Minkowski Space-Time and Thermodynamics

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I. Ever since Minkowski published his four-dimensional representation of space-time, the dominant view in physics and philosophy has been that time is a fourth dimension such that human perception of change and the passage of time is a mere illusion, due to our particular slicing of space-time. But four-dimensional space-time is a block universe. This conclusion takes the form of an inference from the measurable and observable evidence. Traditionally the block universe was inferred from the stipulation of relative simultaneity as a consequence of the Special theory of relativity (STR) (Eddington, Einstein, Gödel). But newer defences infer a static block universe from the well-known relativisitic effects: length contraction, time dilation, the twin paradox. The argument states that such relativistic effects would be impossible in a three-dimensional world. As they occur and are observed, it is legitimate to infer **a)** that the physical world is four-dimensional, and not just a mathematical representation, and **b)** that this four-dimensional world is static and timeless. (Lockwood 2005; Petkov 2005, Ch. 4) Yet it is by no means clear that Minkowski himself was a believer in the block universe. In his 1908 Cologne lecture on 'Space and Time' he speaks of a four-dimensional physics but concedes that a 'necessary' time order can be established at every world point. The conception of the block universe, however, focuses on Minkowski's geometric approach, which is based on his world postulate. But an alternative view has been in circulation since the 1910s according to which the nature of space-time has to be based on the behaviour of light. (Robb 1914, Cunningham 1915, Carathéodorys 1924, Schlick 1917, Reichenbach 1924) These axiomatic approaches constitute a light geometry, according to which the behaviour of signal propagation, under thermodynamic aspects, form histories of trajectories in space-time. It is the assertion of this paper that they give rise to a different inference regarding the nature of space-time. If we built our inferences to the nature of space-time on other aspects of the physical world, which nevertheless fall within the domain of the Minkowski space-time conception – dissipation and energy flows – we arrive at a dynamic conception of Minkowski space-time.

Note that this alternative view does not deny the four-dimensional reality of space-time. If we accept the four-dimensionality of the physical world, and then inquire whether it is 'static' or 'dynamic', it is important to go beyond mere kinematic aspects of the physical world, as enshrined in the equations of the STR, and consider dynamic aspects, related to questions of energy flow, entropy and dissipation.

The paper will explore the compatibility of Minkowski's space-time representation of the Special theory of relativity with a dynamic conception of space-time by investigating axiomatic approaches to the STR, as they were developed by Robb (1914), Carathéodory (1924) and Reichenbach (1924). A central feature of these accounts is to regard the propagation of optical signals as constituting histories of space-time relations. As it turns out this propagation involves invariant sequences between events, which become central for the understanding of time. It will be argued that the roots of a dynamic conception can be located in the thermodynamic and entropic features of the propagation of signals in space-time. If we accept that the geometry and nature of space-time have to be inferred from a range of measurable and observable phenomena (cf. Huggett 2006; Petkov 2005), and that the inference is legitimate on both the axiomatic and geometric approaches, we must conclude that the question of the ontological nature of space-time is at this stage a case of undetermination by the evidence.

II. **Axiomatic Approaches to Space-time**. Let us now consider what effect a chosen representation has on our understanding of space-time. Since Minkowski's introduction of the conception of four-dimensional space-time, a minority view has scraped a meagre existence in the shadows of the majority view. The majority view is the Parmedian block universe, aptly expressed in Einstein's words: 'From a "happening" in three-dimensional space, physics becomes (...) an "existence" in the four-dimensional "world".' (Einstein 1920, 122) Although Einstein's early commitment to the block universe was inspired by Minkowski's world postulate, in his later years Einstein wavered in his support for the Parmedian view. He began to consider thermodynamic aspects of the propagation of signals in space-time. This alternative view, which is notable for its Heraclitean ancestry, had its predecessors in the axiomatic approaches adopted by A. A. Robb (1914), C. Carathéodory (1924) and H. Reichenbach (1924). It avoids the binary choice into which McTaggart's metaphysical speculations seem to lure us: either we accept a dynamic A-series or the static B-series, but in either case time is unreal. The alternative view offers the conceptual possibility of a dynamic space-time, which is nevertheless rooted in the B-series. This view is worth exploring because it allows us to fully accept the consequences of the theory of relativity, without endorsing the Parmedian view of the block universe.

But how is this schematic programme to be cashed in? What does it mean that space-time trajectories have a history? To answer this question we do well to look at some attempts to construct axiomatic accounts of space-time, which do not start from Minkowski's 'absolute world postulate'; in Einstein's words it is a 'four-dimensional continuum described by the "co-ordinates" x_1, x_2, x_3, x_4, (which) was called "world" by Minkowki, who also termed a point-event a "world-point". (Einstein 1920, 122) Reichenbach, Robb and Carathéodory developed, apparently independently of each other, such axiomatic accounts, which start from a basic 'before-after' relation between null-like related events. Although these events are represented in geometric terms, they are crucially based on optical facts, like the emission and absorption of photons. The propagation of these signals constitutes an invariant conical order under the Lorentz transformations. The null-like and time-like trajectories between space-time events form the Minkowski world lines of light signals and material particles, respectively. The propagation of these signals constitutes a history of space-time relations, which may include *both* kinematic and dynamic aspects.[1]

II. 1. A. Robb's Account. These axiomatic attempts reverse the usual tendency to 'spatialize time'. Robb starts with the thesis that 'spacial relations' may be analyzed in terms of the time relations 'before' and 'after' or, as he concludes, 'that the theory of space is really a part of the theory of time'. (Robb 1914, Conclusion) Essential for this conception is the notion of conical order, which is analyzed in terms of the relations of 'before' and 'after' instants of time. An instant (an element of time) is the fundamental concept, rather than the space-time event. Furthermore the 'before/after' relation of two instants is an asymmetrical relation. In this way Robb builds a system of geometry, in which we encounter the familiar light cones of the Minkowski representation of space-time. Robb reverses the Minkowski approach in terms of geometrical relations and starts from physical facts, an approach, which is reflected in Einstein's later reservations about the block universe.

> If a flash of light is sent out from a particle P at A1, arriving directly at particle Q at A2, then the instant A2 lies in the α-subset of instant A1, while the instant A1 lies in the ß-subset of A2. Such a system of geometry will ultimately assume a four-dimensional character or any element of it is determined by four coordinates. (…) It appears that the theory of space becomes absorbed in the theory of time. (Robb 1914, 8-9)

Here the α-subset is the future light cone of instant A1 and the ß-subset is the past light cone of A2. (Figure I) After 21 postulates and over 100 theorems defining the light cone characteristics, Robb eventually defines the familiar conditions of the space-time interval, *ds*. The most interesting aspect of Robb's axiomatic system is that it regards Minkowski's contribution as 'merely analytical' and treats the geometry as a 'formal expression' of optical facts, like the propagation of signals in space-time. Thus Robb unwittingly opens up the possibility of considering kinematic space-time relations with respect to other

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[1] Huggett (2006, 47) defines a 'relational state as a specification of the totality of relations, mass and charges of bodies at a time.' See also Penrose/Percival (1962, §2)

physical aspects of space-time, since his declaration that ‘a before-after relation of two instants is an asymmetrical relation’ (Robb 1914, 5) will be based on thermodynamic aspects of electromagnetic radiation. Robb’s intention is to clarify notions like the conventionality of simultaneity by avoiding attempts to define ‘instants of time at different places’. By declaring that events are instantaneous which occur at the same instant, Robb anticipates the notion of relative becoming and local temporality, which have recently been mooted. ‘The present instant, properly speaking, does not extend beyond here.’ (Nature **107**, 1921, 422) But in the end Robb is still puzzled about time:

> Though space may be analyzable in terms of time relations, yet these remain mysterious; events occur in time, yet any logical theory of time itself must imply the Unchangeable. (Robb 1914, Conclusion)

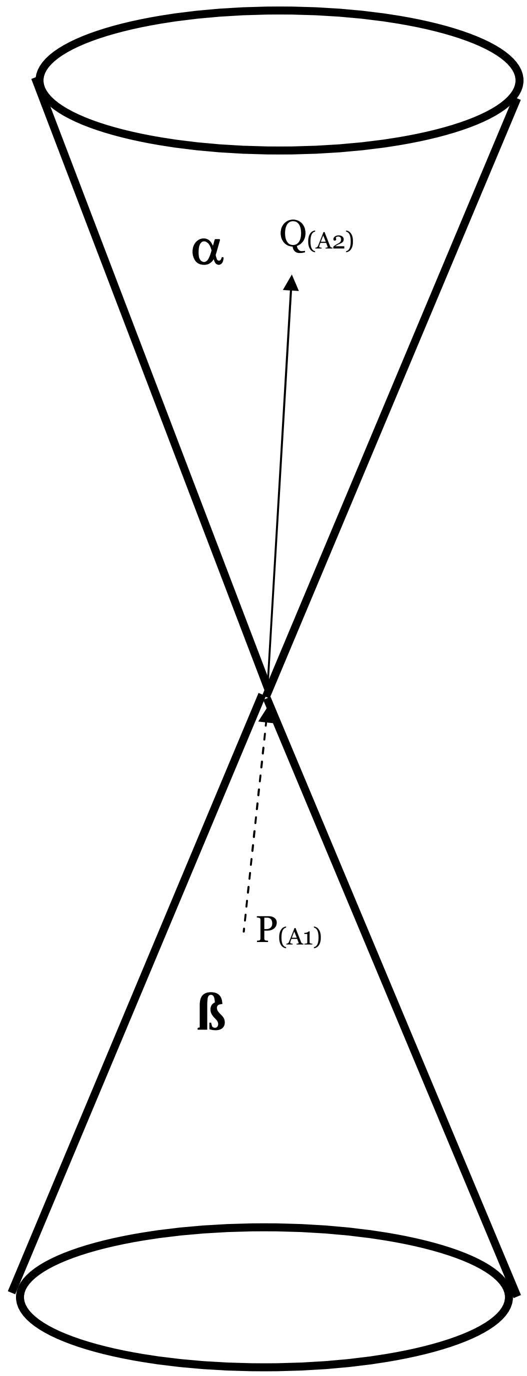

Figure I: ‘Corresponding to any point in space, there is an α-cone of the set having that point as vertex, similarly there is also a ß-cone of the set having the point as vertex.
If A_1 be any point and α_1 the corresponding α-cone, then any point A_2 is after A_1, provided $A_1 \neq A_2$ and A_2 lies either on or inside the cone α_1.

(Robb 1914, 5-6)

II. 2 C. Carathéodory. In 1916 Einstein encouraged Constantin Carathéodory to consider the problem of closed world lines in the General theory. (Hentschel 1990, 352-4) Ten years later, and without referring to Robb, Carathéodory (1924) started with the STR and took a similar approach but with fewer axioms and postulates. Carathéodory aims at a simplification of Einstein's theory: it is to be based on temporal relations (earlier, later, simultaneous) but these temporal relations are based on the behaviour of light signals. Carathéodory proceeds to define axioms of temporal succession and of light propagation. These axioms provide the concept of a 'light clock', which allows to measure time-like relations between events in space-time. These axioms are followed by axioms of topological space, which are reminiscent of Robb's conical order and hence allow the introduction of coordinate systems. Finally, he introduces Einstein's principle of relativity. Thus topological spaces consists of light cones, which are constituted by what Carathéodory calls 'normal light propagation'. As is to be expected Carathéodory defines equivalent topological spaces by the use of normal light propagation, satisfying relativity and symmetry requirements. Carathéodory, in fact, constructs what Reichenbach (1924) calls a 'light geometry', whose axioms are based on empirical facts.

> The propagation of light in (our topological space) $\mathfrak{R}$ is to be called 'normal' if, amongst all possible representation of the space $\mathfrak{R}$ by three parameters, there exists at least one coordinate system *x*, *y*, *z*, which satisfies the following condition:
>
> If we interpret *x*, *y*, *z* as right-angled coordinates of a Euclidean space, then of two simultaneously emitted light signals, which run through the two closed light polygons and whose end points coincide with the origin *O* of the coordinates *x*, *y*, *z* that signal is to arrive earlier, which describes the shorter (in a Euclidean sense) polygon. If the two polygons are of equal length, the signals are to arrive simultaneously.
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> This shows that in a space of normal light propagation there exists a natural measure for both distances and angles, which depends solely on temporal measurements from the light polygons. (Carathéodory 1924, §§9, 10; translated by the author)

As noted earlier, it is one of the advantages of these axiomatic approaches, based as they are on 'optical facts', that they permit an easy transition from kinematic to dynamic considerations. This is reflected in Carathéodory's observation that

Liouville's theorem also applies to the transformation of the topological space with coordinates *x*, *y*, *z*, *t* to primed coordinates. Carathéodory expresses the non-tilting of light cones in Minkowski's presentation, which is a consequence of the constancy of *c* in Minkowski space-time, in the statement:

> If two media **A** and **B** move relative to each other with normal light propagation, then every linear light ray of one medium will be transformed into a linear light ray of the other medium. (Carathéodory 1924, §25; translated by the author)

Liouville's theorem in classical mechanics states that a volume element along a flowline conserves the classical distribution function $f(r,v)drdv$:

$$f(t+dt, r+dr, v+dv) = f(t,r,v) \quad (1)$$

(Kittel/Kroemer 1980, 408; Albert 2000, 73f) In other words, if we consider trajectories in phase space, which include both position and momentum of particles, then the equation of motion of such systems can be expressed in terms of its Hamiltonian, H. H expresses the conservation of total energy of the system. Liouville's theorem then states that the volume of the phase space, which an ensemble of trajectories occupies, remains constant over time. Translated into the language of three-dimensional light cone structure, Liouville's theorem shows that the volume of the phase space regions is invariant over time even though the expansion of the trajectories within this volume can start from different initial states. But an immediate consequence of this theorem is that even though the *volume* is preserved the *shape* of this phase space region is not preserved (see Figure II) and this implies a dynamic evolution of the trajectories within this region. For two shapes cannot differ from each other without an evolution of the trajectories.

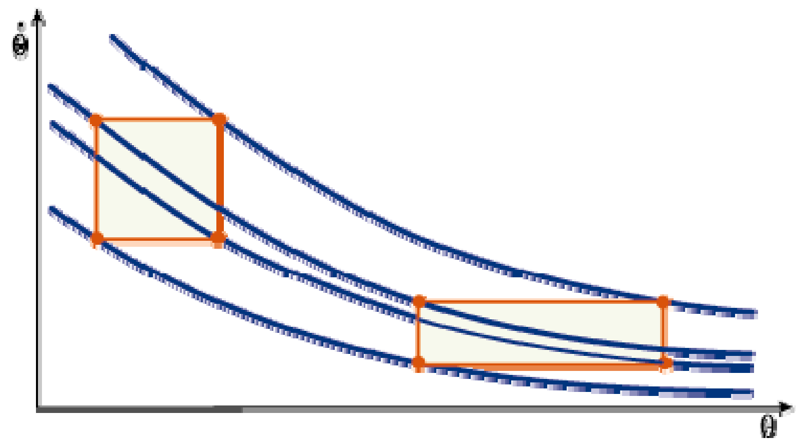


Figure II: Liouville's Phase volume invariance theorem. Source: Stöckler (2000, 206); cf. Davies (1974); Reichenbach (1956, 76); Albert (2000, 103)

The main purpose of these axiomatic approaches is to develop the STR as a light geometry, whose axioms are based on empirical facts. It does not start with an assumption of the existence of the four-dimensional Minkowski 'world' – which is pseudo-Euclidean and in which the linear homogeneous functions x_1, x_2, x_3, x_4 permit a rotation to primed functions x'_1, x'_2, x'_3, x'_4 by the transformation rules of the Poincaré group. The axiomatic approaches start with 'optical facts', like the propagation of light signals. What should be added and investigated is that they are subject to entropic constraints. According to the Robb-Carathéodory representation, the four-dimensional world does not 'exist' but it 'happens' through the propagation of time-like signals between successive events in space-time. These approaches therefore reverse Einstein's famous step from a 'happening' in the three-dimensional world to 'existence' in a four-dimensional world. (Einstein 1920, 122) As the world lines propagate through space-time, they form a history of space-time relations in a conical order. But does this really remove the puzzle about time, so forcefully expressed in Robb's concluding remarks? What did Minkowski mean when he conceded that a 'necessary' time order can be established at every world point? What does it mean that space-time trajectories have a history? In order to answer these questions we must turn from purely kinematic to dynamic considerations. We have two reasons for this transition. As Carathéodory's application of Liouville's theorem to light cone structures shows, we can introduce the thermodynamic language of phase space and speak of the flow of points in phase space. This reminds us that energy considerations are important in the STR and belong to a proper consideration of the four-dimensional world. We need to investigate the implications of this shift in perspective.

III. Towards Dynamics. An essential aspect of the geometric view of STR is that it only deals with kinematic relations. But if the world is four-dimensional and observers only experience a three-dimensional world through their slicing of four-dimensional space-time, it will be important to include some dynamic aspects of this pseudo-Euclidean world.

III.1 Dynamic Aspects. For a consideration of dynamic aspects it is important to introduce some physical grounding to the asymmetric kinematic relations as the axiomatic approaches of Reichenbach, Robb and Carathéodory emphasize. The axiomatic approaches seek a physical grounding to the asymmetric relations between space-time events in 'optical facts'. For the question that needs to be addressed is: Even if the 'before-after' relation, which is central in the axiomatic approaches, constitutes an asymmetric relation between space-time events, how does this linear order lead to a dynamic view of space-time? Here we want to consider some entropic aspects, because light propagation and signal propagation can be characterized in terms of energy flows and dissipation, processes which are subject to such entropic constraints.

III. 2 Provisos. Note that the argument is not to be confused with the usual thermodynamic arguments for or against the arrow of time. Although Eddington held that the increase in entropy established a global, cosmological direction of time, several objections have been raised against the identification of entropic processes with the global arrow of time: **1)** Popper (1956-7) pointed out that the arrow of time cannot have a stochastic character, which it would 'inherit' from an association with the second law of thermodynamics in its probabilistic interpretation. On Boltzmann's probabilistic interpretation of the 2^{nd} law the increase in entropy is merely overwhelmingly likely, and therefore would in principle allow a reversal of the arrow of time. But even without invoking the law of entropy, Popper held that 'it is absurd to link entropy to the arrow of time because of the existence of thermodynamic fluctuations.' (Popper 1957). Such reversible behaviour has been observed in highly viscous liquids (*Physik Journal* June 2008, 21-2) and can be 'engineered' through the recovery of phase correlations in quantum mechanical *which-way* experiments. **2)** The application of the entropy concept to the whole universe is problematic because the entropy concept is best defined for closed systems in thermodynamic equilibrium but the universe as a whole has no environment. (Uffink 2001; Drory 2008) An entropy-free method of obtaining a temporal order is to define a global intrinsic temporal orientability of space-time.

> A relativistic space-time $\langle M, g, \nabla \rangle$ is said to be temporally orientable if there exists a continuous nonvanishing vector field on *M* which is timelike with respect to *g*. (Earman 1974, 17; cf. Cf. Huggett 2006, 234)

The metaphorical arrow of time is then seen as an expression of the geometrical time-asymmetry of the universe. (Aiello *et al.* 2008) **3)** Alternative models for the 'arrow of time' on a global scale have been proposed, for instance the expansion of the universe from the big bang. (Gold 1966; Earman 1974; Earman 2006)
The entropy-free approach may be more satisfactory for a global arrow of time but it has no impact on the interpretation of Minkowski space-time. In fact it shows that we should clearly distinguish between the 'passage' and the 'arrow' of time. Space-time observers may perceive a 'passage' of time even in the absence of a global arrow of time. Concerns about the 'arrow' of time do not address the argument of the block theorist who infers the block universe from the geometric interpretation of space-time phenomena. The definition of temporal orientability appeals to continuous *time-like* vector fields but this does not address the question of time within Minkowski space-time, which is restricted to the behaviour of clocks and light signals, and, as we shall argue, the flow of energy.

These aspects do not involve the 'global' arrow of time, they are concerned with a dynamic conception of Minkowksi space-time.

It is worth noting that in these discussions often implicit presuppositions about the nature of space-time are at work, such as substantival or relational approaches. For the geometric approach to Minkowski space-time implicitly favours a substantival reading of space-time, whilst the axiomatic approaches, introduced above, implicitly favour a relational understanding of space-time. The following considerations will embrace a relational view of space-time, according to which space is the order of coexisting events in space-time and time is the order of the succession of co-existing events. The notion of order is crucial in this context. The Leibnizian view of order is of course pre-relativistic so that the 'order of coexisting events' presupposes absolute simultaneity but not Newtonian absolute space and the 'order of successive events' presupposes a unique temporal axis for all observers but not Newtonian absolute time. To speak of space-time relationism means to subject the order of coexisting events to the condition of relative simultaneity and the constancy of *c* and to speak of the order of successive events means to confine this order to null-like and time-like relations between events in space-time. The Leibnizian order becomes the conical order of events. This move to space-time relationism is possible because, in spite of the notion of relative simultaneity, space-time observers can agree on a number of invariant relationships between events in space-time. As we shall see below such invariant relations are crucial for the appreciation of time.

III. 3 Inferences to the Nature Space-time. The Leibnizian characterization of space and time in terms of the order of events and the relations between them does not restrict us to a consideration of kinematic relations and material bodies. It is a common misunderstanding that relationism is limited to occupied space-time events. (Friedman 1983) A 'liberalized relationism' admits a system of both actual and possible relative trajectories. (Teller 1991; Weinert 2006) It is easy to see an alliance between the axiomatic accounts of four-dimensional space-time and space-time relationism. The axiomatic accounts are based on the fundamental 'before-after' relations between space-time events, whose physical manifestation is the propagation of optical signals. Although the traditional relationist speaks of the order of 'events', 'processes' or 'material objects' in the physical universe, a contemporary relationist is not restricted to purely kinematic relations to constitute physical time. The space-time relationist will consider both kinematic and dynamic 'processes', which will help observers in inertial motion with respect to each other to identify physical time. As the propagation of signals constitutes the grounding of the 'before-after' relation in the axiomatic approaches, it is appropriate to consider entropic aspects of this propagation. The exchange of signals is clearly of great importance in Minkowski space-time, as is well illustrated in the famous twin paradox. As one resolution of the twin paradox in Minkowski space-time shows – it appeals to the relativistic Doppler effect and abstracts from the short periods of acceleration and deceleration of the space-travelling twin – the propagation of signals – their emission and reception – plays an important part in a consideration of four-dimensional space-time. This feature becomes prominent in the axiomatic approaches.

The question of the nature of space-time is a matter of admissible inferences, which inertial observers in space-time would draw from their respective experiences. An influential tradition, from Einstein and Gödel to the present day, has inferred the block universe from the measurable and observational relativistic effects. Such inertial observers, who are attached to reference frames, should also be aware of the propagation of signals, since this is their way of communicating. Such observers would not be far removed from the original concern of Einstein about the coordination of distant clocks. If Reichenbach, Robb and Carathéodory were inertial observers they would direct their attention to thermodynamic properties of signal propagation, which could serve as their basis for inferences about space-time. Whilst the geometric view infers the block universe from the relativity of simultaneity and more recently from other relativistic effects, the axiomatic view will consider dynamic properties of signal propagation, which are considered as the physical basis of the geometric relations. More importantly, as we shall argue below, it will focus on certain invariant relationships between events in space-time.

For the relationist the physical grounding of time is an essential aspect. Apriori it does not matter whether time is measured by heart beats, the orbit of planets around the sun, atomic oscillations, or the anisotropic propagation of electromagnetic signals in space-time. What matters are appropriate regularities *and* the amount of invariance associated with regular processes across different reference frames. For instance, as we shall see below, the temperature of a moving body is relativistically invariant so that a thermostat could in principle serve as a 'clock' to be used by observers in Minkowski space-time. In practical terms, however, some 'clocks' are less likely to succeed than others. Consider the exchange of signals in the famous twin paradox. The twin paradox can be treated in Minkowski space-time because the periods of acceleration and deceleration of the travelling twin can be made arbitrarily small compared to the journey times. As is well-known the respective ages of the twins are subject to relativistic time dilation such that, during the journey time, the earth-bound twin will age more than the travelling twin (and vice versa). Note that in the twin paradox the clock readings of the respective twins are perspectival and yet objective. On the geometric view the differential aging is read as evidence of a static four-dimensional block universe because of the perspectival aspect of the clock reading exercises. (Petkov 2005) But this view neglects that there are invariant features in this situation on which the space-time relationist will want to focus rather than on the perspectival aspects. The exchange of signals is subject to entropic dispersion but entropy is frame-invariant in the STR. This suggests that both twins will 'see' the propagation of their respective light signals as diverging wave fronts whose source is in each case the respective source of emission. The earth-bound twin receives fewer signals from his brother than vice versa. They will agree that the emission event is in each case prior to the reception event: the order of these events, marked by the energy flow, is invariant although they will disagree about the length of the events between emission and absorption, as expressed in the relativistic Doppler formula. Thus the twins will clearly be able to establish earlier-later relationships between events and they will agree on this order for time-like related events.

The twins have every reason to believe that 'earlier-later' relations exist between events in space-time and more generally that space-time trajectories acquire histories in space-time. These histories, as the axiomatic approach has shown, are not confined to kinematic relations between events, but comprise dynamic considerations.

If they focus on the mechanical laws, which hold between events in space-time, they will find these mechanical laws to be time-reversal invariant, which would not be conducive to a dynamic view of space-time. On the other hand, if the world is truly four-dimensional, as many infer from the STR, it is not legitimate to infer assertions about the nature of space-time from a limited range of phenomena. We should not focus on mechanical aspects at the expense of thermodynamic considerations. The latter route was followed by Reichenbach and Grünbaum.

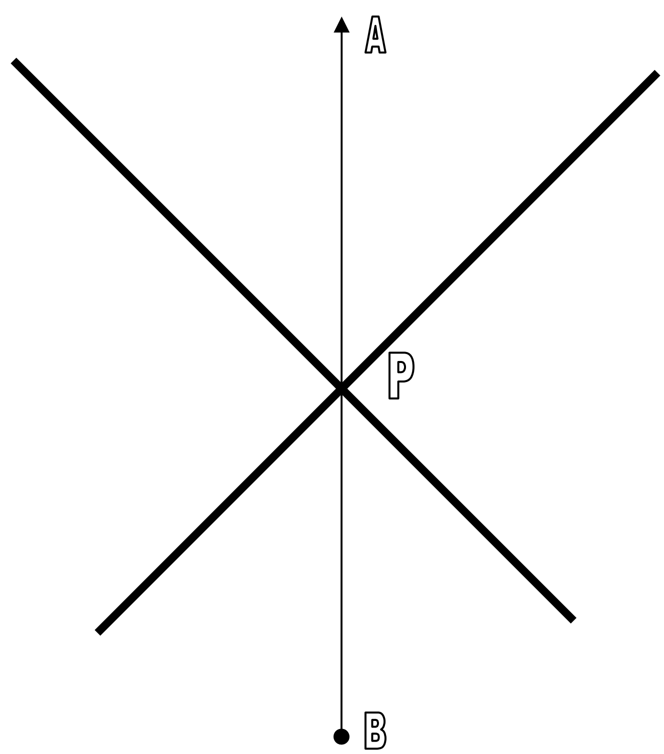
IV. Irreversibility, Regularity and Invariance. In this section we shall consider which inferences about the nature of space-time follow from a shift to dynamic aspects.

IV. 1. Reichenbach & Grünbaum. Reichenbach distinguished the topological question of time order ('before-after') from the dynamic question of time direction. (Reichenbach 1956, 16) He claimed that entropic considerations 'will enable us to solve the problem of the direction of time, a problem that cannot be solved in the framework of Einstein's theory of relativity, because it requires a transition from strictly causal relations to probabilistic relations.' (Reichenbach 1956, 25-6) Reichenbach turns to the statistical interpretation of entropy:

> The direction of physical processes, and with it the direction of time, is thus explained as a statistical trend: the act of becoming is the transition from improbable to probable configurations of molecules. (Reichbach 1956, 55)

Further, Reichenbach points out (1956, 60) that the statistical form of the second law defines a value of S for both equilibrium and non-equilibrium states. This entropic approach has been criticized as 'yielding the wrong result somewhere in space-time'. (Earman 1974, 22) This objection may be justified from the point of view of a global temporal orientability of space-time but it nevertheless harbours some interesting results from the point of view of the axiomatic method and space-time relationism. In his later years Einstein himself grew more aware of dynamic aspects of signal propagation in space-time when he objected to Gödel's interpretation of Minkowski space-time in terms of a block universe and the denial of the objective passage of time. (Figure III)

Figure III: Einstein's consideration of the (local) direction of time in response to Gödel's idealistic interpretation of the special theory of relativity. A *time-like* world line exists between events A and B, which lies within, not outside, the light cone. A and B are linked by an irreversible signal. Einstein (1949), 687



The most interesting result, on Reichenbach's entropic approach, is that it is the majority of branch systems which show an increase in entropy. It is the sectional nature of time direction, which is appealing to the space-time relationist. 'The direction in which most thermodynamic processes in isolated systems occur is the direction of positive time.' (Reichenbach 1956, 127) Grünbaum took up this suggestion but reduced it to *de facto* irreversibility. This weak T-invariance must satisfy the

> requirement that its time inverse (although perhaps improbable) does not violate the laws of the most elementary processes in terms of which it is understood. (Landsberg 1982, 8)

For Grünbaum the direction of physical time is grounded in *de facto* irreversible processes. (Grünbaum 1967; 1955) Grünbaum makes an explicit distinction between physical time and human perception of time. The anisotropy of physical time is not to be confused with a 'transient now' or human perception of becoming ('river of time'). Grünbaum agrees with Reichenbach that the positive direction of physical time is the direction of entropy increase in the majority of branch systems. The emphasis on *de facto* irreversible processes means that they are contingent and compatible with the time reversal symmetry of the basic mechanical laws. He thus rejects Popper's argument that 'thermodynamic behaviour cannot constitute a basis for the anisotropy of time.' But he also distances himself from Reichenbach in 2 ways:

1. Grünbaum does not assume that entropy is defined for the whole universe. To be fair to Reichenbach, he holds that the overall entropy of the universe can only be inferred from the entropic behaviour of branch systems. 'The universal increase of entropy is reflected in the behaviour of branch systems, so to speak; and only this reflection of the general trend in many individual manifestations is visible to us and appears to us as the direction of time.' (Reichenbach 1956, 131)
2. Grünbaum does not assume parallelism of entropy increase in branch systems and the universe. Thus Grünbaum is truly committed to the sectional nature of the passage of time in local neighbourhoods.

Whilst the entropic approach satisfies the space-time relationist's need for physical systems, it also suffers from some weaknesses. For instance, Reichenbach's characterization of branch systems as 'systems that branch off from a comprehensive system and remain isolated from then one for some time'

(Reichenbach 1956, 118) is relatively ill defined and neglects that no subsystem is ever totally isolated from the more comprehensive system. Reichenbach claims that the entropic approach can solve the problem of time. This claim has several important aspects, which should be carefully distinguished: **A)** It indicates dynamic and regular features of signal propagation in Minkowski space-time. Reichenbach points out that the entropic approach confirms common sense in its intuition that 'time flows' and that 'becoming occurs'. (Reichenbach 1956, 17)

> The concept of *becoming* acquires a meaning in physics: The present, which separates the future from the past, is the moment when that which was undetermined becomes determined, and 'becoming' means the same as 'becoming determined.' (Reichenbach 1956, 269; cf. Torretti 2006)

But the language of space ensembles (ensembles of branch systems) no longer refers to the language of world lines and time-like related events. **B)** For this approach to have any chance of succeeding it must be recognized that entropic relations are frame-invariant in the STR (Einstein 1907). This aspect is particularly important because many physical parameters become frame-dependent in the STR and could not serve as a basis for the identification of physical time. **C)** Once we appreciate the importance of invariance for the measurement of time, we realize, as we shall discuss, that there are other invariant relationships between space-time events which could serve as candidates for the identification of physical time.

The emphasis on the sectional nature of time direction in the work of Reichenbach and Grünbaum seems to survive in latter-day attempts to save a notion of 'relational becoming' (Dorato 2006), which regards proper time – time along a world line or local temporality – as the only legitimate notion of time in the STR. (See Dieks 1988; Harrington 2008; Stein 1991) These approaches retain the welcome separation of the notion of becoming from the 'presentism/eternalism debate' (Dorato 2006, §1) but they also neglect the importance of invariant relationships. Even the idea of local time – clock time along a world line as real – prevents us from noticing the invariant features across reference frames. As the axiomatic approach implies, such invariant relationships are essential for the notion of time. For it is not sufficient to register regular pulses in one reference frame, regular pulses must be invariant across reference frames in inertial motion with respect to each other for the notion of physical time to make sense. It is therefore important to consider these aspects of invariance.

IV.2 Time & Invariance. For a reader of the relevant literature, inspired by space-time relationism, it is surprising to find many authors affirming the reality of a static block universe in the same breath as the asymmetric propagation of electromagnetic signals in space-time. (Davies 1974; Lockwood 2005; Petkov 2005) However any association of the arrow of time with entropic processes is regarded with a considerable amount of suspicion, not just for the reasons cited above, but also because it is one of the scandals of modern physics that there is still no consensus on the precise meaning of the 2nd law of thermodynamics. (See Duncan/Semura 2007; Leff 2007; Aiello 2008) On the other hand relationism about time requires a physical grounding, where this physical grounding is a matter of appropriate choice. As Saunders points out, a question that is even more important than objective becoming is whether change is real. (Saunders 1996, 20-1) This depends on an appropriate physical grounding and entropy seems to be a favourite candidate. (See Wald 2006; Davies 1974) But for the 'passage' of time in Minkowski space-time even regular change must have invariant aspects. In other words a symmetry transformation between inertial frames in Minkowski space-time must leave invariant features. For a dynamic view of Minkowki space-time, the entropic aspects of signal propagation are interesting because they offer both dynamic and invariant properties.

We can distinguish several invariant relationships in Minkowski space-time:

- ♦ Traditional replies to the block view have relied on the invariance of c and the space-time interval ds. The invariance of c means that light cones in Minkowski space-time do not tilt, a fact, which Carathéodory related to Liouville's theorem. The invariance of ds means that observers will disagree about spatial and temporal lengths between events in space-time from their respective reference frames, but that the space-time interval, which captures the famous union of space and time, which Minkowski announced in 1908, remains invariant for all time-like related observers.

- Simulations of the molecular dynamics of relativistic gases have shown that the temperature of a moving body does not depend on its state of motion. It is possible to define a relativistic temperature from statistical data (and to construct a thermometer), which respective observers in Minkowski space-time could in principle use to determine time across their respective frames. Bodies appear neither hotter nor cooler if a relativistic temperature $T = (k_B \beta_j)^{-1}$ is adopted [where k_B is the Boltzmann constant and β_j is a numerical distribution parameter, which in these experiments took the value $\beta_j = 0.702(m_1 c^2)^{-1}$]. The experimenters concluded that 'the temperature of classical gaseous systems can be defined and measured in a Lorentz invariant way.' (See Cubero *et al.* 2007) In principle it would be possible to read time off these thermostats but in practice it is inconvient and other methods are preferable.
- But signal propagation offers other possibilities of determining physical time in Minkowski space-time. Signal propagation is a thermodynamic and therefore anisotropic process both for inertial and accelerating observers in flat and curved space-time. (Petkov 2005) It turns out that entropy and the spreading of energy states are also relativistically invariant. (Einstein 1907; Pauli 1981, §46-9) What follows from this invariance is that the convergence and divergence of signals is frame-independent, in local neighbourhoods.

The central aspect in these invariance aspects is that the direction of the energy flow runs in the same direction for all observers. So even though two observers do not agree on the reading of their respective clocks they will agree on the divergence of their signals from their point of origin. They therefore have a physical grounding for their time measurements.

> (...) with the energy flow pointing to the same direction all over the spacetime, we can legitimately say that σ > 0 [σ is entropy production per unit volume] corresponds to a dissipative decaying process evolving from non-equilibrium to equilibrium as $e^{-\gamma t}$ and σ < 0

> corresponds to an antidissipative growing process evolving from equilibrium to non-equilibrium as $e^{\gamma t}$. The two processes, which in principle are only conventionally different, turn out to be substantially different due to the future-directed energy flow that locally expresses the global time-asymmetry of the universe. (Aiello et al. 2008, 287)

In this connection it is helpful to introduce a 'spreading metaphor' to capture the essence of the second law. According to this metaphor the entropy symbol, *S*, is a shorthand for spreading of energy, which includes spatial spreading of energy and temporal spreading over energy states. This entails a picture of dynamic equilibrium in terms of continual shifts from one microstate to another. (Leff 2007, 1748) In order to quantify the spreading metaphor, a spreading function $\Im$ is introduced, which is a function of a system's energy E, its volume V and particle number N. Connecting the spreading function to entropy *S*, Leff writes:

> For a constant-volume heating process that proceeds along a given $\Im$ curve, $dE = \delta Q$ is the (inexact) heat differential. Equation (22) - $\left(\partial\Im/\partial E\right)_{V,N} = 1/T$ - implies that $d\Im = dE/T = \delta Q/T$, in analogy with the Clausius entropy form $dS = \delta Q/T$. Thus, with the temperature definition (22), the spreading function $\Im$ shares the important mathematical property $d\Im = \delta Q/T$ with entropy S. (Leff 2007, 1763-4)

With these considerations in mind we can return to our earlier observation that histories in space-time must include both kinematic and dynamic considerations. If we consider **a)** that time reversal invariance of the dynamic laws is broken by energy flows, pointing in the same direction in local neighbourhoods in space-time and **b)** that the spreading metaphor captures essential aspects of the 2nd law, we notice a longstanding association of time with cosmological regularity. Prior to Einstein, all approaches to time agreed that time was a universal parameter, irrespective of the question of whether it only existed in the mind or in the physical world and irrespective of the question whether it existed in the absence or the presence of physical events. The requirement for regularity in some physical system is well reflected in the relational view and its notion of physical time. It is important to note that the STR obliges us to require that these regularities must possess a certain amount of invariance across coordinate systems. For the importance of STR, under the present perspective, resides in its distinction between frame-dependent and frame-independent parameters. The invariant relationships between space-time events therefore acquire considerable importance for a dynamic view of Minkowski space-time. It is these regular and invariant relationships, which allow for the possibility of measuring objective physical time.

V. Conclusion. The early block theorists held that two observers in Minkowski space-time could not establish the 'march of time' because of the problem of the relativity of simultaneity. Later block theorists held that the well-known relativistic effects do not only establish the reality of the four-dimension space-time but also an eternal block universe, in which the passage of time is a mere human illusion. But clearly if the two observers can identify regular, invariant time directions, even only locally, they can say that time passes and generally that the four-dimensional world evolves into their local future. The identification of these time direction is not based on a global definition of time-orientability of relativistic space-times or the slicing of four-dimensional space-time by conscious observers. It is based on asymmetric physical processes, like the energy flow and dissipation of signals from the source into the future light cones of observers. The observers in Minkowski space-time have access to these phenomena. From the dissipation of signals and entropic invariance the observers will infer that the four-dimensional world is dynamic.

The fact that the axiomatic method implies a different view of space-time – dynamic rather than static – shows that a more inclusive consideration of the history of space-time relations leads to a contrary but equally consistent view of four-dimensional space-time. In fact from the axiomatic point of view the block theorist's inference to a static universe from the relativity of simultaneity and time dilation appears to be premature. The Minkowski space-time representation of the STR seems to be compatible with two incompatible interpretation of space-time. It is a clear case of underdetermination. If this suggestion is correct, the majority view can no longer claim that the passage of time is a human illusion and the only possible inference from the experimental evidence. From a purely geometric point of view of space-time, it is difficult to appreciate the impact of a relationist view of physical time. It is based on the view that temporal relations between events (in space-time) are grounded in the order of succession of events. Whilst Leibniz remained unspecific about the precise physical relations, which could serve as a basis of physical time, the axiomatic approach suggests that purely kinematic relations, based on time-reversal mechanical laws, are insufficient to establish physical time in Minkowski space-time. A space-time relationist will find the axiomatic method more amenable for it suggests that certain thermodynamic processes, like signal propagation, are both invariant and regular. They allow the space-time relationist to infer a dynamic view of four-dimensional space-time.

The following scenario presents itself: if the observers in Minkowski space-time concentrate on the flow of energy and the propagation of signals they will infer that 'local' time has a uniform direction and that space-time is dynamic. The relationist view entitles them to select such energy flows as examples of the invariant order of succession of events in space-time. They will disagree with the block theorist who derive their view from purely geometric and kinematic relations. For the relationist the latter view is mistaken because it is not based on the invariant order of succession of physical events.

Both the block theorist and the space-time relationist can only make inferences from measurable or observable phenomena to the nature of space-time. Are there ways to solve this underdetermination? The opponents would have to show that some relativistic effects are better indicators of the nature of space-time than others. The other strategy is patience: it is possible that some future measurable effect will be able to resolve the stalemate between the block theorist and the space-time relationist. For instance, Saunders (1996) holds that physics can decide between metaphysical views. The writer's own view is that it is unreasonable to suspect that science can be a judge in matters metaphysical. However, it is altogether reasonable to expect that some future observation will show that one metaphysical view is more compatible with the results of relativity than its opponent.

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