Tracking the Evidence

Brian Weatherson

Contribution to symposium on Sherrilyn Roush's *Tracking Truth*

Philosophy of Science Association conference, 2006

In *Tracking Truth*, Sherrilyn Roush puts forward an analysis of knowledge based around the idea that we can know anything where our beliefs track the truth, plus anything entailed by these beliefs. I'm going to argue that this makes knowledge much too easy. In particular, when we consider complex propositions, tracking is too easy to give us knowledge, and this combined with closure leads to very odd results. I won't develop this at all, but one tacit suggestion running through this will be that Roush's tracking theory might do better as part of a theory of evidence than as a theory of knowledge. The idea would be that some of the complicated propositions I'll discuss could not be part of our basic evidence, so my counterexamples won't tell against it.

Let's start with discussing the role of closure in the theory of knowledge. There are two big closure principles that epistemologists have worried about in recent years. They are what I'll call (SPC) and (MPC).

(SPC) If S knows that *p*, and S is able to competently deduce *q* from *p*, then S is able to know that *q*.

(MPC) If S knows each of $p_1, \ldots, p_n$, and S is able to competently deduce *q* from $\{p_1, \ldots, p_n\}$, then S is able to know that *q*.

There are restricted versions of these principles that seem somewhat more compelling.

(RSPC) If S knows that $p \wedge q$, and S is able to competently deduce *q* from $p \wedge q$, then S is able to know that *q*.

(RMPC) If S knows that *p*, and S knows that *q*, and *S* is able to competently deduce $p \wedge q$ from $\{p, q\}$, then S is able to know that $p \wedge q$.

Although many epistemologists have tried to deny (SPC), few have been willing to deny (RSPC). Indeed, a fairly natural way to attack a theory that denies (SPC) is to show that the theory implies denials of (RSPC). For example Kripke, in his unpublished objections to Nozick's tracking theory, does pretty much just this. He shows that according to Nozick, a certain S is able to know that there is a red barn in front of him, but not that there's a barn in front of him. And this seems to be absurd.

Now obviously Roush's theory is not vulnerable to that criticism. She thinks that tracking is important to knowledge, but it is not all there is. Roughly, we can think of her theory as a new and interesting form of foundationalism. The foundations are what we track, and we build on the foundations by drawing out logical consequences. This might look like a nice improvement over more Cartesian versions of foundationalism in three respects. First, it does not require any dubious version of privileged access to phenomenal states. Second, it lets our knowledge start with knowledge of what is in the world, not of our mental states, and this is much more developmentally plausible than Descartes' theory. Third, it says we can restrict the way we build on the foundations to just logical entailments, so we don't have to build a theory of inductive knowledge in as an extra feature of our foundationalism. These are all nice touches, and we should consider very closely whether we can accept a theory that has them all.

Now, Roush endorses both (SPC) and (MPC), and hence both (RSPC) and (RMPC). (In fact she endorses somewhat stronger claims than this, since she merely requires knowledge of implication, not competent deduction. But I think it's pretty clear that (SPC) and (MPC) follow from her views.) But there's a worry that instead of making it too hard to know conjuncts, as Nozick does, she makes it too easy. So what I'll do here is go over three kinds of cases where adding closure, in particular adding (RSPC), seems to lead to it being too easy for subjects to know things. The cases I go over here may appear to be quite disparate, but they have something important in common. The 'foundations', the propositions we do track, are intuitively quite complex. Importantly, they are propositions that could not be the contents of perceptual experience for creatures like us. So perhaps what I say wouldn't tell against a tracking theory of evidence.

1. Lotteries

Assume we have a fair lottery with 100 tickets in it, and a first prize of $10 million. Call the holder of ticket i, h_i, for $i \in \{1, \ldots, 100\}$. The lottery will be won, as it turns out, by h_{100}. For each i, define L_i to be the proposition that h_i will lose the lottery, W_i to be the proposition that h_i will win the lottery, and A_i to be the proposition that h_i will be able to afford a trip later this year on the Virgin Galactic space ship, which costs around $200,000. Assume finally that each of the h_i will survive the year, and will not meet any financial catastrophe so bad that even if they won the lottery, they would not be able to afford the trip.

Now consider an S who knows the lottery is fair, and that it hasn't yet happened. For each i in $\{1, \ldots, 99\}$, S believes L_i. Obviously each of these beliefs do not track the truth. In Nozickian terms, even if they were false, S would still have the belief. Things are a little harder to state in Roush's terms, but it is still pretty clear that these are not tracking. For Roush, S's belief that p tracks the truth iff the following two probabilistic claims is true, where Bp means S believes that p.

$Pr(Bp \mid p) > 0.95$

$Pr(\neg Bp \mid \neg p) > 0.95$

Substituting L_i for *p*, we see easily enough that the first of these is true for each *i*. But the second is not true. Even if W_i, *S* would still believe L_i. So this belief is, in Roush's terms, is adherent but not sensitive. So we cannot directly conclude that *S* knows L_i, for each *i* for which L_i is true.

But let's consider a little more what *S* might know about h_1. She presumably could track the truth that $\sim A_1$. One way in which h_1 could afford to go on the spaceship is to win the lottery. But that's a fairly low chance. It might well be that there are other ways in which h_1 could afford the trip. For instance, h_1 is a candidate for a very high paying job on Wall Street, and *S* has been following her chances closely and accurately. *S* falsely believing that h_1 will get the job is basically out of the question. Since getting the job is by far the most probable way in which A_1 could be true, it is very probable that if A_1 were true, *S* wouldn't believe it to be false. (Not certain, because A_1 could be false because W_1 is true, but probable enough.) And if *S* believes $\sim A_1$, it is probably true, simply because $\sim A_1$ is probably true. So *S* knows $\sim A_1$.

Now as Roush points out, *S* can't conclude from that that L_1 is true, since L_1 & $\sim A_1$ is a logical possibility. But it could well be true that *S* knows $W_1 \supset A_1$, even though this isn't a logical truth. Assume, for example, that the most antecedently probable way for that to be false is for h_1 to gamble all of her lottery winnings away on football. But *S* knows that h_1 has been attending Gamblers Anonymous meetings, hasn't gambled on football for a while, and in general is displaying all of the symptoms of someone cured of their gambling addiction. The most probable scenarios in which $W_1 \supset A_1$ is false are ones where h_1 is not displaying these symptoms. So *S* tracks $W_1 \supset A_1$ and hence knows it. And *S* of course knows $\sim W_1 \supset L_1$.

Putting together all of her knowledge, *S* comes to know L_1. Not *probably* L_1, but L_1 itself. She knows this because it follows from things she knows. Each of the things she knows she doesn't track perfectly, but she tracks them well enough for knowledge. So she knows L_1.

So far the story has been somewhat realistic. The result we get is not intuitively great, but it is something I think we could, if required, live with. Now to make the story unrealistic, and the result intolerable. *S* is in a similar position with respect to each of h_2 through h_{99}. And similarly she comes to know each of L_2 through L_{99}. Using her knowledge of how the lottery is set up, she also knows $(L_1 \wedge \ldots \wedge L_{99}) \supset W_{100}$. And so she comes to know W_{100}. This is a really bad result, though Roush's theory is far from the only theory of knowledge to face troubles around here.

Although we have used (MPC) in general in this argument, and not just (RMPC), it seems that (RMPC) is what is really doing the trouble. It would be tolerable, if a little surprising, to discover that *S* knew each of L_1 through L_{99}. It is only when we add in (RMPC) that we get an utterly intolerable result.

In general, if we allow that agents can know propositions that are not absolutely certain, and we accept (RMPC), we have to do some fancy work to avoid the conclusion that agents can know propositions that are arbitrarily improbable. And unless we say that perfect tracking is required for knowledge, we will end up with the conclusion that agents can know propositions that aren't certain. If we required perfect tracking, then we could block the argument to the conclusion that S knows $W_1 \supset A_1$. It would be surprising, however, if *that* was the weak link in the argument.

2. Nozick-Tracking and Conjunction

If we combine closure with Nozick-tracking, we get the view that it is much too easy to know facts about the world that are counterfactually resilient. By Nozick-tracking, I mean the view that to turn a true belief that p into knowledge, the counterfactuals $p \;\Box\!\!\rightarrow Bp$ and $\neg p \;\Box\!\!\rightarrow \neg Bp$ have to be true, where B again is the agent's belief operator, and $\Box\!\!\rightarrow$ is the Nozickian counterfactual conditional. To see this, let L be any fact about the world such that the world would be *very* different were it true. Let L, for example, be the proposition that there are more flies than pandas in the world. S believes this, but only because has long believed that pandas are extinct, and she (validly) deduced L from this plus her knowledge that some flies exist. Assume that S is playing five-card poker, and that S is perfectly tracking which cards are in her hand. Let C be the proposition that S was just dealt a hand consisting of 8♣, 3♣, J♦, 9♥, 4♠, and assume C is true.

Now S believes that $L \wedge C$ is true. If this proposition weren't true, she wouldn't believe it. That's because all the nearby worlds in which $L \wedge C$ is false are worlds in which C is false. And that's because it is much easier, intuitively speaking, for C to be false than it is for L to be false. If C were false, S wouldn't believe it, so she wouldn't believe $L \wedge C$. Moreover, in all the nearby worlds in which $L \wedge C$ is true are worlds in which S believes it. That's because her belief that pandas are extinct is very counterfactually resilient, and S knows what cards she has. So according to Nozick's theory, S knows $L \wedge C$.

This looks to be a fairly bad result. If we also accept (RSPC), we can infer that S knows that L, even though her belief in L doesn't track the truth. Since S only believes that L because she inferred it from a false proposition, this looks like a bad result. Since S also inferred $L \wedge C$ from a false premise, even the conclusion we get from principles Nozick is committed to looks fairly bad. This seems to be an instance of the general principle that (RSPC) doesn't lead us from acceptable premises to unacceptable conclusions.

3. Probability-Tracking and Conjunction

It isn't clear that this very argument goes through against Roush's theory. That's because it isn't clear just exactly what we should take $Pr(L)$ to be. But I think we can come up with some other results that look fairly implausible by focussing on conjunctions. Roush does discuss this problem for Nozick, but I think that what she says doesn't get to the heart of the problem.

> This is far from enough with conditional probability where in order for scenarios in which q is false to be out of consideration when we judge whether the subject tracks the conjunction $p \wedge q$ it must be that $\Pr(\neg q \mid \neg(p \wedge q))$ is less than 0.05. The only easy ways I can see for this to happen are either if p is a necessary falsehood or q is a necessary truth … So it must be that q is a necessary truth, or at least it is true in some very sturdy sort of way. (110-11)

But our above example should lead us to worry about this. Consider again our subject S playing cards, and as above assume that C is true. Let A be any proposition S truly believes, that has a different subject matter to C, so we can take as given that A and C are probabilistically independent. In order for S to know $A \wedge C$, and hence to be able to know A, the following conditions have to be met.

1. $Pr(B(A \wedge C) \mid A \wedge C) > 0.95$
2. $Pr(\neg B(A \wedge C) \mid \neg(A \wedge C)) > 0.95$

If A is believed in all nearby worlds, then it seems 1 will be satisfied. In that case, we'll say that the belief in A is **resilient**. Assuming that S perfectly tracks whether C is true, then it is sufficient for 2 to be true that $Pr(\neg C \mid \neg(A \wedge C)) > 0.95$. Since $Pr(\neg C) > 0.95$, and presumably $Pr(\neg C \mid \neg(A \wedge C)) > Pr(\neg C)$, it seems to be the case that this will be true. So we get the result that any A that S truly believes, and believes in all nearby worlds, is known. That is to say, the sensitivity requirement does not do any work.

It might be objected that I'm assuming here that we should use the real probability of C when working out whether S's belief that $A \wedge C$ is sensitive. And Roush says that we should allow the probability of certain propositions to vary when working out sensitivity. In particular, we should only hold the probability of a proposition q fixed, when working out the sensitivity of p, if q is an atomic sentence (or disjunction of atomic sentences) such that the following two conditions are met.

$$|Pr(q \mid p) - Pr(q \mid \neg p)| < |Pr(p \mid q) - Pr(p \mid \neg q)|$$
$$|Pr(\neg q \mid p) - Pr(\neg q \mid \neg p)| < |Pr(p \mid q) - Pr(p \mid \neg q)|$$

If we let $p = A \wedge C$, and $q = C$, it is clear enough that the first of these conditions is *not* satisfied. To get around this, assume that S has two opponents, O_1 and O_2. And assume that we have a language such that for each possible distribution of cards to O_1 and O_2, we have an atomic proposition C_i. There will be 5,178,066,751 such C_i, and we will redefine C as the disjunction of each of these. For distinct i, j, it is a fixed point of the story that $\neg(C_i \wedge C_j)$, so we'll assume that $Pr(C_i \wedge C_j) = 0$. For each C_i, if we let $q = C_i$, and $p = A \wedge C$, then the first above equation becomes:

$$|Pr(C_i \mid A \wedge C) - Pr(C_i \mid \neg(A \wedge C)| < |Pr(A \wedge C \mid C_i) - Pr(A \wedge C \mid \neg C_i)|$$

Since C_i entails C, the third term becomes $Pr(A)$, and the fourth term is less than $Pr(A \wedge C)$, since $\neg C_i$ mildly disconfirms C. Since the two terms on the left-hand side are both very small, both less than 1 in 5 billion, as long as A is somewhat probable than that, the left-hand side will be less than the right-hand side. A similar argument shows that the second equation holds, though here we show that the two terms are both extremely close to 1. If $Pr(A)$ is less than $Pr(C_i \mid C)$, this argument won't work, but we can rerun the example with more opponents to find propositions about the card distribution that satisfy Roush's equations. So these propositions about the card distribution should be held fixed when judging sensitivity. Since C is a disjunction of such propositions, and we know antecedently that no two disjuncts hold, the probability of C should also be held fixed, so this argument goes through.

4. Dog the Dogmatist

In the previous section I argued that if S is playing cards, and perfectly tracking the cards, then any of their true, resilient beliefs are knowledge. I'll conclude by looking at a way that we can weaken the resilience requirement. Consider a particular card-player, Dog. Dog is fairly dogmatic, so he is certain that any instance of $Bp \supset p$ is true, where Bp now means *Dog believes that p*. As well as tracking the cards perfectly, Dog also tracks his own belief states fairly well, and knows what he believes. So the probability of $B\neg Bp$ given $\neg Bp$ is extremely high.

Now if Dog is really dogmatic, for a large class of p, Dog's belief $Bp \supset p$ will be one he holds in all nearby worlds. So the probability of Dog believing this conditional, conditional on it being true, is pretty high. So his dogmatic belief, which is of course insensitive, will be resilient.

More or less as above, let C_{Dog} be the proposition that Dog was just dealt a hand consisting of 8♣, 3♣, J♦, 9♥, 4♠, and assume C_{Dog} is true. Let p be any proposition for which the prior probability of Bp is low, but nevertheless Dog truly believes p, and he knows he believes it. Now consider the proposition $(Bp \supset p) \wedge C_{Dog}$. Dog believes this proposition. Since $Bp \supset p$ is resilient, the reasoning of the above section suggests that this belief satisfies all the tracking requirements, so it is a piece of knowledge. By (RSPC), it follows that Dog knows $Bp \supset p$. And by (SPC), and his knowledge of Bp, it follows that he knows p. But we got to this conclusion without any consideration of the epistemic properties of p. All that we required was that p is true, believed, and that Dog is dogmatic in his belief that he believes only truths. Obviously that isn't enough for knowledge.

Equally obviously, $(Bp \supset p) \wedge C_{\text{Dog}}$ is not the kind of proposition that traditional foundationalists would have said can be foundational in a theory of knowledge. And it certainly doesn't look like a proposition that can be the subject of a perceptual experience. (It presumably could be the content of some testimony, so maybe a foundationalist who includes testimony in the foundations would want to allow it foundational status.) So perhaps the recursive tracking theory could avoid these examples if we said that tracking only entails knowledge with regard to perceptual belief. This is an intriguing possibility, possibly one of the most intriguing possibilities to come out of Roush's very rich and interesting book.