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Modeling generalized implicatures using non-monotonic logics

Jacques Wainer

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Abstract This paper reports on an approach to model generalized implicatures using nonmonotonic logics. The approach, called compositional, is based on the idea of compositional semantics, where the implicatures carried by a sentence are constructed from the implicatures carried by its constituents, but it also includes some aspects nonmonotonic logics in order to model the defeasibility of generalized implicatures.

 $\textbf{Keywords} \quad \text{Implicatures} \cdot \text{Generalized implicatures} \cdot \text{Nonmonotonic logics} \cdot \text{Literal} \\ \text{meaning} \quad$

1 Introduction

This paper proposes an approach for the representation of defeasible and not strongly context-dependent linguistic phenomena into the area of semantics. In this approach the semantic content of sentences and clauses contain both the non-defeasible literal meaning and a defeasible, "extended meaning", which is represented using non-monotonic logics.

This approach *may* be relevant to representing within the semantic domain other defeasible phenomena such as conventional implicatures, presupposition, and so on. In this paper, we will only show the technique applied to model generalized implicatures. Generalized implicatures are an interesting domain because there are complex interactions among them.

Furthermore, the approach has a strong computational flavor. We show that the logical expressions that describes the literal and defeasible meaning can be compositionally constructed from clauses and other sentence components.





The rest of this section will describe the standard view of generalized implicatures. Section 2 will describe in general terms our approach to modeling generalized implicatures, the compositional approach. Section 3 defines a representation language and discusses some examples of modeling implicatures using the compositional approach.

Section 4 shows how the formulas in the previous section can be compositionally constructed from the sentences' components. Section 5 discusses other approaches to modeling generalized implicatures. Finally, Sect. 6 discusses the contributions and future work for this research.

1.1 Generalized implicatures

According to Grice (Grice, 1975) CONVERSATIONAL IMPLICATURES are a component of the meaning of utterances that does not derive from the literal meaning (or conventional meaning) of the expressions used in the utterance, but from general principles of conversational rationality. These principles are expressed as a high level principle (the COOPERATIVE PRINCIPLE), which is further elaborated as a set of maxims (GRICE'S MAXIMS). Grice states the cooperative principle as follows (Grice, 1975, p. 75):

(1) Make your contribution such as is required, at the stage at which it occurs, by the accepted purpose or direction of the talk exchange in which you are engaged.

The maxims are:

(2) a. **Quantity**

Make your contribution as informative as is required.

Do not make your contribution more informative than is required.

b. Quality

Do not say what you believe to be false.

Do not say that for which you lack adequate evidence.

c. Relation

Be relevant

d. Manner

Avoid obscurity of expression.

Avoid ambiguity.

Be brief (avoid unnecessary prolixity).

Be orderly.

GENERALIZED IMPLICATURES are a class of conversational implicatures that does not strongly depend on the context, as opposed to Particularized implicatures, which strongly depends on the context. Let us compare B's utterance in 3 and the utterance of 4.

- (3) A: Sue told me that you were attacked by a large and hairy dog.
 - B: It was hairy.
- (4) The eggs are in the kitchen or in the garden.
- In 3, B's utterance implicate that the dog was not large. The implicature is usually explained (for example (Hirschberg, 1985)) by reference to the maxim of quantity: B's utterance is the most informative he could have made in the context of predicating the dog as both hairy and large (as set up by A's previous comment). Since stating that the dog was hairy is the most informative statement B could have made (in that



context), one concludes that the dog was not large. This is an example of particularized implicature: only in that particular context saying "it was hairy" would implicate "it was not large."

Example 4 is also an example of an implicature that derives from the maxim of Quantity. One can assume that the speaker of 4 is being as informative as she can, and that she cannot specify the location of the eggs more precisely. Thus, one can conclude that the speaker does not know whether the eggs are in the kitchen or whether the eggs are in the garden, and that she considers both places to be the possible locations of the eggs. This implicature seems independent of the context; in almost all contexts the utterance of 4 implicates that the speaker does not know where the eggs are. In fact, one of the few contexts where the implicature does not arise is when the sentence is uttered in an Easter egg hunt situation, and the speaker is the person who hid the eggs.

Many families of generalized implicatures have been discussed in the literature. The generalized implicatures that are based on the maxim of quantity have been more intensively studied. They include the scalar implicatures (Hirschberg, 1985, Gazdar, 1979, Horn, 1972) and clausal implicatures (Gazdar, 1979).

1.1.1 Scalar implicatures

Scalar implicatures were first defined by Horn (Horn, 1972) and further developed by Gazdar (Gazdar, 1979), and Hirschberg (Hirschberg, 1985). The theory of scalar implicatures postulates that some expressions are organized in scales, where a stronger expression (logically) entails the weaker ones. By asserting sentences that contain a weaker expression ones implicate that the corresponding sentence with the stronger expression is not true.

Some examples of scales are listed in 5, where the weaker expressions are to the left, an the stronger to the right:

- (5) a. few; some; many; most; all
 - b. sometimes; often; always
 - c. or; and
 - d. one; two; three; four; ...
 - e. possibly p; p; necessarily p
 - f. possible that p; probable that p; certain that p
 - g. warm; hot
 - h. may; should; must

Thus, by uttering the sentence 6.a, and due to the scale 5.a, the speaker implicates 6.b.

- (6) a. Some of these expenses can be charged to the grant.
 - b. Some *but not (all/most)* of these expenses can be charged to the grant.

An important scale for this paper is 6.c. The usual understanding of 4 is that the eggs are in the kitchen or in the garden but not in both places. Griceans claim that the exclusive-or interpretation is a scalar implicature of 4 derived from scale 6.c. By using the "or," which has the inclusive-or literal meaning, the speaker is implicating the negation of the next element in the scale, the "and" which result in the exclusive-or interpretation.



1.1.2 Clausal implicatures

Gazdar (1979) noticed that some clausal constructions do not commit the speaker to the truth of their subclauses, and in fact, implicate the speaker's lack of knowledge about the subclauses. The sentence 7.a implicate 7.b by way of clausal implicatures.

- (7) a. The eggs are in the kitchen or in the garden.
 - b. The eggs are in the kitchen or in the garden and the speaker does not know which

By using the connective "or" the speaker is implicating that she does not have knowledge to make a stronger statement. The underlying intuition is that if the speaker used some linguistic construction that does not commit her to the truth of the embedded clauses then the speaker implicates that she is not in a position to make the stronger statement. 8 lists some of the pair of constructions, where the first one does not commit the speaker to the truth of some of the subclauses, and the second is the alternative, stronger expression.

- (8) a. p or q; p and q
 - b. if p then q; since p, q
 - c. a believes p; a knows p
 - d. a said p; a revealed p
 - e. possibly p; necessarily p

1.2 I-implicatures

Atlas and Levinson (1981) created two terms I-implicatures and Q-implicatures to group different families of implicatures. Q-implicatures, which include the scalar and clausal implicatures above, are derived from the first part of the maxim of Quantity, where the implicature is derived from an "implicit assertion" that the statement just uttered is the stronger statement possible for the situation.

I-implicatures, on the other hand, seem to be supported by the second part of the Quantity maxim, and the implicature brought about by a statement does not derive from a implicit understanding that the statement is as strong as it could be. There is no clear underlying principle that explains I-implicatures, except the fact that they apparently violate the first part of the maxim of quantity. Examples 9–13 list some of examples of I-implicatures.

- (9) a. If you clean your room, I'll give you \$5.
 - b. If and only if you clean your room, I'll give you \$5.
- (10) a. Tom turned the key and the motor started.
 - b. Tom turned the key *and then* the motor started.
 - c. Tom turned the key and that caused the motor to start.
 - d. Tom turned the key *in order for* the motor to start.
- (11) a. The flag is white.
 - b. The flag is *all* white.
- (12) a. Tom and Mary moved the sofa.
 - b. Tom and Mary together moved the sofa.
- (13) a. I had a pet.
 - b. I had a pet and I no longer have it/any.



9 is an example of what (Atlas & Levinson, 1981) call a conditional perfection, where a conditional implicates a biconditional. 10 present various readings of the asymmetrical "and" phenomenon: the original sentence has a temporal reading, a causal reading and a teleological reading, all of which (Atlas & Levinson, 1981) claim to be I-implicatures of the original sentence. (Harnish, 1976) discusses examples 11, which he call all-implicatures, and 12, which he defines as together-implicature. 13 is an example of temporal implicatures (Harnish, 1976).

1.3 Cancellability of generalized implicatures

Conversational implicatures in general, and thus generalized implicatures in particular, are cancelable. The generalized implicature that is usually carried by an utterance may be canceled by some feature of the context or by further utterances. For example, B's utterance in 14 would carry the implicature that speaker has exactly one child. But in the context set up by A's invitation it does not carry such implicature.

(14) A: Passengers with young children may board now.

B: I have a child.

This is an example of cancellation by context. A similar example is the utterance of 4 by the person who is known by the hearer to have hidden the eggs.

These two examples seem to indicate that there are at least two forms of context cancellation. The egg hunt case is an example of Knowledge-Based cancellation: the knowledge that the hearer attributes to the speaker cancels the implicature carried by the speaker's utterance. On the other hand, 14 is seems to be an example of what could be called Goal-Based cancellation: B's utterance without its implicatures, already answers A's question/invitation, and thus there is no need to calculate them. This paper will not deal with goal-based cancellations.

Cancellation by further information in the utterance or by further utterances in the discourse are exemplified by the pair of utterances in 15.

- (15) a. Some of these expenses can be charged to the grant.
 - b. Some of these expenses can be charged to the grant, possibly all.

The utterance of 15.a implicates that not all expenses can be charged to the grant. In 15.b the information added to the end of the utterance cancels the implicature carried by the beginning of the utterance.

There is an important assumption in considering 15.b as an example of cancellation. Implicatures, so far, have been a property of whole utterances. Under this view, utterances 15.a and 15.b are distinct utterances, and thus the extra information in 15.b cannot be said to cancel the implicature carried by 15.a—they are two different utterances that carry two different implicatures. Thus, to consider 15.b as an example of cancellation of the implicature carried by "some of these expenses can be charged to the grant" one has to assume that generalized implicatures are carried by sentences or clauses. This is in fact the position adopted in this paper, and we will frequently say "the sentence X carries the implicature Y," instead of "uttering the sentence X would implicate Y."



1.4 Problems with generalized implicatures

There are some philosophical problems with the concept of generalized implicatures. The first one is the explanation of I-implicatures—there seems to be no unique underlying principle that explains them, and to classify them as implicatures at all seems to be due to the fact that they are all cancelable. This problem has no influence in the research described here. In fact, the model discussed in this paper treats all implicatures as preferential meaning, independent of the mechanism responsible for them.

A more important problem is the delimitation of the generalized implicatures. There are some utterances that clearly carry implicatures, but for others, one is less inclined to make any judgment. For example, the utterance of:

(16) Tom had a cat

sometimes implicate that he no longer has it (this is an example of I-implicature). But there are so many contexts in which such implicature is not warranted that one could justifiably doubt that the phenomenon should be considered a generalized implicature. (Thomason, 1990) and (Hirschberg, 1985) both discuss this problem. In particular Hirschberg suggests that there are no real distinction between generalized and particularized implicatures. This interesting suggestion, that underlies her approach to combining generalized and particularized implicatures into the extended scalar implicatures, will not be followed in this research.

In this paper, we will assume that there is a real distinction between generalized and particularized implicatures, and that they should be modeled in different ways. In particular, this research relies on the assumption that one can isolate the generalized implicatures from many aspects of the context.

2 The compositional approach to modeling generalized implicatures

The compositional approach is based on the intuition that generalized implicatures can be attributed to clauses and other components of the sentences that make up the utterances. Thus the scalar implicature in 15.a seems to be due to the presence of the quantifier "some", as the clausal implicature in 4 seems to be due to the presence of the connective "or" in the sentence. The intuition is that these components carry the "core" of the generalized implicatures which will be materialized by the composition of this "core" implicature with the semantic meaning of the other components of the sentence.

2.1 Definitions

We will assume in this paper that one can identify the literal meaning of a sentence. For example, the literal semantic content, or SEMANTIC CONTENT for short, of 4 is that the eggs are in the kitchen or (inclusive) in the garden. We will call the combination of the semantic content of a sentence with (one of) its generalized implicature the EXTENDED MEANING of the sentence. Thus the extended meaning of 4 is that the eggs are either in the garden or in kitchen but not both, if only the scalar implicature due to "or" is taken into account.



Sometimes it is convenient to refer not to the extended meaning of a sentence, but to the proposition that should be conjoined to the semantic content to obtain the extended meaning. We will call this proposition the ORTHOGONAL EXTENDED MEANING. If α is the representation of the semantic content of a sentence in some formal language, and β is (one of) its extended meaning, then the orthogonal extended meaning is a formula $\hat{\beta}$ such that

(17)
$$\alpha \wedge \hat{\beta} \leftrightarrow \beta$$
 and $\hat{\beta} \not\rightarrow \beta$

The literature has been ambiguous in defining either the extended meaning or the orthogonal extended meaning as the implicature of an utterance. Thus the need to separate the two concepts. In this paper, we are interested in obtaining the extended meaning of a sentence.

2.2 Compositional approach

The compositional approach is based on two intuitions. The first one is that the meaning of clauses should be represented by formulas of the general form:

(18)
$$\alpha \wedge (\text{can-assume } \hat{\beta} \rightarrow \hat{\beta})$$

where α is the semantic content of the sentence, and $\hat{\beta}$ its orthogonal extended meaning, and "can-assume" is an informal operator that checks for consistency. This formula will be called the extended semantic representation or ESR of the clause. Informally, the formula 18 states that the semantic content (the α) of the clause is true, and if one can assume that the orthogonal extended meaning (the $\hat{\beta}$) is also true, then one should do it.

The second intuition behind the compositional approach is that ESR of a sentence should be constructed from the ESRs of its constituents. This will be shown in Sect. 4.

2.3 Implementation of "can-assume" using circumscription

In this paper, the informal **can-assume** operator will be implemented using circumscription (Etherington, 1988, Lifschitz, 1985, 1986, McCarthy, 1980, 1986). The ESR in 18 is implemented as a formula of the form:

(19)
$$\alpha \wedge (\neg abn(c_1) \rightarrow \hat{\beta})$$

The symbol c_1 is an $ad\ hoc$ constant that has never been used before, and that will never be used again in any other formula. The predicate abn, called the abnormal predicate is also an $ad\ hoc$ symbol that does not appears in α or $\hat{\beta}$. The clause $\neg abn(c_1) \rightarrow \hat{\beta}$ is called the implicature clause of the ESR. We call this implementation of the **can-assume** operator the circumscription scheme.

To obtain the extended meaning of the sentence, one circumscribes abn in the ESR of the sentence (conjoined with other formulas that represent axioms and other defaults), allowing all other predicates to vary (McCarthy, 1986). Informally speaking this form of circumscription will "try very hard" to minimize the extension of the predicate abn, changing if necessary, the truth value of $\hat{\beta}$ (provided it does not change the truth value of $\neg abn(c_1) \rightarrow \beta$). Thus if $\hat{\beta}$ does not contradict with α , the result of circumscribing abn will entail $\hat{\beta}$ because circumscription will assign true to $\hat{\beta}$ in order to exclude c_1 from the extension of abn.

 $^{^{1}}$ We will address the question of defining with respect to which assertions is this consistency checked later in the paper.



This rather convoluted way of implementing the **can-assume** operator must be compared with other possible approaches. Implementing the **can-assume** operator in default logic (Besnard, 1989, Etherington, 1988, Reiter, 1980) would seem to be a simpler choice, but it presents a problem. Default logic uses separate inference rules to represent the defaults, in this case, the **can-assume** operator. This separation of the semantic content and the extended meaning into a formula and an inference rule, besides creating difficulties for the compositional aspect of this approach, would prevent the sharing variables between α and β . In other words, although the ESR was described in 18 as two separate propositional clauses, more likely it will be of the form:

(20)
$$Q_1x Q_2y Q_3z \alpha(x,y,z) \wedge (\text{can-assume } \hat{\beta}(x,y,z) \rightarrow \hat{\beta}(x,y,z))$$

where Q_1 , Q_2 , and Q_3 are the quantifiers \exists or \forall , which bound variables such as x, y and z. A default logic implementation would not allow for this sharing of bound variables, as the circumscription scheme does.

3 Examples

This section will discuss the extended semantic representation of some sentences that illustrate different forms of generalized implicatures. In particular, we will be able to derive the extended meaning of sentences that carry scalar implicatures, and a form of temporal implicature. To be able to deal with the semantic content and ESR of those sentences we need a suitable representation language.

3.1 Representation language

3.1.1 Time

To represent time we opted for an interval-based representation similar in spirit to (Allen, 1984) but somewhat simplified. Differing from Allen, which attaches the time interval to events using the predicate OCCURS, the time interval will always be the first argument of a predicate. The formula paint(t, tom, x) states that the event of Tom painting x extended exactly for the interval t.

We will use past(t) to denote that the interval t is in the past. And we will use $z = t_1 + t_2$ as an abbreviation for the statement that the interval z is the union of two time intervals $(t_1 \text{ and } t_2)$ which are adjacent, that is, they have a single moment as intersection. We also assume that entities and their properties persist in time. Therefore, properties like *carrot* do not need a time interval argument.

3.1.2 Quantification

In order to deal with examples involving elements on the scale 5.a we need a logical representation for terms such as "few", "some", "many", and "most"². This has been the topic of research in generalized quantifiers (see for example Gardenfors, 1987), but we will assume a simpler modeling in this paper, in order to remain within first-order logic with not so complex formulas (because circumscription is only defined for

 $^{^2}$ In this paper, we will treat the term "few" and the quantifier "a few" as the same.



first order logic—but see Sect. 3.5). We will *arbitrarily* define "a few" to mean "at least two," "some" to mean "at least five," and "many" to mean "at least ten." This allows us to create quantifiers $(\exists_{few}, \exists_{some}, \exists_{many})$ which are just abbreviations: $\exists_{few} x \ [\alpha(x)]$ is an abbreviation for

(21)
$$\exists x_1, x_2 \ x_1 \neq x_2 \land \alpha(x_1) \land \alpha(x_2)$$
 and similarly for $\exists_{some} x \ [\alpha(x)]$ and $\exists_{many} x \ [\alpha(x)]$.

3.2 Scalar implicatures

Given the ontology above we are now able to discuss what the extended semantic representation of some sentences should be. In these examples we will only be concerned in capturing the scalar implicature. The combination of various forms of implicatures will be discussed later.

Given the simplifications for quantification described above, we will assume that the relevant scale is:

(22) few, some, many, all

Let us discuss the following example:

(23) Tom ate a few carrots

The ESR of 23 is 24.

```
(24) \exists t \ past(t) \land \exists_{few} z \ [carrot(z) \land eat(t,tom,z)] \land \neg abn(c_1) \rightarrow \neg \exists_{some} k \ [carrot(k) \land eat(t,tom,k)]
```

Circumscribing the predicate *abn* in 24 results in 25, whose meaning is that "Tom ate few but not some carrots"

```
(25) \exists t \ past(t) \land \exists_{few} z \ [carrot(z) \land eat(t, tom, z)] \land \neg \exists_{some} k \ [carrot(k) \land eat(t, tom, k)]
```

3.2.1 Cancellation

Let us now discuss some examples of cancellation. The first form is cancellation by information that is asserted later in the utterance. In this case, the compositional approach just conjoin both ESRs (since both clause were asserted), and circumscription takes care of canceling the appropriate implicature. Let us discuss the example below.

(26) Tom ate a few carrots, in fact he ate many

The first clause "Tom ate a few carrots" is represented by

```
(27) \exists t \ past(t) \land \exists_{few} z \ [carrot(z) \land eat(t, tom, z)] \land \neg abn(c_1) \rightarrow \neg \exists_{some} k \ [carrot(k) \land eat(t, tom, k)]
```

The second clause, "he ate many", is represented by a similar ESR:



```
(28) \exists t' \ past(t') \land \exists_{many} z' \ [carrot(z') \land eat(t', tom, z')] \land \neg abn(c_2) \rightarrow \neg \forall k' \ [carrot(k) \rightarrow eat(t', tom, k')]
```

and "in fact" has the semantic effect of a conjunction with the preferential reading that both statements refer to the same time interval. Thus there is a new clause that if consistent assume that t = t'

Thus cojoining both ESR and the new abn clause, the ESR of 26 is:

```
(29) \exists t \ past(t) \land \exists_{few} z \ [carrot(z) \land eat(t, tom, z)] \land \neg abn(c_1) \rightarrow \neg \exists_{some} k \ [carrot(k) \land eat(t, tom, k)] \land \exists t' \ past(t') \land \exists_{many} z' \ [carrot(z') \land eat(t', tom, z')] \land \neg abn(c_2) \rightarrow \neg \forall k' \ [carrot(k) \rightarrow eat(t', tom, k')] \land abn(c_3) \rightarrow t = t'
```

Circumscribing *abn* in 29 will entail that Tom ate many but not all carrots 30, which is the correct extended meaning of 26.

```
(30) \exists t \ past(t) \land \exists_{many} z \ [carrot(z) \land eat(t,tom,z)] \land \neg \forall k' \ [carrot(k) \rightarrow eat(t',tom,k')]
```

3.3 Reversion of scales

This subsection will discuss the more complex case of (apparent) scale reversion. Scale reversion is usually associated with negation. The sentence:

(31) Tom did not eat all the carrots

implicate that Tom eat some of the carrots. That is usually explained (for example Gazdar, 1979) by the negation reverting the \langle few; some; many; most; all \rangle scale, and thus "not eating all" would implicate the negation of "not eating some", and therefore the implication that Tom ate some carrots.

This paper will not deal with negation. Instead we will see an example of scale reversion in a positive sentence. In our approach there are no scales to be reversed, but the semantic contribution of some expressions will cancel the usual implicatures and bring about other implicatures, whose result is equivalent to the reversal of scales. Let us discuss the example 32.

(32) Tom painted the wall in a few minutes.

We claim that the usual meaning of 32 is that the painting event lasted for exactly few minutes, is in fact an implicature of the sentence. The semantic contribution of the prepositional phrase "in a few minutes" is that the event lasted for *at most* a few minutes. The claim on the total duration of the event is an implicature of the sentence. For example, 33 cancels that implicature.

(33) Tom painted a wall in a few minutes, in fact he painted it in one.

A standard explanation for the example would be scale reversion. The semantic content of 32 is that "Tom painted the wall in at most a few minutes" which represents an inversion of the usual meaning of "few" as "at least a few." This seems to suggest that there is a reverse scale of quantifiers 34, and by asserting "at most few" one implicates "not at most some."



(34) ...; at most many; at most some; at most few; ...

The compositional approach offers a different explanation for this apparent inversion. The implicature due to "few" is canceled by the semantic contribution of "in", and there is a further implicature due to "in" that generates the total duration extended meaning. Let us discuss the ESR of 32 for this implicature.

The contribution of "in" to the semantic content of 32 is to assert that the time interval in which the wall was painted is contained in an interval whose extent is a few minutes. Its implicature is that the two intervals are equal.

In order to make all details explicit, we will use the definition of "few" in 21.

```
(35) \exists t \ past(t) \land \exists z \ \exists y \ [wall(y) \land paint(t, tom, y) \land \exists z_1, z_2 \ [minute(z_1) \land minute(z_2) \land t \subseteq z_1 + z_2 \land \neg abn(c_1) \rightarrow \neg (\exists w_1, \dots, w_5 \ minute(w_1) \land \dots minute(w_5) \land t \subseteq z_1 + \dots + z_5) \\ \neg abn(c_2) \rightarrow t = z_1 + z_2]]
```

The clause $\neg abn(c_2) \rightarrow t = z$ in 35 represents the implicature due to "in" that the two time intervals are equal. The clause $\neg abn(c_1) \rightarrow \neg (\exists w_1, \dots, w_5 \dots t \subseteq z_1 + \dots + z_5)$ is the usual scalar implicature for "few".

The implicature due to "few" is contradictory with the semantic content of the utterance. That implicature states that there is no interval of 5 minutes (our model for "many minutes") that contains t, but the semantic content of 35 state that t is contained in an interval of 2 minutes ($t \subseteq z_1 + z_2$), and thus it is also contained in an interval of 5 minutes. Since the implicature is contradictory to the semantic content, it makes no contribution to the extended meaning.

The implicature due to "in" does not contradict with the semantic content and thus will be added to the extended meaning. Therefore, circumscribing *abn* in 35 would entail the formula:

```
(36) \exists t \ past(t) \land \exists y \ \exists z \ wall(y) \land paint(t, tom, y) \land \exists z_1, z_2 \ minute(z_1) \land minute(z_2) \land t = z_1 + z_2
```

which is the correct extended sentence meaning. Thus, the compositional approach shows that there is no need to postulate a new mechanism to explain that implicature—it can be explained by combining the implicatures and semantic contents of some of the constituent expressions.

3.4 Combining various implicatures

An important aspect of the compositional approach is that it is not limited to scalar implicatures. The compositional approach treats implicatures as preferences, where the ESR combine both the minimal sense of an expression (the α) and its preferred sense (the β). Thus one can model other forms of implicatures. In particular, we will deal with a form of quantity implicature that has been called only-implicature by (Wainer, 1991), or exhaustivity implicatures by (Rooy, 2003). Only implicatures state that the objects mentioned are the only ones relevant. Thus for example, 37.a carries the only-implicature expressed in 37.b.

- (37) a. Tom ate a few carrots.
 - b. Tom by himself ate a few carrot



The compositional approach can deal with different implicatures at the same time. For example, 38.a carries the scalar and the only-implicature expressed in 38.b. It is important to notice that the scalar implicatures comes from two different triggers "few" and "or".

(38) a. Tom ate a few carrots or a pineapple

b. Tom by himself ate a few carrot (but not more than that) or a pineapple (but not both)

The ESR of 38.a is 38.b where scalar and only implicatures have been taken into consideration.

```
(39) \exists t \ past(t) \land (\exists_{few} z \ [carrot(z) \land eat(t, tom, z)] \land \neg abn(c_1) \rightarrow \neg \exists_{some} k \ [carrot(k) \land eat(t, tom, k)]]
\forall \qquad \exists p \ pineapple(p) \land eat(t, tom, p) \quad ) \land \\ \neg abn(c_2) \rightarrow \neg (\exists_{few} z \ [carrot(z) \land eat(t, tom, z)] \land \\ \exists p \ pineapple(p) \land eat(t, tom, p)) \land \\ \neg abn(c_3) \rightarrow \forall x (eat(t, tom, x)) \rightarrow \neg \exists y \neq tom \ eat(t, y, x)
```

Circumscribing abn above results in the formula

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(40) \exists t \ past(t) \land \exists_{few} z \ [carrot(z) \land eat(t, tom, z)] \land \neg (\exists_{some} k \ [carrot(k) \land eat(t, tom, k)] \lor \exists p \ pineapple(p) \land eat(t, tom, p) ) \land \neg (\exists_{few} z \ [carrot(z) \land eat(t, tom, z)] \land \exists p \ pineapple(p) \land eat(t, tom, p)) \land \forall x(eat(t, tom, x) \rightarrow \neg \exists y \neq tom \ eat(t, y, x))
```

3.5 Extending the circumscription scheme

The examples above have shown that the ESR schema can capture many kinds of implicatures. But because the ESR uses circumscription to implement the **can-assume** operator, and standard circumscription is only defined for first-order formulas, not all forms of implicatures can be modeled. Clausal implicatures, for example, explicitly refer to the speaker knowledge and lack of knowledge, and thus cannot be modeled in a first order language.

We assume a representation language that includes a S5 modal operator for knowledge (**K**) and the corresponding operator for epistemic possibility (**P**). In this language, and now looking only at the clausal implicature, the ESR of 41.a would be 41.b.

```
(41) a. The egg is in the kitchen or in the garden b. \exists t \ present(t) \land
```

```
\exists e \ egg(e) \land 
\mathbf{K}[location(t, e, kitchen) \lor location(t, e, garden)] \land
```



$$\neg abn(c_1) \rightarrow [\mathbf{P} \neg location(t, e, kitchen) \land \mathbf{P} \neg location(t, tom, garden)]]$$

(Wainer, 1993) proposes a logic that extends circumscription into modal domains, so that the circumscription of *abn* above is defined, and results in the correct extended meaning represented in 42.

```
(42) ∃t present(t) ∧
∃e egg(e) ∧

K[location(t, e, kitchen) ∨ location(t, e, garden)] ∧

P¬location(t, e, kitchen) ∧ P¬location(t, tom, garden)
```

More interestingly, the extended circumscription allows the scheme to deal with sentences like 15.b or 43 below. In sentences of this type, which we call EPISTEMIC CANCELLATION, the speaker refers to his own knowledge state in order to cancel the implicature.

(43) Tom ate a few carrots, possibly many.

The ESR of 43 is the formula in 44, and circumscribing *abn* (under this extended circumscription theory) result in the sentence meaning that Tom has exactly few or exactly many carrots.

(44)
$$\exists t \ past(t) \land \mathbf{K} \begin{bmatrix} \exists_{few} z \ [carrot(z) \land eat(t, tom, z)] \land \\ \neg abn(c_1) \rightarrow \neg \exists_{some} k \ [carrot(k) \land eat(t, tom, k)]] \end{bmatrix} \land \mathbf{P} \begin{bmatrix} \exists_{many} z \ [carrot(z) \land eat(t, tom, z)] \land \\ \neg abn(c_2) \rightarrow \neg \forall k \ [carrot(k) \land eat(t, tom, k)]] \end{bmatrix}$$

3.6 Some conclusions

There are some important facts to be noticed. The examples above and others in (Wainer, 1991) seems to indicate that there is no conflict between implicatures and that there is no need to prioritize among different families of implicatures. Thus, there seems to be no need to define different abnormal predicates for different implicatures.

This is a interesting result since there is no reason to expect that implicatures would not conflict with each other, specially given the fact that I-implicatures tend to go against the reason behind Q-implicatures (Atlas & Levinson, 1981, Horn, 1984), and that we are calculating the extended meaning of sentences that carry both forms. On the other hand, just the few examples discussed here may not reveal the whole picture—more experiments are necessary. But in order to include other forms of implicatures one would need a more expressive representation language. We believe that a Davidsonian approach, where events and states are represented as first-order objects (for example Hobbs, 1985), would allow us to represent a larger set of implicatures, and better test this hypothesis.

Another important point to make is the relation between implicatures and default world knowledge. The utterance of 45 would only-implicate that the speaker's uncle alone build the tower, but it does not.

(45) My uncle build this tower.

The implicature is canceled because it contradicts with a default knowledge, not a logical axiom: usually a single person cannot build alone a normal tower. Thus generalized



implicature should be checked against both true assertions and default assertions, and if it contradicts one of those, the implicature is canceled.

From the point of view of the circumscription scheme, this behavior can be easily achieved by using prioritized circumscription. For example, if the default that no single person can build a tower is represented as:

(46)
$$\forall x, y, t \ [person(x) \land tower(y) \land build(t, x, y) \land \neg ab_1(x, y) \rightarrow \exists x'x' \neq x \land build(t, x', y)]$$

Then to obtain the extended meaning of 45 one first circumscribes all the default abnormal predicates ab_i and then the abn predicates in the ESR of 45. The distinction between general default knowledge and the implicatures themselves is in accordance to the proposal in (Allan, 1999), which distinguishes lexical and encyclopedic knowledge in modeling implicatures.

4 Semantic interpretation

In order to complete the compositional model we need to show that the ESRs postulated in Sect. 3 can indeed be obtained in a compositional manner from the clauses and other components of the sentences. It is important to show that the implicature clauses (the orthogonal extended meaning formulas) can "show up" in the right places. For example in 39 notice that the implicature clause sub-formula for the scalar implicature of "a few carrots" is within the scope of the "or" whereas the sub-formula for the only implicature is cojoined to the main formula. The correct "placement" of these two sub-formulas is essential for the correct modeling of 38.b. This section will show that through the semantic interpretation process the correct formulas are constructed.

This section will deal with the generation of scalar implicatures due to quantification and only implicatures. We restrict to those two forms of implicatures because they represent two basic alternatives from the point of view of semantic interpretation. Scalar implicatures due to quantification are centered in the semantic interpretation of the NPs. Only-implicatures are centered in the semantic interpretation of the VPs.

An important issue for the semantic interpretation process is that the combination of sub-formulas by functional application alone (Dowty, Wall, & Peters, 1981, Montague, 1973) will not be powerful enough for the task. One needs instead a form of unification mechanism. The reason will be clear when we consider sentence 47.a and its ESR in 47.b taking into account just the scalar implicature.

```
(47) a. Tom ate a few carrots
b. \exists t \ [past(t) \land \exists_{few} z \ carrot(z) \land eat(t,tom,z) \land \neg abn(c_1) \rightarrow \neg \exists_{many} w \ carrot(w) \land eat(t,tom,w)]
```

The contribution that the NP "a few carrots" makes to the formula is:

```
(48) \exists_{few} z \land carrot(z) \land \\ \neg abn(c_1) \rightarrow \neg \exists_{many} w \ carrot(w) \land eat(t, tom, w)]
```

The interesting part of the formula above is the underlined portion. To describe the potential implicature of "a few carrots" one needs to refer to the property that will be assigned to it by the rest of the sentence ("be eaten by Tom"). Of course this information is not available for the interpretation of the NP itself, and it is only determined



when the whole sentence is interpreted. In other words, the total contribution of the NP to the ESR will contain components that are not available at the time the NP is being interpreted. These components represent holes in the ESR of the NP, which will be filled (or unified) later. The ESR of the NP "few carrots" is the formula below, where X_0 is a variable that will be later unified with $\lambda x \ eat(t,tom,x)$, which is the contribution of the rest of the sentence.

```
(49) \exists_{few} z \ carrot(z) \land \mathbf{X}_0(z)

\neg abn(c_1) \rightarrow \neg \exists_{many} w \ carrot(w) \land \mathbf{X}_0(w)
```

The need to use unification is not due to the representation language used in this paper, but it is a general characteristic. That can be seen by realizing that the effect of the implicature can be paraphrased by introducing the particle "only", and the semantics of "only" cannot be captured by functional application alone (Lyons & Hirst, 1990).

We will assume that the ESR of a sentence can be determined from its surface structure, in a rule-by-rule fashion, where each production rule of the grammar is annotated with the actions to construct the ESR. We will use a combination of feature unification (Pollard & Sag, 1987, Shieber, 1986) with Prolog-style unification. Extending the feature unification formalism, we also allow for Prolog-style variables, always denoted by symbols like \mathbf{X}_1 , that can be unified with some feature and later used to construct other structures. An example of a production rule is shown below.

```
(50) VP \rightarrow TV NP

NP : referent = \mathbf{X}_1

VP : arg0 = \mathbf{X}_2

TV : predicate = \mathbf{X}_3

VP : time = t

NP : hole = \lambda w \mathbf{X}_3(t, \mathbf{X}_1, w)

VP : plug = \lambda z \mathbf{X}_3(t, z, \mathbf{X}_2)

VP : body = NP : body & \mathbf{X}_3(t, \mathbf{X}_1, \mathbf{X}_2)
```

The annotations in the production rule mean that the value of feature referent of NP should be unified with the Prolog-style variable \mathbf{X}_1 ; the value of the feature argo should be unified with \mathbf{X}_2 ; and so on. These variables are then used to construct new atoms (from the feature unification point of view) like $\lambda z \ \mathbf{X}_3(t,z,\mathbf{X}_2)$. The last line is an abbreviation for stating that the feature body of the VP should be unified with the conjunction of the value of the feature body of the NP with the application of \mathbf{X}_3 to the arguments t, \mathbf{X}_1 , and \mathbf{X}_2 .

This section will describe very simplified grammar. The grammar is sufficient to interpret the example 47.b and similar examples, but it leaves out all syntactic considerations like agreement and the such. The semantic interpretation uses an uniform representation for NPs, VPs and S.

The NP is represented by four features. The REFERENT represents the handler for the NP, a variable that stands for the NP. The HEADER represents the quantification on the referent. The feature hole is a place holder for the contribution that the rest of the sentence makes to the NP. Thus, in the case of 49, the hole is the variable \mathbf{X}_0 . Finally, the fourth feature, BODY, contain the ESR of the NP.

Thus, for the NP "a few carrots" whose contribution to the ESR of the sentence is described in 49, would be represented as the feature structure below.



$$(51) \begin{cases} \text{header} & \exists_{few} z \\ \text{referent } z \\ \text{body} & carrot(z) \land \mathbf{X}(z) \\ & \neg abn(c_1) \rightarrow \neg \exists_{many} w \ carrot(w) \land \mathbf{X}(w) \\ \text{hole} & \mathbf{X} \end{cases}$$

The VP is represented by five features. The feature TIME holds the time variable(s). The feature ARG0 is a place marker for the subject of the sentence, the feature PLUG is a lambda abstraction that represents the semantic contribution of this VP. The feature HEADER holds the headers of the internal NP's, and the feature BODY holds the ESR of the VP.

The VP "ate a few carrots" would be represented by the feature structure below:

(52)
$$\begin{cases} \text{time } t \\ \text{arg0 } \mathbf{X}_0 \\ \text{header } \exists_{few} z \\ \text{plug } \lambda x \ eat(t, x, z) \\ \text{body } cat(z) \land eat(t, \mathbf{X}_0, z) \\ \neg abn(c_1) \rightarrow \neg \exists_{many} w \ carrot(w) \land eat(t, \mathbf{X}_0, q) \land \\ eat(t, \mathbf{X}_0, z) \end{cases}$$

Finally, the S is represented by three features: TIME is a list of time variables, all assumed to be existentially quantified. The feature HEADER represents a list of quantifications on the variables that represents the NP. It is a Cooper storage (Cooper, 1983, Pereira & Shieber, 1987) which allow for a compact representation of NP scope ambiguity. And the feature BODY contains the ESR of the sentence (without the quantifiers for time and for the NPs).

4.1 The grammar

Let us discuss the grammar and the semantic interpreter annotations that would generate the ESR for the sentences with simple scalar implicatures such as 47.b.

The grammar for S.

```
(53) S \rightarrow NP \ VP

S: header = NP: header \& VP: header

S: body = NP: body \& VP: body \ VP: arg0 = NP: referent

NP: hole = VP: plug
```

The grammar for NPs.

```
(54) NP \rightarrow Q N

Q : noun = N : sem

Q : variable = NP : referent = \mathbf{X}_0^3

NP : header = Q : header NP : hole = Q : hole

NP : body = Q : body
```

(55) NP → PropName NP : referent = PropName : name

³ This triple equality is an abbreviation for the mutual unification of Q: variable, NP: referent and the variable \mathbf{X}_0 . In practice it means that \mathbf{X}_0 holds the value resulting from the unification of the two other feature structures.



The grammar for quantification:

```
(56) Q \rightarrow \text{Num}

Q : \text{hole} = \mathbf{X}_1

Q : \text{variable} = \text{Num} : \text{variable} = \mathbf{X}_2

Q : \text{noun} = \text{Num} : \text{noun} = \mathbf{X}_3

Q : \text{body} = \text{Num} : \text{body } \& \neg abn(\bullet) \rightarrow \forall k[\mathbf{X}_3(k) \land \mathbf{X}_1(k) \rightarrow k < \mathbf{X}_2]^4
```

(57) $Q \rightarrow \text{Num}(\text{N1})$ "or" Num(N2) Q : noun = N1 : noun = N2 : noun Q : variable = N1 : variable = N2 : variable Q : hole = N1 : hole = N2 : hole $\text{N1} : \text{body} = \mathbf{X}_1$ $\text{N2} : \text{body} = \mathbf{X}_2$ $Q : \text{body} = \mathbf{X}_1 \vee \mathbf{X}_2$

The grammar for VP.

```
(58) VP \rightarrow TV NP

VP : time = TV : time = \mathbf{X}_t

TV : sem = \mathbf{X}_s

VP : arg0 = \mathbf{X}_0

NP : referent = \mathbf{X}_r

NP : hole = \lambda z \mathbf{X}_s(t, \mathbf{X}_0, z)

VP : plug = \lambda z \mathbf{X}_s(t, z, \mathbf{X}_r)

VP : header = NP : header

VP : body = TV : body & NP : body & \mathbf{X}_s(t, \mathbf{X}_0, \mathbf{X}_r) \land \neg abn(\bullet) \rightarrow \forall k[\mathbf{X}_s(t, k, \mathbf{X}_r) \rightarrow k \leq \mathbf{X}_0]
```

4.2 The lexicon

- (59) PropName → "Tom" PropName : name = tom
- (60) $N \rightarrow$ "carrots" N : sem = carrot
- (61) Num \rightarrow "a few" Num : noun = \mathbf{X}_n Num : variable = \mathbf{X}_1 Num : body = $\exists_{few} \mathbf{X}_1 \ \mathbf{X}_n(\mathbf{X}_1)$
- (62) $TV \rightarrow$ "ate" TV : sem = eat $TV : time = \mathbf{X}_t$ $TV : body = past(\mathbf{X}_t)$

4.3 Discussion

This section is somewhat dry but we feel it was necessary in order to illustrate an important aspect of the compositional approach—that is it compositional, and that

⁴ The symbol "•" represents the extra-logical process of creating a new, unused logical constant.



such process is necessary to make sure that the implicature clauses of the ESR show up in the right places. More details of the semantic interpretation process can be found in (Wainer, 1991). In more complex examples, the semantic interpretation will have to infer characteristics of the various clauses being interpreted, in order to decide if some implicature clause will or not be inserted. For example, the asymmetric readings of "and" (example 10) seems to only appear when each VP being conjoined by the "and" is either an achievement or an accomplishment. Thus, the semantic interpretation should also calculate the aspectual classes of the VPs in order to determine if the temporal implicature of "and" should be present or not.⁵

5 Related work

5.1 The substitutional approach to generalized implicatures

The substitutional approach was first proposed by Horn (1972) and it has been used by most researchers in the field, mainly to model scalar implicatures. (Gazdar, 1979) provided the first formalization of it.

The intuition behind the substitutional approach is that one should substitute in the sentence an expression that belong to some scale by its extended meaning. Intuitively, one substitute the word "few" in 63.a by its extended meaning "a few and not many" in order to obtain the extended meaning of the original sentence, in 63.b.

- (63) a. Tom has a few cats.
 - b. Tom has a few and not many cats.

The formalization, of course, is not about substituting words in sentences but substituting sub-formulas in the formula that represents the semantic content of the sentence. Nevertheless such formalization is very difficult; in particular Gazdar's formalization does not correctly deal with complex sentences such as 64.a, as pointed out by (Chierchia, 2002, Sauerland, 2001). The problem is that in Gazdar's formalization scalar implicatures are not calculated when the implicature generating term is within the scope of certain modifiers, including logical operators. For example, a sentence such as 64.a would not carry the implicature in 64.b since "few" is within the scope of the "or" logical connector.

- (64) a. Tom has a few cats or a few dogs
 - b. Tom has exactly a few cats or a exactly a few dogs

Gazdar's formalization aside, the general problem of the substitutional approach is that the orthogonal extended meaning that carries the implicatures are added to the whole of the semantic content of the sentence. This is a problem in complex sentences, where the implicature is not at the "sentence level", but is at a "sub-sentence level", for example in 64.b. Thus the need for complex rules or mechanisms to get the orthogonal extended meaning to the "right place". For example Sauerland (2004) proposes a cross product of two or more scales to model the scalar implicatures of complex sentences.

⁵ Grasso et al. (1990) discuss an algorithm to determine the aspectual class of VP.

5.2 Relevance approaches to conversational implicatures

An interesting approach to modeling conversational implicatures is based on the notion of relevance, and Grice's relevance maxim. In general, relevance approaches to conversational implicatures assume that there is no real distinction between particularized and generalized implicatures and that all implicatures are context dependent. For example Rooth (1992) develops a model in which scalar implicatures are explained using (Hirschberg, 1985) extended scales and focus. Other models based on relevance are discussed in (Carston, 1995, 1998, van Rooy, 2002) among others.

For example Rooy (2003) proposes an exhaustiveness operator of the form $\lambda R\lambda P[R(P) \land \neg \exists P'[R(P') \land P \neq P' \land \forall x[P'(x) \rightarrow P(x)]]]$ which state that P is the largest predicate for which R also applies. This captures the fact that generalized implicatures seems to be captured by expressions that contain "only" or "just"—the "only" is expressed by the exhaustiveness operator. But to what predicates the operator is applied (which predicates are P and R) is defined from a relevance stand point by considering explicit or implicit questions that the utterance answers.

Relevance based approaches to conversational implicatures have the problem of defining what is "relevance." Some researchers in the area (for example Hirschberg, 1985, Rooy, 2003) discuss implicatures in the context of question answering because in this case the question itself defines what is relevance and what is the relevant context under which to compute the implicatures.

Recently, the computational linguistics community has turn their attention to conversational implicatures, specially regarding the generation of answers (Dale & Reiter, 1995, Green & Carberry, 1994, Oberlander & Lascarides, 1992). One does not want a computer generated answer to carry implicatures that were not intended—a description that is longer than needed would carry (by the brevity maxim) unwanted implicatures. For example (from Reiter (1990)) 65.b would carry strange implicatures in the context where only one table is visible.

- (65) a. Sit by the table
 - b. Sit by the brown, wooden table

Finally, we feel that this research can be added to a growing body of research in non-monotonic logics and language. Language is full of examples of defeasible phenomena, where implicatures are just a somewhat obscure one. Among other research in nonmonotonic logics and language we can point out the work of Marcu and Hirst (1996) who proposes a stratified logic approach to model defeasible pragmatic inferences. Mercer (1988) proposes the use of default logic to model presuppositions. Lascarides and Asher (1991) uses non-monotonic logic to model default temporal relations among discourse components.

6 Conclusions

This paper presented the compositional approach to model generalized implicatures. The intuition behind the compositional approach is reasonable: the extended meaning of a sentence is constructed by combining the contributions from its different constituents in a compositional way, a straight forward extension of the idea of compositional semantics. Since generalized implicatures are cancelable, whatever is constructed by the compositional process must contain non-monotonic capabilities. In the model



we proposed, the compositional process constructs a formula which we called the extended semantic representation, and the extended meaning is derived from that structure through a non-monotonic inference.

We implemented this approach by using circumscription as the non-monotonic inference system, in what we called the circumscription scheme. Furthermore, by defining a representation language for the ESRs, we were able to show the approach does work on different examples, including examples in which different implicatures interact with each other.

A consequence of this approach is that generalized implicatures are *not* derived from conversational principle, and thus this paper assumes an anti-Gricean position. Generalized implicatures are defeasible, preferred meaning of lexical items, of clauses, and so on, which are combined in a compositional way. Such defeasible meaning for whole sentences is then combined with world knowledge (see example 45), with knowledge or lack of knowledge attributed to the speaker (see example 43), in order to finally compute the extended sentence meaning.

Independent of the reader's willingness to accept that all generalized implicatures are just a form of defeasible, preferential meaning, the mechanism discussed in this paper can be useful in explaining I-implicatures. We believe that it is reasonable to assume that I-implicatures are indeed defeasible, preferential meaning associated with lexical items and clauses. Under this view, the I-implicatures of 10 could be modeled as:

```
(66) \exists e_1 \text{ turn-key-event}(e_1) \land time(e_1) = t_1 \land \dots

\exists e_2 \text{ motor-start-event}(e_2) \land time(e_2) = t_2 \land \dots

\neg abn(c_1) \rightarrow before(t_1, t_2) \land

\neg abn(c_2) \rightarrow cause(e_1, e_2) \land
```

Some of the assumptions and conclusions of this paper strongly agree with some of the positions taken by Levinson (2000). In particular

Pragmatics can be avoided if we can find a way of accounting for pragmatic intrusion into truth conditions while maintaining the modularity of a distinct pragmatics (built on nonmonotonic principles) and semantics (built on monotonic principles) (Levinson, 2000, p. 243).

The model discussed in this paper can be seen as a way to realizing that.

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