules – or at least are supposed to be mastered by any user of such representation. Now, should we consider that the transformation rules from Celsius to Fahrenheit are part of the system's rules? If they are, then the graph also says – implicitly – that the temperature in July is 77 degrees Fahrenheit. If not, then the graph does not contain such information. Here, one has to acknowledge that whether these transformation rules are part of the system depends on the context in which the graph is used.

Moreover, the informational content of a representation seems to be context-dependent for (at least) one more reason. As Haugeland [13] suggests, one should distinguish between what he calls the "bare-boned" content of a representation and its "fleshed-out" content. The bare-boned content of a representation needs no further assumption to get extracted. On the other hand, the fleshed-out content is obtained via a deduction which implies some background knowledge. As Haugeland notes, this distinction does not correspond to the implicit/explicit distinction. Some implicit information can be extracted from the graph without any further *factual* knowledge: inferring that the temperature in December is 22 degrees less than in July only requires mastering the rules of arithmetic. However, inferring, for instance, that the temperature in Paris in July is 15 degrees less than in Madrid implies possessing the knowledge that the temperature in Madrid in July is 40 degrees Celsius. In some contexts – and particularly scientific ones –, a considerable amount of background knowledge is indispensable to derive important

[18] Whether one can genuinely speak of representation and informational content when there is no actual referent is a difficult issue, one I shall not tackle here. Since I will not consider issues concerning successful representations or misrepresentations, I shall not use the term "information" as a success term; rather, I use it to refer to any propositional content that an agent who knows how to read the graph – who masters its symbol system – can extract from it, whether or not this agent is mistaken concerning the target, and whether or not there is any such target.

[19] See footnote 5.

information from a representation. Here again, what knowledge is supposed to be possessed by the user of a representation depends on the context. Therefore, the informational content of a representation is context-dependent.

However, acknowledging this fact does not dangerously challenge the characterization of the informational content of a representation in terms of its symbol system. In any case, the very system in which a representation has to be read is settled by the context: to borrow one of Goodman's examples [11, pp. 229-230], whether a "black wiggly line" has to be read as an electrocardiogram or as a drawing of Mt Fujiyama obviously depends on the context (for instance, on the caption). One can therefore state that, for a given set of marks, once what is part of the system under which it functions is settled by the context, this system fully determines the informational content of the set of marks. Although this set of marks' having a content certainly depends on the existence of some agent, since nothing is a representation unless someone uses it as such, this agent is an ideal one, who perfectly masters the system, and whose cognitive abilities are not limited.

So far, nothing has been said about the *actual cognitive processes that agents with limited abilities have to run* when they seek information within a representation, nor about the practical possibility for them to access different pieces of its informational content. Symbol systems are sets of objective syntactic and semantic rules, which determine the informational content of representations. The inferential procedures agents have to follow when they want to extract pieces of this content certainly depend on the symbol system of this representation: the differences in the procedures required in order to read the list and the graph in figure 1 are obviously due to objective differences in the way data are structured within each of them; the graph and the list are not constructed along the same syntactic and semantic rules. In other words, the inferential affordances of a representation obviously depend on its symbol system. However, as we will see in the next section, merely referring to its symbol system is not sufficient to account for the inferential affordances of a representation, and therefore to fully capture the intuition underlying the use of the notion of format.

3.2 The Inferential Affordances of a Representation Are Agent-Relative

As suggested above, the intuitive use of the notion of format relies on an analogy with computer science. Following this analogy, the format of a representation might be said to determine a set of procedures for constructing, transforming, and interpreting this representation. *Prima facie*, there is no reason why one should not describe these procedures by referring to the symbol system of the representation, namely the rules governing the way data are structured within it.

However, as a further analysis of the computer science analogy reveals, these procedures do not only depend on the way data are structured (on the symbol system), but also on the "processor" which is operating on them[20], namely on the agents themselves, and on their cognitive peculiarities. Indeed, a difference in symbol system may induce a difference in the inferential affordances of the representation for one and the same agent, but the same set of marks functioning under the same symbol system can also have different inferential affordances for different agents. Laplace's demon – whose cognitive abilities are unlimited –, a computer, a trained agent and a child do not process data the same way. For these different "agents", the very same representation (the same set of data structured following the same syntactic and semantic rules) may have different inferential affordances.

To be clear, let me take a simplistic example. Imagine two representations, A and B, which have the same informational content, consisting of two pieces of information x and y. Suppose that, in virtue of the way data are structured within A, and of my own cognitive abilities, I need to extract x first if I want to access y. On the other hand, B does not enable me access x unless I extract y first. Obviously, this difference in the inferential procedures needed to access x and y is grounded in a difference in symbol system (though it does not affect the informational content)[21]. However, suppose now that there exist cognitive agents different from me (let's say, Martians) for whom the situation is the other way around. Accessing x and y within A requires the same inferential procedures for me as accessing x and y in B would require for Martians. The operations these fictional agents have to perform in order to access respectively x and y with B are the same as the operations I have to perform if I use A. The symbol systems of A and B are the same for Martians and for me (A and B encode data along the same objective rules). However, their inferential affordances are not the same for Martians and for me.

Without even appealing to fictional agents such as Martians, there exist many inter-individual differences in virtue of which the same representation does not have the same inferential affordances for two different agents. A trained agent may be able to see immediately the form of the solutions of an

[20] As Larkin and Simon [28, p. 67] note, "when we compare two representations for computational equivalence [as opposed to informational equivalence], we need to compare both data and operators. The respective value of sentences and diagrams depends on how these are organized into data structures and on the nature of the processes that operate on them". Larkin and Simon refer to Anderson [1] who argues "that the distinction between representations is not rooted in the notations used to write them, but in the operations used on them" [28, p. 68].

[21] Indeed, the change in symbol system is accompanied by a change in the perceptual properties, therefore canceling the effects on the informational content: consider for example a map of temperature where reddish colors would stand for warm areas, and blueish colors for cold areas. If I both change the system – the rules according to which red stands for warm and blue for cold – and the colors, the content remains unmodified.

equation, while a beginner might need to perform various operations (sometimes with the help of paper and pencil) in order to reach the same conclusion. In some cases, a change in the perceptual properties of a representation (e.g. the addition of colors on a black and white diagram, facilitating the extraction of some information) counts as a change in its inferential affordances for some agents (agents with normal perceptual abilities) though not for others (color-blind persons).

More generally, one can state that the objective rules in virtue of which a certain set of marks has a certain content *underdetermine* the procedures one would actually follow in searching information. Consider again the graph in figure 1, and suppose one wants to calculate the difference of temperature between October and February. The system's rules in virtue of which the graph contains this information allow for a great variety of procedures in order to find it: one can add a graduation to the graph (based on the two numerical values which are given), draw lines from the points of the graph corresponding to February and October to the graduated scale, and then make the subtraction; one can also measure the distance between the height of the graph in february and in october and compare it to the distance between July and December (which we know corresponds to $25 - 3$), etc. Therefore, one cannot say that the syntactic and semantic rules of a representation determines a set of fixed and objective procedures, independently from a particular user in a particular situation. These rules do not correspond to the actual procedures agents have to follow. The procedures one will perform in order to access such or such piece of information also depend on one's cognitive abilities, skills, habits, preferences, background knowledge, etc.

As a consequence, in order to fully capture the intuition underlying the use of the notion of format, the right unit of analysis is not the symbol system of a representation, but the formatted representation insofar as it is used by agents with limited cognitive abilities – the rules according to which data are structured *insofar as they are processed by cognitive agents*. In other words, one should focus on the *cognitive interactions of agents with formatted representations*.

To sum up, the format of a representation determines its inferential affordances (or potential) for *a particular agent*. The inferential affordances of a representation for a particular agent might be defined as the set of procedures this agent, given his/her cognitive abilities and epistemic peculiarities, should follow in order to access the various pieces of the informational content of this representation. In the following section, I propose to draw some consequences of such a view for an analysis of theorizing. Meanwhile, it will appear that such a definition of the inferential affordances of a representation is untenable: not only is it impossible, in practice, to state the explicit algorithms agents have to implement in order to use a representation, but in fact the very idea of a set of procedures to be implemented is a rough idealization, which does not do justice to the complexity of the cognitive interactions of agents with external representations.

4 Theorizing as "Work in Progress"

In section 2, we have seen that formats matter to the use of theoretical representations as well. The analysis of section 3 led me to conclude that the actual processes one has to follow in order to extract information from a representation depend both on the way data are structured and on one's cognitive abilities. In other words, the inferential affordances of a representation are agent-relative. How relevant is such an agent-relative notion to an analysis of the role of theoretical representations in scientific practice?

In subsection 4.1, I argue that the recognition of the agent-relativity of the inferential affordances of a representation might shed light on our understanding of expertise and learning. In subsection 4.2, I propose to go one step further, by reassessing the normative dimension of my analysis of formats – as determining procedures agents *have to* follow in order to access the content of a representation. Finally, in subsection 4.3., I draw a few consequences of my view for an analysis of the content of theoretical representations.

4.1 Agent-Relativity of Inferential Affordances: Consequences on Expertise and Learning

Let me come back to the examples of section 2, which reveal the importance of formats for theoretical representations as well (as opposed to mere data display). As we have seen, the different forms of equations of CM have different inferential affordances for agents. However, one can assess these differences without referring to any particular agent. Newtonian equations are practically useless in describing the motion of a constrained system for *all agents*. Certainly, the differences in inferential potential between the equations of CM would not apply to Laplace's demon, since differences in format do not affect the informational content of a representation. But, in order to assess the inferential differences between the equations of CM, it seems legitimate to assume a *standard agent* without considering inter-individual differences. Unlike idealized agents for whom differences of formats would not matter, standard agents have limited cognitive abilities. Such a standard agent would be the typical user of this kind of representation: in the case of the graph, a human adult with normal cognitive abilities; in the case of the equations of CM, someone mastering the rules of the calculus and having a fair training in physics. For a layman without such training, it makes little sense to compare the Lagrangian and the Hamiltonian formats. If one does not even know how to solve a differential equation, whether integrable or not, one would not gain anything if provided with the Legendre transformations, in addition to a non-integrable Lagrangian equation. Therefore, the agent-relativity of the inferential affordances of representations does not seem, *prima facie* a relevant feature for an analysis of theoretical representations.

In some cases, though, it proves useful to pay attention to inter-individual differences, and therefore to acknowledge that the same representation can be used in different ways by different agents (and thus have different inferential affordances for them). As I will now suggest, this might shed light on expertise and learning (the process of becoming an expert), conceived of as the deepening and sharpening of one's understanding of a theory.

Beginners and experts, trivially, do not use differential equations in the same way. However, that does not mean that beginners always use them in a faulty way: even in conforming to the rules of the calculus, beginners may follow longer and less efficient inferential paths than experts. One could therefore think of learning as a process consisting in progressively modifying the procedures of use of some representations. By acquiring the skills enabling one to use a type of representation in an efficient way, one reduces, so to speak, the inferential path leading to a problem's solution. Learning how to use a certain type of equations, and becoming more and more skillful in it, consists in modifying their inferential affordances by learning new transformation rules – new *inferential paths*[22].

Accordingly, expertise could be thought of as the ability to use certain representations in an optimally efficient way. As suggested by Andrea Woody [33], becoming an expert consists in acquiring an "articulated awareness" of the representations used in this field. The more expert you are, the more easily you draw inferences with these representations. Moreover, in addition to solving problems more quickly, the expert has a deeper understanding of the very content of theories, namely of the deductive relationships between the various hypotheses this theory consists in. Deepening one's understanding of a theory therefore consists in progressively modifying the inferential architecture of its various principles and hypotheses, by developing new inferential paths between them. Consider again the equations of CM: whereas the beginner might find it difficult to understand why Newtonian and Lagrangian equations are equivalent, the trained physicist can "see immediately", so to speak, their equivalence. Indeed, he is able to transform the ones into the others very quickly.

Now, whether one considers the inferential affordances of a representation for a beginner or for an expert, the view of formats I have proposed still has a normative dimension. Indeed, according to the above analysis, the format of a representation for a particular agent determines the set of procedures this agent (given his/her limitations, skills, background knowledge, etc.) *has to* follow in order to access the various pieces of information contained in it. In other words, given a particular agent, there seems to be something as *the right way* of using a representation. Moreover, by suggesting that expertise could be assessed by referring to the efficiency of the way one uses a certain type of representations, I have assumed that there exists an optimal way of using scientific representations in order to access their content. In the next

[22] As noted by Kuhn [22], learning consists in acquiring skills (know-how) rather than learning explicit rules.

subsection, I shall argue that the very idea of an agent's optimally mastering the rules of a certain type of representations, and of the existence of a set of procedures *to be followed*, relies on an illegitimate idealization.

4.2 Who Is the Expert?

Consider again the case of Feynman's diagrams. The interest of this case is not exhausted in the comparison between diagrams and mathematical formulae (which can be made without referring to inter-individual differences). As their letters and personal papers show, Feynman and Dyson explicitly disagreed on the legitimate use and status of the diagrams within QED. Dyson conceives of them as secondary, psychological aids to the performance of mathematical calculations. On various occasions, he claimed that their use would be illegitimate if they had not been proven rigorously derivable from mathematical formulae: "until the rules were codified and made mathematically precise, I could not call [Feynman's method] a theory." [6, p. 127]. For him, diagrams were means to "visuali[ze] the formulae which [he derived] rigorously from field theory" [6, pp. 129-130][23]; they had a meaning only within QED, to which they added nothing except cognitive tractability.

On the other hand, Kaiser reports that Feynman never felt the need to show how to derive diagrams from mathematical expressions, and expressed clearly on various occasions his theoretical preference for diagrams over mathematical formulae: "All the mathematical proofs were later discoveries that I don't thoroughly understand but the physical ideas I think are very simple." (Feynman, Letter to Ted Welton, 16 Nov. 1949, quoted in [19, p. 178])[24]. Hence, unlike Dyson, he thought of diagrams as primary and more important than any mathematical derivation that might be given. In addition to being mnemonic devices, they provided an intuitive dimension to the theory, and Feynman took them as "intuitive pictures" [19, p. 176]. As Dyson notes, Feynman "regard[ed] the graph as a picture of an actual process which is occurring physically in space-time" [6, p. 127][25]. Rather than visualizations of the formulae, they were primary visualizations of the physical processes themselves. Despite their agreement on the in-principle equivalence of the diagrams and the formulae, Feynman and Dyson did not construct and use diagrams in the same way, and in the final analysis did not even "see" the same thing in them.

As Kaiser suggests, this difference in use by the two physicists can be explained by referring to their own theoretical commitments and preferences. Unlike Dyson, who demonstrated how to cast both Feynman's diagrams and

[23] Quoted in [19, p. 190].
[24] Feynman also spoke of the "physical plausibility" of the diagrammatic approach (quoted in [19, p. 177]).
[25] Quoted in [19, p. 190].

Schwinger's equations within a consistent field-theoretic framework[26], Feynman's renormalization approach, from which the diagrammatic method arose, was based on particles, rather than on fields. More generally, as Kaiser notes, Feynman had a preference for a semi-classical approach, and worked almost entirely in terms of particles, trying to remove fields from theoretical descriptions altogether. Such theoretical commitments and interests, together with individual preferences for some kinds of reasoning (Feynman expressed on various occasions his favoring "visualization" over abstract calculation) must have contributed to giving the diagrams different inferential affordances for Feynman and Dyson. Dyson deduces them from mathematical formulae, whereas Feynman draws them intuitively: each one relates them in a different way to other representations, and, finally, to the physical world.

In addition to Feynman and Dyson's using the diagrams according to different rules, Feynman himself, as well as other physicists, continuously modified their rules of use. Kaiser [19] gives an impressive analysis of the "plasticity" of diagrams throughout their "spreading" in theoretical practices in modern physics. He studies their varying uses and interpretations in different contexts and "schools" (Oxford and Cambridge, Japan, Soviet Union). Despite Dyson's efforts, the rules of construction and interpretation of the diagrams have not been strictly followed. Moreover, this is the reason why they were so successful: rather than mere calculation tools, they were genuine discovery tools that contributed important theoretical developments to modern physics, by being used and applied in new fields.

Who is the "standard user" of these diagrams? Feynman? Dyson? Others? Who is the expert who follows the procedures corresponding to the inferential affordances of the diagrams *for experts*? The inferential affordances of diagrams are obviously different for Feynman and for Dyson (since they do not use them the same way). Moreover, these affordances constantly changed for Feynman himself. Stating that the format of the diagrams determines a set of procedures their user must conform to would amount to missing some essential aspects of their role in theorizing.

I suggest that, far from being an exception, this is an exemplary case of the way representations are used in theorizing. Let me come back to the example of CM. In the case of the Newtonian, Lagrangian, and Hamiltonian equations, there seem to exist fixed sets of procedures that any expert masters. Let's suppose that this is so, and that every (trained) physicist today uses them by following the same processes. Reducing them to representations whose inferential affordances are strictly fixed would nevertheless prevent us from noticing essential aspects of scientific invention and discovery. Consider Hamilton's use of the Legendre transformations: this innovation does not rely on any empirical novelty, but rather consists in the introduction of new transformation rules within mechanics, which results in a modification of the

[26] Dyson [7, p. 23] claims that he contributed to allow "people like Pauli who believed in field theory to draw Feynman diagrams without abandoning their principles".

inferential affordances of the various equations. I suggest that theory development often consists in such a process of modifying the procedures followed in using the representations. As a chess player who, knowing the rules, invents a new gambit, the theoretician modifies the inferential processes he/she performs in order to solve problems and sometimes develops novel connexions between different equations. In the case of CM, as well as in the case of Feynman's diagrams, stating that the format of the different equations determines a set of rules agents have to follow in order to extract some pieces of information relies on an illegitimate idealization: *the* right algorithm does not exist.

These considerations imply that we should reassess the normative dimension of my previous analysis of formats and inferential affordances. The consequences of what I intuitively meant by "format" are too context and agent-dependent to be settled in terms of a fixed set of procedures *to be applied*. An agent's use of a representation depends on the particular situation in which he/she is involved and on his/her particular goals. As the case of Feynman's diagrams show, the inferential affordances of a representation for the same user – as well as for the community – change over time: the inferential affordances of diagrams are not the same for Feynman in 1948 and ten years later. Likewise, the inferential affordances of the Lagrangian equations were modified by Hamilton's adding to mechanics new rules of transformation, which changed the role of the various equations of the theory in the scientists' reasoning processes. Therefore, the inferential affordances of a representation are fundamentally dynamical in character; they should to be defined (beside the perceptual properties of the representation and a minimal set of construction and interpretation rules) in reference to a particular situation, involving a particular agent, with particular skills, theoretical commitments, preferences, reasoning habits, as well as interests and intentions in the particular inquiry in which he/she is involved.

By this, I do not mean that a philosophical analysis of the use of representations in theorizing and the importance of formats has to be strictly descriptive. Once the highly agent-dependent and situation-dependent nature of the use of representations has been acknowledged, it is certainly worth trying to find the theoretically interesting regularities in the use of representations in different contexts, and there is certainly room for normative claims. For instance, acknowledging that the inferential affordances of a representation are relative to its users' background knowledge and skills could help us analyze the role and virtue of various kinds of representation in scientific teaching and popularization. In these activities, as well as in theory development by experts, there are definitely successful as well as failing strategies. In analyzing those cases, a certain degree of idealization is required, and it is worth ignoring some inter-individual differences and assuming *types* of agents (therefore speaking of the inferential affordances of a representation for beginners, for experts, etc). According to the kind of question to be studied, different levels of idealization might be justified: if one is interested in popularization, one should pay attention to various inter-individual differences and to the cognitive abilities of the laypersons – although one can assume *types* of laypersons; on the other hand, if one is interested in the inferential differences between Lagrangian and Hamiltonian equations, one should definitely assume a type of agents – experts – who use differential equations the same way. Note, however, that the case of Feynman's diagrams shows that inter-individual differences between experts (or groups of experts[27]) are sometimes also worth taking into account.

4.3 Formats, Theorizing, and the Content of Theoretical Representations

Finally, the view I have been defending enlightens some common features of various theoretical activities, in particular learning and theory development, which are usually not treated under the same heading. Indeed, as suggested above, both might be thought of as processes consisting in the modification of the inferential affordances of some representations – i.e. of the way one uses them. Contrary to what the idea of formats as determining a fixed set of procedures to be followed suggests, experts, as well as students, continue to deepen their understanding of the theoretical hypotheses they develop and use, by inventing new inferential paths between representations. This is what physics students do. This is what Hamilton did, with this (important) difference: he did it first and made his modification publicly available.

Let me clearly state that my point does not amount to saying that scientists' reasoning does not obey any rule and that there exists no difference between a sound inference and a wrong one. Of course, solving a differential equation implies that one conforms oneself to a whole set of calculation rules; if one obeys them, one cannot deduce contradictory results from the different types of equations of mechanics. There is a sense in which these equations are equivalent; one cannot draw just anything from them. Just like a chess game, the inferential affordances of a representation partially depend on a set of rules which are objective, in the sense that these rules do not depend on the users and on the situation. My point is to claim that these rules are highly insufficient to determine the way representations are used; reducing our analysis of theoretical representations to an idealized – positivist-like – image of a

[27] Such a view enables us to characterize scientific communities by referring to their sharing types of representations and using them the same way, as already suggested by Kuhn [22]. As Kuhn emphasized, different communities can use the same "symbolic generalizations" – e.g. Schrödinger's equations – in consistent (i.e. conforming to objective mathematical rules) but *different* ways (applying them to different cases and giving them different interpretations). These equations do not have the same inferential affordances for these different practitioners. Kaiser's study [19] of the uses and interpretations of the diagrams by different schools is an example of the fruitfulness of this view.

fixed set of rules results in a narrow conception of theorizing, which does not enable us to capture the complex processes which are at play in theorizing, and particularly the creative dimension of theorizing[28]. Theoretical representations such as the equations of mechanics or Feynman's diagrams are highly sophisticated tools of calculation and inquiry; as the analogy with the chess-player's inventing new gambits suggests, the process of drawing new bridges between them and inventing new ways to connect them to the phenomena – and therefore of modifying the procedures of use of these representations – are potentially infinite. Theorizing partially consists in developing these rules in order to *explore the content* of representations.

Acknowledging that such exploration is potentially infinite might finally offer a new standpoint to think of the content of theoretical representations, and of their predictive and explanatory power. Indeed, in virtue of the objective rules' underdetermining the inferential procedures one may follow in using them, it is always possible to draw new consequences and "discover" new relations between them and the phenomena. In other words, the famous "theoretician's dilemma" formulated by Hempel [14] as a consequence of the reductionist demand on theoretical terms does not arise: theoretical representations are not theoretical because they seem to refer to some unobservable entities, but rather because they allow theoreticians to create novel connections between them by manipulating them and modifying their inferential affordances.

5 Conclusion

Philosophers of science generally consider theories and models as abstract entities, whose representational relationships with the phenomena have to be elucidated by formal reconstruction. I hope to have shown that one cannot understand the explanatory and predictive fruitfulness of scientific representations without taking into account the particular form of what Humphreys calls the "concrete pieces of syntax" [16], which are used in theory learning, application, and development, and whose rules of construction and interpretation are not fixed. As he suggests, one should give up the "no-ownership perspective" [16] characteristic of most philosophy of science, and pay attention to the *computational* dimension of theorizing. Moreover, I have shown that, once agents and their limited cognitive abilities get into the picture, it becomes impossible to draw a clear-cut frontier between epistemic differences, which would count for all humans, and purely psychological differences.

[28] Note, incidentally, that Kuhn, although he strongly criticized the positivist image of theorizing conceived of as mere application of rules, also missed this creative dimension. He indeed considered the mathematical development of a theory such as CM as a "purely formal" work, as opposed to conceptual innovation. My analysis of theorizing aims at showing that conceptual novelty sometimes arises from formal invention.

This is particularly true in the case of scientific representations, which are complex and sophisticated tools, and not mere displaying of data. Taking into account particular agents, with particular skills, involved in particular situations, finally enables us to enlighten some essential aspects of theorizing, which are often neglected. From this perspective, theorizing partially consists in constructing and manipulating representations, whose role in the agents' reasoning processes – whose inferential affordances – change(s) with the agents' abilities and interests.

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