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Research Article

Inversion of Thermal Conductivity in Two-Dimensional Unsteady-State Heat Transfer System Based on Boundary Element Method and Decentralized Fuzzy Inference

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Based on the boundary element method and the decentralized fuzzy inference algorithm, the thermal conductivity in the two-dimensional unsteady-state heat transfer system changing with the temperature is deduced. The more accurate inversion results are obtained by introducing the variable universe method. The concrete method is as follows: using experimental means to obtain the instantaneous temperature in the material or on the boundary, to determine the thermal conductivity of the material by solving the inversion problem. The boundary element method is used to calculate the regional boundary and internal temperature in the direct problem. With the inversion problem, the decentralized fuzzy inference algorithm is used to compensate for the initial guess of the thermal conductivity by using the difference between the temperature measurement and the temperature calculation. In the inversion problem, the influence of the initial guess of different thermal conductivities, different numbers of measuring points, and the existence of measurement errors on the results is discussed. The example calculation and analysis prove that, with different initial guesses, existence of measurement errors, and the number of boundary measurements decrease, the methods adopted in this paper still maintain good validity and accuracy.

1. Introduction

Inversion heat transfer problems refer to the fact that some of the output information of the heat transfer system is obtained through experimental methods to invert some structural features or input information in the system. For example, the inversion of information such as the shape of the temperature boundary layer, the thermal conductivity of the material, and the heat flux density are all typical inversion heat conduction problems. Inversion heat transfer problems have been widely used in many fields such as nondestructive testing, geometrical shape optimization, aerospace engineering, power engineering, mechanical engineering, constructional engineering, bioengineering, metallurgical engineering, material processing, biological medicine, and food engineering, all of which achieve great success [1–13]. For inversion heat conduction problems, a lot of researches have been done by scholars at home and abroad.

The boundary element method and the complex variable derivation method are applied by Yu to invert the thermal conductivity of heterogeneous materials, which can effectively identify the thermal conductivity of single or multiple parameters [14]. When the heat conduction boundary value of the stability boundary is inverted by Yaparova, Laplace and Fourier transforms [15] are applied. The boundary element method is used to analyze two-dimensional transient conduction problems by Zhou et al. and the conjugate gradient method is introduced to solve the thermal conduction coefficient, which verify the effectiveness and stability of this method [16]. Mera et al. use iterative BEM to generate a stable numerical solution, which increases the number of boundary elements and reduces the amount of noise added in the input data [17]. Chen and Tanaka use a coupling application of the dual reciprocity boundary element method and dynamic programming filter to some inversion heat conduction problems [18]. A new and simple boundary element

method is presented by Gao and Wang; this method is called interface integral boundary element method for solving heat conduction problems consisting of multiple media [19]. Wang et al. apply a nonsingular indirect boundary element method for the solution of three-dimensional inversion heat conduction problems. The exact geometrical representation of computational domain is adopted by parametric equations to eliminate the errors in traditional approaches of polynomial shape functions [20].

The differential transformation is studied and a stable differential calculus method is proposed to solve the inversion heat conduction problem [21] by Baranov et al. A new method to invert the thermal conductivity of material with temperature is proposed Miao et al., by which the temperature of measurement point is obtained by finite element method, and the residual between calculated value and measured value of temperature at the measurement point is minimized to get numerical solution, proving the effectiveness and accuracy of the algorithm [22].

The thermal conductivity of material with temperature changes is piecewise discrete by temperature range by Tang et al., and the genetic algorithms and the adjoint equation are used to carry out the inversion [23] of the thermal conductivity of the full temperature range. Based on the semi-infinite one-dimensional thermal model, the thermal conductivity inversion algorithm is studied by Lei; by changing the mathematical model, different intensity of noise is simulated, and the impact of noise on the accuracy of inversion is observed and studied; the method to improve the accuracy is proposed [24].

The decentralized fuzzy inference algorithm is successfully applied to the unsteady-state heat transfer system by Ran, which shows good anti-ill-posedness and obvious advantages and effectiveness [25]. In this paper, the boundary element method is used to solve the boundary and internal temperature in the two-dimensional unsteady-state heat transfer system, and the decentralized fuzzy inference algorithm is used to compensate for the initial guess of the thermal conductivity in order to minimize the residual between the calculated and the measured values of the temperature, and the true thermal conductivity is obtained.

2. Direct Heat Conduction Problem

2.1. The Boundary Integral Equation. The mathematical model of the two-dimensional unsteady-state heat transfer problem [26]:

$$\begin{aligned}
&\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}=\frac{1}{\alpha}\frac{\partial T}{\partial t} \quad (\in \Omega,\ t>t_0),\\
&T=\overline{T} \quad (\in \Gamma_1,\ t>t_0),\\
&q=-nk(T)\frac{\partial T}{\partial n}=\overline{q} \quad (\in \Gamma_2,\ t>t_0),\\
&T=T_0,\\
&t=0.
\end{aligned} \tag{1}$$

In the mathematic model formula, there are Γ_1, Γ_2 and the boundary of domain Ω, which meets $\Gamma=\Gamma_1+\Gamma_2$. And α is the thermal conductivity coefficient $\alpha=k(T)/\rho c$, ρ is the density of the object, and c is the specific heat capacity of the object. And $k(T)$ is the heat transfer coefficient of the object changing with temperature, T is the temperature, and n is the outer normal vector of the boundary. And T_f is the ambient temperature, and q is the heat flux density. The letter with "–" denotes the known quantity.

Weight function T^* is introduced into the expression of weighted residual of governing equation [27].

$$\begin{aligned}
&\int_{t_0}^{t_1}\int_{\Omega}\left(\nabla^2 T-\frac{1}{\alpha}\frac{\partial T}{\partial t}\right)T^* d_\Omega d_\tau\\
&\quad=\int_{t_0}^{t_1}\int_{\Gamma_2}(q-\overline{q})\,T^* d_\Gamma d_\tau\\
&\qquad-\int_{t_0}^{t_1}\int_{\Gamma_1}\left(T-\overline{T}\right)\frac{\partial T^*}{\partial n}d_\Gamma d_\tau.
\end{aligned} \tag{2}$$

The left side of the equation is decomposed to get:

$$\begin{aligned}
&\int_{t_0}^{t_1}\int_{\Omega}\left(\nabla^2 T-\frac{1}{\alpha}\frac{\partial T}{\partial t}\right)T^* d_\Omega d_\tau\\
&\quad=\int_{t_0}^{t_1}\int_{\Omega}T^*\nabla^2 T\,d_\Omega d_\tau-\int_{t_0}^{t_1}\int_{\Omega}\frac{1}{\alpha}\frac{\partial T}{\partial t}T^* d_\Omega d_\tau.
\end{aligned} \tag{3}$$

In Green's theorem for the Laplacian, $\iint_D(v\nabla^2 u-u\nabla^2 v)d_\Omega=\int_z(v(\partial u/\partial n)-u(\partial v/\partial n))d_s$ of which z is the boundary curve of plane closed Region D and d_s is the arc differential.

According to Laplace Green function, the following is obtained:

$$\begin{aligned}
\int_{t_0}^{t_1}\int_{\Omega}\nabla^2 T T^* d_\Omega d_\tau&=\int_{t_0}^{t_1}\int_{\Gamma}\left(T^*\frac{\partial T}{\partial n}-T\frac{\partial T^*}{\partial n}\right)d_\Gamma d_\tau\\
&\quad+\int_{t_0}^{t_1}\int_{\Omega}T\nabla^2 T^* d_\Omega d_\tau.
\end{aligned} \tag{4}$$

Equation (4) is taken into (3) and (2) becomes

$$\begin{aligned}
&\int_{t_0}^{t_1}\int_{\Gamma}\left(T^*\frac{\partial T}{\partial n}-T\frac{\partial T^*}{\partial n}\right)d_\Gamma d_\tau+\int_{t_0}^{t_1}\int_{\Omega}T\nabla^2 T^* d_\Omega d_\tau\\
&\quad-\int_{t_0}^{t_1}\int_{\Omega}\frac{1}{\alpha}\frac{\partial T}{\partial t}T^* d_\Omega d_\tau\\
&=\int_{t_0}^{t_1}\int_{\Gamma_2}(q-\overline{q})\,T^* d_\Gamma d_\tau\\
&\quad-\int_{t_0}^{t_1}\int_{\Gamma_1}\left(T-\overline{T}\right)\frac{\partial T^*}{\partial n}d_\Gamma d_\tau.
\end{aligned} \tag{5}$$

It is further simplified, and $\int_{t_0}^{t_1}\int_\Omega (1/\alpha)(\partial T/\partial t)T^* d_\Omega d_\tau$ is transformed to get

$$\begin{aligned}\int_{t_0}^{t_1}\int_\Omega \frac{1}{\alpha}\frac{\partial T}{\partial t}T^* d_\Omega d_\tau &= \int_{t_0}^{t_1}\int_\Omega \frac{1}{\alpha}\frac{\partial T}{\partial t}T^* d_\Omega d_\tau \\ &+ \int_{t_0}^{t_1}\int_\Omega \frac{1}{\alpha}\frac{\partial T^*}{\partial t}T\, d_\Omega d_\tau \\ &- \int_{t_0}^{t_1}\int_\Omega \frac{1}{\alpha}\frac{\partial T^*}{\partial t}T\, d_\Omega d_\tau.\end{aligned} \tag{6}$$

One Integration by parts ($\int u\,dv + \int v\,du = uv$) is carried out in the equation for t to get

$$\begin{aligned}\int_{t_0}^{t_1}\int_\Omega \frac{1}{\alpha}\frac{\partial T}{\partial t}T^* d_\Omega d_\tau &= -\int_{t_0}^{t_1}\int_\Omega \frac{1}{\alpha}\frac{\partial T^*}{\partial t}T\, d_\Omega d_\tau \\ &+ \left[\int_\Omega \frac{1}{\alpha}T^* T\, d_\Omega\right]_{t=t_0}^{t=t_1}.\end{aligned} \tag{7}$$

Equation (6) is taken into (5) to get

$$\begin{aligned}&\int_{t_0}^{t_1}\int_\Omega T\nabla^2 T^* d_\Omega d_\tau + \int_{t_0}^{t_1}\int_\Omega \frac{1}{\alpha}\frac{\partial T^*}{\partial t}T\, d_\Omega d_\tau \\ &\quad - \left[\int_\Omega \frac{1}{\alpha}T^* T\, d_\Omega\right]_{t=t_0}^{t=t_1} \\ &= \int_{t_0}^{t_1}\int_{\Gamma_2} (q-\overline{q})\,T^* d_\Gamma d_\tau \\ &\quad - \int_{t_0}^{t_1}\int_{\Gamma_1} \left(T-\overline{T}\right)\frac{\partial T^*}{\partial n} d_\Gamma d_\tau \\ &\quad - \int_{t_0}^{t_1}\int_\Gamma \left(T^*\frac{\partial T}{\partial n} - T\frac{\partial T^*}{\partial n}\right) d_\Gamma d_\tau.\end{aligned} \tag{8}$$

Because of $\Gamma = \Gamma_1 + \Gamma_2$ and $\partial T^*/\partial n = q^*$, $\partial T/\partial n = q$, $\int_{t_0}^{t_1}\int_\Gamma (T^*(\partial T/\partial n) - T(\partial T^*/\partial n))d_\Gamma d_\tau$ is discomposed and taken into (8) to get

$$\begin{aligned}&\int_{t_0}^{t_1}\int_\Omega T\left(\nabla^2 T^* + \frac{1}{\alpha}\frac{\partial T^*}{\partial t}\right) d_\Omega d_\tau - \left[\int_\Omega \frac{1}{\alpha}T^* T\, d_\Omega\right]_{t=t_0}^{t=t_1} \\ &= \int_{t_0}^{t_1}\int_{\Gamma_2} (q-\overline{q})\,T^* d_\Gamma d_\tau - \int_{t_0}^{t_1}\int_{\Gamma_1} \left(T-\overline{T}\right) q^* d_\Gamma d_\tau \\ &\quad - \int_{t_0}^{t_1}\int_{\Gamma_1+\Gamma_2} \left(T^* q - T q^*\right) d_\Omega d_\tau.\end{aligned} \tag{9}$$

The corresponding basic solution to this formula is

$$T^* = \frac{1}{\left[4\pi\alpha\left(t_1-t\right)\right]^{d/2}}\exp\left(-\frac{r^2}{4\alpha\left(t_1-t\right)}\right), \tag{10}$$

where d is the dimensionality of space, for two-dimensional problem, $d = 2$ $r = \sqrt{[(x-x_i)^2 + (y-y_i)^2]}$.

Differential derivation of (10) is done to get

$$q^* = \frac{\partial T^*}{\partial n} = -\frac{D}{8\pi\alpha^2\left(t_1-t\right)^2}\exp\left(-\frac{r^2}{4\alpha\left(t_1-t\right)}\right). \tag{11}$$

In the formula, D is the vertical distance from the source point "i" to the boundary element line.

The basic solution has the following characteristics:

$$\begin{aligned}\nabla^2 T^* + \frac{1}{\alpha}\frac{\partial T^*}{\partial t} &= 0, \\ \int_\Omega T T^*\, d_\Omega &= T_i\left(t = t_1\right).\end{aligned} \tag{12}$$

Equations (12) and (11) are taken into (10) and a good merger of similar items is done to get

$$\begin{aligned}\frac{1}{\alpha}C_i T_i &= \int_{t_0}^{t_1}\int_\Gamma q T^* d_\Gamma d_\tau - \int_{t_0}^{t_1}\int_\Gamma T q^* d_\Gamma d_\tau \\ &+ \left[\int_\Omega \frac{1}{\alpha}T^* T\, d_\Omega\right]_{t=t_0}.\end{aligned} \tag{13}$$

2.2. The Boundary Element Equation. The change of the function T, q over time is small enough to be negligible compared to that of T^*, q^*, which can be reasonably approximated as a constant over small time intervals, and (13) can be segmented into time integration [28].

$$\begin{aligned}&C_i T_i + \alpha\int_\Gamma T\int_{t_0}^{t_1} q^* d_\tau d_\Gamma \\ &\quad = \alpha\int_\Gamma q\int_{t_0}^{t_1} T^* d_\tau d_\Gamma + \left[\int_\Omega T^* T\, d_\Omega\right]_{t=t_0}.\end{aligned} \tag{14}$$

And the interval integral for t is

$$\begin{aligned}\int_{t_0}^{t_1} q^* d_\tau &= -\frac{D}{2\pi\alpha r^2}\exp\left(-\frac{r^2}{4\alpha\left(t_1-t_0\right)}\right), \\ \int_{t_0}^{t_1} T^* d_\tau &= \frac{1}{4\pi\alpha}E_i(b), \quad b = \frac{r^2}{4\alpha\left(t_1-t_0\right)}.\end{aligned} \tag{15}$$

In the formula, $E_i(b)$ is the exponential integral function, which can be calculated by the series, which is

$$E_i(b) = -C - \ln b + \sum_{k=1}^{\infty}(-1)^{(k-1)}\frac{b^k}{k\cdot k!}. \tag{16}$$

In the formula, C is Euler function, $C = 0.57721566$, for $0 \le b \le 1$; generally the first five approximations are taken.

According to the above formula, (17) can be written as

$$\begin{aligned}C_i T_i^{t_1} + \alpha\int_\Gamma T^{t_1} q_t^*\, d_\Gamma &= \alpha\int_\Gamma q^{t_1} T_t^*\, d_\Gamma \\ &+ \left[\int_\Omega T^* T\, d_\Omega\right]_{t=t_0}.\end{aligned} \tag{17}$$

For the spatial domain division, the boundary Γ is divided into N units and the domain Ω is divided into M units. Equation (17) becomes

$$C_i T_i^{t_1} + \alpha \sum_{j=1}^{N} \int_{\Gamma_j} T_j^{t_1} q_t^* \, d_\Gamma = \alpha \sum_{j=1}^{N} \int_{\Gamma_j} q_j^{t_1} T_t^* \, d_\Gamma + \sum_{l=1}^{M} \left[\int_{\Omega_m} T^* T \, d_\Omega \right]_{t=t_0}. \tag{18}$$

Linear element interpolation is adopted and the interpolation function of linear element is $\{\varphi_1(\xi) = (1-\xi)/2,\ \varphi_2(\xi) = (1+\xi)/2\}$. Therefore, the boundary curve is approximated by a straight line. The values of T and q in the unit are approximated by the linearity with two endpoint values.

Equation (18) is done as

$$C_i T_i^{t_1} + \sum_{j=1}^{N} \left[h_{ij}{}^{(1)} T_j^{t_1} + h_{ij}{}^{(2)} T_{j+1}^{t_2} \right] = \sum_{j=1}^{N} \left[g_{ij}{}^{(1)} q_j^{t_1} + g_{ij}{}^{(2)} q_{j+1}^{t_1} \right] + \sum_{l=1}^{M} \left[\int_{\Omega_m} T^* T \, d_\Omega \right]_{t=t_0}. \tag{19}$$

In the formula, $\{h_{ij}^{(e)} = \alpha \int_{\Gamma j} \varphi_e q_t^* \, d_\Gamma,\ g_{ij}^{(e)} = \alpha \int_{\Gamma j} \varphi_e T_t^* \, d_\Gamma\}$ $e = (1, 2)$. The formula is written in matrix form:

$$HT^{t_1} = GQ^{t_1} + p^{t_1}, \quad p = \sum_{l=1}^{M} \left[\int_{\Omega_m} T^* T \, d_\Omega \right]_{t=t_0}. \tag{20}$$

T and q at the boundary node can be obtained by (21). Take $C_i = 1$ and the inner point temperature is obtained by (13) and (18).

3. Decentralized Fuzzy Inversion

3.1. Inversion of Thermal Conductivity. The thermal conductivity inverted in this paper varies with the temperature of the material, the function of which is known. By the known measured temperature at a particular measurement point, the inversion algorithm is used to determine the constant coefficients of the function.

The difference between the temperature measurement and the temperature calculation is taken as the objective function, which is minimized as

$$J(X) = \sum_{b=1}^{B} \sum_{r=1}^{R} \left[T_b{}^{k+r-1}(X) - Y_b^{k+r-1} \right]^2. \tag{21}$$

X is the inversion parameter in the objective function; $T_b{}^{k+r-1}$ and Y_b^{k+r-1}, respectively, represent the temperature measurement and the temperature calculation at the measuring point T_{Cb} at time t^{k+r-1}; B is the number of temperature measuring points; R is the number of future time steps; and the minimum value of the objective function $J(X)$ is calculated as the parameter vector X in the inversion problem.

3.2. Decentralized Fuzzy Inference Method. The difference between the temperature measurement and the temperature calculation is used to correspondingly compensate for the initial guess of the thermal conductivity. Therefore, a multiple-input multiple-output fuzzy inference system is established. Each independent measurement point is a single fuzzy inference unit (FIU). Fuzzy inference unit is shown in Figure 1. The independent fuzzy inference controller is divided into four parts: fuzzy interface, knowledge base, inference engine, and fuzzy decision interface (defuzzy).

The input to fuzzy inference unit FIU_b^r ($b = 1, 2, 3, \ldots, B$; $r = 1, 2, 3, \ldots, R$) is the error e_b^r of temperature calculation and measurement at the time of $t^k, t^{k+1}, \ldots, t^{k+r-1}$ at the measuring point T_{Cb} on the known boundary condition T_{Cb}:

$$e_b^r = T_b{}^{k+r-1}\left(X^k\right) - Y_b^{k+r-1}. \tag{22}$$

The independent fuzzy inference unit FIU_b^r output Δu_b^r is a fuzzy inference result corresponding to that of inputting e_b^r and is a numerical value for compensating for the guess value of the inversion parameter with only one independent measurement point T_{Cb}.

Linguistic values of each linguistic variable are defined: seven fuzzy sets are defined on the universe of inputting variable e_b^r and output variable Δu_b^r of FIU_b^r and these are $\{A_1, A_2, \ldots, A7\}$ and $\{D_1, D_2, \ldots, D7\}$. The linguistic values corresponding to the fuzzy sets are, respectively, {PB, PM, PS, ZO, NS, NM, NB}. There are a lot of files and the rules are formulated flexibly and detailed. However, rules are too many and complex, and the programming is difficult, accounting for more internal storage; rare files correspond to less rules, which can be easy to implement. The disadvantage is that control function becomes less detailed, whose effect cannot be satisfactory. So setting the fuzzy rule base is to take into account both the simplicity and accuracy.

The membership function of each language value and triangle membership function are defined; the shape and distribution of membership functions are shown in Figures 2 and 3.

The knowledge base is mainly composed of two parts, the database and language control rule base. The language control rules are based on the difference between the temperature calculation and the temperature measurement. If $e_b^r < 0$, it is proved that the temperature calculation is smaller than the temperature measurement; it is necessary to raise the guess value of the inversion parameter to eliminate the temperature error e_b^r, and the larger the $|e_b^r|$ is, the larger the range ability of the guess value of the inversion parameter will be. When $e_b^r > 0$, it is proved that the temperature calculation is larger than the temperature measurement; it is necessary to reduce the guess value of the inversion parameter to eliminate the temperature error e_b^r. The fuzzy control rules are shown in Table 1.

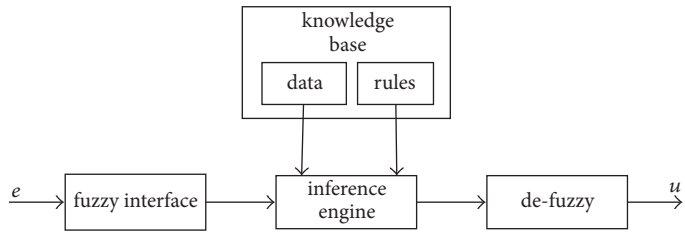

Figure 1: Fuzzy inference unit.

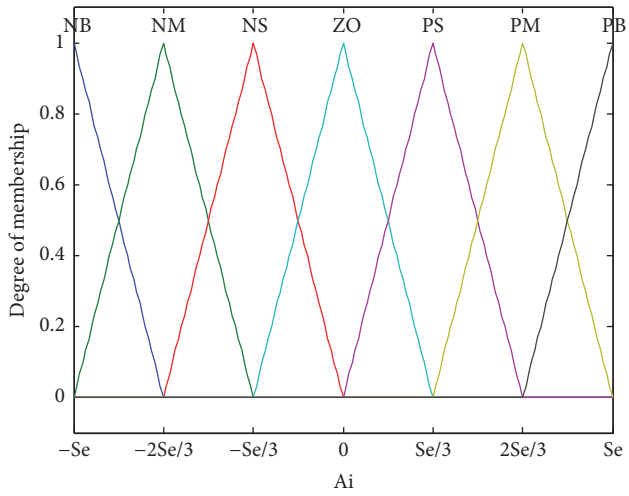

Figure 2: The degree of membership of fuzzy set Ai.

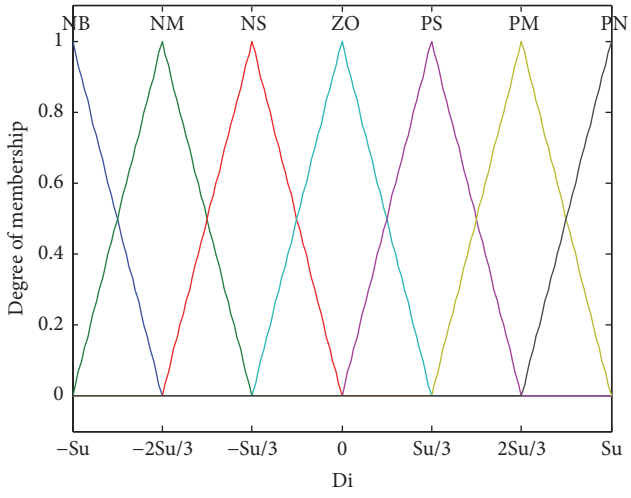

Figure 3: The degree of membership of fuzzy set Di.

Table 1: Fuzzy control rules state.

e_b^r	NB	NM	NS	ZO	PS	PM	PB
Δu_b^r	PB	PM	PS	ZO	NS	NM	NB

The fuzzy inference engine is based on the fuzzy input and language control rules and the fuzzy relational equation is solved to obtain the fuzzy output. Mamdani Maximum - Minimum Fuzzy Inference Algorithm is used to determine the fuzzy set D of output variables. The set D of output variables Δu_b^r is from the following formula:

$$\mu_D\left(\Delta u_{b,0}^r\right) = \max_{l=1}^{7}\left\{\min\left[\mu_{Al}\left(e_{b,0}^r\right), \mu_{Bl}\left(\Delta u_{b,0}^r\right)\right]\right\}. \quad (23)$$

In the fuzzy decision interface, the fuzzy output is done with defuzzification to get a precise control. In the fuzzy set D output by the fuzzy inference engine, the center of gravity is used to solve defuzzification:

$$\Delta u_b^r = \frac{\int \Delta u_{b,0}^r \mu_D\left(\Delta u_{b,0}^r\right) d_{\Delta u_i}}{\int \mu_D\left(\Delta u_{b,0}^r\right) d_{\Delta u_i}}. \quad (24)$$

3.3. Variable Universe. In variable universe, the appropriate universe extension factor is to be selected; the error is changed and some changes are also made to the universe, and through the universe changing with the error changes, the precise control effect can be achieved [29, 30].

In this paper, the thermal conductivity changing with temperature is inverted; the error of the input information has a more important influence on the inversion result. Therefore, effectively reducing the sensitivity of the inversion results to the error information e_b^r is the prerequisite for obtaining a stable inversion result. Universe S increases with e_b^r decreasing, which makes the partition of universe S rough, and the corresponding output becomes more detailed. Therefore, the following variable universe formula is applied:

$$S_i = \eta\left(\frac{S}{|e_b^r|}\right)^{\alpha}, \quad (25)$$

in which $[-S, S]$ is the initial input universe of e_b^r; $[-S_i, S_i]$ is the universe of e_b^r after being changed. In this paper, two different sets η, α are taken based on different assumptions.

3.4. Inversion Process. The process of inversing thermal conductivity is as follows.

(1) Set the number of iterations $h = 0$ and take the initial guess of thermal conductivity X.

(2) Calculate the direct problem to get temperature calculation at the temperature at point b.

(3) From (23), the deviation e_b^r can be calculated to determine whether the convergence condition $J(q) \leq \varepsilon$ is satisfied. If it is satisfied, the iteration is stopped. The value X is assumed to be the thermal conductivity; otherwise, the next calculation is performed.

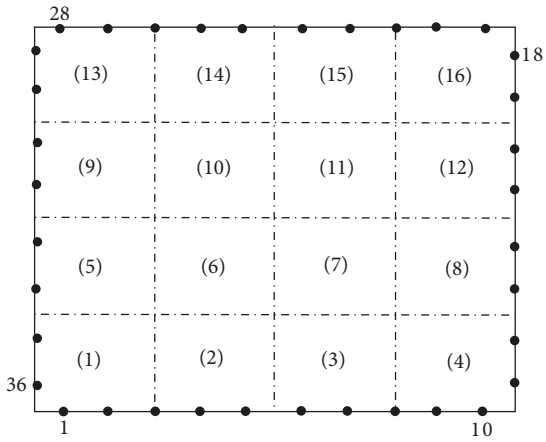

Figure 4: Schematic diagram.

(4) Calculate the real-time universe $[-S_i, S_i]$ from (25).

(5) Determine Δu_b^r by one-dimensional fuzzy inference unit FIU_i.

(6) Calculate the new guess of thermal conductivity X^{h+1} and return to step (2).

4. The Instance Calculation and Analysis

The schematic diagram is shown in Figure 4. A transient heat conduction problem in a 10×8 quadrilateral region is considered and the thermal conductivity meets

$$K(T) = K_0 + \lambda(T - T_0). \tag{26}$$

T_0 is the initial temperature, K_0 is the thermal conductivity at temperature T_0, and λ is an experimentally determined constant. For simplicity of description, the physical properties of the material are set to $\rho = 1\ c = 1$. The temperature of the four boundaries is 1°C, the initial temperature in the domain is 0°C, the boundary is divided into 36 boundary elements, and the domain is divided into 16 quadrilateral elements. The material thermal conductivity inversion is done under the premise of the temperature at the measuring point in the domain is known. The function of thermal conductivity is known, $K_0 = 1\ \lambda = 1/10$, the correct thermal conductivity is introduced into the direct problem to get the temperature at the measuring point at different time, and the correct temperature is defined as the temperature measurement. In the inversion process $K(T)$, T is the temperature at this position at the previous moment. The number of measuring points is selected as $B \in \{3, 6, 9\}$, at each measuring four point temperature calculations of different time are taken, and by inversion the three groups of data of the temperature and the corresponding thermal conductivity are obtained.

When there is a measurement error, temperature measurement at the measuring point is

$$Y_b^{k+r-1} = Y_b^{k+r-1} + \omega\sigma. \tag{27}$$

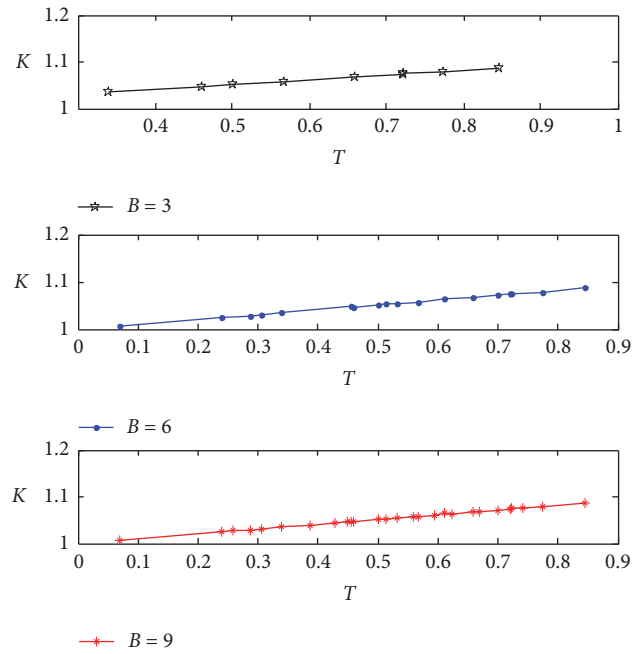

Figure 5: The inversion result of the thermal conductivity.

In this formula, ω is the random number of the normal distribution $N(0, 0.01)$ and σ is the standard deviation of the measurement.

4.1. The Impact of the Number of Measuring Points. The initial guess value is taken as $K(T) = 1.5$, the standard deviation of measurement is taken as $\sigma = 0$, the number of temperature measurement points is taken as $B = 3$, $B = 6$, $B = 9$, and the temperature at the measuring point and the corresponding thermal conductivity are obtained by inversion. The inversion result is shown in Figure 5.

At the measuring point $B = 3$, $B = 6$, $B = 9$, by the least square method K_0 and λ are calculated, and the average relative errors are shown in Table 2. When $B = 3$, the average relative errors of K_0 and λ are 0.28% and 3.5%; when $B = 6$, the average relative errors of K_0 and λ are 0.14% and 3.0%, respectively; when $B = 9$, the average relative errors of K_0 and λ are 0.05% and 2.1%, respectively. And thus it is shown that, by increasing the number of measuring points, the average relative error decreases and the inversion accuracy improves.

4.2. The Impact of Initial Guess. The number of measuring points is $B = 9$, the standard deviation of measurement is $\sigma = 0$, and three different initial guesses, $K(T) = 1$, $K(T) = 1.5$, and $K(T) = 2.0$ are used, respectively, for numerical test. The inversion result is shown in Figure 6.

$K(T) = 1$, $K(T) = 1.5$, and $K(T) = 2.0$ are used, respectively, by the least square method to calculate the value of K_0 and λ; the average relative errors are shown in Table 3. When $K(T) = 1$, the average relative errors of K_0 and λ are 0.02% and 4.4%; when $K(T) = 1.5$, the average relative errors of K_0 and λ are 0.05% and 2.1%; when $K(T) = 2.0$, the average relative errors of K_0 and λ are 0.06% and 3.0%. It can be seen

Table 2: The value of K_0, λ and the average relative error.

Test points	Calculated value of K_0	Calculated value of λ	Relative error of K_0 (%)	Relative error of λ (%)
3	0.9972	0.1035	0.28	3.5
6	0.9986	0.1030	0.14	3.0
9	0.9995	0.1021	0.05	2.1

Table 3: The value of K_0, λ and the average relative error.

Hypothetical value	Calculated value of K_0	Calculated value of λ	Relative error of K_0 (%)	Relative error of λ (%)
1	1.0002	0.0966	0.02	4.4
1.5	0.9995	0.1021	0.05	2.1
2.0	1.0006	0.1030	0.06	3.0

Table 4: The value of K_0, λ and the average relative error.

σ Standard deviation	Calculated value of K_0	Calculated value of λ	Relative error of K_0 (%)	Relative error of λ (%)
$\sigma = 0$	0.9995	0.1021	0.05	2.1
$\sigma = 0.2$	0.9980	0.0970	0.20	3.0
$\sigma = 0.4$	0.9860	0.1361	1.4	36.1

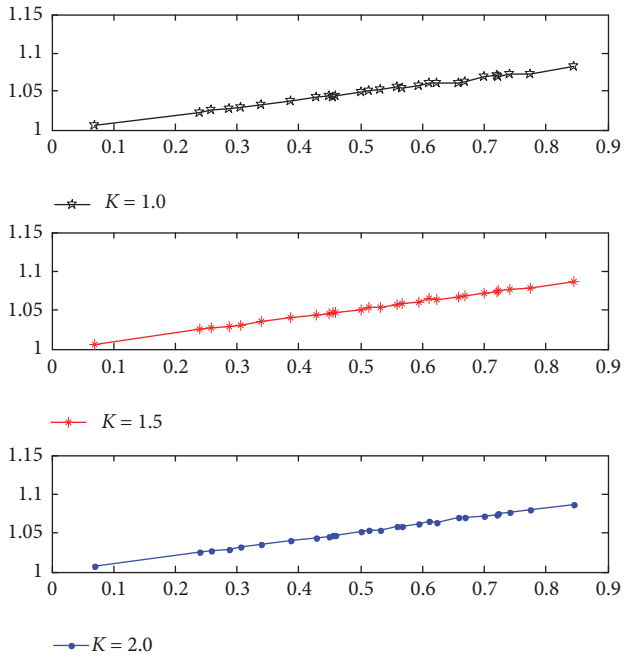

Figure 6: The inversion result of the thermal conductivity.

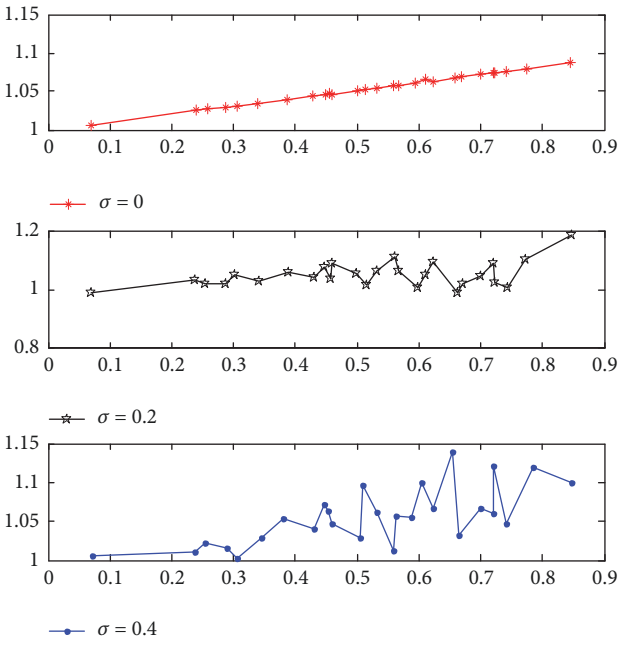

Figure 7: The inversion result of the thermal conductivity.

that the initial guess has a certain effect on the result, but the satisfactory result can be obtained within a reasonable range.

4.3. The Impact of Measurement Error. The initial guess is $K(T) = 1.5$, the number of measuring points is $B = 9$, and three groups of standard deviation, $\sigma = 0$, $\sigma = 0.2$, $\sigma = 0.4$ are used, respectively, for numerical test. The inversion result is shown in Figure 7.

The standard deviations of measurement are $\sigma = 0$, $\sigma = 0.2$, $\sigma = 0.4$; the values of K_0, λ are calculated by the least square method; the average relative errors are shown in Table 4. When $\sigma = 0$, the average relative errors of K_0 and λ are 0.05% and 2.1%; when $\sigma = 0.2$, the average relative errors of K_0 and λ are 0.20% and 3.0%; when $\sigma = 0.4$, the average relative errors of K_0 and λ are 1.40% and 36.1%. It can be seen that there are some measurement errors under the premise of large amount of measurement data, and the inversion results can still be satisfactory. However, the larger the standard deviation of measurement is, the more distorted the inversion results will be.

5. Conclusion

In this paper, the boundary element method and decentralized fuzzy inference algorithm are used to invert the thermal conductivity changing with temperature in two-dimensional unsteady-state heat transfer system. The effects of initial guess, different number of measuring points, and measurement errors on the results are discussed. It proves that if the initial guess is taken within a reasonable range and when there is some measurement error, inversion results can be satisfactory. Through the calculation and analysis of examples, the accuracy and stability of the thermal conductivity inversion algorithm are verified.

Data Availability

In order to better share our research results, we agree to distribute the data publicly. The data relating to this research is available upon request by contacting the corresponding author, Shoubin Wang (wsbin800@126.com, +86-022-23773022).

Disclosure

The founding sponsors had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript; and in the decision to publish the results.

Conflicts of Interest

The authors declare no conflicts of interest.

Authors' Contributions

Shoubin Wang and Li Zhang proposed mathematical description of boundary element method; Xiaogang Sun proved the decentralized fuzzy inference method; Li Zhang and Huangchao Jia designed the experiments and analyzed the results. Li Zhang wrote part of code.

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