Hindawi Complexity Volume 2017, Article ID 2513815, 7 pages https://doi.org/10.1155/2017/2513815



Research Article

RBF Nonsmooth Control Method for Vibration of Building Structure with Actuator Failure

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Received 13 July 2017; Accepted 30 October 2017; Published 20 December 2017

Academic Editor: Junpei Zhong

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In order to accommodate the actuator failure, the finite-time stable nonsmooth control method with RBF neural network is used to suppress the structural vibration. The traditional designed control methods neglect influence of actuator failure in structural vibration. By Lyapunov stable theory, the designed control method is demonstrated to suppress the building structural vibration with actuator failure. Finally, there are some examples to numerically simulate the three-layer building structure which is affected by El Centro seismic wave. Control effect of nonsmooth control is compared with no control and LQR control. The simulation results demonstrate that the designed control method is great for vibration of building structure with actuator failure and great antiseismic effect.

1. Introduction

How to reduce the severe and persistent vibration for structure under the earthquake vibration and wind resistance is a hot topic. In the last few decades, the structural vibration control was proved as an active and effective measure to suppress vibration. And international and domestic academics have brought up many kinds of effective control algorithm such as LQR (Linear Quadratic Regulator), LQG (Linear Quadratic Gaussian), ILC (Iterative Learning Control), and pole placement [1–5]. The aforementioned results do not consider the case of actuator failure. However, actuator failure is inevitable in the real project.

Thus, more and more researchers make contributions to control strategy in the case of actuator failure. And many effective ways have some development in respect of compensation of actuator failure. The study of actuator failure is started with dealing with linear system fault [6, 7]. Nevertheless, the control of actuator failure is very limited in the application of building structure.

In addition, the convergence of control system is an important index [8]. However, many linear control methods are to make system Lyapunov stable. What is more, they

belong to asymptotic stable research field that motion track is converged to the system's equilibrium point in the case of time that tends to infinity. In the view of making control system of structural vibration rapidly stabilize, it is necessary to study control methods making closed-loop system converge in finite time.

With the study and development of Lyapunov stable theory [9] and theorem of homogeneity, continuous nonsmooth control has made certain breakthrough [8–10]. Nonsmooth control has been widely applied [11–15] such as attitude control of spacecraft [12], high-precision guidance laws [14], and position control of permanent magnet synchronous motors [15]. Nevertheless, this control method is not applied on the building structure. At the same time, it cannot approximate well for uncertain part. The problem causes difficulty and challenge for design and analysis of control method. By learning literature [16–25], it is not hard to find out that neural network has wide prospect. And the neural network has great approximation effect for unknown model. Meanwhile, RBF neural network has great generalization and approximates any nonlinear function at random.

The paper carries out mathematical modeling and analysis for a building structure. According to the RBF neural

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network, the seismic wave is made by autoadaptable approximation. Then, according to finite-time stable theory and analysis of actuator failure, the finite-time stable nonsmooth algorithm is designed for the problem of structural vibration. Finally, the control system is under seismic wave called El Centro. And numerical analysis of the strong nonlinear model is studied. The control effects of nonsmooth control and LQR control are analyzed contrastively.

The main contributions of this dissertation are as follows: The impact of uncertain actuator failures on building structure vibration is considered. Meanwhile, the actuator failure is compensated with RBF neural network. Building structure vibration is suppressed in a fast speed by applying the method of finite-time nonsmooth vibration control, which prevents building structure from vibration in a long time.

2. The Modeling and Analysis of Building Structure

Interlaminar shear model is used. The *n* layers' building structure is simplified into building structure *n* degrees of freedom. Effected under one-dimensional horizontal earthquake, the equation of motion is as follows [1]:

$$M\ddot{D} + C\dot{D} + KD = F\ddot{x}_a + EU. \tag{1}$$

In this equation, $D = [d_1, d_2, \ldots, d_n]^T$ is displacement vector of the structure relative to the ground, where d_i ($i = 1, 2, \ldots, n$) is displacement of the building structural ith floor relative to the ground. M is mass matrix. C is damping matrix. K is stiffness matrix. F = MI is transform matrix of the ground seismic acceleration where $I = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}^T$ is the unit column of $n \times 1$. \ddot{x}_g is the ground seismic acceleration. E is a matrix denoting the location of actuators. U is the control input.

We define a state-space vector $S = [S_1, S_2]^T$, where $S_1 = [s_{11}, s_{12}, \dots, s_{1n}]^T = D$ and $S_2 = [s_{21}, s_{22}, \dots, s_{2n}]^T = \dot{D}$. Space state equations of (1) can be formulated as [26]

$$\dot{S} = A_r S + W_r \ddot{x}_a + B_r U, \tag{2}$$

where

$$A_{r} = \begin{bmatrix} 0 & I_{n \times n} \\ -M^{-1}K & -M^{-1}C \end{bmatrix},$$

$$W_{r} = \begin{bmatrix} 0 \\ M^{-1}F \end{bmatrix},$$

$$B_{r} = \begin{bmatrix} 0 \\ M^{-1}E \end{bmatrix}.$$
(3)

According to the rank criterion, the system (see (2)) is controllable. Hence, structural vibration can be suppressed effectively via designing control variable.

According to the finite-time stability theory, considering the motion equation of the structure, an actuator has been installed in each layer. E is $n \times n$ full rank matrix called

invertible matrix. We use variable $V = [u_1, u_2, ..., u_n]^T$ and choose

$$U = E^{-1} \left(C\dot{D} + KD - F\ddot{x}_q + MV \right). \tag{4}$$

Equation (4) is plugged into (2) as

$$\dot{S} = \begin{bmatrix} 0 & I_{n \times n} \\ 0 & 0 \end{bmatrix} S + \begin{bmatrix} 0 \\ I_{n \times n} \end{bmatrix} V. \tag{5}$$

The system can be decomposed into *n* mutual independent subsystems as

$$\dot{s}_{11} = s_{21},
\dot{s}_{21} = u_1,
\dot{s}_{12} = s_{22},
\dot{s}_{22} = u_2,
\vdots
\dot{s}_{1n} = s_{2n},
\dot{s}_{2n} = u_n.$$
(6)

The ith actuator failure mathematic model can be modeled as

$$u_i = \rho_i v_i + u_{ki}, \quad \forall t \ge t_{iF},$$

$$\rho_i u_{ki} = 0,$$
 (7)

where $0 \le \rho_i \le 1$ and u_{ki} and t_{iF} are uncertain constants. When the constants $\rho_i = 1$ and $u_i = v_i$, this indicates that the ith actuator works normally (i.e., the actuators work in the failure-free case). Thus, the following 2 patterns of failures are considered.

- (1) $0 \le \rho_i \le 1$: this case indicates that the systems lose partial performance during the operation, which is known as Partial Loss of Effectiveness (PLOE); that is, $u_i = \rho_i v_i$.
- (2) $\rho_i = 0$: this case implies that actuator output u_i is no longer affected by v_i . v_i indicates Total Loss of Effectiveness (TLOE); that is, $u_i = u_{ki}$.

According to the above analysis, system mathematical model is rewritten as follows:

$$\dot{s}_{1i} = s_{2i},$$
 $\dot{s}_{2i} = u_i,$
 $\forall t \ge t_{i:E}, \ i = 1, 2, \dots, n.$
(8)

3. Design of Control Algorithm

In the failure of the period, the controller is designed as $v_i = k^T w$, $w = (\alpha, 1)$, and $k = (k_{1i}, k_{2i})$ for any *i*th subsystem,

where α is designed as the finite-time stable nonsmooth control law [27].

$$\alpha = -k_1 \cdot \text{sign}(s_{1i}) \cdot |s_{1i}|^{\alpha_1} - k_2 \cdot \text{sign}(s_{2i}) \cdot |s_{2i}|^{\alpha_2},$$
 (9)
where $k_1 > 0$, $k_2 > 0$, $0 < \alpha_1 < 1$, $\alpha_2 = 2\alpha_1/(1 + \alpha_1)$, and $i = 1, 2, ..., n$.

On the basis of the above controller design, in order to guarantee the system stability, we must design k to meet the following formula:

$$\rho_i k^T w = \alpha - u_{ki}. \tag{10}$$

However, the failure model parameters of the actuator are unknown so that \hat{k} is defined as the estimated value for k and $\hat{k} = k - \hat{k}$ is defined.

The above analysis includes unknown seismic wave \ddot{x}_g disturbance, so subsequent analysis faces difficulty and challenge. Thus, the paper uses RBF neural network to approximate \ddot{x}_g .

RBF network has characteristics of universal approximation. We use theory that uses RBF network to approximate f(x). The network algorithm is as follows:

$$h_{j} = \exp\left(\frac{\left\|x - c_{j}\right\|^{2}}{2b_{j}^{2}}\right), \quad j = 1, 2, \dots, N,$$

$$f = W^{*T}h(x) + \varepsilon,$$
(11)

where x is an input of network, j is the jth joint of network's hidden layer, N is the number of network's hidden layers, $h = [h_1, h_2, \ldots, h_N]^T$, W^* is the desirable permission of network, ε is an approximation error of network, and $\varepsilon \leq \varepsilon_N$.

The input of network is $x = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T$. Then the output of network is as follows:

$$\widehat{f}(x) = \widehat{W}^T h(x). \tag{12}$$

According to RBF theory, we make the following definitions: $s = [D \ \dot{D}], \ \hat{f}(s) = \widehat{W}h(s), \ \text{and} \ f(s) = \ddot{x}_g = W^*h(s).$ It is proved accordingly that the nonsmooth control law can make system globally finite-time stable. We can prove the following: the *i*th subsystem of Lyapunov function is built as

$$V_{i} = \frac{\rho_{i}}{2} \widetilde{k}^{T} \Gamma^{-1} \widetilde{k} + \frac{k_{1}}{1 + \alpha_{1}} \left| s_{1i} \right|^{1 + \alpha_{1}} + \frac{1}{2} s_{2i}^{2} + \frac{1}{2\lambda} \widetilde{W}^{T} \widetilde{W}, \quad (13)$$

where Γ is definite matrix. $\lambda > 0$; $\widetilde{W} = W^* - \widehat{W}$.

Setting
$$\hat{k} = -s_{2i}\Gamma^{-1}w$$
 and $\widetilde{W} = \lambda h(s)$, then

$$\dot{V}_{i} = k_{1} |s_{1i}|^{\alpha_{1}} |\dot{s}_{1i}| + \alpha s_{2i} = -k_{2} |s_{2i}|^{1+\alpha_{2}}. \tag{14}$$

Obviously, \dot{V}_i is half negative. Therefore, the system is stable. According to the invariance principle, subsystem $\{\dot{s}_{1i} = s_{2i}, \ \dot{s}_{2i} = \rho_i v_i + u_{ki}, \ \forall t \geq t_{iF}\}$ is asymptotically stable globally in the equilibrium point.

According to the theory of finite-time stability [19], when $\rho_1 = 2 - \alpha_2$ and $\rho_2 = 1$, the subsystem is homogeneous system and the system's degree of homogeneity is $\eta = \alpha_2 - 1 < 0$. In other words, the *i*th subsystem $\{\dot{s}_{1i} = s_{2i}, \ \dot{s}_{2i} = \rho_i \nu_i + u_{ki}, \ \forall t \geq t_{iF}\}$, is globally finite-time stable. Similarly, other subsystems are globally finite-time stable and the system (see (8)) is globally finite-time stable after combination.

4. Analysis of Numerical Simulation

The effectiveness of the finite-time stable nonsmooth control algorithm based on the building structural vibration of actuator failure is verified. A three-layer building structure is simulated by three control methods including nonsmooth control, LQR control, and no control. Each floor is equipped with actuators to provide control force resisting earthquake action for structure. And the system subjected to the earthquake wave called El Centro of external disturbance signal and 15% of the actuator failure after 3 seconds is assumed. Maximum of earthquake acceleration is $a_{\rm max}=3.417~{\rm m/s^2}$. The parameters of finite-time stable nonsmooth control are $\alpha_1=0.3, \alpha_2=0.46, k_1=4,$ and $k_2=3.$

The mass matrix, damping matrix, stiffness matrix, and position matrix of example 1 are as follows:

$$M_{1} = 10^{3} \times \begin{bmatrix} 53.2 & 0 & 0 \\ 0 & 61.3 & 0 \\ 0 & 0 & 54.5 \end{bmatrix} \text{ (kg)},$$

$$C_{1} = 10^{5} \times \begin{bmatrix} 32.569 & -9.0571 & 0 \\ -9.0571 & 22.666 & -7.6198 \\ 0 & -7.6198 & 13.662 \end{bmatrix} \text{ (Ns/m)},$$

$$K_{1} = 10^{6} \times \begin{bmatrix} 230.6 & -115.3 & 0 \\ -115.3 & 190.2 & -74.9 \\ 0 & -74.9 & 74.9 \end{bmatrix} \text{ (N/m)},$$

$$E_{1} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix}.$$

In example 1, the contrast simulation curves of the displacement, velocity, acceleration response, and the control force for each floor are shown in Figures 1–4 under no control, LQR control, and nonsmooth control.

As is shown in Figures 1–4, nonsmooth control algorithm has been more effective than LQR control algorithm with actuator failure. The required control forces of two control methods have a little difference. However, nonsmooth control algorithm has been improved more than LQR control algorithm. In order to further analyze the effect of nonsmooth control, LQR control, and no control, the maximum displacement and maximum acceleration of each layer in the above simulation results are counted. The results are shown in Tables 1 and 2.

As is shown in Tables 1 and 2, compared with no control, the maximum displacement of the first, second, and third floor decreased by 85%, 88%, and 91% in LQR control. The maximum acceleration is also reduced by 23%, 8%, and 9%. Nevertheless, compared with LQR control, the maximum displacement of the first, second, and third floor is decreased by 74%, 77%, and 77% in nonsmooth control, respectively. And the maximum acceleration values are all reduced by 93%.

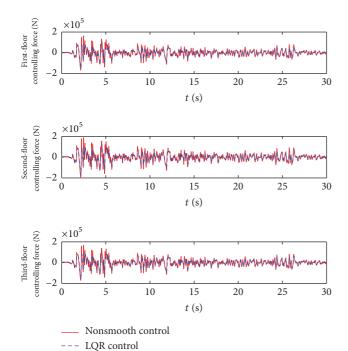


FIGURE 1: Control force of example 1.

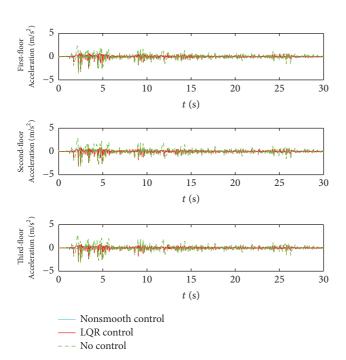
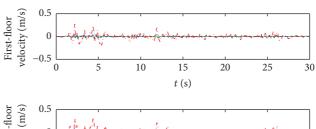
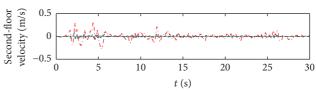


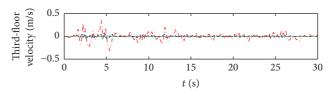
FIGURE 2: Acceleration response of example 1.

TABLE 1: Maximum displacement of each layer for example 1 (mm).

Control strategy	Nonsmooth control	LQR control	No control
First floor	1.8	6.8	44.0
Second floor	1.9	8.3	71.9
Third floor	1.9	8.3	91.3

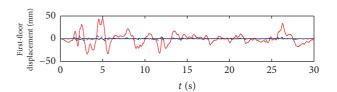


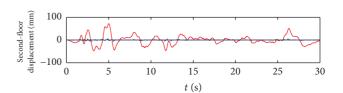


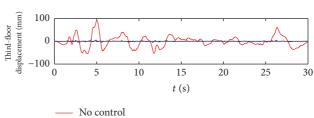


Nonsmooth controlLQR controlNo control

FIGURE 3: Velocity response of example 1.







Nonsmooth control
 LQR control

FIGURE 4: Displacement response of example 1.

TABLE 2: Maximum acceleration of each layer for example 1 (m/s²).

Control strategy	Nonsmooth control	LQR control	No control
First floor	0.2117	2.8595	3.6929
Second floor	0.2131	3.0286	3.3022
Third floor	0.2131	2.9183	3.2125

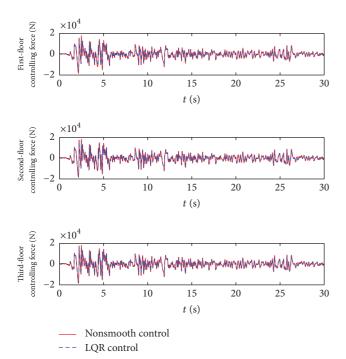


FIGURE 5: Control force of example 2.

The model parameters of example 2 are as follows:

$$M_{2} = \begin{bmatrix} 5897 & 0 & 0 \\ 0 & 5897 & 0 \\ 0 & 0 & 5897 \end{bmatrix} (kg),$$

$$C_{2} = 10^{3} \times \begin{bmatrix} 125 & -58 & 0 \\ -58 & 115 & -57 \\ 0 & -57 & 57 \end{bmatrix} (Ns/m),$$

$$K_{2} = 10^{3} \times \begin{bmatrix} 72825 & -39093 & 0 \\ -39093 & 67714 & -28621 \\ 0 & -28621 & 28621 \end{bmatrix} (N/m),$$

$$E_{2} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix}.$$

$$(16)$$

In example 2, the contrast simulation curves of the displacement, velocity, acceleration response, and the control force for each floor are shown in Figures 5–8 under no control, LQR control, and nonsmooth control.

As is shown in Figures 5–8, nonsmooth control algorithm is also more effective than no control and LQR control algorithms. At the same time, the maximum displacement and acceleration of each layer from the results of the second example are counted. The results are shown in Tables 3 and 4.

As is shown in Tables 3 and 4, compared with no control, the nonsmooth control declined 96%, 98%, and 98% in the maximum displacement of the first, second, and third floor. The maximum acceleration values are all reduced

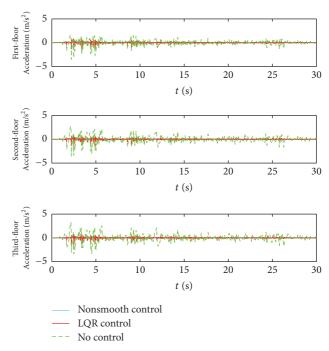


FIGURE 6: Acceleration response of example 2.

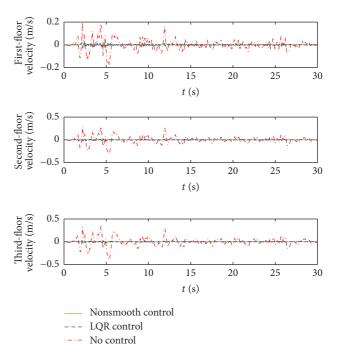


FIGURE 7: Velocity response of example 2.

by 94%. And, compared with LQR control, the maximum displacement of the first, second, and third floor is decreased by 18%, 24%, and 27% in nonsmooth control, respectively. At the same time, the maximum acceleration values are all reduced by 85%.

According to the above two examples, with the case of external distraction and actuator failure, two control methods can give good control force for displacement. However, with

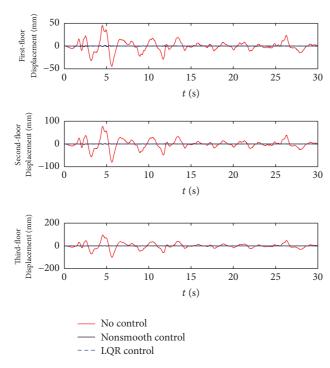


FIGURE 8: Displacement response of example 2.

TABLE 3: Maximum displacement of each layer of example 2 (mm).

Control strategy	Nonsmooth control	LQR control	No control
First floor	1.8	2.2	45.3
Second floor	1.9	2.5	80.2
Third floor	1.9	2.6	100.8

TABLE 4: Maximum acceleration of each layer of example 2 (m/s²).

Control strategy	Nonsmooth control	LQR control	No control
First floor	0.2090	1.4286	3.5473
Second floor	0.2135	1.4604	3.5473
Third floor	0.2137	1.4598	3.3957

nonsmooth control, structural vibration is suppressed effectively better than LQR control. And interstory displacement is controlled within a small range. The displacement, velocity, and acceleration tend to a small range of vibration better and to be stable lastly. Thus, nonsmooth control algorithm can better protect the building structure from damage of the earthquake compared with LQR control algorithm.

5. Conclusion

Aiming at the problem restraining nonlinear vibration of the building structure, a structure is mathematically modeled and analyzed. Then, according to the theory of finite-time stability and the analysis of actuator failure, nonsmooth control with RBF neural network is designed for the problem of structural vibration. And the stable analysis of the system is demonstrated. Finally, nonsmooth control, LQR control,

and no control are compared by analysis. The control system is affected by seismic wave called El Centro. At the same time, the numerical simulation of the model with strong nonlinearity is studied. The above works verified the feasibility and effectiveness of nonsmooth control algorithm. In this paper, uncertainty and external perturbation estimation of the parameters are taken into account in the simulation. On this basis, further analyses of the systematic robustness and antijamming have theoretical and practical significance, which are worth studying further.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This work is supported by National Natural Science Foundation (NNSF) of China under Grant no. 51478132, Guangzhou City College Scientific Research Project under Grant no. 1201630173, and Science and Technology Planning Project of Guangdong under Grant no. 2016B090912007.

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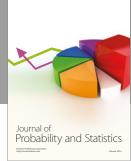
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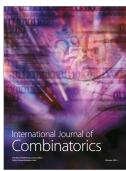








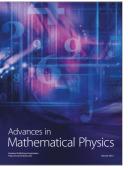


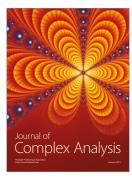




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