Old Evidence and New Explanation II*

Carl G. Wagner†‡
Department of Mathematics, The University of Tennessee

Additional results are reported on the author's earlier generalization of Richard Jeffrey's solution to the problem of old evidence and new explanation.

1. Introduction. Several years ago Richard Jeffrey (1991, 1995) devised a probability revision method that confirms (i.e., raises the probability of) hypothesis H when it is discovered that H implies previously known evidence E. This method, called *reparation,* furnished a solution to a well-known problem in Bayesian epistemology first posed by Clark Glymour (1980). The author recently generalized Jeffrey's method to the case of old probable evidence and new probabilistic explanation (Wagner 1997) and identified certain conditions entailing the confirmation of H in this more general context. The present paper delineates further conditions of this type, deriving these, as well as those formerly identified, from new, more transparent representations of the prior and revised odds on H.

2. Preliminaries. The aforementioned generalization of Jeffrey's method takes as its starting point a probability distribution p on the algebra generated by H and E. Empirical investigation (*observation*) has given us a certain measure of confidence in the truth of E, as reflected in the value $p(E)$. We subsequently discover, quite apart from the observation underlying p, theoretical considerations that, taken alone, indicate that the truth of H would confer probability v on E, and its falsity would confer probability u on E(*explanation*). Taken

*Received July 1998.

†Send requests for reprints to the author, Department of Mathematics, The University of Tennessee, Knoxville, TN 37996-1300.

‡This research was supported by the National Science Foundation (SBR-9528893).

Philosophy of Science, 66 (June 1999) pp. 283–288. 0031-8248/99/6602-0006$2.00

alone, these considerations would not have tended to alter whatever probability might have been ascribed to H prior to the assessment of p. How should p be revised in light of this theoretical discovery?

Adapting a key feature of Jeffrey's approach to this problem, we resurrect a notional *ur-distribution* p_0 predating both observation and explanation, with p assumed to have come from p_0 by probability kinematics (Jeffrey 1983, 1988) on the partition $\{E, \bar{E}\}$. In logical effect, it is the conceptual state captured by p_0 in which we make the aforementioned theoretical discovery. This discovery would have led us to revise p_0 to the unique distribution p_1 satisfying (i) $p_1(E|H) = v$, (ii) $p_1(E|\bar{H}) = u$, and (iii) $p_1(H) = p_0(H)$. Using the explanation-based revision p_1 of p_0 as a paradigm, we revise p to a distribution p^*, so that p^* bears the same relation to p as p_1 does to p_0, in the sense that

$$\frac{p^*(A)}{p(A)} \propto \frac{p_1(A)}{p_0(A)}, \; A = HE, H\bar{E}, \bar{H}E, \bar{H}\bar{E}.^{1} \qquad (1)$$

In effect, Jeffrey treated the special case of the above in which $p(E) = 1$, $p_1(E|H) = 1$, and $p_1(E|\bar{H}) = p_0(E|\bar{H})$. In that case, it turns out that $p^*(H) > p(H)$, i.e., that H is always confirmed. In the general case it is of course not necessarily true that $p^*(H) > p(H)$, and so it is of obvious interest to identify conditions under which this inequality obtains.

In the next section we identify two such conditions which, roughly speaking, ensure that H is confirmed if (1°) observation resulting in *any* increase in our confidence in the truth of E is combined with a *sufficiently strong* probabilistic explanation of E or (2°) observation rendering E *sufficiently probable* is combined with explanation for which the likelihood ratio $p_1(E|H)/p_1(E|\bar{H})$ exceeds the corresponding ratio $p_0(E|H)/p_0(E|\bar{H})$ by *any* amount.

3. Confirmation. It is useful to formulate the confirmation of H in terms of the inequality $p^*(H)/p^*(\bar{H}) > p(H)/p(\bar{H})$ rather than the equivalent inequality $p^*(H) > p(H)$. We begin by deriving useful expressions for these odds.

1. The symbol $\propto$ denotes proportionality. There are several equivalent formulations of (1), which I have labeled collectively *the uniformity principle*. Alternatively, p^* can be shown to come from p_1 by probability kinematics on $\{E, \bar{E}\}$ with the ratio of p^*-odds on E to the corresponding p_1-odds being identical to the ratio of p-odds on E to the corresponding p_0-odds (*the commutativity principle*). See Wagner 1997, Theorems 2 and 4.

Lemma 1. *With p, p_0, p_1, and p^* as defined above,*

$$\frac{p(H)}{p(\bar{H})} = \frac{p_0(H)}{p_0(\bar{H})}\left[\frac{p_0(E|H)\,\dfrac{p(E)}{p_0(E)} + p_0(\bar{E}|H)\,\dfrac{p(\bar{E})}{p_0(\bar{E})}}{p_0(E|\bar{H})\,\dfrac{p(E)}{p_0(E)} + p_0(\bar{E}|\bar{H})\,\dfrac{p(\bar{E})}{p_0(\bar{E})}}\right] \tag{2}$$

and

$$\frac{p^*(H)}{p^*(\bar{H})} = \frac{p_0(H)}{p_0(\bar{H})}\left[\frac{p_1(E|H)\,\dfrac{p(E)}{p_0(E)} + p_1(\bar{E}|H)\,\dfrac{p(\bar{E})}{p_0(\bar{E})}}{p_1(E|\bar{H})\,\dfrac{p(E)}{p_0(E)} + p_1(\bar{E}|\bar{H})\,\dfrac{p(\bar{E})}{p_0(\bar{E})}}\right]. \tag{3}$$

Proof. Formula (2) follows from the fact that for $A = H, \bar{H}$,

$$\begin{aligned} p(A) &= p(E)p_0(A|E) + p(\bar{E})p_0(A|\bar{E}) \\ &= p_0(A)\left[p_0(E|A)\,\frac{p(E)}{p_0(E)} + p_0(\bar{E}|A)\,\frac{p(\bar{E})}{p_0(\bar{E})}\right]. \end{aligned}$$

Note that (2) reduces simply to the odds form of Bayes' rule when $p(E) = 1$. Formula (3) follows from (1), and the fact that $p_1(H) = p_0(H)$, $p(HE)/p_0(HE) = p(\bar{H}E)/p_0(\bar{H}E) = p(E)/p_0(E)$ and $p(H\bar{E})/p_0(H\bar{E}) = p(\bar{H}\bar{E})/p_0(\bar{H}\bar{E}) = p(\bar{E})/p_0(\bar{E})$.[2] □

It is clear from (2) and (3) that the probabilities $p(E)$ and $p_0(E)$ and the likelihoods $p_1(E|H)$, $p_1(E|\bar{H})$, $p_0(E|H)$, and $p_0(E|\bar{H})$ are crucial determinants of whether H is confirmed. It is also clear that it would be vain to expect *simple* necessary and sufficient conditions for such confirmation (the simplest such probably being that the bracketed ratio in (3) is larger than its counterpart in (2)). In the following two theorems, however, some relatively simple and intuitively reasonable sufficient conditions for confirmation are identified.

Theorem 1. *If $p(E) > p_0(E)$ and either (1°) $p_1(E|H) > p_0(E|H)$ and $p_1(E|\bar{H}) \leq p_0(E|\bar{H})$ or (2°) $p_1(E|H) \geq p_0(E|H)$ and $p_1(E|\bar{H}) < p_0(E|\bar{H})$, then H is confirmed.*

Proof. Since $p(E) > p_0(E)$, we have $p(E)/p_0(E) > 1 > p(\bar{E})/p_0(\bar{E})$. Clearly, $p_1(E|H) - p_0(E|H) = p_0(\bar{E}|H) - p_1(\bar{E}|H)$ and so if (1°) holds, then $(p_1(E|H) - p_0(E|H))p(E)/p_0(E) > (p_0(\bar{E}|H) -$

2. The latter identities are well-known consequences of (indeed, equivalent to) the fact that p comes from p_0 by probability kinematics on $\{E, \bar{E}\}$. See, e.g. Jeffrey 1988, 29(11).

$p_1(\bar{E}|H))p(\bar{E})/p_0(\bar{E}))$, from which it follows that the numerator of the bracketed ratio in (3) is larger than its counterpart in (2). By a similar argument, the denominator of the bracketed ratio in (3) is no larger than its counterpart in (2). Hence $p^*(H)/p^*(\bar{H}) > p(H)/p(\bar{H})$. The obvious variant of this argument leads from condition (2°) to the desired result.

Remark 1.1. By an argument similar to the above, one can show that if $p(E) \leq p_0(E)$ and either (1°) or (2°) holds, then $p^*(H) \leq p(H)$. Thus, in the presence of (1°) or (2°), the inequality $p(E) > p_0(E)$ is both necessary and sufficient for H to be confirmed.

Remark 1.2. Roughly speaking, Theorem 1 says that H will be confirmed if observation resulting in *any* increase in our confidence in the truth of E is combined with a *sufficiently strong* (in the sense of (1°) or (2°)) probabilistic explanation of E. It is, by the way, possible for (1°) or (2°) to hold even if $p_1(E|H) < p_1(E|\bar{H})$. This would of course necessitate that $p_0(E|H) < p_0(E|\bar{H})$ as well, with the interval $[p_1(E|H), p_1(E|\bar{H})]$ being contained in the interval $[p_0(E|H), p_0(E|\bar{H})]$.

Remark 1.3. In order for (1°) or (2°) to hold, it does not suffice simply that $p_1(E|H) - p_1(E|\bar{H}) > p_0(E|H) - p_0(E|\bar{H})$, although the latter inequality is a consequence of (1°) or (2°). On the other hand, it is easy to see that if the difference between the p_1-likelihoods of H and $\bar{H}$ on evidence E is sufficiently large, in the sense that

$$p_1(E|H) - p_1(E|\bar{H}) > \max\{p_0(E|H), p_0(\bar{E}|\bar{H})\},$$

then both (1°) and (2°) hold.

The following theorem (which appears, with a more involved proof, in Wagner 1997, 64) complements Theorem 1 by demonstrating that H is also confirmed whenever observation rendering E sufficiently probable is combined with explanation in which the p_1-likelihood ratio of H on evidence E exceeds the corresponding p_0-likelihood ratio by any amount. In what follows,

$$\lambda_{p_i}(H, E) := p_i(E|H)/p_i(E|\bar{H}), \ i = 0, 1.$$

Theorem 2. *If* $\lambda_{p_1}(H, E) > \lambda_{p_0}(H, E)$, *then* H *is confirmed for sufficiently large values of* $p(E)$.

Proof. From (2) it is clear that

$$\lim_{p(E)\to 1} \frac{p(H)}{p(\bar{H})} = \frac{p_0(H)}{p_0(\bar{H})} \lambda_{p_0}(H, E),$$

and from (3) that

$$\lim_{p(E)\to 1} \frac{p^*(H)}{p^*(\bar{H})} = \frac{p_0(H)}{p_0(\bar{H})} \lambda_{p_1}(H, E),$$

from which it follows that if the likelihood ratio inequality $\lambda_{p_1}(H, E) > \lambda_{p_0}(H, E)$ holds, then $p^*(H)/p^*(\bar{H}) > p(H)/p(\bar{H})$ for $p(E)$ sufficiently large. □

Remark 2.1. Theorem 2 is misleadingly paraphrased in Wagner 1997 as stating that when $p(E)$ is sufficiently large and $p_1(E|H)$ is sufficiently larger than $p_1(E|\bar{H})$, then H is confirmed. While this is not false, it fails to capture the full force of Theorem 2 by obscuring the fact that the key likelihood ratio inequality $\lambda_{p_1}(H, E) > \lambda_{p_0}(H, E)$ can hold even if the p_1-likelihood of H on evidence E is smaller than (i.) the p_1-likelihood of $\bar{H}$ on E, (ii.) the p-likelihood of H on E, and (iii.) the p_0-likelihood of H on E. The following example illustrates this possibility:

	HE	$H\bar{E}$	$\bar{H}E$	$\bar{H}\bar{E}$
p:	8/35	4/25	4/7	1/25
p_0:	1/6	1/3	5/12	1/12
p_1:	1/8	3/8	1/4	1/4
p^*:	4/19	21/95	8/19	14/95

Here $p_1(E|H) = 1/4$, which is smaller than (i.) $p_1(E|\bar{H}) = 1/2$, (ii.) $p(E|H) = 10/17$, and (iii.) $p_0(E|H) = 1/3$, but $\lambda_{p_1}(H, E) = 1/2 > 2/5 = \lambda_{p_0}(H, E)$, and $p(E) = 4/5$ is large enough for H to be confirmed ($p^*(H) = 41/95 > 68/175 = p(H)$).

Remark 2.2. It is easy to verify that if $p(E) = 1$ (the case of certain evidence and probabilistic explanation), then the inequality $\lambda_{p_1}(H, E) > \lambda_{p_0}(H, E)$ is both necessary and sufficient for H to be confirmed (cf. Remark 1.1).

Remark 2.3. That the hypotheses of Theorem 1 posit a stronger degree of explanation than those of Theorem 2 (as suggested above) is borne out by the fact that the inequality $\lambda_{p_1}(H, E) > \lambda_{p_0}(H, E)$ is entailed by either of the conditions (1°) or (2°) of Theorem 1.

As described more fully in Wagner 1997, the above can be generalized to the case in which p comes from p_0 by probability kinematics on a partition $\{E_1, \ldots, E_n\}$, and the explanation-based revision p_1 of p_0 is defined by the likelihoods $p_1(E_i|H)$ and $p_1(E_i|\bar{H})$ and the assumption $p_1(H) = p_0(H)$. We leave it as an exercise for interested readers to extend Lemma 1 and Theorems 1 and 2 to this situation.

REFERENCES

Glymour, C. (1980), *Theory and Evidence.* Princeton: Princeton University Press.

Jeffrey, R. (1983), *The Logic of Decision,* 2nd ed. Chicago: University of Chicago Press.

———. (1988), "Conditioning, Kinematics, and Exchangeability", in B. Skyrms and W. Harper (eds.), *Causation, Chance, and Credence,* vol. 1. Boston: Kluwer Academic Publishers, 221–255.

———. (1991), "Postscript 1991: New Explanation Revisited", in Jeffrey 1992, 103–107.

———. (1992), *Probability and the Art of Judgment.* Cambridge: Cambridge University Press.

———. (1995), "Probability Reparation: the Problem of New Explanation", *Philosophical Studies* 77:97–102.

Wagner, C. (1997), "Old Evidence and New Explanation", *Philosophy of Science* 64:677–691.