IMPERATIVES, LOGIC OF*

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Peter B. M. Vranas vranas@wisc.edu University of Wisconsin-Madison 26 August 2010

Suppose that a sign at the entrance of a restaurant reads: "Do not enter these premises unless you have a reservation and you are properly attired". You see someone who is properly attired and is about to enter, and you tell her: "Don't enter these premises if you don't have a reservation". She asks why, and you reply: "It follows from what the sign says". It seems that you made a valid inference from an imperative premise to an imperative conclusion. But it also seems that imperatives cannot be true or false, so in what sense is your inference valid? Its validity cannot consist in the *truth* of its premise guaranteeing the truth of its conclusion. One is thus faced with "Jørgensen's dilemma" (Ross 1941: 55-6): apparently imperative logic *cannot* exist because logic deals only with entities that, unlike imperatives, can be true or false, but apparently imperative logic must exist. It must exist not only because inferences with imperatives can be valid, but also because imperatives (like "Enter" and "Don't enter") can be inconsistent with each other, and also because one can apply logical operations to imperatives: "Don't enter" is the *negation* of "Enter", and "Sing or dance" is the *disjunction* of "Sing" and "Dance". A standard reaction to this dilemma consists in basing imperative logic on *analogues* of truth and falsity. For example, the imperative "Don't enter" is satisfied if you don't enter and is violated if you enter, and one might say that an inference from an imperative premise to an imperative conclusion is valid exactly if the *satisfaction* (rather than the truth) of the premise guarantees the satisfaction of the conclusion. Imperative logic may matter for metaethics: if one understands moral claims as disguised imperatives and thus as lacking truth values, then imperative logic may help explain how inferences involving moral claims can be valid. But before getting into the details of imperative logic, more needs to be said on what exactly imperatives are.

<u>1. The nature of imperatives</u>

The word "statement" is arguably ambiguous between a declarative sentence and what (an utterance of) such a sentence typically expresses, namely a *proposition*. Similarly, the word "imperative" is arguably ambiguous between an imperative sentence and what (an utterance of) such a sentence typically expresses, namely a *prescription*. Some people deny that propositions exist, and similarly some people may deny that prescriptions exist. But if one accepts that propositions and prescriptions exist, the following is natural to say. A proposition can be expressed by performing the speech acts of asserting, conjecturing, admitting, predicting, and so on. Similarly, a prescription can be expressed by performing the speech acts of commanding, advising, requesting, suggesting, and so on. Distinct imperative sentences, for example the English sentence "Entrer" and the French sentence "Entrez", can express the same prescription. A prescription can be expressed by an imperative sentence (for example, "You will now leave"), and a proposition can be expressed by an imperative sentence (for example, "Marry in haste and repent at leisure"). Propositions are the primary (and declarative sentences are secondary) bearers of the semantic values of truth and falsity; similarly, prescriptions are the primary (and imperative sentences are secondary) bearers of the semantic values of whatever semantic values they have. Now three questions arise. What are the semantic values of

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prescriptions? What kinds of prescriptions are there? How are prescriptions to be modeled for the purposes of logic? The rest of this section considers these three questions in order.

1.1. The semantic values of prescriptions

There is widespread-though not universal-agreement that prescriptions cannot be true or false (for references see Vranas 2008: 552). Two main analogues of truth have been proposed for prescriptions (and for imperative sentences): bindingness and satisfaction. The prescription expressed by (addressing to you the imperative sentence) "Go" is (all-things-considered) binding exactly if there is some (undefeated) reason for you to go and is satisfied exactly if you go (and violated exactly if you don't go). The prescription expressed by "If it rains, run" is satisfied if it rains and you run, is violated if it rains and you don't run, and is avoided (i.e., neither satisfied nor violated) if it doesn't rain. (Some people deny that prescriptions can ever be avoided, and would say that the prescription expressed by "If it rains, run" is satisfied if it doesn't rain; cf. Dummett 1959: 150-1.) One might want to distinguish satisfaction from obedience (i.e., intentional satisfaction), saying for example that, if you get out not because I ordered you to get out (suppose you did not hear me) but rather because you wanted to get out anyway, then my order is satisfied but not obeyed. But the imperative sentence "Get out" might express a prescription that is satisfied exactly if you get out, no matter for what reason (and thus is satisfied in the above example), and might also express a prescription that is satisfied exactly if you get out because I order you to do so (and thus is not satisfied in the above example). So there is no need to invoke a distinction between satisfaction and obedience in response to the above example; one can instead invoke a distinction between two prescriptions that the imperative sentence "Get out" might express (cf. Vranas 2008: 534).

1.2. Kinds of prescriptions

Call a prescription (or an imperative sentence) *personal* exactly if its satisfaction requires the performance of an action, and *impersonal* otherwise. For example, the prescription expressed by "Someone turn off the light" is personal (it is satisfied only if someone performs the action of turning off the light), but the prescription expressed by "Let there be no light in the room" is impersonal (it is satisfied even if no one performs any action but the room goes dark due to a power failure). (Personal imperative sentences are also known as *directives*, and impersonal imperative sentences are also known as *fiats* (Hofstadter & McKinsey 1939: 446); some people deny that fiats exist.) Unlike "Someone turn off the light", some personal prescriptions are satisfied only if a *particular* agent (or group of agents) performs an action; for example, "Max, turn off the light". Some people (for references see Vranas 2008: 555) deny the existence of *unsatisfiable* prescriptions (e.g., "Disprove the Pythagorean theorem"), of *unviolable* prescriptions (e.g., "Open the door yesterday").

1.3. Models of prescriptions

(1) Some people propose identifying prescriptions with propositions (or imperative sentences with declarative ones), but such proposals are subject to numerous objections (cf. Hamblin 1987: 97-136). For example, one reason why "Run" cannot be identified with the *deontic* proposition (or sentence) "You must run" (and thus imperative logic differs from deontic logic) is that "You must run", unlike "Run", entails that there is a (normative) reason for you to run. (This is not to say that whenever someone—or the law—*says* "You must run" there is a reason for you to run; it is rather to say that, necessarily, if it is *true* that you must run, then there is a reason for you to run. Still, some people deny the entailment.)

(2) Other people propose modeling prescriptions (or imperative sentences) in terms of two factors: a factor—sometimes called a *mood indicator*—indicating that something is being prescribed, and a factor—sometimes called a *sentence radical*—indicating what is being prescribed (cf. Hare 1952;

Stenius 1967). An objection to such proposals is that they do not adequately model *conditional* prescriptions: it seems that "If it rains, run" should be modeled in terms of *two* "sentence radicals", one corresponding to "It rains" and one corresponding to "Run". (This is because the prescription "If it rains, run" is avoided if it doesn't rain, but a prescription modeled in terms of a single sentence radical is never avoided: it is satisfied or violated, depending on whether the sentence radical is true or false.)

(3) Vranas (2008) models a prescription as an ordered pair of logically incompatible propositions, namely the *satisfaction proposition* and the *violation proposition* of the prescription; he defines the *context* of a prescription as the disjunction of those two propositions, and the *avoidance proposition* as the negation of the context. Vranas also defines a prescription to be *unconditional* exactly if its avoidance proposition is (logically) impossible (equivalently, its violation proposition is the negation of its satisfaction proposition), and to be *conditional* exactly if it is not unconditional. For example, the prescription expressed by "Run" is unconditional and the prescription expressed by "If it rains, run" is conditional; the satisfaction proposition is the proposition that it rains and you run, the violation proposition is the proposition is the proposition that it rains, and the avoidance proposition is the proposition that it rains. Vranas's account would be rejected by those who deny that prescriptions can ever be avoided (see §1.1).

2. Logical operators and inconsistency

This section considers (i) negations, (ii) conjunctions and disjunctions, (iii) conditionals and biconditionals, and (iv) inconsistency.

2.1. Negations

It is natural to say that the negation of the unconditional prescription expressed by "Dance" is the unconditional prescription expressed by "If you sing, dance", and that the negation of the conditional prescription expressed by "If you sing, dance" is the conditional prescription expressed by "If you sing, dance" is the conditional prescription expressed by "If you sing, don't dance". (One might suggest that the negation of "If you sing, dance" is "Don't both sing and dance", but this is the negation of "Sing and dance".) These examples motivate defining the negation of a prescription I as the prescription that is satisfied exactly if I is violated and is violated exactly if I is satisfied—and thus is avoided exactly if I is avoided (cf. Storer 1946: 31). (Assume that specifying satisfaction and violation conditions—and thus also avoidance conditions—uniquely specifies a prescription.) This definition has the consequence that the negation of the negation of a prescription I is just I. If, following Vranas (2008), one models I as the ordered pair $\langle S, V \rangle$ of its satisfaction proposition S and its violation proposition V, then the negation of I is modeled as $\langle V, S \rangle$.

2.2. Conjunctions and disjunctions

It is natural to say that the conjunction of the unconditional prescriptions expressed by "Sing" and by "Dance" is the unconditional prescription expressed by "Sing and dance". This example suggests defining the conjunction of two prescriptions as the prescription that is satisfied exactly if both conjuncts are satisfied and is violated otherwise (cf. Hofstadter & McKinsey 1939: 448). Although this common definition works when both conjuncts are unconditional, it does not always work: it is natural to say that the conjunction of the conditional prescriptions expressed by "If you sing, dance" and by "If you don't sing, dance" is the unconditional prescription expressed by "Dance (whether or not you sing)", or just by "Dance", and this conjunction is satisfied even in some cases in which not both conjuncts are satisfied (e.g., if "If you sing, dance" is satisfied—i.e., you sing and you dance—but "If you don't sing, dance" is avoided—i.e., you sing). To avoid such counterexamples,

Vranas (2008: 538-41) proposes the following definition: *the conjunction of two prescriptions is the prescription that is avoided exactly if both conjuncts are avoided and is violated exactly if at least one conjunct is violated* (and, as for every prescription, is satisfied exactly if it is neither violated nor avoided). For example, on this definition the conjunction of "If you sing, dance" and "If you don't sing, dance" is "Dance" because "Dance" is avoided exactly if both conjuncts are avoided (i.e., exactly if you both don't sing and sing, so never) and is violated exactly if at least one conjunct is violated (i.e., exactly if you sing and you don't dance or you don't sing and you don't dance, so exactly if you don't dance). (Vranas (2008: 559) notes that this definition differs from usual definitions of conjunction in three-valued logic.) Having defined conjunction and negation, one can define the disjunction of two prescriptions so that analogues of de Morgan's laws hold, namely as the negation of the conjunction of the negations of the disjuncts (Vranas 2008: 541-3).

2.3. Conditionals and biconditionals

The conditional prescription expressed by "If you are happy, smile" can be thought of as a conditional whose antecedent is the proposition expressed by "You are happy" and whose consequent is the prescription expressed by "Smile". This example motivates defining the *conditional* whose antecedent is a proposition P and whose consequent is a prescription I as the prescription that is satisfied exactly if both P is true and I is satisfied, and is violated exactly if both P is true and I is violated (cf. Storer 1946: 31). Following Castañeda (1970: 441-3), one can then define biconditionals by noting that, for example, the prescription expressed by "Smile if and only if you are happy" is the conjunction of the prescriptions expressed by "Smile if you are happy" and by "Smile only if you are happy", and these two prescriptions are the conditionals that can be expressed respectively by "If you are happy, smile" and by "If you are not happy, don't smile".

2.4. Inconsistency

A common suggestion is that two prescriptions are inconsistent (with each other) exactly if they are mutually satisfaction-exclusive; i.e., necessarily, they are not both satisfied. Although this suggestion works when both prescriptions are unconditional (e.g., "Dance" and "Don't dance"), it does not always work: the conditional prescriptions expressed by "If you sing, dance" and by "If you don't sing, don't dance" are mutually satisfaction-exclusive (necessarily, it is not the case that both (i) you sing and you dance and (ii) you don't sing and you don't dance) but are clearly not inconsistent (cf. Castañeda 1970: 443). To avoid this problem, Vranas (2008: 545-6) suggests that two prescriptions are inconsistent exactly if they are mutually nonviolation-exclusive; i.e., necessarily, they are not both nonviolated (equivalently: necessarily, at least one of them is violated). (Mutual nonviolation-exclusion entails mutual satisfaction-exclusion and, for unconditional prescriptions, is entailed by it.) On this suggestion, the prescriptions expressed by "If you sing, dance" and by "If you don't sing, don't dance" are not inconsistent: if you sing and you dance, then neither prescription is violated. Similarly, however, on this suggestion the prescriptions expressed by "If you sing, dance" and by "If you sing, don't dance" are not inconsistent, although the one is the *negation* of the other. Vranas (2008: 547) suggests that this result is correct because one can "comply" with both prescriptions by not singing. Still, there is a tension between the two prescriptions, and this can be captured by saying that they are *conditionally inconsistent*: some (logically) possible proposition (e.g., the proposition that you sing) entails that at least one of the two prescriptions is violated.

3. Imperative arguments and imperative inferences

Distinguish imperative *arguments* from imperative *inferences*. (1) An *argument* can be defined as an ordered pair whose first coordinate is a nonempty set of propositions or prescriptions or both (the *premises* of the argument) and whose second coordinate is either a proposition or a prescription (the

conclusion of the argument). (Alternatively, an argument can be defined in terms of declarative and imperative sentences.) Say that an argument is declarative exactly if its conclusion is a proposition, and that it is *imperative* exactly if its conclusion is a prescription. Say that an argument is *pure* exactly if its premises and its conclusion are either all propositions or all prescriptions, and that it is mixed otherwise. (2) An (endorsing) inference can be defined as a (token) process of reasoning (carried out by a given agent) that starts by endorsing certain propositions or prescriptions or both (the premises of the inference) and ends by endorsing either a proposition or a prescription (the conclusion of the inference). (Say that to endorse a proposition is to believe that it is true, and that to endorse a prescription is to believe that it is (all-things-considered) binding (see §1.1). One can similarly define a hypothetical inference by specifying that the premises and the conclusion are not endorsed but are rather entertained only hypothetically.) To any given inference corresponds the argument whose premises and conclusion are the premises and the conclusion of the given inference. Say that an inference is declarative or imperative and pure or mixed exactly if its corresponding argument is. Given these definitions, the rest of this section considers (i) the validity of pure imperative arguments, (ii) the validity of mixed arguments, and (iii) attacks on the possibility and the usefulness of imperative inferences.

3.1. The validity of pure imperative arguments

A reasonable requirement for a definition of validity for pure arguments is that a *multiple*-premise pure argument *A* be valid exactly if the corresponding *single*-premise pure argument is valid whose single premise is the conjunction of the premises of *A* and whose conclusion is the conclusion of *A*: one should be able to combine multiple premises by conjunction into a single premise and to split a single premise into conjuncts without affecting validity. So in what follows only single-premise pure imperative arguments are considered.

(1) By analogy with the standard definition of validity for pure declarative arguments, one might suggest that a pure imperative argument is valid exactly if some appropriate property is "transmitted" from its premise to its conclusion. The appropriate property cannot be truth (which is "transmitted" in valid pure declarative arguments), but can be (i) satisfaction, (ii) nonviolation, or (iii) bindingness. So say that a pure imperative argument is *satisfaction-valid* exactly if, necessarily, its conclusion is satisfied if its premise is (Hofstadter & McKinsey 1939: 452), and define nonviolation-valid and bindingness-valid similarly. (i) Against satisfaction-validity, it can be shown that the argument from "If your GRE score is at least 600, apply" to "If your GRE score is at least 700, apply" (which is intuitively valid: think about instructions on a graduate admissions website) is not satisfaction-valid, and that the argument from "If your GRE score is at least 600, apply" to "If your GRE score is at least 500, apply" (which is intuitively invalid) is satisfaction-valid. (ii) Against nonviolation-validity, it can be shown that the arguments from "Apply" to "If you don't apply, kill yourself' and from "If you have a perfect score, apply" to "If you don't apply, don't have a perfect score" (which are arguably intuitively invalid) are nonviolation-valid. (iii) Against bindingnessvalidity, note that in the absence of a formal account of bindingness it is unclear how to find out whether any given pure imperative argument is bindingness-valid.

(2) Vranas (forthcoming) defines when a reason "weakly" or "strongly" *supports* a prescription, and defines a pure imperative argument to be *weakly valid* exactly if, *necessarily, every reason that weakly supports the premise also weakly supports the conclusion* (and defines *strongly valid* similarly). Vranas argues that there is an unresolvable conflict of intuitions about the validity of the argument (which corresponds to "Ross's paradox"; see Ross 1941: 62) from "Post the letter" to "Post the letter or burn it", and that this conflict is largely explained by a divergence between strong and weak validity: the argument is weakly but not strongly valid. Against weak validity, it can be

shown that the argument from "Apply" to "If you don't apply, kill yourself" (which is arguably intuitively invalid) is weakly valid.

(3) For further definitions of validity for pure imperative arguments see Hare 1952: 25, Rescher 1966: 82-91, Sosa 1966: 232, and Makinson & van der Torre 2000.

3.2. The validity of mixed arguments

(1) Concerning mixed *declarative* arguments (namely arguments whose conclusion is a proposition and whose premises include a prescription), it is natural to adopt the principle that *a mixed declarative argument is valid if and only if its conclusion follows from its declarative premises alone*. (The "only if" part of this principle corresponds to "Hare's Thesis"; see Rescher 1966: 73.) This principle has the consequences that (i) if a pure declarative argument is valid, then the argument that one gets by "adding" any imperative premises is also valid, and (ii) no argument from only prescriptions to a (non-necessary) proposition is valid. Against (ii), one might claim that the arguments from "Kiss Paul's sister" to "Paul has a sister" (cf. Rescher 1966: 92-7) and from "If smoking is permitted, smoke" and "Don't smoke" to "Smoking is not permitted" (cf. Geach 1958: 52), are valid. In reply one might claim that "Kiss Paul's sister" *presupposes* (but does not *entail*) "Paul has a sister", and that the conjunction of "If smoking is permitted, smoke" and "Don't smoke", which on Vranas's definition of conjunction (see §2.2) is "Let it be the case that smoking is not permitted and that you don't smoke", entails "Let it be the case that smoking is not permitted", not "Smoking is not permitted".

(2) Concerning mixed *imperative* arguments with *only* declarative premises, namely arguments from only propositions to a prescription, it is natural to adopt the principle (which corresponds to "Poincaré's Principle"; see Rescher 1966: 74) that no such argument is valid (except perhaps trivially, if its conclusion is *vacuous*—namely necessarily satisfied, like "Run or don't run"—or its premises are inconsistent). This principle is similar to—but different from—the is/ought thesis attributed to Hume. Against this principle, one might claim that the argument from "Jupiter is the largest planet" to "Either go to Jupiter or don't go to the largest planet" is valid (cf. Geach 1958: 52-6).

(3) Finally, concerning mixed *imperative* arguments with *both* declarative and imperative premises, no satisfactory definition of validity exists in the literature. One might suggest that such an argument is valid exactly if, necessarily, if its declarative premises are true and its imperative premises are satisfied, then its (imperative) conclusion is satisfied; but this suggestion has the implausible consequence that the argument from "Smile" and "You will run" to "Run" is valid. For further suggestions concerning the validity of such arguments see Clarke 1970 and Sosa 1970.

3.3. Attacks on imperative inference

(1) Although imperative logic deals with imperative *arguments* rather than imperative *inferences*, if imperative inferences were impossible then what would be the point of imperative logic? Contesting the possibility of imperative inferences, Bernard Williams (1963) gives an argument that can be reconstructed as follows: distinct prescriptions have conflicting permissive presuppositions (e.g., "Sing" does not permit dancing without singing but "Sing or dance" does permit this), so successively endorsing the premises and the conclusion of a pure imperative argument (e.g., the argument from "Sing" to "Sing or dance") amounts to changing one's mind and thus cannot amount to carrying out an inference. Williams's reasoning is widely discussed in the literature (for references see Vranas 2010: 66) and is subject to several objections. For example, Vranas (2010: 67) argues that one can carry out an inference even if while doing so one changes one's mind, as illustrated by an examiner who says "Answer exactly two out of the three questions—*any* two questions, at your choice" and a minute later says "I changed my mind. Don't answer both the first

and the last question. Therefore, answer either the first two or the last two questions (not both), at your choice".

(2) Even if one grants that imperative inferences are possible, one might argue that they are useless because they can always be *replaced* with declarative inferences, in the sense that the validity of an imperative argument always amounts to the validity of a corresponding declarative argument (cf. Harrison 1991: 116); for example, the validity of the imperative argument from "Sing" to "Sing or dance" amounts to the validity of the declarative argument from "You will sing" to "You will sing or dance". Vranas (2010: 69) replies with an analogy: even if geometric reasoning can always be replaced with algebraic reasoning (by using Cartesian coordinates), geometric reasoning is often useful because it is often more straightforward than corresponding algebraic reasoning. Similarly, Vranas argues that imperative reasoning is often more straightforward than corresponding declarative reasoning.

(3) For further attacks on imperative inference see Wedeking 1970, Harrison 1991, and Hansen 2008; for replies see Castañeda 1971 and Vranas 2010.

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