

Patterns in the Fabric of Nature

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Abstract

From classical mechanics to quantum field theory, the physical facts at one point in space are held to be independent of those at other points in space. I propose that we can usefully challenge this orthodoxy in order to explain otherwise puzzling correlations at both cosmological and microscopic scales.

“Nature uses only the longest threads to weave her patterns, so that each small piece of her fabric reveals the organization of the entire tapestry.”

The Character of Physical Law, Richard Feynman

1 Introduction

Despite radical differences in their conceptions of space, time, and the nature of matter, all of the physical theories we presently use — non-relativistic and relativistic, classical and quantum — share one assumption: the features of the world at distinct points in space are understood to be independent. Particles may exist anywhere, independent of the location or velocity of other particles. Classical fields may take on any value at a given point, constrained only by local constraints like Gauss’s law. Quantum field theories incorporate the same independence in their demand that field operators at distinct points in space commute with one another.

The independence of physical properties at distinct points is a theoretical assumption, albeit one that is grounded in our everyday experience. We appear to be able to manipulate the contents of a given region of space unrestricted by the contents of other regions. We can arrange the desk in our office without concern for the location of the couch at home in our living room.

Yet there are realms of physical theory, more remote from everyday experience and physical manipulation yet accessible to observation, in which there appear to be striking correlations between the values of physical properties at different points in space. Quantum theory predicts (and experiment confirms) the existence of strongly correlated measurement outcomes apparently inexplicable by classical means. I refer, of course, to the measurements of the properties of pairs of particles originally envisioned by Einstein, Podolsky and Rosen (EPR) [7], measurements that suggested to EPR the incompleteness of the theory. Bell (1964) showed that no theory satisfying two seemingly natural conditions could possibly account for these correlations.

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Quantum mechanics itself violates one of the conditions, known as *Bell locality*, *strong locality* or *factorizability*, leading to the measurement paradoxes that so troubled Einstein and his collaborators. The other condition, *statistical independence*, has only rarely been questioned. It is tantamount to the rejection of the assumption of local degrees of freedom, or the existence of nonlocal constraints.

On a completely different scale, the electromagnetic radiation that pervaded the early universe – the remnants of which form the cosmic microwave background – appears to have been extraordinarily homogeneous. It is strikingly uniform, yet the theories that describe the early universe – classical electrodynamics (for the radiation) and general relativity (for the expanding spacetime the radiation fills) – do not stipulate any sort of restrictions or correlations that would go anywhere near explaining this. To the extent that they have been explained at all, it has been through the postulation of an as-yet unobserved field known as the inflaton field.

What I want to do here is raise the possibility that there is a more fundamental theory possessing nonlocal constraints that underlies our current theories. Such a theory might account for the mysterious nonlocal effects currently described, but not explained, by quantum mechanics, and might additionally reduce the extent to which cosmological models depend on finely tuned initial data to explain the large scale correlations we observe. The assumption that spatially separated physical systems are entirely uncorrelated is a parochial assumption borne of our experience with the everyday objects described by classical mechanics. Why not suppose that at certain scales or certain epochs, this independence emerges from what is otherwise a highly structured, nonlocally correlated microphysics?

2 Nonlocal constraints

All physical theories in current use assume that the properties of physical systems at different points in space are independent. Correlations may emerge dynamically – many liquids crystallize and develop a preferred orientation when cooled, for example – but the fundamental theories permit any configuration as an initial condition.

For example, consider the straightforward and simple theory of the free massless scalar field $\phi(\vec{x})$. A scalar field is simply an assignment of a single number (a “scalar” rather than a vector) to every point in space and time. The evolution of the field is given by the well-known wave equation

$$\frac{\partial^2 \phi(\vec{x}, t)}{\partial t^2} = c^2 \nabla^2 \phi(\vec{x}, t) ,$$

in conjunction with initial data $\phi(\vec{x})$ and $\partial\phi(\vec{x})/\partial t$ giving the value of the field and its rate of change at some initial time. This initial data can be specified arbitrarily — it is unconstrained.

A more realistic field theory is the classical electrodynamics of Maxwell, which *does* feature constraints. In Maxwell’s theory, we have a pair of coevolving fields, the electric field \vec{E} and the magnetic field \vec{B} . The fields are described by vectors at each point rather than scalars. The significant difference between the electromagnetic field and the free scalar field is that the electric and magnetic fields may not be specified arbitrarily. They are subject to constraints $\nabla \cdot \vec{E}(\vec{x}) = 4\pi\rho(\vec{x})$ and $\nabla \cdot \vec{B}(\vec{x}) = 0$ which hold at every point \vec{x} in space. The divergence of the electric field at any given point with coordinates must be equal to a multiple of the charge density $\rho(\vec{x})$ at that point, and the divergence of the magnetic field must be zero. The divergence is a measure of the outflow of the field in the neighborhood of a point, and the two constraints tell us respectively that any such outflow of the electric field is due to the presence of a charge at that point acting as a source, while the magnetic field can have no sources (there are no

magnetic charges). These constraints are *local* in that they provide a constraint on values of the field at each point that does not depend on values of the field or the charge distribution at other points.

What would a nonlocal constraint look like? Here's a candidate: $\nabla \cdot \vec{E}(\vec{x}) = 4\pi\rho(\vec{x} - (1, 1, 1))$. This says that the divergence of the electric field at one point is equal to a constant times the charge density at a point which is one unit away in each of the x, y and z directions. But this constraint is hardly worthy of the name, since it only holds at a single time: unlike the constraint $\nabla \cdot E(\vec{x}) = 4\pi\rho(\vec{x})$, it is not preserved by the equations of motion (Maxwell's equations for the field and the Lorentz force law for the charge distribution). I.e., it may hold at one time, but will not continue to hold as the field evolves. Since it is not preserved, it does not hold at arbitrary moments of time, hence it is not a true regularity or law.

Let's return to simple mechanics for an example of a true nonlocal constraint, one that is conserved in time. The particles are characterized by their positions and their momenta. The constraint we will impose is that the total momentum (the sum of the momenta of each of the particles) is zero. This is a constraint because we cannot specify the momentum freely for each particle; if we know the momentum of all but one of the particles, the momentum of the other particle is fixed. It is nonlocal, because the momentum of that particle depends on the momenta of particles some distance away. Unlike the first nonlocal constraint we considered, it is conserved, since total momentum is a conserved quantity in particle mechanics. But it is not a particularly interesting constraint, because all but one of the momenta may be freely specified. Whereas the two constraints in electromagnetism reduce the number of degrees of freedom from six to four at each point in space (so that there are only two-thirds the number of degrees of freedom), this constraint only reduces the total number of degrees of freedom by one.

A more interesting nonlocal constraint may be obtained by considering once more the wave equation, this time in one space dimension (for simplicity). Suppose that the spacetime on which the field takes values is compactified in the time direction, so that the entirety forms a cylinder (see Figure 1).

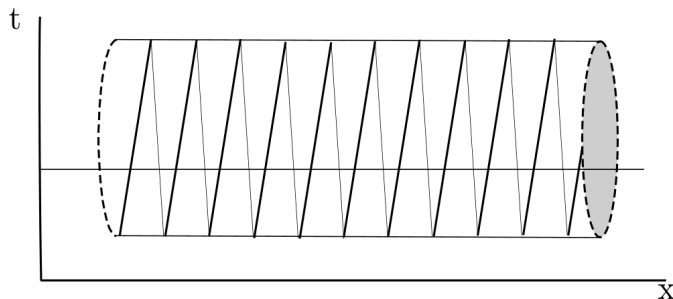


Figure 1: Timelike compactification

The solutions must clearly be periodic, and this amounts to imposing a nonlocal constraint. More specifically, whereas in the ordinary initial value problem, initial data may be any smooth functions $\phi(x, 0)$ and $\phi_t(x, 0)$ (where ϕ_t stands for $\partial\phi/\partial t$), we now require that $\phi(x, 0) = \phi(x, T)$ and $\phi_t(x, 0) = \phi_t(x, T)$, where T is the circumference of the cylinder. This is just to say that the time evolution from 0 to T must return us to the same starting point. What are the constraints, then, on this initial data? They are essentially those data that can be written as sums of sine or cosine waves with wavelength $\frac{T}{2\pi n}$ (for any integer value of n).¹

The restriction to a discrete (though infinite) set of plane waves means that initial data do not have compact support; they are periodic. However, for sufficiently large T or sufficiently

small Δx , the local physics is indistinguishable from the local physics in ordinary spacetime. Only at distance scales on the order of T does the compact nature of the direction become evident in the repetition of the spatial structure. Thus we have here an example of a nonlocal constraint which might give the appearance of unconstrained local degrees of freedom.

Now, this spacetime obviously has closed timelike curves, and it is interesting to note that under such conditions, classical computers are as powerful as quantum computers [1]. Thus there is some reason to think that a nonlocal constraints might allow one to mimic other quantum phenomena using classical physics. In any event, we will now proceed to a discussion of the way in which the presence of nonlocal constraints opens the door to a little-explored loophole in Bell’s theorem, in that their presence undermines the *statistical independence* assumption required for the proof of the theorem.

3 Bell’s theorem

Einstein believed quantum theory to be an incomplete description of the world, and he and his collaborators Podolsky and Rosen attempted to show this in their 1935 paper [7]. The argument involves a pair of particles specially prepared in an entangled state of position and momentum.² Quantum mechanics makes no definite predictions for the position and momentum of each particle, but does make unequivocal predictions for the position or momentum of one, given (respectively) the position or momentum of the other. EPR argued that this showed that quantum mechanics must be incomplete, since measurement of the position (or momentum) of one particle could not simultaneously give rise to a definite position (or momentum) of the other particle, on pain of violation of locality. They concluded that quantum mechanics, because it did not assign a position (or momentum) to the other particle beforehand, must be incomplete.³

In 1964, John Bell proved a result based on David Bohm’s streamlined version of the EPR experiment [2][4]. Instead of positions and momenta, Bohm focuses on the spins of a pair of particles (either fermions or bosons). Prepared in what has come to be known as a Bell state,

$$\psi = \frac{1}{\sqrt{2}}(|+x\rangle_A |-x\rangle_B - |-x\rangle_A |+x\rangle_B), \quad (1)$$

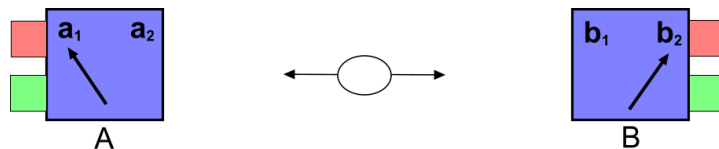
quantum mechanics predicts that a measurement of the component of spin of particle A in any direction (e.g., \hat{z}) is as likely to yield $+1$ as -1 (in units of $\hbar/2$), and so the average value \bar{A} is 0. However, quantum mechanics also indicates that an outcome of $+1$ for a measurement of the spin of A in the \hat{z} direction is guaranteed to yield an outcome of -1 for B for a measurement of the spin of B in the \hat{z} direction, etc. This is directly analogous to the correlations between position and momentum measurements in the original EPR experiment.

In and of themselves, these phenomena offer no barrier to a hidden-variable theory, since it is straightforward to explain such correlations by appealing to a common cause – the source – and postulating that the particles emanate from this source in (anti)correlated pairs. However, one must also account for the way that the anticorrelation drops off as the angle between the components of spin for the two particles increases (e.g., as A rotates from \hat{x} toward \hat{z} while B remains oriented along the \hat{x} direction). It was Bell’s great insight to note that the quantum theory implies that the anticorrelation is held onto more tightly than could be accounted for by any “local” theory — that is, any theory satisfying the seemingly natural condition known in the literature variously as *strong locality*, *Bell locality*, or *factorizability*. Bell showed that the predictions of a local theory must satisfy a certain inequality, and that this inequality is violated by quantum theory for appropriate choices of the components of spin to be measured.

Bell's result was widely understood to provide a barrier to the sort of "completion" of quantum mechanics considered by Einstein. That is, Einstein's hope for a more fundamental theory underlying quantum theory would have to violate *strong locality*, of which more below.

However, there is a further assumption known as the *statistical independence* assumption that is necessary for Bell's result. This assumption is quite closely related to the assumption of local degrees of freedom, or the absence of nonlocal constraints. Without the assumption, Bell's result does not go through, and the possibility re-emerges of a local completion of quantum theory after all.⁴

Rather than repeat the derivation of Bell's result, let me just focus on the meaning of the two crucial assumptions of *strong locality* and *statistical independence*. The physical situation we are attempting to describe has the following form:



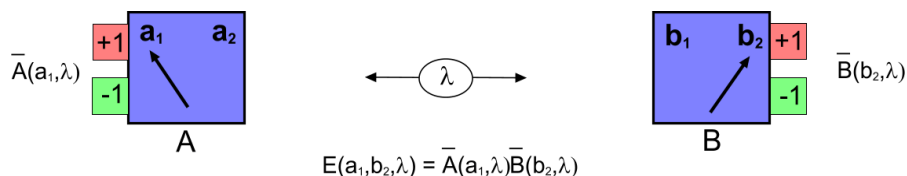
A source (represented by the ellipse) emits a pair of particles, or in some other way causes detectors A and B to simultaneously (in some reference frame) register one of two outcomes. The detectors can be set in one of two different ways, corresponding, in Bohm's version of the EPR experiment, to a measurement of one of two different components of spin.

Let us now suppose that we have a theory that describes possible states of the particles and which gives rise to either probabilistic or deterministic predictions as to the results of various measurements one might make on the particles. The state of the particle will be represented by a discrete or continuous parameter λ , describing either a discrete set of states $\lambda_1, \lambda_2, \dots$ or a continuous set. The expressions $\bar{A}(a, \lambda)$ and $\bar{B}(b, \lambda)$ correspond to the expected (average) values of measurements of properties a and b at detectors A and B (respectively) in a given state λ . (The appeal to *average* values allows for stochastic theories, in which a given λ might give rise to any number of different outcomes, with various probabilities.)

In general, one might suppose that \bar{A} also depends on either the detector setting b or the particular outcome B (i.e., $\bar{A} = \bar{A}(a, \lambda, b, B)$), and one might suppose the same for \bar{B} . That it does not, that the expectation value \bar{A} in a given state λ does *not* depend on what one chooses to measure at B , or on the value of the distant outcome B (and vice-versa) is Bell's *strong locality* assumption. Given this assumption, one can write the expression $E(a, b, \lambda)$ for the expected product of the outcomes of measurements of properties a and b in a given state λ as

$$E(a, b, \lambda) = \bar{A}(a, \lambda) \bar{B}(b, \lambda). \quad (2)$$

This strong locality is also known as 'factorizability', deriving as it does from the fact that the joint probability of a pair of outcomes can be factorized into the product of the marginal probabilities of each outcome. We can thus represent the analysis of the experimental arrangement in this way, where the expression for $E(a, b, \lambda)$ in the center encodes the assumption of strong locality:



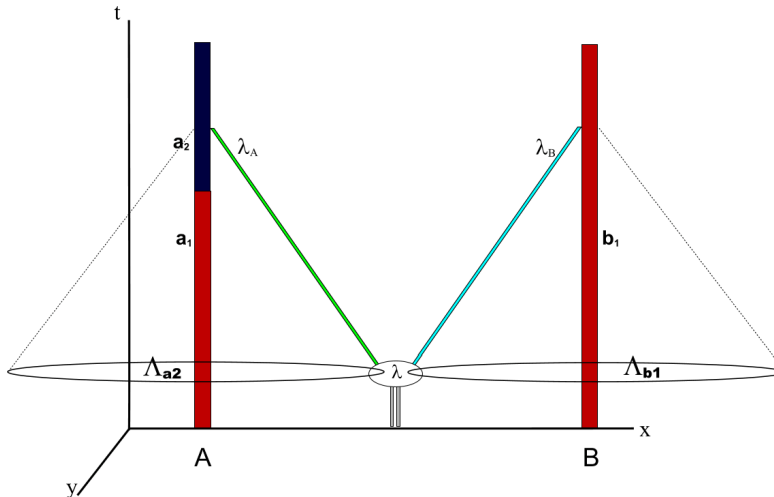


Figure 2: EPR: Spacetime diagram

Now the further assumption required for Bell’s result is that the probability of a given state λ is independent of the detector settings. In other words, Bell assumes that the theory will be one in which

$$P(\lambda|a, b) = P(\lambda). \tag{3}$$

This is *statistical independence*. For example, we might suppose that the theory tells us that one of three states $\lambda_1, \lambda_2, \lambda_3$ will be generated by our particle preparation procedure. This condition tells us that the likelihood of obtaining any one of these states is independent of how the detectors will be set at the time of detection. In other words, knowledge of the future settings of those detectors (their settings at the time the particles arrive) does not provide any further information as to which of the three states was emitted.⁵

The assumption of *statistical independence* has been called into question only infrequently, but when it has, the critique has often been motivated by an appeal to the plausibility of Lorentz-invariant “backward causation”, whereby the change of detector settings gives rise to effects that propagate along or within the backward lightcone and thereby give rise to nontrivial initial correlations in the particle properties encoded in λ (e.g., [6],[16],[13]). In my [18] I offer a critique of this way of thinking. Here instead I would like to offer a rather different sort of motivation for thinking that statistical independence might be violated, coming as promised from the possibility of nonlocal constraints.

Depicted in Figure 2 is a run of the EPR-Bohm experiment in which the setting of A is changed from a_1 to a_2 while the particles (or whatever it is that emanates from the source) are in flight. What we have here is a pair of particles traveling toward detectors A and B, with detector A switching from setting a_1 to a_2 while the particles are in flight, and detector B simply set to b_1 .

Let’s again suppose that the particles are in one of three states $\lambda_1, \lambda_2, \lambda_3$. According to classical, relativistic physics, the detector settings a_2 and b_1 are determined by the goings-on in their past lightcones, which include the particle preparation event but also far more. Suppose that setting a_2 is compatible with a variety of initial data at the time of preparation, and the same for b_1 . Let Λ_{a_2} be the presumably large subset of microscopic states (in the past lightcone of the detection event) consistent with a final detector setting of a_2 , and let Λ_{b_1} be those states compatible with b_1 . Though they reside in the past lightcone of the detection events,

let us suppose that the state of the particles, λ_1, λ_2 , or λ_3 , does not play a *dynamical* role in determining the setting of either detector. The question at hand is whether there is any reason to think that, nevertheless, the state of the particles is *correlated* with the detector setting, which is to say whether the theory constrains the state of the particles on the basis of Λ_{a_2} and Λ_{b_1} .

Now, if the underlying theory is one in which local degrees of freedom are independent, there is no reason to think that knowledge of Λ_{a_2} and Λ_{b_1} should tell us anything about which of $\lambda_1, \lambda_2, \lambda_3$ are realized. On the other hand, if there are nonlocal constraints, then it may well be otherwise. Suppose that Λ_{a_2} is compatible with λ_1 and λ_2 but incompatible with λ_3 . In other words, suppose that there are no microstates which generate a_2 which are consistent with the particle pair starting in state λ_3 . Then we already have a violation of the statistical independence condition, without even bothering yet to consider correlations with the other detector B .

Of course, there may be, and typically are, many things going on in the past lightcone of a detection event at the time the particle pair is produced. Most of these will at least have the appearance of being irrelevant to the final setting of the detector. There is certainly no guarantee that a nonlocal constraint will generate the kind of correlations between detector settings and specially prepared particles that we are talking about. The precise nature of the nonlocal constraint or constraints that could explain quantum correlations is a decidedly open question.

4 Superdeterminism, conspiracy and free will

The idea that the rejection of *statistical independence* involves preexisting and persisting correlations between subsystems has been broached before, under the terms ‘conspiracy theory’, ‘hyperdeterminism’, and ‘superdeterminism’. From here on, let us adopt ‘superdeterministic’ as a generic term for this way of thinking about theories that violate this condition. Bell [3], Shimony [15], Lewis [11] and others have suggested that superdeterministic theories imply some sort of conspiracy on the part of nature. This is frequently accompanied by the charge that the existence of such correlations is a threat to “free will”. Let me briefly address these worries before returning to the big picture.

The idea that postulating a correlation between detector settings and particle properties involves a “conspiracy” on the part of nature appears to derive from the idea that it amounts to postulating that the initial conditions of nature have been set up by some cosmic conspirator in anticipation of our measurements. It seems that the conspiracy theorist is supposing that violations of *statistical independence* are not lawlike, but rather are *ad hoc*. But nonlocal constraints are lawlike, since (by definition), we require them to be consistent with the dynamical evolution given by the laws of motion. If they exist, they exist at every moment of time. This is no more a conspiracy than Gauss’s law is a conspiracy.

Another worry about giving up *statistical independence* and postulating generic nonlocal, spacelike correlations has to do with a purported threat to our “free will”. This particular concern has been the subject of renewed debate in the last couple of years, prompted in part by an argument of Conway & Kochen [5]. The core of the worry is that if detector settings are correlated with particle properties, this must mean that we cannot “freely choose” the detector settings. However, as ’t Hooft [17] points out, this worry appears to be based on a conception of free will which is incompatible with ordinary determinism, never mind superdeterminism. Hume [10] long ago argued that such a conception of free will is highly problematic, in that it is essential to the idea that we freely exercise our will that our thoughts are instrumental in

bringing about, which is to say determining, our actions.

5 The Cosmos

So much for the possible role of nonlocal constraints in underpinning quantum phenomena. The other point of interest is early universe cosmology. Our universe appears to have emanated from a big bang event around 14 billion years ago, and to have been highly homogeneous for quite some time thereafter. The cosmic microwave background radiation is a fossil remnant of the time, around 400,000 years into the universe's existence, when radiation effectively decoupled from matter, and this radiation appears to be quite evenly distributed across the sky, with slight inhomogeneities which presumably seeded later star and galaxy formation.

The task of explaining the homogeneity of the early matter distribution is known as the horizon problem. This, along with the flatness problem and monopole problem, were for some time only explained by fine-tuning, which is to say that they were not really explained at all. Later, inflationary models entered the picture, and these provide a mechanism for generating inhomogeneity in a more generic fashion. However, these models are still speculative – there is no direct evidence for an ‘inflaton’ field – and moreover inflation itself requires rather special initial conditions[12].

The existence of a nonlocal constraint on the matter distribution and on the state of the gravitational field might address one or more of these problems without recourse to inflation. Certainly, a detailed description of the very early universe requires few variables, since the universe looks essentially the same from place to place with respect to both matter distribution (high temperature, homogeneous) and spatial structure (flat). A reduction in the number of variables is what we would expect from a constrained system, and any constraint demanding that the matter distribution is identical from place to place is indeed nonlocal. However, it is evidently not preserved under dynamical evolution because of the action of gravity. One might speculate, though, that the constraint holds between matter and gravitational degrees of freedom, and that the early universe is simply a demonstration of one way to satisfy it. The interplay of gravity and matter mix up the degrees of freedom as time goes on, and the current remnant of these correlations are the quantum correlations discussed above.

6 Conclusion

The idea of using nonlocal constraints to account for the large-scale matter distribution in the universe and the large-scale spacetime structure of the universe is interesting but highly speculative, and the idea that these same constraints might account for quantum correlations as well is even more speculative. The most conservative strategy of exploration would be to ignore cosmological scenarios and instead focus on the persistent and experimentally repeatable correlations in the quantum realm. But I think it is worth considering a connection between the two, if for no other reason than the fact that it has proven difficult to construct a testable and sensible quantum theory of gravity, suggesting that the relation between gravitation and quantum phenomena might be different from anything heretofore explored.

A more conservative approach focusing just on quantum phenomena might ponder the way in which the ordinarily superfluous gauge degrees of freedom of modern gauge theories might serve as nonlocal hidden variables. The vector potential in electrodynamics, for example, ordinarily plays no direct physical role: only derivatives of the vector potential, which give rise to the electric and magnetic fields, correspond to physical “degrees of freedom” in classical and

quantum electrodynamics. The Aharonov-Bohm effect shows that the vector potential does play an essential role in the quantum theory, but the effect is still gauge-invariant. One might nevertheless conjecture that there is an underlying theory in which the potential *does* play a physical role, one in which the physics is *not* invariant under gauge transformations. It may be impossible for us to directly observe the vector potential, and the uncertainties associated with quantum theory may arise from our ignorance as to its actual (and nonlocally constrained) value. From this perspective, quantum theory would be an effective theory which arises from “modding out” over the gauge transformations, with the so-called gauge degrees of freedom being subject to a nonlocal constraint and accounting for the correlations we observe in EPR-type experiments

I would conclude by reminding the reader that the sort of nonlocality under discussion in no way violates either the letter or the spirit of relativity. No influences travel faster than light. The idea is simply that there are correlations between spatially separate degrees of freedom, and thus that the fabric of nature is a more richly structured tapestry than we have heretofore believed.

Notes

¹Solutions to the wave equation can be written as sums of plane waves, with Fourier space representation $\hat{\phi}(k, t) = \hat{F}(k)e^{-ikt} + \hat{G}(k)e^{ikt}$. Since these plane waves must have period T (in the preferred frame dictated by the cylinder), we have a constraint $k = \frac{2\pi n}{T}$ (where n is a positive or negative integer), so that initial data are no longer arbitrary smooth functions of k

$$\begin{aligned}\hat{\phi}(k, 0) &= \hat{F}(k) + \hat{G}(k) \\ \hat{\phi}_t(k, 0) &= -ik(\hat{F}(k) - \hat{G}(k))\end{aligned}$$

but are rather constrained by the requirement $k = \frac{2\pi n}{T}$. Thus the initial data are the functions

$$\begin{aligned}\phi(x, 0) &= \frac{1}{\sqrt{T}} \sum_{n=-\infty}^{\infty} \hat{\phi}\left(\frac{2\pi n}{T}, 0\right) e^{i\frac{2\pi n}{T}x} dk \\ \phi_t(x, 0) &= \frac{1}{\sqrt{T}} \sum_{n=-\infty}^{\infty} \hat{\phi}_t\left(\frac{2\pi n}{T}, 0\right) e^{i\frac{2\pi n}{T}x} dk\end{aligned}$$

i.e., they consist of arbitrary sums of plane waves with wave number $k = \frac{2\pi n}{T}$, for any integer value of n .

²The state used by EPR is an eigenstate of the operators representing the sum of the momenta and the difference of the positions of the two particles.

³The argument of the EPR paper is notoriously convoluted, but I follow [9] in regarding this as capturing Einstein's understanding of the core argument.

⁴A more detailed discussion of Bell's derivation and the role of the Statistical Independence assumption can be found in [18].

⁵Actually, a slightly weaker condition than SI is sufficient to derive the CHSH inequality. See [8] and the discussion thereof in section 3.3.1 of [14].

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