GENERAL RELATIVITY AND QUANTUM THEORY—ONTOLOGICAL INVESTIGATIONS

1. INTRODUCTION

It is common to note that the two great theoretical frameworks of twentieth-century physics, quantum theory and general relativity, are prima facie incompatible. Within the physics community, the incompatibility is largely discussed on a technical level. In this paper, I would like to frame the problem more conceptually, focusing on the respective ontologies of the two frameworks.

When I speak of ontology, I have in mind the idea that theories make use of a formal (often mathematical) language for talking about the world. The ontology of the theory then just consists of the objects and properties to which the names and predicates of the language refer. Although this ontology may vary according to different ways of regimenting the language, there is frequently a common or standard formulation. Thus the ontology of astronomy is roughly understood to be planets, stars and galaxies (the objects), which have their various masses and shapes ("internal" properties) and positions and velocities ("external" properties).

For the most part, fundamental physical theories refer only implicitly to the objects themselves, for it is the properties that are mathematically quantified. The language of Newton's mechanics and his theory of gravitation enables one to refer to any object having a mass, a position, and a velocity. Thus Newton's theory is as much about planets as it is about apples. Maxwell's theory of electromagnetism refers to objects that have the property not only of mass but charge as well. In that respect it is not significantly different from Newton's theory. But it also refers to something called the electromagnetic field. The values of the field are properties, but they are properties not of Newtonian objects but of spacetime itself.

Historically, Maxwell's theory may be the first theory in which spacetime plays the role of an "object", a bearer of properties. Previously, space and time had simply been thought of as the background in which objects lived. (Of course, one can anachronistically reconstruct, e.g., Newtonian theory from the modern perspective.) As we shall see, general relativity expands on this development, while quantum theory retains a largely passive view of spacetime while suggesting entirely new ways to think about properties.

2. GENERAL RELATIVITY

General relativity is among other things a theory of gravitation, which is to say it is a theory about the interactions of massive bodies. The sources of gravitational attraction include not only the mass and energy inherent in "bodies" but the energy inherent in various fields, as well.

Classical (non-quantum) general relativity is almost universally formulated as a geometric theory, in which gravitation is manifested as curvature of spacetime. This geometric approach builds on Minkowski's formulation of special relativity as a theory of "flat" 4-dimensional spacetime, where spacetime is represented by a 4-dimensional differential manifold with Minkowski (flat) metric. General relativity extends this, using the tools of pseudo-Riemannian geometry to talk about more general spacetimes, ones with a Lorentz metric g_{ab} (of which the Minkowski metric is a special case). The metric carries information about the distance between points on the manifold, and thus encodes the geometric structure of the manifold. The relevant curvature measures are given by the Riemann tensor R^a_{bcd} , and the related Ricci tensor $R_{ab} = R^c_{acb}$ and scalar curvature $R = g^{ab}R_{ab}$. These curvature tensors are all made up of derivatives of the metric; they describe curvature by describing how the metric changes when one moves in various directions.

The curvature of spacetime is constrained by the stress-energy-momentum distribution T_{ab} , which in addition to encoding the mass-energy at each spacetime point also encodes the flow of mass-energy. Einstein's equation quantifies the way in which T_{ab} constrains the metric g_{ab} :

$$G_{ab} := R_{ab} - \frac{1}{2}g_{ab}R = 8\pi T_{ab} . {1}$$

Thus we have a sort of relativistic generalization of Newton's theory of gravity, in which the spatio-temporal distribution of mass and energy determines the gravitational field, here represented by curved spacetime.

The language of general relativity is the language of tensors on a manifold, the language of differential geometry. Insofar as one regards the points of the manifold as representing spacetime points, one must regard the stress tensor T_{ab} , the metric tensor g_{ab} , and the latter's associated curvature tensors, as assignments of physical quantities (energy, momentum, spacetime curvature) to the various points. This language of

properties at points is suggestive of a field ontology, but the stress tensor need not be constructed purely out of fields. Bulk matter will do as well—one simply assigns a matter density and velocity to each point in spacetime occupied by the massive object.

The implicit assumption involved in construing tensors as fields in spacetime is that the points of the differential manifold are points of spacetime. But stripped of the metric, the bare manifold has very little of the structure one associates with spacetime. Among other things, there is no causal structure, no notion of whether two points are space-like or time-like related. This suggests that it is only in the presence of a metric tensor that it makes sense to refer to points as points in spacetime. In that spirit, we would say that the statement that a tensor field such as T_{ab} represents a distribution of properties in spacetime makes sense only in the presence of a metric tensor.

Now, the idea that the stress tensor T_{ab} represents a spacetime property distribution only in the presence of a metric is not really problematic, since the metric is already built into the stress tensor. However, what are we to say about the metric itself—does it represent a spacetime distribution of properties? This is a tricky question. On the one hand, once we have a metric, we have spacetime points, in the sense that the metric defines spatial and temporal distances between points. These points have definite physical properties attributed to them—the derivatives of the metric describe the gravitational field. On the other hand, from a strictly mathematical perspective, the metric is an attribution of properties to points on the manifold, and the manifold does not in itself represent spacetime. From the former perspective, the metric is a field like any other tensor field, and one might think it should be treated as such. From the latter perspective, the metric plays a rather special role, and it is not clear that it should be regarded as a conventional field, in the sense of a spacetime property distribution. Support for the former view comes mainly from practical concerns—after all, one can measure the gravitational field just as one can measure the electromagnetic field. Support for the latter view comes primarily from formal concerns, in particular the role the metric plays in the definition of other fields and their stress tensors. We shall not settle this issue here!⁴

See [1], though, for an algebraic approach to general relativity.

For example, the stress tensor for a massless scalar field is given by $T_{ab} = \nabla_a \phi \nabla_b \phi - \frac{1}{2} g_{ab} (\nabla_c \phi \nabla^c \phi)$.

It must be said, however, that the strength of the gravitational field at a point is not well-defined, unlike the strength of the electromagnetic field. This despite the fact that the Riemann tensor $R^a_{\ bcd}$ is well-defined at a point.

⁴The interested reader will find a lucid discussion of this issue in [2], which contains

3. QUANTUM THEORY

There are various quantum theories. There is quantum mechanics, by which I mean the quantum theory of non-relativistic particles. Then there are the quantum field theories, which are theories of systems with infinitely many "degrees of freedom": these include theories of the fundamental forces such as electroweak theory and QCD, as well as theories of various many-body systems, such as superconducting matter, superfluids, plasmas, electrical conductors, etc.

Let us consider quantum mechanics first. In quantum mechanics, one works with fixed numbers of objects, such as elementary particles, atoms, or protons. A collection of these objects constitutes a "system." The "state" of a system is technically a map from the configuration space of the system to the complex numbers. For instance, two unconstrained massive, spinless particles in three-dimensions have a configuration space of \mathbb{R}^6 , and the map is given by a (normalized) "wave-function" that assigns a complex-valued "amplitude" to each point in the configuration space. The square of the amplitude, integrated over a region of the space, gives the probability of finding the system in one of the configurations in that region. For a single, spinless particle, the configuration space is typically three-dimensional physical space, and the integral of the squared amplitude evaluated over a given region of this space yields the probability of finding the particle in that region.

Note that the complex nature of the amplitude enables the wave-function to encode not only position information, but momentum information as well. The momentum information is extractable in various ways, among them taking the Fourier transform of the original wave-function, and evaluating the squared amplitudes of the resulting "momentum-space" wave-function. What is important for our purposes is simply that a maximally specific description of the state of any quantum system encodes probabilistic distributions of position and momentum (among other properties), and that these distributions resist being "squeezed" past a certain point. What this means is that, after a certain point, a system may have its position specified more closely only at the cost of specifying its momentum less closely. Thus the precise properties of any quantum system are inherently indefinite.

Despite the uncertainty in the *properties* of atoms, electrons, or what have you, one can still speak of localized, particle-like objects which *possess* the properties, even though they only "possess" them probabilistically. Thus the ontological situation in quantum mechanics is one in which there are a definite number of objects at any given time, and

further references to the literature.

in which these objects persist over time. So far, so classical. What is unclassical, again, is that the objects do not have definite properties at a given time (of position, momentum, spin, etc.) and do not have definite trajectories over time. Furthermore, if the systems have interacted, these properties will be "entangled", so that, e.g., the position of one object will in general be correlated with the position of another.

The ontological situation in quantum field theory is somewhat different. Indeed, the formal apparatus of the theory was developed to accommodate physical situations in which the number of particles can change. The general treatment is thus one of systems with "infinite degrees of freedom." The idea of infinite degrees of freedom is suggestive of classical fields, and indeed there are many cases in which quantum fields are analogous to classical fields, in that at each point in spacetime, one has operators (technically, operator-valued distributions) that represent various field properties (such as the electric field in the x direction), and that allow one to extract the probability of observing various values for those properties at the various points in space. The possibility of describing such quantum systems either in terms of an indefinite number of particles, or in terms of probabilistic values of a field, is the essence of wave-particle duality.⁵

So we have a situation in which there is a sort of wave-particle duality. But because the number of particles is indefinite, it would seem odd to say that the ontology is *particles*. One of the founders of quantum field theory, P.A.M. Dirac, shared this view:

If one can create particles, then the question of which are the fundamental constituents of matter ceases to have a definite meaning. Previously, one could say that one only had to analyze a piece of matter as far as possible, and get at the ultimate constituents in that way. But if one can create particles by atomic interactions, then one cannot give a definite definition for an elementary particle. [3, p. 19]

In an important sense, a sort of field ontology appears to be forced on us in relativistic quantum theory in a way that it is not in classical, relativistic particle theory, for not only the properties of the particles but their very number is indeterminate.

One might bite the bullet at this point and argue that, just as prop-

Note that the "wave–particle duality" often attributed to the non-relativistic quantum theory of particles is a bit of a misnomer, since the wave-function for a system with more than one particle cannot be thought of as a wave in three-dimensional space. (In an n particle system the wave function typically exists in a space of 3n dimensions.)

erties of objects can be indefinite, so can the objects themselves, and therefore quantum field theory can be construed as being a theory about particles. However, one runs into real trouble when considering quantum fields in curved spacetime, which is to say in the presence of gravity. In such situations, the vacuum ("no particle") state is observer-dependent, and since the particle content of the theory turns out to depend on the vacuum state, one has no preferred notion of particles. One observer's vacuum turns out to be another observer's "bath" of particles. (See [4] for further discussion.)

It would appear, then, that quantum field theory might be better said to simply be about fields. The field representation of a massless scalar field, without spin, involves an algebra of operators $\widehat{\phi}(\vec{x},t)$ and canonically conjugate operators $\widehat{\pi}(\vec{x},t)$. As the label indicates, the position and time serve to parametrize these operators. As noted above, they describe a property distribution about points in space at a given time, and consequently invite the interpretation that they represent a quantum version of a field. On this interpretation, the principle difference between a quantum and a classical field is that the property distribution is indeterminate and probabilistic.

There are two main problems with this interpretation of quantum fields as probabilistic versions of classical fields. The first is that there is no particular reason to privilege the parametrization in terms of \vec{x} and t. One can equally well Fourier-transform, and represent the quantum field by an algebra of operators parametrized by the momentum \vec{p} and the time. This suggests that the field interpretation is at best optional. The second problem is that any field with internal degrees of freedom will have an operator algebra parametrized by those degrees of freedom as well. So for instance a field whose degrees of freedom include position, the z component of spin and the third component of isospin would have an operator algebra parametrized as $\widehat{\phi}(\vec{x}, \sigma_z, I_3, t)$. Now, this *could* be interpreted as a spatio-temporal distribution of properties à la the classical field concept, though note that for every point in space at a given time one will have four operators $\hat{\phi}$ and four operators $\hat{\pi}$ (since spin and isospin have two possible values each). But what is worse for the field point of view is that it is perfectly plausible to construct a quantum field theory without any reference to space at all, simply because the spatial degrees of freedom play no privileged role in the formalism of the theory.

There are undoubtedly those who will argue that the idea of a quantum

field theory which is not in any sense a spacetime theory is absurd, the argument being that because observations take place at points in space at particular times, any theory must have spatial degrees of freedom in order for it to yield predictions for observations. Though this argument has some appeal, it is highly problematic, for it rather begs the question of how to represent observations in quantum theory. In general, I would suggest that we be wary of it, for one of the fundamental problems in physics is the reconciliation of quantum theory and gravity, and it may be just such prejudices regarding the nature of observation that obstruct the way toward a reconciliation of the two theoretical frameworks.

4. GENERAL RELATIVITY AND QUANTUM THEORY

Let me briefly rehearse the conclusions regarding the ontologies associated with general relativity and with quantum theory. General relativity, it will be recalled, accommodates both particles and fields as the material constituents, though it describes both in a field-like way, ascribing properties to points of the spacetime manifold. Its classical nature consists in the fact that all of these properties of the particles or fields are *definite* properties—at any point in space at any time, the stress-energy tensor takes on definite values. The primary ontological point of contention in general relativity is whether one ought to characterize the spacetime metric itself as some sort of field. If it is a field, then is it the same sort of field as other fields?⁷

Quantum theories, on the other hand, are characterized in part by the indefiniteness of the properties they describe. In quantum mechanics, the indefinite properties may be ascribed to underlying objects called "particles," but a full relativistic treatment involves a progression to the notion of quantum fields. Here a particle ontology is essentially untenable. Moreover, the internal (non-spatial) degrees of freedom which go along with quantum field theories suggest that it may furthermore be inappropriate to characterize the theories as theories of fields, in the sense of spacetime property distributions.

To begin to grasp the tension between general relativity and quantum theory, consider again Einstein's equation $G_{ab}=8\pi T_{ab}$. The stress tensor, on the right, invariably requires a specification of the properties of matter such that the matter has well-defined values for various quantities at each point in spacetime, and this is simply not obtainable from any quantum-theoretic treatment of matter. The four-momentum of a particle, for example, is not well-defined, both because it is impossible

This is a mild abuse of language. Technically, the degrees of freedom of the quantum field are the various (classical) functions $\phi(\vec{x}, \sigma_x, I_3, t)$, not the properties such as position, spin, and isospin, that parametrize them.

One argument against treating the gravitational field as "just another field" is that it is difficult to provide a notion of local gravitational energy.

to completely localize a quantum-mechanical particle, and furthermore because one must ultimately trade-off increased definiteness in position (at a time) with decreased definiteness in 3-momentum (or vice-versa).

How might one go about incorporating quantized matter into general relativity? One might try to "quantize" the stress tensor, the idea being that just as the classical tensor T_{ab} is essentially a function of the classical field variables ϕ and π , its replacement should be an operator \widehat{T}_{ab} which is a function of $\widehat{\phi}$ and $\widehat{\pi}$. This makes a fair amount of intuitive sense, since if one thinks that the physical matter under consideration has a quantum uncertainty associated with it, one would expect that its stress-energy-momentum properties would reflect this.

Quantizing the stress tensor, however, presents numerous difficulties. One, it immediately raises the heretofore postponed question of whether or not to quantize the metric, for the stress tensor is a function not only of the matter variables ϕ (reverting for simplicity to the covariant formalism and thus effectively incorporating π), but of the metric g_{ab} as well. I.e., do we want an equation of the form

$$\widehat{G}_{ab}\psi = 8\pi \widehat{T}_{ab}(\widehat{\phi}, \widehat{g}_{ab})\psi \tag{2}$$

or

$$G_{ab} = 8\pi \left\langle \widehat{T}_{ab}(\widehat{\phi}, g_{ab}) \right\rangle_{\psi} \tag{3}$$

or perhaps something else?8

The first equation (2) couples quantized matter to quantized spacetime. This implies a modified ontology for general relativity, whereby matter has indefinite properties, and whereby spacetime has indefinite properties as well. Note that the notion of a quantized spacetime, a quantized gravitational field, is suggestive of the more general notion of a quantum field theory as a specification of certain families of properties which are *not* necessarily spatio-temporal. In treating the metric as something to be quantized, as "just another field", we end up treating it as the most general sort of quantum field, as a field which is not essentially a spatio-temporal distribution of properties, since the quantization of the metric deprives one of a well-defined spacetime background.

Despite the compelling ontological unity suggested by (2), in which "everything" is a quantum field, the problems are legion. It perhaps suffices to note that no one has yet succeeded in quantizing vacuum

gravity, i.e., an equation of the form $\hat{G}_{ab}\Psi=0$, much less couple it to a quantized stress tensor, of which more below. 9 Part of the difficulty involved in doing so is to select an appropriate representation of the relevant operators. If one did have such a representation, then one could try to give (2) a physical interpretation. On the face of it, if one were told that the "system" was in some eigenstate of the stress tensor, then this equation would appear to allow one to determine the probability of observing various metrics given a certain stress-energy distribution. But note that an eigenstate of the stress tensor cannot be a state in which the stress-energy-momentum of the system has a definite classical value in the usual sense because it is quite unclear what it even means to say that the stress-energy-momentum is "definite" if the background metric is quantized. Definite at every point in space at some time? What is meant by "space" in the context of a quantized metric? In short, an equation such as (2) is riddled with both technical and conceptual difficulties.¹⁰ If something like it actually turns out to be mathematically viable, it will no doubt require a major conceptual change to make physical sense of it. There is every reason to think that a fully quantum theory of gravity would be incompatible with any ontological structure, be it that of general relativity or non-relativistic quantum theory, which makes essential reference to a background spacetime. (See [11] for discussion.)

What about equation (3)? This is known as the semi-classical Einstein equation; it couples quantized matter to classical spacetime. It is occasionally used when doing quantum field theory in curved spacetime, as a way of getting an approximate handle on the way in which the evolution of a quantum field affects the structure of the spacetime in which the field is embedded. However, its use in even this limited role is heavily circumscribed [4], and virtually no one takes it seriously anymore as a candidate for a fundamental theory [12]. One can see one aspect of the difficulty as follows. Suppose we have a massive object which is in a superposition of states, such that there is equal probability of finding it on either side of some room, with vanishing probability anywhere in the middle. Then according to (3), the gravitational field associated with this object would be the field one would classically obtain were the mass in the center of the room, since that is the expectation value for the position of the object (and one expects in this situation that the

Equation (2) is simply a heuristic. It is most closely analogous to the constraint equations of canonical quantum gravity. See [5] for a clear introduction to this research program.

See [6] and [7] for excellent reviews of quantum gravity and discussions of the sorts of problem one encounters. See [8] and references therein for news of recent progress in this area.

My quick dismissal notwithstanding, Rovelli (see [9], [10]) has recently proposed a way of dealing with some of the issues raised here.

expectation value of the stress tensor will be a simple function of the expectation value for the position). Now perform a position measurement. One will find the object on one side of the room or the other. So at the moment of measurement, when the object becomes localized, we must say that the metric changes discontinuously. Besides the fact that this seems intuitively unphysical, such a theory represents a *major* departure from the framework of general relativity!

Now, this quick example by no means constitutes a refutation of the semi-classical approach. For one thing, it could be that introducing measurement and reduction of the state vector is not playing the game fairly, that one must deal only with closed systems when considering the coupling of quantized matter to classical spacetime. However, there are equally serious difficulties for a closed system treatment. The conceptual problems are less obvious simply because the quantum theory of closed systems is problematic. But consider what happens if one adopts the many-worlds/decoherent histories approach to closed systems (see [13] for a review), in which the system continuously "branches". If one were to invoke the semi-classical equation (3) in this context, one would have a situation in which an observer would in theory be able to detect the branching, because the (classical) state of the gravitational field would be a function of the (quantum) matter distribution in all the branches, not just her own. The gravitational field would not, in general, have anything to do with the observed (within the branch) matter distribution. This is of course in contradiction with what we observe, and so it seems as if the closed system approach to semi-classical gravity fares no better than the open system approach. In short, semi-classical gravity seems to be a rather implausible halfway-house.

In the face of the apparent difficulties with putting together classical general relativity and quantum theory in the ways sketched above, it is notable that no one has yet come up with a formal argument that the quantization of the sources of the gravitational field implies that the field itself *must* be quantized, though such an argument does exist for the quantization of the electromagnetic field [14], [15]. This is related to the fact that in electromagnetism, the field strength couples to the charge of the source, whereas in gravity, it couples to the mass. If one considers the limit in which the mass goes to zero, the uncertainty principle predicts an enormous indeterminacy in the position or momentum of the source particle, and a correspondingly large indeterminacy in the value of the field. However, for gravity, the field *vanishes* as the mass goes to zero [16].

5. CONCLUDING REMARKS

The prima facie incompatibility of the general relativistic and quantum theoretic ontologies does not necessarily indicate an insuperable incompatibility. The discussion in this paper is offered more in the spirit of indicating that something has to give. In general relativity, we see some room for flexibility in the question of whether or not we take the spacetime metric to be a field in its own right, and in quantum theory we see the possibility, suggested by quantum theories with internal degrees of freedom, of conceiving of a quantum field as something quite unlike a classical, spatiotemporal field.

Of course, despite this flexibility, we still have to face the apparent incompatibility between the definite physical properties which are the province of general relativity, and the indefinite, probabilistic properties of quantum theory. It is notable, however, that general relativity is a typical classical theory in that the properties (of particles, of spacetime points, etc.) are *objectively* possessed, whereas in quantum theory, properties are indeterminate and become determinate (but *not* necessarily objective (see [17])) only on observation. An investigation of the notions of observation and objectivity in general relativity and quantum theory would be a useful complement to the ontological issues raised here.

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