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The Signature of Risk: Agent-based Models, Boolean Networks and Economic Vulnerability

Ron Wallace, Department of Anthropology, University of Central Florida, USA
ronald.wallace@ucf.edu

Abstract

Neoclassical economic theory, which still dominates the science, has proven inadequate to predict financial crises. In an increasingly globalised world, the consequences of that inadequacy are likely to become more severe. This article attributes much of the difficulty to an emphasis on equilibrium as an idealised property of economic systems. Alternatively, this article proposes that actual economies are typically out of balance, and that any equilibrium which may exist is transitory. That single changed assumption is central to complexity economics, a view which is presented in detail. It is suggested that economic crises will be most effectively avoided when economists utilise methods, grounded in complexity theory, which can identify threat in an early stage. As a programmatic example, the use of Agent-Based Models (ABMs) combined with Boolean networks (BNs), is defended as a promising method for recognising vulnerability.

Keywords: economic models, complexity economics, computational modelling, Agent-based modelling, Boolean networks, LZ complexity

JEL Classification Codes: B410, C530, C630

Abbreviations: ABM: Agent-based models; BD: Banker/Dealer; BN: Boolean Network; CP: Cash Provider; GN: Graphic Network; HF: Hedge Fund; EWS: Early Warning System; IMF: International Monetary Fund; KL: Kullback-Leibler; LZ: Lempel-Ziv; RBN: Random Boolean Network; RET: Rational Expectations Theory; VaR: Value at Risk

1. Introduction

That the 400-year history of Western capitalism has been punctuated by crises has been extensively noted (Galbraith, 1990; Reinhart, 2011). Analyses addressing the trajectories of financial collapses have considered a range extending from Tulipomania (1637), to the Panic of 1857 – arguably the original global economic crash – to the onset of the Great Depression (1929), to the dot-com debacle (1997) and to the 2008 burst of the subprime-mortgage bubble. Many recent studies have faulted neoclassical economic theory, and its governing assumption of equilibrium, for an alleged lack of usefulness in predicting bubbles and crashes (Stiglitz, 2010; Bresser-Pereira, 2010; Colander et al., 2010). Yet circumstantial factors may have also played a role. Tularam and Subramaniam (2013) note that crisis models were adapted to specific situations to explain the financial challenge at hand rather than adopting a visionary or systematic approach. Similarly, *The Economist* (April 12, 2014) observes that economic models motivated by emergencies were 'cobbled together at the bottom of financial cliffs. Often what starts out as a post-crisis sticking plaster becomes a permanent feature of the system. If history is any guide, decisions taken now will reverberate for decades.' Exacerbating the effect of problematic theory is the instability of an increasingly securitised global economy. Thus John Edmunds (1997) has emphasised the amplifying effect of telecommunications on volatility in global securities: for a 'botched devaluation or an attempted coup d'état, the punishment is quick and dire'; 'hundreds of billions of dollars can be lost in a matter of hours and days' (p. 15). Together these studies suggest that global economic crises may become more sweeping, frequent, and difficult to foresee, underscoring the need for improved vulnerability models.

This article proposes a computational method synthesising Agent-Based Models (ABMs) and Boolean networks (BNs) to reliably determine when an economy is rapidly approaching a 'meltdown' (Buchanan, 2009). The proposed method is viewed through the lens of complexity economics. This viewpoint, relatively new, was described by W. Brian Arthur (2013) as one which construes the economy as continually in motion, perpetually reconstructing itself. Key features of this approach are examined in detail. Emphasis is placed on multiple equilibria and expectation indeterminacy, two properties not easily accommodated in neoclassical economics. Following Arthur (2013) and, earlier, Kuhn (1962) it is suggested that these anomalies may signal a paradigm shift in economic thought. The discussion forms a conceptual bridge to an examination of ABMs and BNs: bottom-up, nonlinear computational strategies closely linked to complexity economics. The main structure of ABMs and BNs is developed in outline, but our objective is not to present a primer of either approach, a task which has been accomplished competently and extensively elsewhere (see Bonabeau, 2002; Helikar et al., 2011, among many others). Rather, it is our purpose to suggest how the two approaches may be linked, and used together as a powerful computational tool to identify vulnerability within an economy. The novel method is motivated by Bookstaber (2012) and Bookstaber et al. (2014) who noted: 'For the analysis of market dislocations and crises, this dynamic generation of networks [from an ABM model] is critical' (p. xxx). A key feature of this hybrid method is that successive BN 'snapshots' permit the use of the Lempel-Ziv (LZ) complexity algorithm – which we will present in some detail – to determine whether the economy is stable, weakly chaotic, or hovering somewhere in between: i.e., in a poised, critical regime (Lempel and Ziv, 1976). Through the application of LZ to an ABM-based BN, it should be possible to determine if an economic system is on the edge of a meltdown. Put somewhat differently, we propose a computational method for identifying an economic crisis when it is still at its inception, or metaphorically, mapping the 'fault lines' (Rajan, 2010) just before they become a collapse. We conclude that the framework of complexity economics, and its closely related strategies of ABM and BN modelling, may prove more valuable for reliably identifying economic vulnerability, and pointing the way towards policy, than the equilibrium-based neoclassical approaches that remain in widespread use.

2. Complexity Economics: Historical Anticipations

The claim that modern economies are highly dynamic systems, rarely in equilibrium, and thus ineffectively captured by the assumptions (and mathematics) of physics-inspired static models, did not begin with complexity theory (Galbraith, 1987). The idea was anticipated in the 19th-century schemes of Friedrich List and Karl Marx who, like their anthropological counterparts, Lewis Henry Morgan and John McLennan, envisioned societies as moving through a series of historical stages, each derived from, and more complex than, its predecessor (Carneiro, 2003). Yet not until Joseph Schumpeter's (1883-1950) descriptions in *Capitalism, Socialism, and Democracy* (1942) of entrepreneurs incessantly generating novelty while destroying old forms of wealth do we find the unmistakable expression of the complexity view:

> 'Capitalism, then, is by nature a form or method of economic change and not only never is but never can be stationary… The fundamental impulse that sets and keeps the capitalist engine in motion comes from the new consumers' goods, the new methods of production or transportation, the new markets, the new forms of industrial organization that capitalist enterprise creates' (Schumpeter, 1947 [1942], p. 82).

A generation later, Joan Robinson (1903-1983) endorsed a similarly dynamic view – an economic 'arrow of time' – which echoed, almost verbatim, the words of Zeno of Elea (5th century B.C.E.):

> 'Any movement must take place through time, and the position at any moment of time depends on what it has been in the past… The point is that the very process of moving has an effect on the destination of the movement, so there is no such thing as a position of long-run equilibrium which exists independently of the course which the economy is following at a particular date' (Robinson, 1964, pp. 222-238; also see the valuable historical discussion in Turk, 2010).

A similar understanding – of economics and much else – was emerging in Robinson's day (1960s), in the relatively new, computer-inspired information sciences (Kurzweil, 1999). Highly visible among those theorists was Magoroh Maruyama (1963), who vigorously questioned the key assumption of self-regulatory informational loops (i.e. homeostasis) that had defined the cybernetics of mathematician Norbert Wiener (1894-1964) and psychiatrist W. Ross Ashby (1903-1972). Announcing a 'second cybernetics', he argued that

> '[b]y focusing on the deviation-counteracting aspects of the mutual causal relationships however, the cyberneticians paid less attention to the systems in which the mutual causal effects are deviation-amplifying. Such systems are ubiquitous: accumulation of capital in industry, evolution of living organisms, the rise of cultures of various types, interpersonal processes which produce mental illness, international conflicts, and the processes that are loosely termed as "vicious circles" and "compound interest"; in short, all processes of mutual causal relationships that amplify an insignificant or accidental initial kick, build up deviation and diverge from the initial condition' (Maruyama, 1963, p. 164).

A full examination of this major theoretical shift would require a separate article (see, for example, Boulding, 1985). Here we will emphasise the emergence, within that context, of the two computational strategies with which we will be concerned: agent-based models (ABMs) and Boolean networks (BNs).

ABM and BN models began as the almost inevitable outgrowth of the 'second cybernetics' approach. If, as was increasingly claimed, most – if not all – systems were only momentarily in balance, and vulnerable at all times to 'accidental kicks', it followed in the case of economics, that the neoclassical framework – inspired by classical physics, and emphasising stability – would require extensive revision. This would not – and historically did not – entail the simple substitution of one theory for another. Rather, it was held that an inductive approach was needed, proceeding in 'bottom-up' fashion from raw economic data sets to novel overarching views. ABM and BN approaches, amplified by the rapidly changing 'digital revolution', were highly consistent with this critical view. What distinguished these two methodologies was that they both configured complex artificial worlds comprised of interactive components defined by explicit rules; once the system was set in motion, the micro-level components would generate macro-level patterns. Users of both methods thus sought to avoid the pitfall – no stranger to the history of science – of imposing an outmoded theory upon recalcitrant data. In early ABM modelling, the economist Thomas Schelling (1921-2016) demonstrated that individuals (agents) in an artificial community following simple threshold rules regarding satisfaction with neighbours would generate, on the macro level, pronounced segregation along some specified social dimension (Schelling, 1971). Similarly, theoretical biologist Stuart Kauffman – building on earlier threshold models of 'satisficing' behaviour pioneered by Nobel Laureate and polymath Herbert Simon (1916-2001) – showed in the first BNs how a system of binary units interacting through algebraic rules would display self-organisation (Simon, 1956; Kauffman, 1969). As is evident, the two approaches both generate global patterns from local interactions, and share an inductive philosophy. Nonetheless, they have followed largely separate paths since their beginnings in the 1960s. Perhaps their surface dissimilarity – ABMs' descriptive richness versus BNs' Spartan simplicity – has been sufficient to discourage attempts at hybridisation. But must we so choose? Below we will first examine the fundamental concepts of complexity economics, and then defend a synthesis of ABM and BN approaches (Borshchev and Fillipov, 2004; Bookstaber et al., 2014).

3. Complexity Economics: Addressing the Limitations of the Neoclassical View

Closely identified with the Santa Fe Institute and the writings of W. Brian Arthur, complexity economics extends the tradition of Schumpeter and Robinson, and builds on the contributions of information theory, through its construal of an economy that is typically in disequilibrium (Arthur, 1999; 2005; 2013). In contrast to the neoclassical viewpoint which nominates agent behaviours, strategies and historical assumptions believed to be logically consistent with some specified aggregate pattern – thus proceeding in a top-down fashion, and yielding a static model – complexity economics views agents as dynamic and reactive: they respond to economic patterns by transforming their beliefs and behaviours; these changes, in turn, give rise to a transformed and transient, pattern. This reflexivity is the essence of a non-equilibrium economy. Importantly, agent reactions are frequently creative; they are novelty-generating. Like talented chess-players in an artificial tournament tracked through time (recall Robinson's 'arrow'), their strategies call upon memory and, at least temporarily, display increasing complexity (Lindgren, 1991); players utilise mixtures of several proven gambits or, occasionally, revert to a devastatingly simple – because unexpected – manoeuvre. Similar innovative responses can often accelerate market dynamics (see Fox, 2009 or any history of speculation), yet are poorly accommodated in the static, neoclassical view. As a final but critical point, the complexity approach is not a retreat from theory. A wide range of models and related computational experiments, frequently involving statistical study of system properties and their causes, can be accommodated within the complexity framework (examples abound; see the special issue of *Journal of Economic Dynamics & Control*, 33, 2009 devoted to the topic). Indeed, in larger perspective, complexity thinking appears consistent with the 'semantic view' of science, traceable to Patrick Suppes (1922-2014) and

Bas van Fraassen, in which models are investigatory tools or instruments which mediate between a theory and the world at which the model is 'targeted' (Morrison and Morgan, 1999).

Acknowledging that the assumption of an economy that is normally in flux makes a substantial claim on his audience, Arthur (2005, p. 4) adds that

> 'a natural question to ask is what it delivers. What novel phenomena do we see when we do economics out of equilibrium? Are there questions that equilibrium economics can not [sic] answer, but that this more general form of economics can? In Kuhnian language, are there anomalies that this new paradigm resolves?'

The candidate challenges are *multiple equilibria* and *expectation indeterminacy*. The first of these – which posits that a dynamic economy may settle into one of multiple possible stable states, and that the state selected is difficult to predict in advance – is by no means new; indeed it was recognised as problematic by the founders of neoclassical theory. Léon Walras (1834-1910) in his *Elements of Pure Economics* (1874) had envisioned highly idealised market behaviour, specified by simultaneous equations, for which there were as many unknowns as equations; he believed that under these conditions, the system would (probably) yield a unique equilibrium (Ingrao, 1989). This formal argument has been analogised to a grand, auction-like setting, involving a single commodity, in which sellers call out prices, and consumers respond by purchase or by refusal to buy at that price: i.e., the 'Walrasian auction'. The aggregate effect is a trial-and-error 'groping' (*tâtonnement*) toward a unique equilibrium. (For valuable alternate views of the auction metaphor – which Walras himself never used – see Kirman, 2010 and De Vroey, 2003). But Walras himself acknowledged that if one chose to evaluate a more elaborate, two-commodity 'auction', there may, in fact, be multiple possible equilibria. Later generations of economists have sought to specify the mathematical conditions under which a unique equilibrium would be guaranteed. A full consideration of these controversies extends beyond the scope of this article. Perhaps the best summation was provided by Timothy Kehoe (1998), who raised the eminently practical issue of computational difficulty:

> 'It may be the case that most applied models have unique equilibria. Unfortunately, however, these models seldom satisfy analytical conditions that are known to guarantee uniqueness, and are often too large and complex to allow exhaustive searches to numerically verify uniqueness' (Kehoe, 1998, pp. 38-39).

In contrast to this tradition, complexity economics essentially cuts the Gordian Knot: the quest for an inerrant method for guaranteeing a unique equilibrium is abandoned as a sterile exercise. In its place, the dynamic properties – especially positive feedback – of a simulated economic system are observed from their origin as random perturbations: Maruyama's 'insignificant or accidental initial kick[s]' (Maruyama, 1963, p. 167). Because identical initial conditions, in a series of simulations, can often generate different equilibria, probabilities reflecting these alternative possible outcomes can be assigned to selected features:

> 'Sometimes one equilibrium will emerge, sometimes (under identical conditions) another. It is impossible to know in advance which of the candidate outcomes will emerge in any given unfolding of the process, but it is possible to study the probability that a particular solution emerges under a

> certain set of initial conditions. In this way the selection problem can be handled by modeling the situation in formation, by translating it into a process with random events' (Arthur, 2005, pp. 5-6).

The alternative approach has been applied to actual economic systems. Woods and Vayssières (2002) describe the effects of a price-unit change on the Internet-provider market:

> 'They operated as if the price unit in the market was a connection hour. The pricing system now is much different. People are not generally charged by the minute, but by the bandwidth, the maximum instantaneous capacity of the connection. The differing pricing systems produced different incentives for modem and other technology' (Woods and Vayssières Kandel, 2002, p. 6.319).

Citing this and other perturbations (e.g., the introduction of product bundling), they conclude that a small policy change can drive a market to 'a different equilibrium' (Woods and Vayssières Kandel, 2002, p. 6.319).

Unlike multiple equilibria – a concept proximately tethered to the quantitative revolution in modern economics – the second of the anomalies, expectation indeterminacy, is deeply anchored in Western philosophy. Plato (c. 429-347 B.C.E.) argued in *Symposium* (c. 385-370 B.C.E.) that any discussion of expectations was made problematic by changes in the self, especially in values and tastes, that occur over the lifespan. Much later, David Hume (1711-1776) in his *Treatise on Human Nature* (1738-1740), advanced an almost identical argument. In modern economics, the construct of the multifaceted, evolving individual did not disappear so much as it was elegantly dismissed. In Léon Walras' static model, people inhabit a timeless world, are endowed with perfect information regarding market performance, and interact 'in an unending round of robotic behavior' (Rothbard, 2005, p. 97). Because time is not part of the model, expectation is rendered meaningless. Although recent scholarship suggests that Walras may have intended to later revise the model to include realistic dynamics – an issue explored by the author in a forthcoming article – his project was never realised (Ingrao, 1989). Instead, the 'unending round' was mathematically refined by Kenneth Arrow and Gérard Debreu (1921-2004), by which point 'expectations' had all but vanished from economic thought (Arrow and Debreu, 1954). In 1961, however, the omission was addressed with far-reaching consequences by John F. Muth's (1930-2005) presentation of rational expectations theory (RET): investors make, and update decisions based on all relevant information regarding market variables of interest (e.g., performance of a particular stock). Given that the investors are on a time-path – here a departure from Walras – and are seeking to maximise profits, they modify their forecasts in the light of new information, and in the direction of greater reward (utility). There is thus a feedback loop between the signals of market performance and the adjustments in investor decisions; as a result, expectations – in the aggregate, and in the long run – *will not systematically differ from actual market outcomes* (Muth, 1961). It is difficult to underestimate the impact of RET, both on academic economics and on actual economic behaviour (for a valuable critique see Kirman, 2014). In academia, the model's proponents – most conspicuously, Thomas Sargent and Robert E. Lucas – ignored or understated its most destructive weakness: a body of investors is not an idealised set of isolates, but an interacting, randomised, and self-referential system (Arthur, 2005); investors' expectations are rarely independent but are more often 'coordinated' (Kirman, 2014): they constantly form and evolve with regard to the expectations of others. This self-referential property is not only the crux of the indeterminacy problem but, given that humans have imperfect knowledge and are notoriously error-prone, has played a critical role in outbreaks of 'financial euphoria' (Galbraith, 1990; see also Fox, 2009). Its absence from RET, and hence from the outlook of policymakers, may have therefore significantly contributed to the recent financial crisis. Ben Bernanke, Chairman of the Federal Reserve (2006-2014), candidly acknowledged as much to the International Herald Tribune (2010):

> 'I just think that it is not realistic to think that human beings can fully anticipate all possible interactions and complex developments. The best approach for dealing with this uncertainty is to make sure that the system is fundamentally resilient and that we have as many fail-safes and back-up arrangements as possible' (Bernanke cited in Kirman, 2014, p. 36).

Whether Bernanke's admonition will be adequately answered by complexity economics only later events can show. But the treatment of expectations in the complexity approach is promising, and differs markedly from that of Muth and Walras. Like multiple equilibria (to which it is systemically related) expectation indeterminacy is not a problem to be avoided by positing unrealistic constraints, but accepted as a property of a non-equilibrium system. Arthur (2005) contrasts the two approaches:

> 'To suppose [as in RET] that a solution to a given problem would we reached in a one-off non-repeating problem, we would need to assume that agents can somehow deduce in advance what model will work, that everyone 'knows' this model will be used, and everyone knows that everyone knows this model will be used, ad infinitum' (Arthur, 2005, p. 7).

This critique is markedly similar to Muth's candid acknowledgment:

> 'The rational-expectations hypothesis assumes all individuals in the economy have unlimited computational ability and know how to make use of the information they possess' (Muth, 1994, pp. 101-102).

In a more parsimonious strategy, complexity economics views expectations 'in formation': 'To do this,' Arthur proposes,

> 'we can assume that agents start each with a variety of expectational models, or forecasting hypotheses, none of these necessarily "correct". We can assume these expectations are subjectively arrived at and therefore differ. We can also assume agents act as statisticians: they test their forecasting models, retain the ones that work, and discard the others. This is inductive behavior. It assumes no *a priori* "solution" but sets out merely to learn what works. Such an approach applies out of equilibrium… as well as in equilibrium, and it applies generally to multi-agent problems where expectations are involved' (Arthur, 2005, pp. 7-8).

A growing body of experimental studies suggest that, while RET is often consistent with the behaviour of more experienced traders, over successive trials, and at a slower rate of investment, bubbles and crashes also occur. Computerised studies conducted by Smith et al. (1988) with experienced and inexperienced subjects found that fourteen of 22 experiments exhibited price bubbles followed by crashes relative to intrinsic dividend value. When traders were experienced, however, this factor reduced, but did not eliminate, the probability of a bubble. (But see also the qualifying remarks regarding this experimental tradition provided by Joyce and Johnson, 2012). The inadequacy of RET was additionally indicated by an artificial-market study conducted by Arthur et al. (1996). In their analysis, virtual investors formed and discarded hypotheses in a recursive 'ecology of beliefs'; i.e., their decisions were based on their expectations of others' expectations. Their major finding may be compared to phase transitions in chemistry: when investor decisions proceeded at a relatively low rate, RET was supported, but when the rate was increased, the market self-organised into complexity: technical models were transiently vindicated, and 'bubbles and crashes' occurred – a regime remarkably similar to actual market behaviour.

4. Agent-based Models and Boolean Networks: Toward a Signature of Risk

Can computational models predict economic crises? Tularam and Subramaniam (2013), as we have seen, vigorously criticised proposed methods for forecasting crashes, concluding that,

> '[T]here is little evidence of a longer term vision and therefore financial crises modeling has not been dealt with by researchers with any level of visionary approach' Tularam and Subramaniam, 2013, p. 118).

Alternatively, Andrew Berg and colleagues at the International Monetary Fund (IMF) – writing in the wake of the Asian currency collapses (1997), but before the Great Recession (2008) – were cautiously sceptical:

> 'Overall, … EWS [Early Warning System] models are not accurate enough to be used as the sole method to anticipate crises. However, they can contribute to the analysis of vulnerability in conjunction with more traditional surveillance methods and other indicators' (Berg et al., 2005 p. 491).

So can it be done? Or is the endeavor Quixotic? Richard Bookstaber (2012) has offered a more restricted objective. He notes that any attempt to predict a systemic shock is 'a task that in itself is fraught with uncertainty' (clearly increasing in relation to the futurity of the event). Citing the 2008 crisis, he emphasises that 'central banks took unprecedented actions that would have been difficult to anticipate, as would the effects of those actions' (Bookstaber 2012, p. 4). (This view can be easily amplified: In the years preceding the 2008 crash, the 1999 repeal of the Glass-Steagall Act, the rise of subprime 'robo-lending', and the invention of credit default swaps were all without precedent, and likely contributed to the event. There is a burgeoning literature on these machinations. For an irreverent insider account, emphasising credit default swaps, see Lewis (2010). For an apologia regarding the repeal of the Glass-Steagall Act, see Binder (2013, pp. 266-267). Accordingly, Bookstaber and his group endorse an ABM model, linked to a network approach, which could identify, and possibly avert, a crisis at its inception. Rather than predicting it well in advance – the weakness of EWS approaches – the model can 'trace the path a shock follows as it propagates through the financial system' (Bookstaber et al., 2014, p. 4). Moreover, shocks can be artificially induced as a means of policy testing.

The defining attribute of an ABM is the individual actor or *agent* which is a computer program or, more commonly, a distinct part of a program, which can represent a person, a household, a firm, or even a nation (Gilbert, 2007). Agents exchange information both directly and indirectly, as well as within and between levels; they modify their behaviour with regard to what they have learned. Expressed somewhat differently, each agent is assigned a set of updatable rules which regulate its changing responses to the inputs from other agents. In the case of individuals, the rules might be simple heuristics; for a bank or investment firm, they may be formal policies adhered to in a stable economy, but modified or even abandoned in the event of a financial crisis. As this artificial economy evolves, it will often display emergent properties, i.e. features that arise from a system of interacting agents, but are not present in any agent within the system. This remarkable natural process is also encountered in actual economies (see the special issue of the *Journal of Economic Behavior & Organization*, 82, May 2012 devoted to 'Emergence in Economics'). Thus Paul Templet has shown that the diversity of industrial economies emerges over time from the number of energy flows which circulate through energy-consuming sectors and from the allocation of energy use across those sectors (Templet, 2004).

The Bookstaber ABM is focused on hedge-fund investing, but its underlying concepts could be readily applied to other scales, including that of the global economy. Indeed, similar approaches have been used to analyse 'housing and mortgage prepayments, payment systems, market microstructure, and macroeconomics' (Bookstaber, 2012, p. 11). The model is comprised of three types of agents:

1) *Hedge funds* (HF), financial vehicles which pool the capital of individuals and institutions to invest in a wide range of assets using risk-management techniques. This focus is defended as immediately relevant to crisis modelling because significant HF leverage – its borrowed capital and cash which is used for investment – combined with increasing investor redemptions, can lead to an HF funding risk during a sudden economic downturn.

2) *Cash provider* (CP), a funding source which pools the investors' assets. The CP agent may be an asset manager, or a range of financial institutions; e.g., a pension fund, an insurance company or, most often, a money-market fund. The latter typically provides only short-term funding which, importantly, may be redeemed in the event of a financial crisis, thereby increasing the vulnerability of the HF.

3) *Banker/dealer* (BD), a hub in the HF network, comprised of multiple sub-agents (e.g., financial desk, trading desk, derivatives desk), which coordinates flows of funding and collateral between the HF and the CP. Agents and sub-agents in the model are described by decision rules that govern their responses to inputs; e.g., CP constraints on the amount that will be lent to the finance desk in the BD system. The ABM, as described, is subjected to a price shock, and – in what is arguably the most innovative feature of the model – the spreading destabilisation is depicted in a series of snapshots via a complementary graphic network (GN), which receives periodic output from the ABM. In this approach, agents are represented as nodes, and their interactions are edges (links). A coloured area within a node depicts the amount of capital; an empty node represents a default; the thickness of an edge denotes the intensity of the impact of one node (agent) upon another. The Bookstaber GN, when put into motion, pictorially expressed spreading financial disorder. Dramatically falling prices forced a default of HF and BD, resulting in the near-collapse of funding within the system. Importantly, conventional stress analyses (e.g., Value-at-Risk, or VaR methods) would have identified only the early stages of the crisis. (For an overview of VaR, see Linsmeier and Pearson, 2000).

The architecture of Bookstaber's prototype could be made even more informative if linked to a BN, and in turn expressed as a complexity metric. Each picture in the GN would have its mathematical equivalent. It would be possible, through such an approach, to identify a 'tipping point' in the GN progression where a change in one or more nodes (e.g., a massive

HF liquidation) may be sufficient to generate chaos. A BN, introduced briefly above, is comprised of a set of components or nodes $\{\sigma_1,\ \sigma_2, \ldots\ \sigma_n\}$ which can assume only two values, ON (1) or OFF (0) corresponding, respectively, to the active or inactive state of the variable, or to its above- or below-threshold value. Nodes are linked by a wiring diagram in a first approximation which may be somewhat speculative, especially if the variables are not yet well-understood (Helikar et al., 2011). The binary output of each node is specified by logical operations utilising AND, OR, and NOT; the input-output relations, or Boolean functions $\{B_1, B_2, \ldots B_n\}$, are represented in a 'truth table'. Under the best of conditions – i.e., when educated guesswork is minimal – the BN approach may be regarded as an approximation technique in which successive applications of updated model variants frequently yield a result with high predictive power. On a fundamental conceptual level, BNs would appear to have a strong affinity with ABMs: In both approaches computational objects interact via simplified rules relating to incentives or information. Converting the output of the ABM-based GN into a BN model would therefore be plausible but, like any BN application, would require trial-and-error manipulation (of the wiring diagram, the truth table, and the choice of node values (0,1) during multiple simulations) until BN activity would correspond to the GN picture.

How does the BN yield a signature of risk? Ilya Scmulevich's team developed a method for quantifying the dynamic state of a BN through the Lempel-Ziv (LZ) metric (Schmulevich, 2005; Lempel and Ziv, 1976). The approach was developed by Abraham Lempel and Jacob Ziv in 1976, and first used in the informational sciences as a data-compression algorithm. Variations in LZ were subsequently developed, including one by Terry Welch (1984) for use in high-performance computing. Compatible with BN modelling, LZ has been applied in molecular biology (Schmulevich et al., 2005), medicine (Zhou et al., 2011), evolutionary studies (Yu et al., 2014), and a variety of other sciences. (There are several complexity metrics in addition to LZ which may prove potentially useful for evaluating economic systems. The state of the art is fluid, and new methods are emerging all the time. For an overview, see Lloyd, 2001).

The mathematical basis of LZ is given in Lempel and Ziv (1976), which those wishing a formal presentation are urged to consult. This article offers instead an illustration of the algorithm by means of an example (Schmulevich et al., 2005; Wallace, 2015). Consider a finite and binary alphabet in which complexity is defined as the number of unique substrings, with the possible exception of the last one (which may not be unique), as the sequence evolves from left to right. Importantly, in the version of LZ presented here, the search for previous occurrences of a substring may bridge previously seen substring boundaries. Thus, in the sequence 01100101101100100110, the first digit, 0, is new; it has not been encountered before. The (evolving) LZ value is therefore 1. Proceeding to the next digit, 1, it is clear that this digit is also new. Thus the LZ value increases to 2. Continuing from left to right, the next (third) digit is 1, which has been previously encountered. Therefore, the length of the substring is increased by one digit (i.e., the fourth digit, 0), yielding a new substring 10, and an evolving LZ value of 3. Next, beginning with the fifth digit, 0, and continuing to move from left to right, the next new substring is 010, yielding an evolving LZ value of 4. The process is continued until the sequence is parsed as follows: 0-1-10-010-1101-100100-110, where dashes indicate the boundaries between substrings. The LZ complexity of this sequence is thus 7. It should be noted that in this LZ variant, the final substring 110, although it was counted, was not new (the reader should bridge the second dash; recall that, in this variant, bridging is permitted).

In applying this method to a BN, LZ values are calculated for binary time-series readouts at clocked moments as the BN simulation is running. The LZ values of the successive BN snapshots are then displayed as a probability distribution. The latter is then compared with three probability distributions of LZ values for Random Boolean Networks (RBNs) representing ordered, critical, and chaotic states. In an RBN, inputs to nodes are randomly chosen, and 'each node is assigned an update function that prescribes the state of the node in the next time step, given the state of its input nodes. The update function is chosen from the set of all possible update functions according to some probability distribution' (Drossel, 2008). Although simple visual examination of superposed actual and random LZ probability distributions could itself be informative, several mathematical approaches exist for evaluating their similarity. We may represent the LZ probability distribution corresponding to actual data as $P = [p_1, \dots, p_m]$ and each of the RBN mock-data probability distributions as $Q = [q, \dots, q_m]$. The Euclidean distance, a similarity measure, is then given by

$$E(P, Q) = \left(\sum_{i=1}^{m} (p_i - q_i)^2\right)^{1/2},$$

and the Kullback-Leibler (KL) divergence or relative entropy, an alternate measure, is given by

$$D\ (P, Q) = \sum_{i=1}^{m} p_i \log(p_i / q_i)$$

In a comparison, the more different the distributions of P and Q, the larger will be the values of the Euclidean and KL measures. (It follows that if the P and Q probability distributions are identical, $E(P, Q) = D(P, Q) = 0$.) Thus, in an economic application, if the modelled system were approaching a meltdown (e.g., the subprime market shortly before the fall), E and D values for successive comparisons between the actual distribution P and the RBN distribution Q for a critical regime would increasingly approach 0. Schmulevich et al. (2005) recommend that more than one metric be applied to insure that 'the results do not depend on the particular measure used' (p. xx). Consistent with this view, the continuing development of new complexity metrics and related divergence measures, as documented by Lloyd (2011) should reduce the likelihood of artefacts.

5. Conclusion

Economists, generations hence, will likely regard this period as a difficult time in their science: an uneasy historical juncture between the inadequate and the untried. But the episode is not without precedent. It is useful to remember that neoclassical economics, against which the critical mood is predominantly directed, itself began as a conscious reaction against the passions of its day. The austere and elegant theories of Léon Walras and his school were intended to overcome the acrimonious, and occasionally violent, confrontations within and between Political Economists, Marxists, anarchists, and Romantics, themselves a philosophic reflection of the wider social disorder (Tuchman, 1966; Burleigh, 2006). No less disorderly is our own era. Yet there is an important difference: Today's global marketplace is linked by information technology which can propagate economic crises in hours or sometimes minutes (Edmunds, 1997). Moreover, financialisation, not only of the US economy, but of much of the industrial world (e.g., OECD countries), deploys this same technology, thus propagating unemployment, negative growth, and inequality (Kus, 2012). The development of novel approaches in computational economics for identifying crises and responding with effective policies – essentially matching speed with speed – is thus far more crucial today than it was in Walras' era. The stakes are global, not regional. In this time of competing persuasions, ABM and BN models show promise primarily because they are both inductive strategies. At the onset of a crisis, it is surely reckless to be guided by preconceived formulae which, in turn, are correlated with an inadequate theoretical view. On the other hand, there is relative safety in an exploratory approach. The Procrustean temptation is made more difficult when no one – not even, in many cases, the modellers themselves – can predict the simulation, even though the initial rules have been precisely specified (Amigoni and Schiaffonati, 2008). This would amount, nontrivially, to a scientific shift: structured but unpredictable, ABM and BN models would not only be representations (Morgan and Morrison, 1999). They would occupy centre stage in computational discovery procedures, perhaps bringing economics closer to the elusive understanding of risk.

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