Hindawi
Complexity
Volume 2018, Article ID 7692375, 11 pages
https://doi.org/10.1155/2018/7692375

Research Article

Exponential Stabilization for a Class of Nonlinear Switched Systems with Mixed Delays under Asynchronous Switching

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Received 3 April 2018; Revised 4 August 2018; Accepted 5 August 2018; Published 30 August 2018

Academic Editor: Lingzhong Guo

This paper deals with the exponential stabilization problem for a class of nonlinear switched systems with mixed delays under asynchronous switching. The switching signal of the switched controller involves delay, which results in the asynchronous switching between the candidate controllers and subsystems. By constructing the parameter-dependent Lyapunov-Krasovskii functional and the average dwell time approach, some sufficient conditions in forms of linear matrix inequalities are presented to ensure the exponential stability of the switched nonlinear system under arbitrary switching signals. In addition, through the special deformation of the matrix and Schur complement, the controllers with asynchronous switching are designed. Finally, a numerical example and a practical example of river pollution control are provided to show the validity and potential of the developed results.

1. Introduction

The switched system is a relatively complex and typical dynamic system. It is composed of a group of discrete or continuous dynamic subsystems and a switching rule that coordinates the operation of each subsystem. In recent years, the research achievements of a switched system are widely used in aerospace, artificial intelligence, and biochemical and industrial manufacturing [1–3]. Moreover, the research of switched systems has attracted more and more domestic and foreign scientific research workers' attention [4, 5]. Thus, switched systems are of significant interest not only for their applicability in practice but also for their interesting theoretical properties. This is motivated by the need for systematic approach to investigate switched systems.

It is well known that time delay, perturbation, and stochastic term are inevitable in some practical control systems, which are often the main cause for instability or undesirable system performance of a control system [6–8]. In the past few years, the subject of switched systems with time-varying delays has attracted considerable attention due to a strong engineering background. For instance, Bingi et al. [9] investigate the phenomenon of time-dependent transmittance of evanescent Bloch modes (EBM) in ZnS random photonic crystal (RPC) which forms the basis for photonic delay switching. Exponential stability and L_1-gain analysis for positive time delay Markovian jump systems with switching transition rates subject to average dwell time have been investigated in [10]. Among these studies, stability is a crucial and fundamental problem for a switched system. Therefore, the research on stability of system has attracted a large number of domestic and foreign scientific researchers' attention [11, 12]. However, the problems of stability for switched nonlinear systems are a great challenge and few results have been reported for a switched nonlinear system with time delay. Hence, many researchers have paid more attention to studying nonlinear switched systems in the past few years and some stability results related to switched nonlinear systems have been reported in the literature [13, 14]. Moreover, Sun et al. and Daafouz et al. [15, 16] obtain sufficient conditions guaranteeing the exponential stability by a common Lyapunov functional (CLF). However, a common Lyapunov functional approach might become too conservative when stability is assessed. To address this issue, scholars investigate the systems by using multiple Lyapunov functional (MLF) [17, 18] and average dwell time (ADT) method [19, 20] in recent years. In particular, average dwell time technique plays an important role in switched system analysis and control synthesis. In this paper, our main goal is to provide a novel multiple Lyapunov-Krasovskii functional to study the exponential stabilization of switched nonlinear uncertain systems with mixed delays by an average dwell time method under asynchronous switching.

On the other hand, it is noted that the majority of the results mentioned above were based on an ideal assumption that the switching between the controller and the system is synchronous. In fact, since it is inevitable that some time is needed to identify the system mode and apply the matching controller, there is an asynchronous phenomenon between the system mode switching and the controller switching. Therefore, it is significant to study the problem of asynchronous switching and such system has gradually become a hot research field. With the deepening of research, some valuable achievements of asynchronous switching have emerged. Xiang and Wang [21] investigate robust control for uncertain switched systems with time delay under asynchronous switching, and some new delay-dependent exponential stabilization criteria for the system were established. Wang et al. [22] study the influence of random interference on the switching system by average dwell time technique under asynchronous switching, and H_∞ control issue for a class of switching system is considered in [23]. However, based on the above discussion, the problem of exponential stabilization for a class of nonlinear switched systems with mixed delays under asynchronous switching has not been well reported.

The core of this paper adds to the further development of switched nonlinear systems under asynchronous switching. Compared with the existing results on switched systems, the results of this paper have four contributions. Firstly, the switching signal of the Lyapunov-Krasovskii functional constructed in the paper is dependent on the controller switching signal, which is convenient for the analysis of the proposed issue. Secondly, the problem of the asynchronous switching between the subsystems and the candidate controllers is considered. We derive some sufficient conditions for exponential stability of the switched nonlinear systems with mixed delays by an average dwell time approach. Thirdly, based on matrix deformation technique and Schur complement, the state feedback controllers of switched nonlinear systems are designed under asynchronous switching, while on the existing work, the controller design problem was not considered. Finally, the results of this paper are extended to a practical example of river pollution control.

The rest of the paper is organized as follows. The problem description and preliminary knowledge are presented in Section 2. In Section 3, a novel multi-Lyapunov-Krasovskii functions related to parameters are constructed and a sufficient condition for exponential stabilization of a class of nonlinear switched systems with mixed delays under asynchronous switching is given by using average dwell time and matrix inequality. Moreover, the controllers of the switching system are designed through a special matrix deformation method, which is the important conclusion of this paper. Section 4 gives a numerical example and a practical example of river pollution control to show the validity and potential of the developed results. A conclusion is shown in Section 5.

2. Problem Description and Preliminaries

This paper studies a class of nonlinear switching systems as follows:

$$\begin{aligned}\dot{x}(t) &= \mathbf{A}_{1\sigma(t)}x(t) + \mathbf{A}_{2\sigma(t)}x(t-\tau(t)) + \mathbf{B}_{\sigma(t)}u(t) \\ &\quad + f(t, x(t-\tau(t))) + g(t, x(t-h(t))), \\ x(t) &= \varphi(t), \quad t \in [\max(-\tau, h), 0],\end{aligned} \tag{1}$$

where $u(t) \in R^m$ is control input, $x(t) \in R^n$ denotes a system state, and $\varphi(s) \in R^n$ is the initial condition. $\mathbf{A}_{1i}$, $\mathbf{A}_{2i}$, and $\mathbf{B}_i$, $i \in L$, are constant matrices, and $\sigma(t)$: $[0, \infty] \to L = \{1, 2, \ldots, n\}$ is the switching signal, which is a piecewise continuous function, n is the number of subsystems. Specially, switching sequence of the system is expressed as $\{(t_0, \sigma(t_0)), (t_1, \sigma(t_1)), \ldots, (t_k, \sigma(t_k))\}$, where t_0 and t_k represent the initial and the kth switching time, respectively. $\tau(t)$ and $h(t)$ represent the mixed delays of the switched system and satisfy the following cases:

$$\begin{aligned} 0 &\le \tau(t) \le \tau, \\ \dot{\tau}(t) &\le d < 1, \\ 0 &\le h(t) \le h, \\ \dot{h}(t) &\le \upsilon < 1. \end{aligned} \tag{2}$$

$f(t, x(t-\tau(t)))$ and $g(t, x(t-h(t)))$ are nonlinear perturbation functions, which satisfy the following condition:

$$\begin{aligned} \|f(t, x(t-\tau(t)))\| &\le \varepsilon \|x(t-\tau(t))\|, \\ \|g(t, x(t-h(t)))\| &\le \rho \|x(t-h(t))\|, \end{aligned} \tag{3}$$

where $\varepsilon > 0$ and $\rho > 0$ are known constants, noting that the nonlinear perturbations are widely applicable in practice and considered by many researchers.

When the controller synchronizes with the switching subsystem, the state feedback controller is often designed as

$$u(t) = \mathbf{K}_{\sigma(t)}x(t), \tag{4}$$

where $\mathbf{K}_i$, $i \in L$, denote the feedback gain matrix. However, in actual operation, the controller switching time lags behind the subsystem switching time. In other words, the controller and the switching subsystem are asynchronous. At this point, we assume that the delay of the switching signal of the controller is τ_d, where the lag time $\tau_\mathrm{d} < c$ and c is a known constant. Then, the state feedback controller is designed as

$$u(t) = \mathbf{K}_{\sigma(t-\tau_\mathrm{d})}x(t). \tag{5}$$

Remark 1. This article studies switched systems under asynchronous switching. We suppose that the jth subsystem is activated at the switching instant t_{k-1} and the ith subsystem is activated at the switching instant t_k, then the corresponding switching controllers are activated at the switching $t_{k-1}+\tau_d$ and $t_k+\tau_d$, respectively. Therefore, the closed-loop system of system (1) in the interval $[t_k, t_{k+1})$ can be represented as follows:

$$\begin{aligned}\dot{x}(t) &= (A_{1i}+B_iK_j)x(t)+A_{2i}x(t-\tau(t)) \\ &\quad + f(t,x(t-h(t)))+g(t,x(t-\tau(t))), \\ &\qquad \forall t\in[t_k, t_k+\tau_d) \text{ (mismatched periods)}, \\ \dot{x}(t) &= (A_{1i}+B_iK_i)x(t)+A_{2i}x(t-\tau(t)) \\ &\quad + f(t,x(t-h(t)))+g(t,x(t-\tau(t))), \\ &\qquad \forall t\in[t_k+\tau_d, t_{k+1}) \text{ (matched periods)}.\end{aligned} \tag{6}$$

To facilitate the calculation, define $\bar{A}_{1ij}=A_{1i}+C_iK_j$ and $\bar{A}_{1i}=A_{1i}+C_iK_i$. Then, the closed-loop system is abbreviated as follows:

$$\begin{aligned}\dot{x}(t) &= \bar{A}_{1ij}x(t)+A_{2i}x(t-\tau(t)) \\ &\quad + f(t,x(t-\tau(t)))+g(t,x(t-h(t))), \\ &\qquad \forall t\in[t_k, t_k+\tau_d) \text{ (mismatched periods)}, \\ \dot{x}(t) &= \bar{A}_{1i}x(t)+A_{2i}x(t-\tau(t)) \\ &\quad + f(t,x(t-\tau(t)))+g(t,x(t-h(t))), \\ &\qquad \forall t\in[t_k+\tau_d, t_{k+1}) \text{ (matched periods)}.\end{aligned} \tag{7}$$

In order to prove the next statements, the following definitions and lemmas are introduced.

Definition 1 [24]. The equilibrium $x^*=0$ of the closed-loop system (7) is called to be exponential stabilization under the switching signal σ and feedback control (5), if the solution of the closed-loop system (7) satisfies the following inequality.

$$\|x(t)\| \le \omega \sup_{-\max(\tau,h)\le\theta\le 0} \|x(t_0+\theta)\| e^{-\lambda(t-t_0)}, \quad \forall t\ge t_0, \omega\ge 1, \lambda>0. \tag{8}$$

Definition 2 [25]. For given $N_0\ge 0$, $\tau_a\ge 0$, and any $t_2>t_1\ge 0$, let $N_\sigma(t_1,t_2)$ be the number of switching signals $\sigma(t)$ in the interval (t_1,t_2), if the following equation

$$N_\sigma(t_1,t_2)\le N_0+\frac{(t_2-t_1)}{\tau_a}, \tag{9}$$

holds, then τ_a is said to be average dwell time. The authors choose $N_0=0$ in this paper.

Lemma 1 [25]. $\mathbf{S}_1$, $\mathbf{S}_2$, *and* $\mathbf{S}_3$ *are symmetric matrices of appropriate dimensions with* $\mathbf{S}_1=\mathbf{S}_1^T<0$, $\mathbf{S}_3=\mathbf{S}_3^T>0$, *then* $\mathbf{S}_1+\mathbf{S}_2\mathbf{S}_3^{-1}\mathbf{S}_2^T<0$ *if and only if*

$$\begin{bmatrix}\mathbf{S}_1 & \mathbf{S}_2 \\ \mathbf{S}_2^T & -\mathbf{S}_3\end{bmatrix}<0. \tag{10}$$

3. Main Results

In this section, we derive some sufficient conditions of the exponential stabilization for switched nonlinear systems by an average dwell time approach under asynchronous switching. Moreover, the state feedback controllers of switched nonlinear systems are designed.

Proposition 1. *For given positive constants α, β, τ, d, h, υ, and $\mu\ge 1$, if there is $\mathbf{P}_i$, $\mathbf{Q}_i$, and $\mathbf{R}_i$, which are symmetric and positive definite matrices, such that*

$$\begin{aligned}&\mathbf{P}_i\le\mu P_j, \\ &\mathbf{Q}_i\le\mu Q_j, \\ &\mathbf{R}_i\le\mu R_j, \\ &\quad \forall i,j\in L,\end{aligned} \tag{11}$$

$$\Pi_i=\begin{pmatrix}\varphi_{11}^i & \varphi_{12}^i & 0 & P_i & P_i \\ * & \varphi_{22}^i & 0 & 0 & 0 \\ * & * & \varphi_{33}^i & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -I\end{pmatrix}<0, \tag{12}$$

$$\Xi_i=\begin{pmatrix}\omega_{11}^i & \varphi_{12}^i & 0 & P_i & P_i \\ * & \omega_{22}^i & 0 & 0 & 0 \\ * & * & \omega_{33}^i & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -I\end{pmatrix}<0, \tag{13}$$

where

$$\begin{aligned}\varphi_{11}^i &= P_i\bar{A}_{1i}+\bar{A}_{1i}^TP_i+Q_i+R_i+\alpha P_i, \\ \varphi_{22}^i &= \varepsilon^2-(1-d)e^{-\alpha\tau}Q_i, \\ \varphi_{12}^i &= P_iA_{2i}, \\ \varphi_{33}^i &= \rho^2-(1-\upsilon)e^{-\alpha h}R_i, \\ \omega_{22}^i &= \varepsilon^2-(1-d)Q_i, \\ \omega_{33}^i &= \rho^2-(1-\upsilon)R_i, \\ \omega_{11}^i &= P_i\bar{A}_{1ij}+\bar{A}_{1ij}^TP_i+Q_i+R_i+\alpha P_i.\end{aligned} \tag{14}$$

Then, the closed-loop system (7) is exponentially stable under the feedback controller (5) for arbitrary switching signal with the average dwell time satisfying

$$\tau_a>\tau_a^*=\frac{\ln\mu+(\alpha+\beta)\tau_d}{\alpha}. \tag{15}$$

Proof 1. There exist two periods during the whole running time of stability for switched systems (7): matched and mismatched periods.

Firstly, when $t \in [t_k + \tau_d, t_{k+1})$, $\sigma(t_k) = i \in L$, the closed-loop system (7) is active within the ith subsystem and the corresponding ith switching controller is also activated. The Lyapunov-Krasovskii functional is constructed as follows:

$$V_{1\sigma(t)}(t) = x^T(t)P_{\sigma(t)}x(t) + \int_{t-\tau(t)}^{t} e^{\alpha(s-t)}x^T(s)Q_{\sigma(t)}x(s)ds + \int_{t-h(t)}^{t} e^{\alpha(s-t)}x^T(s)R_{\sigma(t)}x(s)ds. \quad (16)$$

We can easily obtain the following inequalities:

$$\begin{aligned}\dot{V}_{1i} &= 2x^T(t)P_i\dot{x}(t) - \alpha\int_{t-\tau(t)}^{t} e^{\alpha(s-t)}x^T(s)Q_ix(s)ds \\ &\quad - \alpha\int_{t-h(t)}^{t} e^{\alpha(s-t)}x^T(s)R_ix(s)ds \\ &\quad - (1-\dot{\tau}(t))e^{-\alpha\tau(t)}x^T(t-\tau(t))Q_ix(t-\tau(t)) \\ &\quad - \left(1-\dot{h}(t)\right)e^{-\alpha h(t)}x^T(t-h(t))R_ix(t-h(t)) \\ &\quad + x^T(t)R_ix(t) + x^T(t)Q_ix(t) \\ &\le x^T(t)\left[P_i\bar{A}_{1i} + \bar{A}_{1i}^TP_i + Q_i + R_i\right]x(t) \\ &\quad - (1-d)e^{-\alpha\tau}x^T(t-\tau(t))Q_ix(t-\tau(t)) \\ &\quad + x^T(t)P_iA_{2i}x(t-\tau(t)) + x^T(t)P_if(t,x(t-\tau(t))) \\ &\quad + x^T(t-\tau(t))A_{2i}^TP_ix(t) + f^T(t,x(t-\tau(t)))P_ix(t) \\ &\quad - (1-\upsilon)e^{-\alpha h}x^T(t-h(t))R_ix(t-h(t)) \\ &\quad + x^T(t)P_ig(t,x(t-h(t))) + g^T(t,x(t-h(t)))P_ix(t) \\ &\quad - \alpha\int_{t-\tau(t)}^{t} e^{\alpha(s-t)}x^T(s)Q_ix(s)ds \\ &\quad - \alpha\int_{t-h(t)}^{t} e^{\alpha(s-t)}x^T(s)R_ix(s)ds.\end{aligned} \quad (17)$$

The inequality (3) can be rearranged as follows:

$$\begin{aligned}&\varepsilon^2x^T(t-\tau(t))x(t-\tau(t)) \\ &\quad - f^T(t,x(t-\tau(t)))f(t,x(t-\tau(t))) \ge 0, \\ &\rho^2x^T(t-h(t))x(t-h(t)) \\ &\quad - g^T(t,x(t-h(t)))g(t,x(t-h(t))) \ge 0.\end{aligned} \quad (18)$$

Combining (17) with (18), we can get

$$\begin{aligned}\dot{V}_{1i} + \alpha V_{1i} &\le x^T(t)\left[P_i\bar{A}_{1i} + \bar{A}_{1i}^TP_i + Q_i + R_i + \alpha P_i\right]x(t) \\ &\quad - g^T(t,x(t-h(t)))g(t,x(t-h(t))) \\ &\quad + x^T(t)P_if(t,x(t-\tau(t))) \\ &\quad + x^T(t-\tau(t))A_{2i}^TP_ix(t) \\ &\quad + f^T(t,x(t-\tau(t)))P_ix(t) \\ &\quad + x^T(t-h(t))\left(\rho^2I - (1-\upsilon)e^{-\alpha h}R_i\right)x(t-h(t)) \\ &\quad + x^T(t)P_ig(t,x(t-h(t))) \\ &\quad + x^T(t)P_iA_{2i}x(t-\tau(t)) \\ &\quad + x^T(t-\tau(t))\left(\varepsilon^2I - (1-d)e^{-\alpha\tau}Q_i\right)x(t-\tau(t)) \\ &\quad + g^T(t,x(t-h(t)))P_ix(t) \\ &\quad - f^T(t,x(t-\tau(t)))f(t,x(t-\tau(t))).\end{aligned} \quad (19)$$

Let

$$\zeta(t) = \left[x^T(t) \quad x^T(t-\tau(t)) \quad x^T(t-h(t)) \quad f^T(t,x(t-\tau(t))) \quad g^T(t,x(t-h(t)))\right]^T. \quad (20)$$

Then,

$$\dot{V}_{1i} + \alpha V_{1i} \le \zeta^T(t)\Pi_i\zeta(t) < 0, \quad (21)$$

where Π_i is given by (12). Therefore, the following formula is established.

$$\dot{V}_{1i} < -\alpha V_{1i}. \quad (22)$$

Secondly, when $t \in [t_k, t_k + \tau_d)$, the closed-loop system (7) is active within the ith subsystem and the corresponding jth switching controller is also activated. We choose the Lyapunov-Krasovskii functional candidate as follows:

$$\begin{aligned}V_{2\sigma(t)}(t) &= x^T(t)P_{\sigma(t)}x(t) \\ &\quad + \int_{t-\tau(t)}^{t} e^{\beta(t-s)}x^T(s)Q_{\sigma(t)}x(s)ds \\ &\quad + \int_{t-h(t)}^{t} e^{\beta(t-s)}x^T(s)R_{\sigma(t)}x(s)ds.\end{aligned} \quad (23)$$

We can get the time derivative of $V_{2\sigma(t)}$ as follows:

$$\begin{aligned}\dot{V}_{2i} &= 2x^T(t)P_i\dot{x}(t) + x^T(t)Q_ix(t)\\&\quad+\beta\int_{t-\tau(t)}^{t} e^{\beta(t-s)}x^T(s)Q_ix(s)ds\\&\quad-(1-\dot{\tau}(t))e^{\beta\tau(t)}x^T(t-\tau(t))Q_ix(t-\tau(t))\\&\quad+\beta\int_{t-h(t)}^{t} e^{\beta(t-s)}x^T(s)R_ix(s)ds\\&\quad-\left(1-\dot{h}(t)\right)e^{\beta h(t)}x^T(t-h(t))R_ix(t-h(t))\\&\quad+x^T(t)R_ix(t)\\&\le x^T(t)\left[P_i\bar{A}_{1ij}+\bar{A}_{1ij}^TP_i+Q_i+R_i\right]x(t)\\&\quad-(1-d)x^T(t-\tau(t))Q_ix(t-\tau(t))\\&\quad+x^T(t)P_iA_{2i}x(t-\tau(t))+x^T(t)P_if(t,x(t-\tau(t)))\\&\quad+f^T(t,x(t-\tau(t)))P_ix(t)\\&\quad-(1-\upsilon)x^T(t-h(t))R_ix(t-h(t))\\&\quad+x^T(t)P_ig(t,x(t-h(t)))+g^T(t,x(t-h(t)))P_ix(t)\\&\quad+x^T(t-\tau(t))A_{2i}^TP_ix(t)+\beta\int_{t-h(t)}^{t} e^{\beta(s-t)}x^T(s)R_ix(s)ds\\&\quad+\beta\int_{t-\tau(t)}^{t} e^{\beta(s-t)}x^T(s)Q_ix(s)ds.\end{aligned} \tag{24}$$

Recalling (18), it follows that

$$\begin{aligned}\dot{V}_{2i}-\beta V_{2i} &\le x^T(t)\left[P_i\bar{A}_{1ij}+\bar{A}_{1ij}^TP_i+Q_i+R_i-\beta P_i\right]x(t)\\&\quad+x^T(t-\tau(t))\left(\varepsilon^2I-(1-d)Q_i\right)x(t-\tau(t))\\&\quad+f^T(t,x(t-\tau(t)))P_ix(t)\\&\quad+x^T(t)P_if(t,x(t-\tau(t)))\\&\quad+x^T(t-\tau(t))A_{2i}^TP_ix(t)\\&\quad+x^T(t-h(t))\left(\rho^2I-(1-\upsilon)R_i\right)x(t-h(t))\\&\quad+x^T(t)P_ig(t,x(t-h(t)))\\&\quad+g^T(t,x(t-h(t)))P_ix(t)\\&\quad-g^T(t,x(t-h(t)))g(t,x(t-h(t)))\\&\quad+x^T(t)P_iA_{2i}x(t-\tau(t))\\&\quad-f^T(t,x(t-\tau(t)))f(t,x(t-\tau(t))),\end{aligned} \tag{25}$$

that is,

$$\dot{V}_{2i}-\beta V_{2i}\le\zeta^T(t)\Xi_i\zeta(t)<0. \tag{26}$$

where Ξ_i is given by (13). Therefore, the following formula is established.

$$\dot{V}_{2i}<\beta V_{2i}. \tag{27}$$

By recalling (2), we have

$$\begin{aligned}&\int_{t-\tau(t)}^{t} e^{\alpha(s-t)}x^T(s)Q_ix(s)ds+\int_{t-h(t)}^{t} e^{\alpha(s-t)}x^T(s)R_ix(s)ds\\&\quad\le\int_{t-\tau(t)}^{t} x^T(s)Q_ix(s)ds+\int_{t-h(t)}^{t} e^{\alpha(s-t)}x^T(s)R_ix(s)ds\\&\quad\le\int_{t-\tau(t)}^{t} e^{\beta(t-s)}x^T(s)Q_\sigma x(s)ds+\int_{t-h(t)}^{t} e^{\beta(t-s)}x^T(s)R_\sigma x(s)ds.\end{aligned} \tag{28}$$

Thus,

$$V_{1i}(t)\le V_{2i}(t). \tag{29}$$

In the entire interval $[t_0,t)$, the Lyapunov-Krasovskii function is expressed as

$$V(t)=\begin{cases}V(t), & \forall t\in[t_k+\tau_{\mathrm{d}},t_{k+1}),\\ V_{2\sigma}(t), & \forall t\in[t_k,t_k+\tau_{\mathrm{d}}),\end{cases} \tag{30}$$

where $k=0,1,2,\ldots,n$.

When $t\in[t_k+\tau_{\mathrm{d}},t_{k+1})$, integrating both sides of (22) from $t_k+\tau_{\mathrm{d}}$ to t, we get

$$\begin{aligned}V(t)&\le e^{-\alpha(t-(t_k+\tau_{\mathrm{d}}))}V_{1i}((t_k+\tau_{\mathrm{d}})^+)\\&\le e^{-\alpha(t-(t_k+\tau_{\mathrm{d}}))}V_{2i}((t_k+\tau_{\mathrm{d}})^-)\\&\le e^{-\alpha(t-(t_k+\tau_{\mathrm{d}}))}e^{\beta\tau_{\mathrm{d}}}V_{2i}((t_k)^+)\\&\le\mu e^{-\alpha(t-(t_k+\tau_{\mathrm{d}}))}e^{\beta\tau_{\mathrm{d}}}V_{1i}((t_k)^+)\\&\le\cdots\le\mu^ke^{(k+1)\beta\tau_{\mathrm{d}}}e^{-\alpha(t-t_0-(k+1)\tau_{\mathrm{d}})}V(t_0)\\&\le e^{(\alpha+\beta)\tau_{\mathrm{d}}}e^{\{[\ln u+(\alpha+\beta)\tau_{\mathrm{d}}]/\tau_{\mathrm{a}}-\alpha\}(t-t_0)}V(t_0).\end{aligned} \tag{31}$$

When $t\in[t_k,t_k+\tau_{\mathrm{d}})$, integrating both sides of (27) from t_k to t, we get

$$\begin{aligned}V(t)&\le e^{\beta(t-t_k)}V_{2i}((t_k)^+)\\&\le\mu e^{\beta(t-t_k)}V_{1i}((t_k)^-)\\&\le\mu e^{-\alpha(t-(t_{k-1}-\tau_{\mathrm{d}}))}e^{\beta\tau_{\mathrm{d}}}V_{1i}((t_{k-1}+\tau_{\mathrm{d}})^-)\\&\le\cdots\le\mu^ke^{(k+1)\beta\tau_{\mathrm{d}}}e^{-\alpha(t-t_0-(\mathrm{k}+1)\tau_{\mathrm{d}})}V(t_0)\\&\le e^{(\alpha+\beta)\tau_{\mathrm{d}}}e^{\{[\ln u+(\alpha+\beta)\tau_{\mathrm{d}}]/\tau_{\mathrm{a}}-\alpha\}(t-t_0)}V(t_0).\end{aligned} \tag{32}$$

Using the equality (16) and (23), we obtain

$$\begin{aligned}&V(t)\ge a\|x(t)\|^2,\\&V(t_0)\le b\sup_{-\max(\tau,h)\le\theta\le0}\|x(t_0+\theta)\|^2,\end{aligned} \tag{33}$$

where

$$a = \min_{i \in L} \lambda_{\min}(P_i),$$
$$b = \max_{i \in L} \lambda_{\max}(P_i) + \tau e^{\beta\tau} \max_{i \in L} \lambda_{\max}(Q_i) + h e^{\beta h} \max_{i \in L} \lambda_{\max}(R_i). \quad (34)$$

So,

$$\|x(t)\| \le e^{1/2(\alpha+\beta)\tau_d} \sqrt{\frac{b}{a}}_{-\max} \sup_{(\tau,h)\le\theta\le 0} \|x(t_0+\theta) e^{\iota}\|, \quad (35)$$

where $\iota = -1/2\{\alpha - [\ln u + (\alpha+\beta)\tau_d]/\tau_a\}(t-t_0)$.

According to Definition 1, it is easily to prove that the closed-loop system (7) is exponential stabilization.

Remark 2. In [17, 21], stabilization of nonlinear switched systems with delay was investigated under asynchronous switching and some criteria of stability for nonlinear switched systems were obtained. However, the nonlinear term in [17, 21] did not consider time delay. Liu et al. [8] focus on the problem of stability for a class of switched nonlinear systems with time-varying delay, but they [8] do not contain asynchronous switching. In this paper, we consider nonlinear switched systems and the nonlinear term in our system contains time-varying delay. In addition, some exponential stabilization criteria are obtained under asynchronous switching. Compared with [8, 17, 21], we have overcome the situation where the controller switching instant lags behind the subsystem switching instant and we have a greater advantage when dealing with complex systems in practice.

Remark 3. It is worth noting that the parameter-dependent Lyapunov-Krasovskii functional constructed in the form of (30) has three main features. On the one hand, the Lyapunov-Krasovskii functional depends on the switching signal of the controller, which is convenient for the analysis of the controller design under asynchronous switching. On the other hand, the Lyapunov-Krasovskii functional is incremental both at switching instants and during the mismatched periods, but it is decreasing as a whole and the stability of the system is guaranteed. Finally, the subsystems are allowed to be unstable during mismatched periods resulted from asynchronous switching.

Proposition 2. *For given α, β, τ, d, h, υ, and $\mu \ge 1$, if there is $\mathbf{X}_i$, $\mathbf{G}_i$, $\mathbf{O}_i$, and $\mathbf{Y}_i$, which are symmetric and positive definite matrices, such that*

$$\begin{aligned} &\mathbf{X}_j \le \mu X_i, \\ &\mathbf{G}_j \le \mu G_i, \\ &\mathbf{O}_j \le \mu O_i, \\ &\qquad \forall i, j \in L, i \ne j, \end{aligned} \quad (36)$$

$$\begin{bmatrix} \overline{\varphi}_{11}^i & \overline{\varphi}_{12}^i & 0 & I & I & \varepsilon X_i & 0 & X_i & X_i \\ * & \overline{\varphi}_{22}^i & 0 & 0 & 0 & 0 & \rho X_i & 0 & 0 \\ * & * & \overline{\varphi}_{33}^i & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & * & -O_i & 0 \\ * & * & * & * & * & * & * & * & -G_i \end{bmatrix} < 0, \quad (37)$$

$$\begin{bmatrix} \overline{\omega}_{11}^i & \overline{\varphi}_{12}^i & 0 & I & I & \varepsilon X_i & 0 & X_i & X_i \\ * & \overline{\omega}_{22}^i & 0 & 0 & 0 & 0 & \rho X_i & 0 & 0 \\ * & * & \overline{\omega}_{33}^i & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & * & -O_i & 0 \\ * & * & * & * & * & * & * & * & -G_i \end{bmatrix} < 0, \quad (38)$$

where

$$\begin{aligned} \overline{\varphi}_{11}^i &= A_{1i}X_i + B_iK_iX_i + (A_{1i}X_i + B_iK_iX_i)^T + \alpha X_i, \\ \overline{\varphi}_{22}^i &= (1-d)e^{-\alpha\tau}(G_i - 2X_i), \\ \overline{\varphi}_{12}^i &= A_{2i}X_i, \\ \overline{\varphi}_{33}^i &= (1-\upsilon)e^{-\alpha h}(O_i - 2X_i), \\ \overline{\omega}_{22}^i &= (1-d)(G_i - 2X_i), \\ \overline{\omega}_{11}^i &= A_{1i}X_i + B_iK_jX_i + (A_{1i}X_i + B_iK_jX_i)^T - \beta X_i, \\ \overline{\omega}_{33}^i &= (1-\upsilon)(O_i - 2X_i). \end{aligned} \quad (39)$$

Then, system (1) is exponentially stable if average dwell time $\tau_a > \tau_a^* = (\ln\mu + (\alpha+\beta)\tau_d)/\alpha$ holds. Furthermore, the controller can be designed by the following formula:

$$\mathbf{K}_i = Y_i X_i^{-1}. \quad (40)$$

Proof 2. From $G_i > 0$ and $O_i > 0$, we can get

$$\begin{aligned} (G_i - X_i)^T G_i^{-1} (G_i - X_i) \ge 0, \\ (O_i - X_i)^T O_i^{-1} (O_i - X_i) \ge 0. \end{aligned} \quad (41)$$

Then,

$$G_i - 2X_i \geq -X_i G_i^{-1} X_i,$$
$$O_i - 2X_i \geq -X_i O_i^{-1} X_i. \quad (42)$$

Both sides of (37) multiplies simultaneously $\{X_i^{-1}, X_i^{-1}, X_i^{-1}, I, I, I, I, I, I\}$, and we have the following inequality:

$$\begin{bmatrix} \tilde{\varphi}_{11}^i & \tilde{\varphi}_{12}^i & 0 & X_i^{-1} & X_i^{-1} & \varepsilon & 0 & I & I \\ * & \tilde{\varphi}_{22}^i & 0 & 0 & 0 & 0 & \rho & 0 & 0 \\ * & * & \tilde{\varphi}_{33}^i & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & * & -O_i & 0 \\ * & * & * & * & * & * & * & * & -G_i \end{bmatrix} < 0, \quad (43)$$

where

$$\tilde{\varphi}_{11}^i = X_i^{-1} A_{1i} + X_i^{-1} B_i Y_i X_i^{-1} + \left(X_i^{-1} A_{1i} + X_i^{-1} B_i Y_i X_i^{-1}\right)^T + \alpha X_i^{-1},$$
$$\tilde{\varphi}_{12}^i = X_i^{-1} A_{2i},$$
$$\tilde{\varphi}_{22}^i = -(1-d) e^{-\alpha\tau} G_i^{-1},$$
$$\tilde{\varphi}_{33}^i = -(1-\upsilon) e^{-\alpha h} O_i^{-1}. \quad (44)$$

Let

$$Y_i = K_i X_i,$$
$$X_i^{-1} = P_i,$$
$$G_i^{-1} = Q_i,$$
$$O_i^{-1} = R_i. \quad (45)$$

Using Schur complement in (43), we can get that (12) holds and the same method can be used to prove that formula (13) holds. From (45), the controller gains are given by (40). The proof is completed.

4. Simulation Examples

In this section, a numerical example and a practical example are given to illustrate the effectiveness and applicability of the proposed approach.

Example 1. We consider nonlinear switched systems with mixed delays, which is consisted of two subsystems with the following parameters:

$$A_{11} = \begin{bmatrix} -0.5 & 0 \\ 0 & -0.6 \end{bmatrix},$$
$$A_{21} = \begin{bmatrix} -0.4 & 0 \\ 0 & -0.6 \end{bmatrix},$$
$$A_{12} = \begin{bmatrix} -0.8 & 0 \\ 0 & -0.6 \end{bmatrix},$$
$$A_{22} = \begin{bmatrix} -0.6 & 0 \\ 0 & -0.7 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 0.3 \\ 0.5 \end{bmatrix},$$
$$B_2 = \begin{bmatrix} 0.2 \\ 0.6 \end{bmatrix}. \quad (46)$$

We choose $\alpha = 1.05$, $\beta = 0.4$, $h = 0.9$, $\tau = 0.8$, $d = 0.4$, $\upsilon = 0.3$, $\tau_{\mathrm{d}} = 0.3$, $\mu = 1.9$, $\varepsilon = 0.3$, $\rho = 0.4$,

$$f(t, x(t-\tau(t))) = \begin{cases} 0.1 \cos(x_1(t)), \\ 0.2 \sin(x_2(t-\tau(t))) - 0.1, \end{cases}$$
$$g(t, x(t-h(t))) = \begin{cases} 0.1 \sin(x_1(t-h(t))), \\ 0.2 \cos(x_2(t)) - 0.1. \end{cases} \quad (47)$$

Through calculation, we get the average dwell time:

$$\tau_{\mathrm{a}} > \tau_{\mathrm{a}}^* = \frac{\ln \mu + (\alpha+\beta)\tau_{\mathrm{d}}}{\alpha} = 1.0256. \quad (48)$$

According to (38), (40), and (42), we can obtain

$$X_1 = \begin{bmatrix} 0.2777 & 0.1202 \\ 0.1202 & 0.0866 \end{bmatrix},$$
$$X_2 = \begin{bmatrix} 0.2686 & 0.1179 \\ 0.1179 & 0.0841 \end{bmatrix},$$
$$G_1 = \begin{bmatrix} 0.4163 & 0.0270 \\ 0.0270 & 0.3832 \end{bmatrix},$$
$$G_2 = \begin{bmatrix} 0.4109 & 0.0245 \\ 0.0245 & 0.3827 \end{bmatrix},$$
$$O_1 = \begin{bmatrix} 0.4240 & 0.0340 \\ 0.0340 & 0.3813 \end{bmatrix},$$
$$O_2 = \begin{bmatrix} 0.4177 & 0.0315 \\ 0.0315 & 0.3809 \end{bmatrix},$$
$$Y_1 = \begin{bmatrix} 0.9355 & 0.9169 \end{bmatrix},$$
$$Y_2 = \begin{bmatrix} 0.4103 & 0.8936 \end{bmatrix}. \quad (49)$$

Controller gains of the system can be calculated by (40) and be expressed as

$$K_1 = [-2.6819 \quad 13.9815],$$
$$K_2 = [-8.1572 \quad 22.0690]. \tag{50}$$

Figure 1 shows the switching signals of the system and controller, while Figure 2 shows the state response with an initial state $x(0) = (-1 \quad 1)^T$ of the considered switched nonlinear system under the designed switching signal depicted in Figure 1. It is easy to find that the system is exponentially stabilizable under asynchronous switching.

Example 2. We will illustrate the effectiveness of our approach through river pollution control issues.
In a reach of a polluted river, the concentrations per unit volume of biochemical oxygen demand and dissolved oxygen are denoted as $z(t)$ and $q(t)$, respectively. Let z^* and q^* corresponding to some measure of water quality standards denote the desired steady values of $z(t)$ and $q(t)$, respectively. Define $x_1(t) = z(t) - z^*$, $x_2(t) = q(t) - q^*$, and $x(t) = [x_1^T(t)\, x_2^T(t)]^T$. Then, the dynamic equation for $x(t)$ can be written as [18, 19]:

$$\dot{x}(t) = Ax(t) + \tilde{A}x(t - \tau(t)) + Bu(t) + \omega(t), \tag{51}$$

where

$$A = \begin{bmatrix} -k_{10} - \eta_1 - \eta_2 & 0 \\ -k_{30} & -k_{20} - \eta_1 - \eta_2 \end{bmatrix},$$
$$\tilde{A} = \begin{bmatrix} \eta_2 & 0 \\ 0 & \eta_2 \end{bmatrix}, \tag{52}$$
$$B = \begin{bmatrix} \eta_1 \\ 1 \end{bmatrix}.$$

where $u(t) = [u_1^T(t)\, u_2^T(t)]^T$ is the control variables of river pollution, $k_{i0} (i = 1, 2, 3)$, η_1 and η_2 are known constants, and $\omega(t)$ is the disturbance input of the system. The physical meaning of these parameters can be found in [26, 27]. In accordance with the actual situation, we assumed that the system actuators are subject to good performance or failure in this paper. Therefore, the model is divided into two subsystems for discussion. Then, system (51) can be described as the following switched system:

$$\dot{x}(t) = \begin{cases} A_{11}x(t) + A_{21}x(t - \tau(t)) + B_1u(t) + f(t, x(t - \tau(t))) + g(t, x(t - h(t))) \text{ (no failures occur)}, \\ A_{12}x(t) + A_{22}x(t - \tau(t)) + B_2u(t) + f(t, x(t - \tau(t))) + g(t, x(t - h(t))) \text{ (failures occur)}. \end{cases} \tag{53}$$

For the simulation of our purposes, we choose $k_{10} = 1.6$, $k_{20} = 1$, $k_{30} = 1.6$, $\eta_1 = 0.3$, and $\eta_2 = 0.7$ and we can get

$$A_{11} = \begin{pmatrix} -2.6 & 0 \\ -1.6 & -2 \end{pmatrix},$$
$$A_{21} = \begin{pmatrix} 0.7 & 0 \\ 0 & 0.7 \end{pmatrix}, \tag{54}$$
$$B_1 = \begin{pmatrix} 0.3 \\ 1 \end{pmatrix}.$$

Let $\tau(t) = 0.3 \sin(t)$, $h(t) = 0.3 \sin(t)$, $\omega(t) = \omega_1(t) + \omega_2(t)$,

$$\omega_1(t) = f(t, x(t - \tau(t))) = \begin{pmatrix} 0.1 \sin(x_1(t)) \\ 0.2 \sin(x_2(t - \tau(t))) \end{pmatrix},$$
$$\omega_2(t) = g(t, x(t - h(t))) = \begin{pmatrix} 0.2 \cos(x_1(t)) \\ 0.1 \cos(x_2(t - h(t))) \end{pmatrix}. \tag{55}$$

Then, we will use the above parameters to design a set of switching sequences to stabilize the above system (53). At the same time, we choose

$$A_{12} = \begin{pmatrix} -2.9 & 0 \\ 1.2 & -1.8 \end{pmatrix},$$
$$A_{22} = \begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \end{pmatrix}, \tag{56}$$
$$B_2 = \begin{pmatrix} 0.2 \\ 0.8 \end{pmatrix}.$$

$\alpha = 0.3$, $\beta = 0.5$, $\mu = 1.7$, $h = 0.8$, $\tau = 0.9$, $\tau_d = 0.4$, $d = 0.3$, $\upsilon = 0.3$, $\tau(t) = 0.3 \sin(t)$, $h(t) = 0.3 \cos(t)$, we get the average dwell time:

$$\tau_a > \tau_a^* = \frac{\ln \mu + (\alpha + \beta)\tau_d}{\alpha} = 2.8354. \tag{57}$$

By solving (38), (40), and (42), we have

$$X_1 = \begin{bmatrix} 0.4161 & 0.0690 \\ 0.0690 & 0.4290 \end{bmatrix},$$

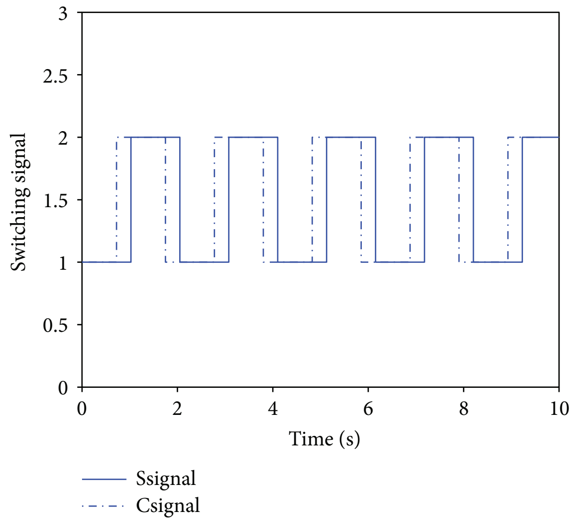

Figure 1: The switching law.

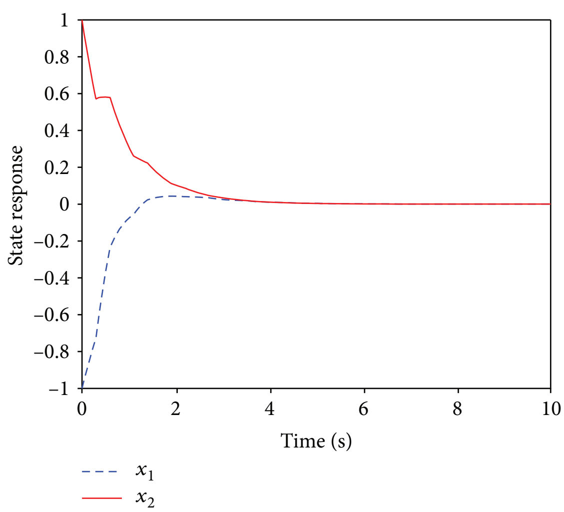

Figure 2: State response of the system.

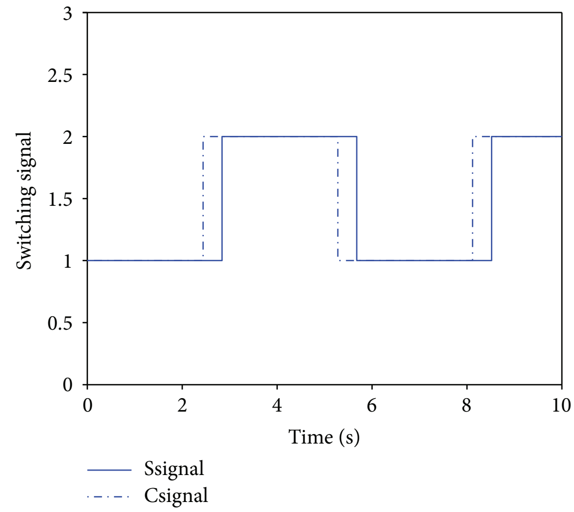

Figure 3: The switching law.

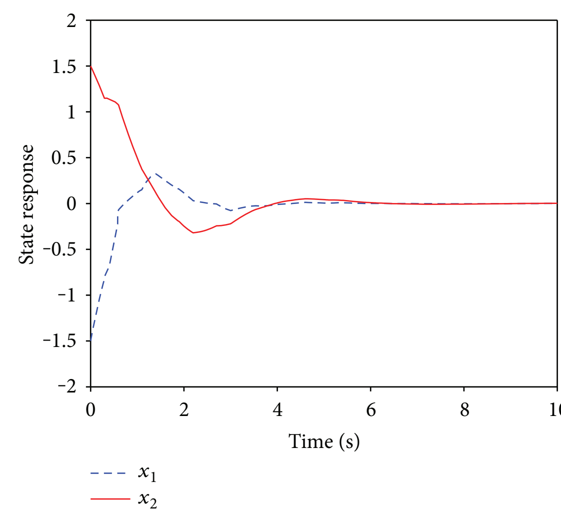

Figure 4: State response of the system.

$$X_2 = \begin{bmatrix} 0.2293 & 0.2241 \\ 0.2241 & 0.4260 \end{bmatrix},$$

$$G_1 = \begin{bmatrix} 1.3080 & 0.0244 \\ 0.0244 & 1.2977 \end{bmatrix},$$

$$G_2 = \begin{bmatrix} 1.3263 & 0.6489 \\ 0.6489 & 1.3533 \end{bmatrix},$$

$$O_1 = \begin{bmatrix} 0.7389 & 0.0799 \\ 0.0799 & 0.7482 \end{bmatrix},$$

$$O_2 = \begin{bmatrix} 0.5590 & 0.2137 \\ 0.2137 & 0.7364 \end{bmatrix},$$

$$Y_1 = \begin{bmatrix} 0.9501 & 0.2311 \end{bmatrix},$$

$$Y_2 = \begin{bmatrix} 0.6068 & 0.4860 \end{bmatrix}. \quad (58)$$

Then, the controller gains constructed by (40) are

$$K_1 = \begin{bmatrix} 2.2541 & 0.1760 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} 3.150 & -0.5166 \end{bmatrix}. \quad (59)$$

Figure 3 shows the switching signals of the system and controller, while Figure 4 shows the state response with an initial state $x(0) = (-1.5, 1.5)^T$ of the considered switched nonlinear system under the designed switching signal depicted in Figure 3. Therefore, the effectiveness of our approach is verified by its application in the control of the river pollution process.

Remark 4. Zheng et al. [27] have investigated the pollution problem of a single reach river modelled by the dynamics of water quality subject to uncertainty in system parameters; an adaptive controller is developed based on linear matrix inequality technique, and it is shown that the controller can guarantee the closed-loop system to converge, globally and exponentially. However, the performance of the actuator for the river pollution model may be deviated in practical applications. This article has solved this situation by changing the original system model to a switched system for processing. Compared with [26, 27], we have stronger application value in practice.

5. Conclusion

In this paper, the problem of exponential stabilization for a class of a nonlinear switched system with mixed time delays has been studied. The switching signal of the switched controller involves delay, which results in the asynchronous switching between the candidate controllers and subsystems. Based on a novel parameter-dependent Lyapunov-Krasovskii functional, some sufficient conditions for the exponential stability of the switched system under asynchronous switching are obtained by the average dwell time approach. Moreover, the controllers of the switched system are designed through a special matrix transformation method. Finally, a numerical example and a practical example of river pollution control are provided to show the validity and potential of the developed results.

Through the research of this paper, we learned that different piecewise Lyapunov functionals may lead to different conservatism. It deserves further study to choose an improved piecewise Lyapunov functional so as to reduce the conservativeness. In order to better study the asynchronous switching problem in multiple aspects, we will further optimize Lyapunov functionals and the dual controller design for better performance. In this article, we did not consider stochastic term. It is well known that stochastic term is inevitable in some practical control systems [28], which is often the main cause for instability or undesirable system performance of a control system. Specifically, the stabilization of stochastic switched nonlinear systems with Markov jumps will be taken as a main direction of our future research.

Data Availability

No data were used to support this study.

Conflicts of Interest

The author declares no conflict of interest.

Acknowledgments

The author would like to express sincere gratitude and appreciations to the institutions and people who have worked collaboratively to evaluate that the aims and objective to complete this research are attained. The author is grateful to the reviewer for comments and suggestions. The support of the National Natural Science Foundation for Young Scientists of China (Grant no. 71502050), the Key Scientific Research Projects of Colleges and Universities of Henan Province (nos. 18A630001, 18A110008, and 18A110010), and the 2017 Teaching Research Project of Anyang Normal University (no. ASJY-YB-047) is gratefully acknowledged.

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