# *"In Nature as in Geometry": Du Châtelet and the Post-Newtonian Debate on the Physical Significance of Mathematical Objects*

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**Abstract:** *Du Châtelet holds that mathematical representations play an explanatory role in natural science. Moreover, things proceed in nature as they do in geometry. How should we square these assertions with Du Châtelet's idealism about mathematical objects, on which they are 'fictions' dependent on acts of abstraction? The question is especially pressing because some of her important interlocutors (Wolff, Maupertuis, and Voltaire) denied that mathematics informs us about the properties of real things. After situating Du Châtelet in this debate, this chapter argues, first, that her account of the origins of mathematical objects is less subjectivist than it might seem. Mathematical objects are non-arbitrary, public entities. While mathematical objects are partly mind-dependent, so are material things. Mathematical objects can approximate the material. Second, it is argued that this moderate metaphysical position underlies Du Châtelet's persistent claims that mathematics is required for certain kinds of general knowledge, including in natural science. The chapter concludes with an illustrative example: an analysis of Du Châtelet's argument that matter is continuous. A key but overlooked premise in the argument is that mathematical representations and material nature correspond.*

## 1 Introduction

A familiar early modern doctrine is that mathematical objects aptly represent the physical world, such that physics can and should be mathematical. But this was contested. One source of vigorous resistance was the difficulty in giving a *metaphysical* account of mathematical objects. It was widely agreed that mathematical objects, unlike the subject matter of physics, are not substances that wield causal powers. Difficulties also arise if mathematical objects, with their definitional exactness, are regarded as accidents or modes of physical things. A related *epistemological* question concerns how inferences from mathematics to the natural sciences might be justified. Third, there were *methodological* worries: it was often thought that science must deal with substances and offer causal demonstrations. Since mathematics does not seem to deal with primary substances or offer causal demonstrations, it is not clear how it meets these criteria for proper science.[1]

These controversies persisted in the wake of Newton's *Principia*. In the first half of the eighteenth century, this work was often treated, both by critics and sympathizers, as a work in pure mathematics. Many German and French thinkers adopted a broadly anti-realist approach to Newton's physics, holding that despite its impeccable mathematics, the *Principia* did not constitute proper, causal physics. Yet it was difficult to dismiss Newton's arguments,

[1] For example, in a 1669 letter to Thomasius, Leibniz endorses these two broadly Aristotelian criteria for science, and adds that the "*scholastici*"—though not Leibniz himself—deny they can be met by geometry (1875–90, IV.168–70).

which have causal conclusions, as in his famous defense of universal gravitation. And his empirical results in optics and orbital mechanics were widely accepted. François De Gandt has called this a "crisis of causality" in the reception of Newton (2001, 129).[2]

Emilie Du Châtelet develops an intriguing and still neglected position on these issues.[3] As I show, three of her contemporary interlocutors—Maupertuis, Voltaire, and Wolff—are skeptical about inferences from mathematics to physics, despite their broadly Newtonian sympathies. Du Châtelet has been read as likewise regarding the *Principia* as merely mathematical, rather than genuinely causal (Shank 2018, 273–74). But in fact, I'll argue, she is confident about these inferences from mathematics to physics, and therefore about the standing of Newtonian physics as she understands it.

This stance regarding the epistemic bona fides of mathematics is compatible with her idealist metaphysics of mathematical objects, which are fictions and products of abstraction. The objects of physics are also ideal and partly mind-dependent, for Du Châtelet, if to a lesser degree. This may help with the problem of understanding the success of inferences from mathematics to physics. This problem is still much discussed by philosophers of science and mathematics, but usually in a physicalist and metaphysically realist framework.[4]

I conclude by considering Du Châtelet's frequently overlooked arguments for the continuity of matter and change. These arguments, which are important for her physics, rely on the correspondence between nature and mathematics.

## 2 The Ambivalent Reception of Newton's Mathematical Physics

Du Châtelet's Preface to the *Institutions* frames the work as an exposition of Newton's "system" (Du Châtelet 1742, 7). That system is presented as mathematical physics. On her telling, Newton played a critical role in a scientific "revolution" by which modern physics acquired "solid foundations" through a combination of "geometry" and "observations" (12; 5; 9; cf. 1738, 535, 1759, 7–8).

Geometry, she holds, is "the key to all discoveries" in science, without which "progress in the study of nature" is impossible (1740, 3–4; 2018, I:500). Much remains unexplained in physics, but this is because of a failure to make sufficient use of geometry, which here includes post-Cartesian algebraic geometry (3–4; 14). Geometrical demonstrations, moreover, are maximally rigorous and link necessary truths (14; 17; 20–21; 26).

The Preface also makes clear that while the *Institutions* is intended to be understood by a reader who knows only ordinary geometry, more advanced physicists make use of "algebra" (3). The task of "making Newton accessible" to a wider public, especially the "mystery" of his mathematical techniques, is already stated as a goal in her earlier, anonymous review of a work on Newton by Voltaire (Du Châtelet 1738, 534–5; 538; 541). The *Institutions* also refers to calculus, though usually in passing (e.g. 1742, 308). Du Châtelet later

[2] See further Gingras (2001), Janiak (2007), and Shank (2008, 49–104). Thanks to Anat Schechtman for insightful comments on these issues.

[3] I focus here on Du Châtelet's *Institutions de Physique*, and still more narrowly on the foundational and methodological discussions early in the *Institutions*, as opposed to more specific discussions of physics later in the work. It is undisputed that mathematics plays a pivotal role in the *Institutions*. The puzzle is why and how Du Châtelet takes this to be justified.

[4] One recent statement: "there must be some kind of correspondence between the mathematics in which we formulate our theories and the nature of the physical world, a correspondence that helps explain, on the one hand, the effectiveness of the mathematics in describing the world, and on the other, the success of our inferences about the world on the basis of that mathematics" (North 2021, 5).

calls calculus the "geometry of the infinite" (1759, 9).[5] She considers it to be just as certain as classical geometry.

But how exactly is mathematics a "means," as she writes, to foundations for physics (1742, 12)? In working out the answer to this question, it's helpful to consider Du Châtelet's historical context, and especially three figures in the reception of Newton's *Principia* who were of particular significance for her: Maupertuis, Voltaire, and Wolff. While these men influenced Du Châtelet in some ways, she disagrees with them about the role of mathematics in physics.

Maupertuis's short treatise on the *Figures des Astres* (1732) is a work that Du Châtelet knew well (Du Châtelet 2018, I.350). Maupertuis presents the work as defending Newtonianism. The second chapter is an excursus on the metaphysics of Newtonian attraction or gravitational force. Maupertuis notes that Newton's theory was accused of being unparsimonious, and of reviving "the doctrine of occult qualities" (Maupertuis 1732, 11).

Let's consider the second of these accusations. Maupertuis does not say what he means by occult qualities. But epistemological worries about occult qualities often turned on their alleged causal efficacy (Pasnau 2011, 540–46). The problem was not just positing unobserved qualities, but taking these to be causal powers that explain what is observed, for example when fire is allegedly explained by a power to burn.

The identification of occult qualities with unobserved causal powers sheds light on Maupertuis's response to this objection. He argues that Newton never advanced attraction as a causal explanation ("*explication*") of the behavior of bodies, such as the "heaviness [*pesanteur*] of some bodies towards others" (1732, 11). Instead, Newtonian theory gives a merely mathematical account of certain regular effects that are "susceptible" to being considered as greater or lesser in quantity ("*de plus & de moins*") (12). A mathematical account is within our "means," whatever the "nature" of the underlying cause turns out to be ("*quelle que soit sa nature*") (12).

It is clear from Maupertuis's examples that the 'mathematicians' he has in mind include thinkers we'd now consider natural scientists or physicists. In addition to Newton, Maupertuis discusses Galileo. He presents the theory of falling bodies in Galileo as mathematically "explaining the phenomena," but leaving an account of causes to "more sublime philosophers" (1732, 12–13). Maupertuis does not elaborate on just what this would involve, and there is room for interpretive debate here. But Maupertuis favors the merely mathematical, acausal approach. He deflates the question of whether Newtonian force is an "inherent" property of matter: it can be usefully regarded "as if it were [*comme si elle étoit*]" such a property, but no metaphysical commitment is supposed to follow from this (10; cf. Maupertuis 1735, 343).

A helpful point of comparison here is classical geometrical optics. Euclid's *Optics* precisely described many observed aspects of vision, even though the causal mechanism of sight remained unknown for centuries.[6] In the eighteenth century, optics was often still distinguished from the proper causal science of nature. Leibniz used the analogy with

[5] This follows eighteenth-century usage. For example, in 1702 Leibniz presents his infinitesimal calculus as a type of "geometrical reasoning" (Leibniz 1863, IV:91–92), and in 1743 D'Alembert places calculus within pure geometry (D'Alembert 1743, vi–viii).

[6] Aristotle's *Physics* distinguishes between pure geometry and optics, which is "*hupó*" or subordinate to it (194a6–11). But optics is still a mathematical rather than natural science: the *Meteorology* expounds observations about reflection drawn "from the theory of optics" (1984, 372a29), then separately lays out *causal* natural-scientific theses concerning halos, rainbows, and so on (372b13–378b6). Early modern optical texts continued to classify optics as applied mathematics, citing Aristotle as an authority (Dear 1995, 41–58).

classical optics to stress the explanatory limitations of the Newtonian project, which he described as merely abstract and geometrical.[7]

Voltaire's 1738 *Eléments de la Philosophie de Newton* went through twenty-six editions in fifty years. Du Châtelet was closely involved in its composition and may have even written portions of it, although she raised criticisms of the results, both publicly and in private letters.[8]

Voltaire is explicitly indebted to Maupertuis's *Figure des Astres*, and especially to its chapter on the metaphysics of attraction. He praises it as perhaps the best philosophical work in French (Voltaire 1738, 84). In fact, already in 1732, Voltaire had turned to Maupertuis for help in understanding the *Principia* (De Gandt 2001, 136–39).

Like Maupertuis, Voltaire seeks to defend Newton's system broadly speaking, while also driving a wedge between mathematics and causal physics. Many chapters are devoted to "proofs" and "demonstrations" of Newton's theory of gravitation (Chapters XVIII–XXII), or to refutations of its Cartesian rivals (Chapters XV–XVI). But in a set of 'Eclairissements' added to the second edition, Voltaire defends the reality of gravity, while denying that Newton revealed its "principle" or "cause" (xiv–xv; cf. Voltaire 1734, 65; Du Châtelet 1738, 539–40). Newton is depicted as a mere mathematician, lacking the time for the kinds of deeper metaphysical investigations Voltaire takes to be required in natural philosophy (1738, vii). Voltaire raises comparisons to classical optics, which as "simple geometry" cannot seriously apply to empirical problems (iii–v).[9]

These remarks suggest that for Voltaire, natural science ought to give an account of causes, and that Newton fails to do so. This is hard to reconcile with Voltaire's repeated claims that Newtonian theory is strictly proven.

One case where Voltaire's position is clear, however, is that of inferences from mathematics to physics. In Chapter X, Voltaire criticizes the Newtonian John Keill. For Keill geometry is foundational for all philosophy, and there is no other way to obtain knowledge of "the forces of nature" (1733, viii). Voltaire denies both these claims, consigning geometry to a more limited role.

Voltaire objects with particular zeal to Keill's arguments for the infinite divisibility of matter. Keill's basic move is to claim that, because space can be proven to be infinitely divisible within Euclidean geometry, matter must be infinitely divisible as well (Keill 1733, 20–32). This argument, then, moves from mathematical premises—Keill gives five examples of how indivisibles conflict with Euclidean assumptions—to conclusions about the physical world.[10] We will see below that Du Châtelet could endorse Keill's argument, so long as it is not taken to show that matter is made up of an infinity of *actual* parts (Du Châtelet 1742, 191–92).

Consider one of Keill's examples, which was discussed in thirteenth-century thinkers such as Roger Bacon in his *Opus Maius* (1962, 173), and originally found in Book X of Euclid's *Elements*. The assumption of indivisibles, Keill points out, would allow one to build

[7] See for example a letter to Hartsoeker of 8 February 1712 (Leibniz 1875–90, III:534–5; cf. VII:452).

[8] On the publication history, see De Gandt (2001, 126). Zinsser (2006, 145–51) argues for Du Châtelet's involvement. See Du Châtelet (1738, 1740, 7, 2018, I:345–6; I:353) for her mixed assessments of the work, as well as Lu-Adler's (2018, 183–88) examination of divergences between Voltaire and Du Châtelet.

[9] Instead, Voltaire praises the psychological account given in Berkeley's *New Theory of Vision*. Voltaire presents Berkeley's theory as both empirical and *metaphysical*: the intended contrast is with the merely mathematical and acausal style of classical optics. Du Châtelet seems sympathetic to this point (1738, 537). But given the broad scope of the term 'metaphysics' here, we cannot assume that Voltaire influenced Du Châtelet's shift towards the a priori metaphysical systems of Leibniz and Wolff in the *Institutions*, as Gessell (2019, 868–71) has argued.

[10] Kant, in 1790, still cites Keill as giving an authoritative "demonstration" of matter's infinite divisibility (2004, 8:202).

up both a square and its diagonal out of some finite number of spatial parts. The lengths of the side of the square and its diagonal would then be commensurate (i.e., have a common divisor) (Keill 1733, 30). But as a consequence of the Pythagorean Theorem, the side and diagonal of a unit square are incommensurable.[11]

Voltaire rejects Keill's argument. He is willing to accept that within geometry, it can be proven with complete certainty that a line is infinitely divisible (Voltaire 1738, 102–3). But he also takes it to be proven a priori that there are indivisible physical atoms.[12] Granting that this might seem "contradictory," Voltaire insists that it is not: "geometry has as its object the ideas of our mind [*esprit*]" (102). He spells out the point on the following page. Geometrical objects have only a mental existence, and geometrical points, lines, and planes cannot exist in the natural world (103). Lines and planes are indefinitely divisible in thought ("*en idée*"), and Voltaire hints that we can indefinitely divide matter in thought, as well (103). But mere divisibility in thought "hardly prevents" physical atoms from existing (1738, 102–3).

It follows that geometrical reasoning is not a reliable guide to the physical world. Geometrical objects are divisible, but merely mental. Real, extramental matter is discrete. Voltaire does not, however, consider what this might mean for the accuracy of mathematical physics.

Most of Christian Wolff's 'German Metaphysics,' first published in 1720, was available to Du Châtelet in translation by early 1737. She had some knowledge of Wolff's Latin works as well, and his system influenced some aspects of her *Institutions*.

Wolff takes metaphysics to study more fundamental entities than does mathematics. Metaphysics is concerned with fundamental substances "in themselves," which include finite souls, fundamental simple substances, and God (Wolff 2001/1731, 336; cf. 1720, §593). Such substances are causally active, and in turn, causal activity is the criterion of substantiality (1720, §116). The objects of mathematics, by contrast, are not causally active (1965/1726, §18). Since causal activity is the criterion of substantiality, mathematical objects are not substances, but indeterminate and ideal entities, essentially dependent on our imagination (2001/1730, §§110–11).

Wolff takes philosophy to be the study of possible *things*, in the strict sense of substances or *res*.[13] Mathematical objects are not substances or *res*, and do not fall within the domain of philosophy. That is, mathematical statements do not have a place in the ideal system of philosophical "truths" (1965/1726, §36). They instead serve to make the contents of that system clearer and more distinct. For Wolff, even Euclid's first principles are not primitive, but have an ontological basis (Buchenau 2013, 32–33).

Accordingly, Wolff's account of mathematics focuses on its applications. The goal of mathematics is, through measurement, to make clearer and more distinct our knowledge of qualities and quantities in substances, such as forces (1713, Vorbericht, XV, 2001/1730, §442; §742). Even here, Wolff is cautious about regarding forces as literally quantifiable. While he does endorse degrees of force for which arithmetical operations seem to be well-defined,

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[11] Voltaire comments that this example shows how the mathematics of the infinitely large and small, despite its apparent insanity or absurdity ("*déraison*"), is "founded on simple ideas" (1961/1734, 71; 76). Yet he denies that we can directly draw any physical consequences from it.

[12] If the division of matter were actually completed, there would only be empty "pores," not matter; thus the actual existence of matter contradicts its infinite divisibility (1738, 102). Clarke's Fourth Reply to Leibniz presents a similar argument that also identifies pores with void (Leibniz & Clarke 2000, 35). But Aristotle already discusses the gist of this argument, attributing it to the Presocratic atomists (1984, 324b25–5a16). In the same passage, Aristotle discusses a theory of pores (*poroi*)—though as it appears in the Empedoclean theory of causation and change, rather than an atomist argument.

[13] Wolff famously claims to follow a mathematical method in philosophy, though critics soon objected that his method has little to do with mathematical practice (Basso 2008). Wolff presents the method as in the first instance logical: it could be grasped "even if mathematics did not exist" (Wolff 1965/1726, §139).

these degrees are merely "imaginary" (2001/1730, §752–54; §747). His idea is that magnitudes for which operations such as addition are well-defined—often called extensive magnitudes—must consist of homogeneous parts that can be added together. Forces have no such parts: they are merely intensive magnitudes.

For Wolff, then, we cannot infer from the properties of mathematical objects to the properties of things (1965/1726, §6; §17; 2001/1730, §110). Such inferences can only lead to contradictions, preventing progress in both physics and metaphysics (2001/1730, §110). For example, although Wolff accepts that geometrical objects are continuous, he holds that the world is made up of simple, discontinuous elements (1720, §76; §81).

In a 1731 essay on the differences between mathematical and metaphysical concepts, Wolff draws on this account in assessing Newton's *Principia* (Wolff 2001/1731, 286–348). As Katherine Dunlop (2013) has emphasized, Wolff dismisses Newtonian forces as "imaginary" (Wolff 2001/1731, 316). For Wolff, the core concepts and causal inferences of physics need to be grounded in metaphysics (1965/1726, §§94–95). But Newton is "no metaphysician" (2019, 1:209). The *Principia*'s propositions refer only to phenomena, and describe their regularities. They are merely mathematical (Dunlop 2013, 466). So they do not get at underlying causes, and cannot yield genuine explanations without metaphysical supplementation.[14]

**3 Du Châtelet on the Metaphysics of Mathematical Objects**

I now turn to surveying a family of claims Du Châtelet makes in the *Institutions* about the metaphysical status of mathematical objects, which are non-fundamental and mind-dependent. I discuss the location of mathematical objects in her global metaphysics, her general account of magnitude, and her claims that mathematical objects are fictions and products of abstraction. While I cannot treat these topics in full detail, I hope to provide enough information to set up a later discussion of Du Châtelet's confidence in the role of mathematics in physics.

**3.1 Mathematical Objects and Metaphysical Idealism**

The metaphysical status of mathematical objects must be understood in terms of Du Châtelet's multi-level idealist metaphysics. She holds that the fundamental level of created reality consists of simple, active substances (1742, 141–46; 155–58). Some simple substances are souls (45; 128; 133–34; 149–50; 156–58). Souls represent the whole universe, although confusedly. Other simple substances do not seem to be endowed with representational capacities at all.

Simple substances lack spatiotemporal properties and are not directly perceived. But we can indirectly infer their existence and activity (138–41; 185–87). Matter is non-mereologically grounded in these simples. As we'll see in section 4.2, matter has no lowest level of mereological structure because it is indefinitely divisible.

Apparently, Du Châtelet is committed to genuine causal interaction and dependence among at least some fundamental created substances (147–48; Stan 2018). It seems that souls represent the whole universe partly in virtue of being causally affected. If the rest of the universe was different, their representational states would differ as well (151–52). Simple substances themselves, however, cannot depend just on other finite substances. Their existence can be shown to depend on God (142).

[14] The Wolffian Samuel Formey, secretary of the Berlin Academy, took this position to extremes. He argued in a reply to Euler that quantity, as an essentially imaginary concept, cannot be applied to "real and existing" objects at all (1754/1747, 281). For Formey, then, attempts at mathematical physics, including Newton's, issue in "absurdity," "occult qualities," and contradictions (282; 285; 294). He rejects not only geometrical arguments for the indivisibility of body (284), but all quantitative conceptions of motion and force, taking these to be based on "*notions imaginaires & confuses*" (292).

Du Châtelet's views may be reminiscent of Leibniz's multi-level idealist metaphysics.[15] But there are important differences. Leibniz's simple substances or monads are all conceived as perceiving and hence as mind-like in at least a minimal sense (Leibniz 1875–90, II:270; IV:479; VI:598). It is in virtue of expressing or representing the rest of the world from a unique point of view that simple substances are individuated (II.47).[16] Furthermore, Leibniz famously forbids efficient causation among created substances: monads have no windows (II:264; II:271; IV:509–10; VI:607–8). On these points, Du Châtelet diverges from Leibniz.

Like Leibniz, however, Du Châtelet does not take space and time to be fundamental, mind-independent entities. Instead, she regards them as mind-dependent (1742, 101–4; 120–22). Material beings are essentially spatiotemporal, so their existence and properties are also mind-dependent (1742, 155–56; 176–82). Thus the properties of material things relevant for physics are at least partly mind-dependent: extension, motion, and force are "phenomena" (176). Such phenomena are not wholly mind-dependent, however, because they are also grounded in non-mind-like simples (143–44).

This brings us to mathematical objects, which Du Châtelet presents as still more mind-dependent than material things (Carson 2004, 170). This is a complex claim that will need to be unpacked.

We can begin with a contrast Du Châtelet draws between material things as "real and determined" and mathematical objects as ideal and indeterminate (1742, 112). An important application of this point I'll return to later is that only real things have determinate, actual parts. The number of parts in a geometrical object, by contrast, is "absolutely indeterminate": these parts are merely potential (190; cf. Leibniz 1875–90, II:282; IV:491). Their determinacy is one indication that material things are more real than mathematical objects, along with the fact that material things are endowed with force (163–64). These differences between material things and mathematical objects are presumably explained by their standing in different grounding relations to simple souls and non-mind-like simple substances.

The indeterminacy of mathematical objects also means they are not subject to a principle of the identity of indiscernibles that is based on qualitative properties. According to this principle, neither fundamental substances nor material things can be alike in all of their qualitative properties, such that they differ only numerically (30). The individuality of a part of geometrical extension, however, is grounded solely in its numerical distinctness, and this holds for other mathematical objects as well (103). Du Châtelet is not a strict Cartesian mechanist, and does not seek to reduce qualitative properties to mereological properties. Thus the qualitative indeterminacy of mathematical objects seems to be an additional claim, over and above the thesis that they have an indeterminate number of parts.

Du Châtelet concludes that no mathematical object can be identical to ("*la même chose que*") a real, material thing (1742, 112). A mathematical object, as indeterminate, must lack the determinate qualitative properties that make a material thing what it is, so it cannot have the same qualitative properties as a material thing. By the principle of the identity of indiscernibles, she argues, it cannot be identical to a material thing, since identity requires sharing all qualitative properties. Nevertheless, as we'll see in the following section, Du Châtelet apparently holds that magnitudes are *in* material things, even if these magnitudes are not identical to mathematical objects proper.

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[15] Leibniz also distinguishes between mereological and non-mereological grounds. Monads are not literally parts of bodies (Leibniz 1875–90, II:268–69; II:436). See further Rutherford (1990).

[16] The 1740 edition of the *Institutions* suggests that souls (but not all simple substances) are individuated by their unique representational states, and cites Leibniz (§128). However, Du Châtelet deletes this discussion from the 1742 edition.

### 3.2 The Metaphysics and Epistemology of Magnitude

Considering some of Du Châtelet's remarks on magnitudes will shed light on the metaphysical status of mathematical objects. In brief, she describes magnitudes as *internal properties* of material things. Du Châtelet further distinguishes between magnitudes as such and the conditions under which we fully understand magnitudes and communicate differences between them. Paradigmatic objects of mathematics, such as numbers or geometrical lines, plausibly depend on the conditions for understanding magnitudes. Communicating about magnitudes, in turn, requires units, and often an element of convention.

An especially detailed discussion of magnitudes appears only in the longer manuscript version of Chapter 1 of the *Institutions*. These remarks are echoed in various parts of the published work, however, and are not canceled out in the manuscript, so they may have remained unpublished for reasons of space.

The manuscript defines magnitude (*grandeur*) as an "internal" property of a "thing" (*chose*) in virtue of which it can differ from other entities that are in other respects "similar" to it (Du Châtelet 1738–40, ff. 33*r*–33*v*).[17] The things in question are, paradigmatically, material bodies. For example, "particles of matter" have a volume, and this is a magnitude (f. 32*v*). Elsewhere, Du Châtelet refers to forces standing in proportion to one another. Thus forces are also magnitudes or quantities (1742, 83).

Quantitative equality is something above and beyond being qualitatively "similar" or "alike" (*semblable*) (Du Châtelet 1738–40, ff. 33*r*–33*v*; 1742, 103–4; cf. Aristotle 1984, 6a26–35; 1021a). Given this distinction, two objects may differ in magnitude even if they are qualitatively alike, as in two similar triangles that are different sizes and hence not congruent. For two qualitatively identical things, a "real difference" in magnitude can metaphysically individuate those things (1738–40, f. 43*v*).[18]

Magnitudes per se, then, exist in concrete, physical things. They serve to metaphysically individuate things from qualitatively similar things. So magnitudes do not have a merely mental existence. This is significant for assessing the metaphysical status of mathematical objects.

Having answered the metaphysical question of what magnitudes are, the manuscript turns to further questions about how we can fully understand magnitudes and communicate about them. First, regarding understanding: even though magnitude is an internal property of things, "it can only be understood [*comprise*] through the comparison our senses make of one object to another" (f. 33*r*). For two magnitudes to be compared, they must have some degree of qualitative similarity (f. 33*v*). The volumes of two particles can be compared, for example, because each particle is extended in three dimensions.

Even if magnitudes are internal properties of things, then, they can only be fully understood through comparison with other objects. For example, a token length is an internal property of a thing that does not metaphysically depend on acts of comparison. When we wish to understand the *size* of this length, for example, we must compare it with a class of lengths. This class defines a type of magnitude. (In Du Châtelet's French there is just one word, *grandeur,* for both magnitude and size, but she nevertheless seems to be drawing a distinction here.) While the epistemological details are not spelled out, the reference to understanding

[17] By an *internal* property, in turn, I take Du Châtelet to mean one that reliably appears intrinsic to a thing. Because of Du Châtelet's idealist commitments, there is an important sense in which magnitudes are *not* intrinsic properties of material things. Material magnitudes are partly mind-dependent, and so can't be instantiated in worlds without minds. But for practical purposes, one can abstract from the dependent status of material beings and draw a contrast between the intrinsic and the relational at the level of what Du Châtelet calls *substantial phenomena*.

[18] Wolff also emphasizes that qualitatively similar things can differ in quantity (1720, §20–22, 2001/1731, §196). He denies, however, that things can be *individuated* by difference in quantity (1720, §20; 1973/1710, 118). It is debatable whether this is a merely epistemic point, or also concerns metaphysical individuation.

implies that cognitive faculties are constitutively involved in grasping size. By contrast, a token magnitude seems to be an object of mere perception.

Du Châtelet does not claim that understanding a size requires apprehending it in terms of units. This tracks the Euclidean conception of unit-free comparison between magnitudes, which can afford a grasp of ratios. However, units of measure do play a role in Du Châtelet's account. Here we can turn to the condition she lays out for communicating a magnitude to someone. To do this, "it is required to tell him the relation [*raport*] it has [to] a measure that is known [*connuëe*] to him" (f.33*v*).[19]

Our choice of unit may turn out trivially conventional or up to us. Du Châtelet mentions various more or less arbitrary units of time, for example. However, magnitudes themselves, as internal features of things, are not conventional. This partly explains why some *ways* of measuring time are objectively better than others. A well-made pendulum clock, for example, is more accurate than a sundial (1742, 132–3).

This account of measurement differs from a traditional Euclidean theory. For Euclid, measurement was defined in terms of the composition of rational magnitudes out of *aliquot* parts that are multiples of the whole that they compose.[20] The magnitudes of the parts can then be used to measure the magnitude of the whole directly. Euclid calls magnitudes that permit this type of measurement homogeneous, or "of the same kind" (Euclid 1908 II.114).

This Euclidean conception of measurement appears in many early modern mathematics texts (e.g. Arnauld 1683, 5). And it seems to inform Wolff's approach to mathematics. While Wolff denies that forces are literally made up of homogeneous parts, for example, he thinks they must be treated *as* composed of such parts, even if these are strictly speaking imaginary. Plausibly, Wolff is trying to get forces to satisfy Euclid's criteria for measurement. It is not so clear how Wolff can account for the soundness of mathematical reasoning relating heterogeneous quantities, as in algebra.

Du Châtelet, meanwhile, endorses the (early) modern conception of algebra as the science of magnitude in general. Although the *Institutions* officially uses geometrical rather than algebraic methods for pedagogical purposes, she is willing to use algebraic definitions, and also stresses the power of algebraic geometry, where unknowns are computed from given quantities (14).[21]

In turn, Du Châtelet claims that her conception of magnitude is especially apt for algebra (1738–40, f.33*v*). Algebra presumably works at the level of *sizes* and magnitude-types that arise from comparison, and only indirectly applies to token concrete magnitudes. As Descartes, Leibniz, Wallis, and others argued, algebra relates magnitudes of many kinds,

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[19] Compare Wolff: "If I am supposed to tell someone how large something is, I must tell him what relation it has to a certain measure that he is familiar with" (1720, §20; 1713, I, 38). Wolff concludes from this that magnitude cannot be grasped by the understanding, but must be sensibly "given" (*gegeben*) (§20; cf. Sutherland 2005, 147). Du Châtelet does not explicitly draw this epistemological consequence.

[20] This is the sense of 'part' (*méros*) at issue in Definition 1 of Book V of Euclid's *Elements*: "A magnitude is a part of a magnitude, the less of the greater, when it measures the greater" (Euclid 1908, II.113). In turn, Definition 4, the so-called Axiom of Archimedes, stipulates that magnitudes in ratio to one another must be "capable, when multiplied, of exceeding one another" (II.114; White 1992, 148–154). Thus, infinitely small or large magnitudes are incommensurable with finite ones. In Commandino's widely used edition of the *Elements* (1572) this was presented not just as a definition of magnitudes in ratio, but as a genuine axiom, ranging necessarily over *all* magnitudes whatsoever (De Risi 2016, 626). The point was contested even before the development of calculus, however. Cavalieri and Torricelli, among others, treated infinite collections of indivisibles as summing to finite magnitudes, thus violating the Axiom of Archimedes (608).

[21] For example, she defines the "measure" of dead force algebraically, as the product of mass and initial velocity, rather than in terms of parts of a homogeneous magnitude (1742, 438–39).

whether continuous or discrete.[22] It is unclear, meanwhile, how Wolff would account for sound algebraic reasoning that relates heterogeneous quantities.

In discussing basic arithmetic, however, Du Châtelet hews closer to the traditional Euclidean account. In discussing the integers, she takes ultimate "units," that is, the number 1, to be "combined" in order to form larger numbers (1742, 70). Numbers can thus be regarded as composed of units, and as homogeneous with those units (cf. Aristotle 1984, 1057a3–4). It is "hardly necessary" that numerical units actually get composed into a given number (1742, 70). Numbers are partly mind-dependent, and some large numbers will never be reached by finite minds like ours. Yet Du Châtelet attributes a kind of necessity to arithmetical units themselves, comparing them to the simple positive properties that make up the essences of possible things (70). Obscure as this comparison may be, it indicates that at least in the case of arithmetic, we do not have a choice of the unit of measure.

Numbers strictly speaking, however, are not internal properties of things, like the magnitudes discussed in the manuscript. Numbers are partly dependent on "things numbered," but their existence also depends on an active "power to form" mathematical objects through abstraction (1742, 112–13; 110). As discussed further in sections 3.3 and 3.4, mathematical objects such as numbers are presented as ideal, abstract, and fictional. While matter and its properties are partly mind-dependent, they are not described as ideal, abstract, or fictional. So in the end, the ontological status of the integers seems comparable to that of algebraic quantities.

These points may help understand Du Châtelet's claims that extension, number, and other mathematical objects are products of mental abstraction, and therefore fictions. Not just mathematics but "all the sciences" are "full of…fictions" and crucially rely on them, even if mathematics is most of all dependent on fictions (1742, 111–12). One reason for this is the prevalence of mathematical representations in science. Before turning to her account of fictions as such, I briefly discuss the faculty of abstraction, which Du Châtelet thinks makes fictions possible.

### 3.3 The Power of Abstraction

Du Châtelet would have been aware of a wide range of theories of abstraction—from the Port-Royal *Logic*, Locke, Wolff, and others—but she does not commit to any one of these in detail.[23] Instead, she focuses on the applications of this mental faculty. Some basic features of her account emerge in the following passage.

> This power that our mind has of forming by abstraction imaginary Beings that contain just the determinations we want to examine—and of excluding from these Beings all the other determinations, by means of which they could be conceived in another way—is of great use in meditation, for the imagination then rescues the understanding, and helps it to contemplate its idea. (Du Châtelet 1742, 111; cf. 182–83; 343–45; 352–53)

Abstraction, then, is characterized as an active or creative power of our faculty of imagination. This already distinguishes Du Châtelet's account from a purely negative conception on which abstraction merely strips away irrelevant properties from objects. For Du Châtelet, abstraction does, negatively, ignore select features of things, notably "internal" qualitative "determinations" (1742, 113). But it also, positively, creates new intentional objects.

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[22] See e.g. Leibniz (1863, IV:451–2; V:178–9) and the discussions in Hill (1996) and De Risi (2021, 15–17).

[23] The Port-Royaliens define abstraction in terms of analysis into parts or aspects (Arnauld and Nicole 1996, 37). Du Châtelet does not define abstraction this way. Locke and Wolff introduce abstraction to give a nominalist account of how we acquire general ideas (Locke 1975, 2.11.9; Wolff 1713, I, 26). Du Châtelet does not explicitly share this aim, or their nominalism. Abstractionism about Lockean general ideas need not go together with abstractionism concerning mathematical objects. Kant and the Frege of the *Grundlagen*, for example, retain an abstractionist account of concepts such as <tree> or <cat>, but deny that it applies to mathematical objects. Conversely, one could endorse mathematical abstractionism without applying the theory to other general ideas.

As we've already seen, the intentional objects of mathematics are indeterminate, with only potential parts, whereas physical objects have fully determinate, actual parts. Hence mathematical objects are designated abstract entities ("*les Abstraits*"), and contrasted with spatiotemporal, concrete entities ("*les Concrets*") (111).

For Du Châtelet, then, abstracta are partly dependent on acts of abstraction. But they are also grounded in material things. An abstract entity "cannot subsist without a concrete entity [*ne peut subsister sans un Concret*]" (112). That is, every actual abstract entity *y* counterfactually depends on some concrete entity or entities *x*: if *x* does not exist, then necessarily, *y* does not exist.

While the passage quoted above links acts of abstraction to the faculty of imagination, it also emphasizes that the imagination assists the *understanding* in contemplating its proper ideas or concepts. In fact, some "geometrical truths" defy our mere imagination, understood as a capacity to "represent our ideas by way of sensible images" (1742, 158). By contrast, algebra "speaks only to the understanding," as its objects cannot be represented in images (3; cf. 158). Cases such as the employment of algebraic techniques in geometry and calculus highlight how the imagination on its own is insufficient for mathematics (159). Moreover, since Du Châtelet's account of mathematics is set in a broader rationalist framework, it presupposes a priori rational principles that are not themselves produced by abstraction, as they would be for empiricists.

Du Châtelet does not make very explicit what role these principles play in the origins of our mathematical concepts. Nevertheless, this rationalist background illuminates the above passage's claim that we can abstract whatever "we want to examine." Elsewhere, Du Châtelet writes that "to make a number [*pour faire un nombre*], one combines some units, of which the combination is not at all necessary, but only possible" (1742, 70). These passages need not be read as implying that mathematical truths themselves are in the voluntary control of agents. There can be constraints on the way a process of mathematical abstraction proceeds, and on the objects it yields, even if token acts of abstraction are performed by an agent.

The context of the passage, which is a genealogy of our ideas of space, provides further evidence on this point (Du Châtelet 1742, 101–10). Throughout this discussion, Du Châtelet uses expressions of *constraint*. Extension "must" (*doit*) be conceived as uniform and homogeneous (103); space "must" seem empty to us (107); we "must represent" space to ourselves as immutable (108), and so on. Sources of these constraints may include the content of the understanding's ideas or concepts, the aims or goals for which these concepts are employed, and psychological facts about our abilities to mentally represent space and its objects. A drawn geometrical diagram can be seen as constrained in one sense by concepts such as <circle>, but also by our goals in using these concepts and by our representational capacities.

The example of absolute space, however, raises a question about the ontological status of abstract entities. Not only does nothing correspond to our abstracted idea of absolute space, but "*nothing* similar [*semblable*] to this idea *could* exist" (Du Châtelet 1742, 110; emphasis added). This is an idea of an impossible object.

The question is then: are all abstract objects impossible objects, on Du Châtelet's view? Not at all. While material absolute space is on her view incoherent, geometrical extension is properly founded in actual physical things that ground or "constitute" it (1742, 112). Numbers, likewise, are partly grounded in the actual concrete things that they number (113).

As we've seen, not only mathematics but "all the sciences" are full of "abstractions" or fictional entities, on her view (111). Here 'abstractions' include positive products of our mental activities that more recent philosophers of science often call *idealizations*.[24] As discussed further in section 4.1, Du Châtelet's understanding of the positive role of abstraction appeals to practices in physical science (notably in the work of Galileo and Newton). The success of abstraction in physics can allay worries that abstraction is up to the whims of individuals.

Du Châtelet's views on the modal status of abstract objects are further developed through a surprising distinction between real or actual abstract entities and those that are merely possible or conceivable. For numbers to be "real and existent," they must be grounded in concrete, countable things (Du Châtelet 1742, 113). Even if no such grounding relation obtained, however, it would still be coherent to speak of "possible numbers," apparently grounded in the divine intellect (113).

Such merely possible mathematical objects would be less dependent on our minds than actual mathematical objects. Appeal to possible mathematical objects also suggests a way around a common objection to mathematical abstractionism, namely that there are far more mathematical objects than can be produced by finite minds and their activities of abstraction. Moreover, it allows Du Châtelet to do justice to the intuition that mathematical truths are necessary. Necessary truths hold for any possible mathematical objects, and do not depend on actualization.

Invoking possible mathematical objects does raise problems of its own. Some deny that the distinction between possibility and actuality applies for mathematical objects, and Du Châtelet does not make it explicit why she thinks such a distinction does hold. One could also ask how there can be knowledge of merely possible mathematical objects. Du Châtelet compares mathematical objects to the essences of possible things (1742, 70–71). In both cases, our knowledge relies in part on intellectual insight into possibilia, as something like *ante rem* universals grounded in the divine intellect (63–64). This insight is fairly mysterious. But Du Châtelet is committed to insight into essences anyway, independent of her stance on merely possible mathematical objects. So there need not be any special epistemological problem raised by her endorsement of merely possible mathematical objects.

### 3.4 Abstraction and Fictions

Du Châtelet, as we've seen, describes the products of abstraction as "fictions" (1742, 111). Fictionalism is now a much-discussed position in the philosophy of mathematics. Recent articulations of fictionalism often rely on an analogy to literary fictions such as novels. Nor is this analogy new. Du Châtelet would likely have known, for example, Jean-Pierre Crousaz's 1714 comparison of Newtonian physics to a mere novel (*roman*) (Crousaz 1714, 112–14; Shank 2008, 110–12).[25] Du Châtelet herself considers the "fabulous world" depicted in "fairy tales" an instructive contrast with properly grounded scientific explanation (1742, 26). Novels also provide her with examples of merely logically possible worlds (46).

But we cannot presume that Du Châtelet relies on an analogy between mathematical fictions and literary works, and we'll see that there are important differences between literary creation and mathematical abstraction. In an early modern context, the French term '*fiction*' often had broad, non-literary connotations of creation, fashioning, or establishment by convention (Rey et al. 2011, s.v. "*fiction*"). And as a Scholastic term of art, *ficta* were mind-

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[24] In the recent literature, idealizations standardly "characterize ideal cases that do not, perhaps cannot, occur in nature" (Elgin 2004, 118). However, the verb *idéaliser* and its derivatives did not enter French until the end of the eighteenth century (Rey et al. 2011, s.v. "*idéal*").

[25] Crousaz also objected to Leibnizianism, and initiated a correspondence on this point with Du Châtelet in 1741 (Du Châtelet 2018, II.43–47). Du Châtelet commented to Maupertuis two months later that "*les* Institutions *m'ont encore attiré une drôle d'adversaire, c'est Crousaz*" (II.54), and sent a cutting reply to Crousaz the next day (II.56).

dependent concepts or objects of thought, introduced to solve a general epistemological problem.[26]

An earlier historical reference point is Aristotelian abstractionism about mathematical objects. Aristotle often emphasizes *eliminating* or subtracting features from an object (Aristotle 1984, 1061a28–b2). What remains after this process has a non-arbitrary basis in the object. The nature of the soul places further constraints on the process of abstraction, such that abstraction yields the same, publicly accessible results for different individual thinkers. As for existence, Aristotle states that mathematical objects can be said "without qualification" to "exist …with the character ascribed to them by mathematicians" (Aristotle 1984, 1077b33–34).[27]

In turn, Leibniz came to hold that some mathematical objects, namely infinitely small and large magnitudes, are mere "fictions of the mind" (Leibniz 1875–90, II:305; V:157–58).[28] He connects them to acts of mental abstraction (Leibniz 1863, VI:234–54). Unlike truths of arithmetic and classical geometry, which for Leibniz hold true at every possible world in virtue of the divine intellect, such fictions exist only at worlds where there are finite minds of a certain kind (Leibniz 1875–90, II:262–65; Jauernig 2010, 175–76). Yet infinitesimals are of great use in discovering truths, even though claims about them are strictly speaking false (Rabouin & Arthur 2020, 406–7). Their usefulness is not up to us but stems from general features of our cognitive faculties.

I take Du Châtelet, like these predecessors, to reject at least three possible implications of conflating fictions in general with literary or artistic fictions. One implication is that mathematical claims are more or less arbitrarily dreamed up by mathematicians. This is how Du Châtelet describes novels. A novelist plays with possibilities in a way that lacks constraint from the actual world (1742, 45–46).

A second implication she rejects is that, because mathematical objects do not exist, mathematical claims are essentially falsehoods (Field 1989, 2; Elgin 2004, 123). In her view, a fiction is not a thing (*chose*) or substance. Nevertheless, it can count as a "being" (*être*), so long as it has a logically consistent essence (1742, 61). So, unlike in contemporary fictionalism, fictional beings exist and claims about them can be true.

Third, whereas she holds that some works of literature are like dreams, in that both "furnish us with the idea of a fabulous world" that is private and depends on our individual psychological constitution, she does *not* consider mathematical fictions in the same class as dreams (1742, 25). Mathematical objects, unlike dreams, are public and shared.

Amie Thomasson (1999, 2007) has advanced a more neutral analytical framework for thinking about fictions. Literary fictions, she suggests, are one species in a genus of abstract artifacts, which also include legal systems and scientific theories (xii; 147–53). Such artifacts are "jointly dependent" on physical things and on human mental states (150). Literary works illustrate some features of abstract artifacts, but they are not the paradigm for all abstract artifacts. If mathematical fictions fall under the genus of abstract artifacts, they need not share the specific features of novels or fables.

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[26] For the early Ockham, *ficta* exist in the mind, but lack the reality of the soul's powers or dispositions: they have intentional existence. Ockham went on to eliminate *ficta* from his epistemology. Pasnau (1997, 82) argues that Ockham found a more parsimonious alternative to *ficta*, but never had "doubts about the concept of fictive existence" itself.

[27] Nonetheless, the *manner* in which mathematical objects exist is "special" and qualified (1984, 1077b16). Substances exist in an unqualified way. So Aristotle's mathematical objects are not substances: they are *in* substances. Interpreters disagree on further details (Hasper 2021).

[28] A letter from Leibniz to Varignon—published in 1702 and perhaps known to Du Châtelet—sketches this position, though Leibniz is cautious about labelling infinitesimals as fictions (Leibniz 1863, IV:91–95; Reichenberger 2016, 152n54). Further details were not publicly available, and remain controversial (Jesseph 2015; Rabouin and Arthur 2020).

Many concrete artifacts are end-directed in a way that distinguishes them from works of literature. To be fit for purpose, such artifacts must have specific and non-arbitrary features (Thomasson 2007). Thomasson's conception of an abstract artifact permits us to view mathematical fictions along similar lines, in contrast with literary fictions. Furthermore, Thomasson thinks we should "postulate" literary fictional objects (111–14; 147). That is, we should take them to exist, albeit in a dependent and non-fundamental way. Finally, abstract artifacts such as legal systems or scientific theories are patently public and shared, unlike dreams.

Thomasson's framework is apt insofar as Du Châtelet develops her account of mathematical fictions through analogies with general grammar, rather than artistic or literary creation. The *Institutions* notably never uses the word 'fiction' to directly describe works of literature. It is mainly used for mathematical and grammatical entities, though also, with a more negative valence, for speculative scientific hypotheses (1742, 79; 93). While cautioning against speculative hypotheses, she stresses that they are advanced in order to rationally explain observed facts, are sometimes historically inevitable (as in Descartes), and do not turn philosophy into "a heap of fables" (88; 79; 93). So here too, fictions in general are not equated with literary fictions.

"Substances by fiction" supply Du Châtelet with an example of fictions in a grammatical context (1742, 76). These are substantive terms that may not refer to anything substantial. For example, the substantive 'whiteness' refers to a mere accident: "whiteness can never be a true substance" (76). Arguably, whiteness could not be an accident of any fundamental, simple substance. It is only ever an accident of bodies, which are mere substantial phenomena. Bodies are apparent rather than genuine and primary substances (181).

Frequently, grammatical fictions are not even approximately true of the material world. I will argue in section 4 that in this respect many mathematical fictions have epistemic advantages over grammatical fictions. But even grammatical fictions have a universality and (quasi-)objectivity that distinguishes them from literary fictions.[29] Grammatical facts, though in some sense products of human activity, are not arbitrary or false. For while the distinctive features of individual languages are contingent, all languages are founded in an implicit, humanly universal "natural logic" (*logique naturelle*) (Du Châtelet n.d., f. 133*r*).[30]

Falsehood results not from grammatical fictions as such, but from confusing grammar with a metaphysical account of real things. Thus she cautions against taking (for example) grammatical substantive terms to reliably pick out "true substances of nature" (76). In the background is her view that "error" is the result of mistaken judgments, rather than stemming from any particular type of singular representation, including false representation (1742, 112; 124; 206). A similar point appears in the Port-Royal *Logic*, where error is said to arise "only from judging badly" (Arnauld and Nicole 1996, 59).[31]

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[29] Du Châtelet wrote, but never published, a *Grammaire Raisonée*. The known surviving chapters discuss substantive terms in detail, stating that the grammar of substantives is based on a generalization from the metaphysics of substance (Du Châtelet n.d., ff. 133v–135*v*; 138*r*; 145*r*).

[30] Similarly, when the Port-Royal *Grammaire* turns from linguistic signs themselves to their signification, it asserts that understanding the latter requires an account of "what occurs in our minds" (Arnauld and Lancelot 1975, 65–68). These mental operations include conceiving concepts and objects of perception, judging propositions, and reasoning through syllogisms, which are all discussed in the Port-Royal *Logic*. Humanly universal logical principles, then, underlie all languages *qua* "signifying…thoughts" (41). We find similar assumptions in Maupertuis's ([1740]) essay on the origin of language, which treats mental capacities expressed in all language, rather than the features of any specific language. For general discussion of early modern general grammar as irreducible to either formal logic or empirical linguistics, see Foucault (1966, 92–136).

[31] Descartes makes the still stronger claim that ideas "cannot strictly speaking be false" (1984, 26). Leibniz, for his part, stresses that an "*abstraction n'est pas une erreur*," including in mathematics, so long as one "knows"

Du Châtelet's discussion of grammar and fiction also plausibly draws on the Port-Royal *Grammar*, which treats at length the case of substantive terms such as 'whiteness.' For the Port-Royaliens, substantive terms originate from the need to refer to real substances. In standard Cartesian fashion, they contrast these substances with attributes and modes. Many substantive terms, such as 'soul,' do refer to substances in this way. But the usage of such terms goes "beyond" this (Arnauld and Lancelot 1975, 69–70):

> Since substance is that which exists by itself, people came to call all those words which exist by themselves in discourse without requiring another noun *substantive* nouns, even though they in fact signified accidents.

Grammatical substantives reflect genuine facts about grammar that are not up to any particular speaker. 'Whiteness,' for example, can appear alone in a sentence while 'white' must be accompanied by a subject term designating some subject that is white. Moreover, these facts are straightforwardly expressible by truths about the term 'whiteness.' It is true that 'whiteness' is a substantive term, for example, even though 'whiteness' does not refer to a substance.

What lessons can we draw from these grammatical examples for Du Châtelet's conception of fictions more generally? First, fictions can be seen as jointly dependent on human mental states and on mind-independent things. Though fictions are partly dependent on our activities, they stand under robust constraints grounded in universal features of our mental faculties (notably the understanding and imagination).

Second, this joint dependence raises the possibility that the mind-dependence of fictions is a matter of degree. There may be a spectrum ranging from highly mind-dependent fairy tales, through grammatical fictions, to mathematical fictions.

Third, Du Châtelet distinguishes the accuracy of fictional representations from the further question whether they bring about error. Error arises from misusing these representations and confusing them with fundamental, mind-independent entities, rather than from sheer inaccuracy of representation.

The fact remains that both literary and grammatical fictions are typically false, and are more or less arbitrary. In the following section I further discuss how, especially in geometry, mathematical fictions can approximately represent physical entities. On Du Châtelet's account, these fictions are not sheer falsehoods.

## 4 Du Châtelet's Defense of Inferences from Mathematics to Material Nature

Recall that for Du Châtelet, the use of mathematics is a necessary condition for scientific progress. Mathematics is needed to explain what, for physics, has so far been "inexplicable" (1742, 3). Moreover, she praises general inferences from mathematics to the physical world: mathematics is a source of physical knowledge.[32]

So it may be puzzling that Du Châtelet is also an avowed fictionalist and abstractionist about mathematical objects. In section 3.4, I began to dispel this puzzle by stressing how for Du Châtelet, abstract and fictional objects can be public and subject to robust constraints. I now examine some epistemic advantages that mathematical fictions enjoy even over grammatical fictions. Mathematical fictions may not be exactly true of the physical world, but even so, they can and ought to *approximately* represent the physical world. By contrast, fictions in grammar are more or less arbitrary and do not seek to represent the physical world,

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that the material world is not really as the abstraction presents it (1875–90, V.50; these *New Essays* were only published after Du Châtelet's death).

[32] Du Châtelet's later commentary on the *Principia* confirms this point. She holds for example that the mathematical Lemmas in Book I of the *Principia* not only lay out Newton's "method of first and last ratios," but also establish "general solutions" for Newton's "entire theory" of gravitation (1759, 9; 32). These solutions then "explain astronomical phenomena" in *Principia* Book III (9).

even approximately. Literary fictions represent the physical world but do not *quantitatively* approximate it.

**4.1 Mathematical Fictions and Approximate Truth**

Du Châtelet's position is summed up in the dictum that "the same thing happens in nature as in geometry" (1742, 34). She does not claim that nature corresponds in this way to grammar or works of literature. I'll now explicate this claim, and note some important qualifications on it. In brief, we'll see that this sort of inference works only for certain general facts about nature: physics cannot be spun out of pure geometry.

For Du Châtelet, 'nature' sometimes refers the actual world in general. It is in this broad sense that she describes God as the author of nature (1742, 472). However, her "same thing" dictum does not range over nature in this broad sense. She does not hold that simple substances and souls correspond to, or are describable in, geometry.

So by "nature," she means the subject matter of physics: matter and its properties, such as force.[33] This is confirmed by her claim that, *because* things are in nature as they are in geometry, one can use the law of continuity to "find and demonstrate the true laws of motion" (1742, 35). Recall that Du Châtelet is an idealist about matter and force: they are partly mind-dependent. This mind-dependence is linked to the spatial and temporal properties of matter and force. But geometrical objects are essentially spatial, and numbers are connected to succession in time. Events in material nature thus have something in common with mathematical facts: they are essentially spatial and temporal.

Du Châtelet's metaphysics of quantity, as we've seen, allows for quantities to exist in physical things: quantitative properties are not confined to mathematical entities. Therefore, the objects of nature and of mathematics can have the same kinds of properties, although they are not "the same" in the sense of being numerically identical. Du Châtelet's idealism and her conception of mathematical objects thus provide a framework in which mathematical representations may be approximately true of the material world. Approximate truth, in turn, suffices for at least some kinds of scientific explanation, on Du Châtelet's view (1742, 203–5).

To take a rudimentary example, a body in motion has a volume, and so does a geometrical solid. We can therefore use geometry to represent a property of bodies, even if bodies do not behave exactly as geometrical figures do. To use an example made famous by Galileo, a geometrical sphere in contact with a plane would touch it at exactly one point, but this does not hold for a physical sphere resting on a surface (cf. 1742, 111). Still, geometrical proofs can legitimately apply to physical nature, even if perfect geometrical spheres and planes are never physically realized. By contrast, volume is not even a possible property of Du Châtelet's non-spatial, non-composite substances (177; 141).

To illustrate the correct use of fictional representations, she considers the case of the Ptolemaic system. Even though it is false, it yields sufficiently accurate solutions for some problems in astronomy (1742, 111–12). More broadly, false assumptions can be made in physics in such a way that they do not lead to "error" in "experiments and…explications" (206). Here too, she may rely on the Port-Royal idea that error properly speaking is the product of judgment.

This point can apply to any mathematical representation used in physics, insofar as it is fictional and never exactly realized in material nature. Du Châtelet considers, for example, the mathematical consequences of the theory of universal gravitation. The theory predicts observations such as the moon's orbital period to a very high degree of accuracy, so much so

[33] The law of continuity could also be applied to mental changes, insofar as these take place in time (cf. Kant 1998, A208–209/B253–54; 2004, 4:471). Du Châtelet may well do this (1742, 152). But my focus is on the *physical* significance of mathematics, so I leave this issue aside

that "calculations" can be "taken for observations" (1742, 332–33).[34] This is consistent with the theory's being "more or less [*à peu près*]" rather than exactly correct (1759, 66).

More positively, Du Châtelet contends that because of the limitations of our cognitive faculties, abstract and fictional representation is unavoidable for us. Without it, we would need to represent far more particulars than we could effectively reason about (1742, 111). To take one of her examples, a token iron bar has a vast array of properties (103–4). Describing all of these properties is a potentially infinite task. For many purposes, we will need to characterize the iron bar in mathematical terms, as a three-dimensional object in Euclidean space. We can then hold this characterization fixed even if some of the bar's internal properties change, for example, if it loses or gains some microscopic parts. So even if mathematical representations are a step removed from material things, they provide a privileged way of knowing the general properties of matter.

### 4.2 From Mathematical to Physical Continuity

One important case where Du Châtelet draws conclusions about the material world from principles of mathematics is her treatment of the continuity of matter and of material change. Since her principle of the continuity of change draws on Leibniz, it would be all too easy to portray Du Châtelet as upholding a 'Leibnizian' material continuum against the 'Newtonian' atomism of Voltaire's *Eléments*. We'll see, however, that Du Châtelet cannot be seen as simply recapitulating Leibniz on this point. Nor did Newton and Leibniz have simple positions on these issues.[35]

A further complication is that Du Châtelet's views on this topic were not static. In the manuscript of the *Institutions* and the 1740 first edition—but not in the second edition of 1742—some parcels of matter are treated as if they were atoms or at least natural *minima*, for practical purposes in physics (1738–40, f. 216*r*; 1740, 185–6; 1742, 194–5). Here I focus on outlining Du Châtelet's position in the 1742 second edition, which is altered to stress the infinite divisibility of time and matter (e.g. 1742, 265). Reichenberger (2016, 146–69) provides a broader discussion of Du Châtelet's development and sources on the topic.

In brief, Du Châtelet holds that matter is continuous, but her account of material parts is potentialist. She does not hold that between any two material parts, there actually exists another part. Rather, matter can always be further divided into parts, and is never actually divided into ultimate parts.[36]

She also holds that all material change is continuous. Her official statement of this point is that if a thing is changing from state *A* to state *B*, it necessarily passes through all conceivable intermediate states (1742, 32). What this means in practice is that changes in material things, such as motions, can always be further divided into smaller units of change or motion.

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[34] Other examples Du Châtelet considers include treating the moon as a point-mass, comets as subject solely to gravitational forces, outer space as if it were a void, and so on. These ideas are further developed in her commentary on the *Principia*.

[35] In the early manuscript "Of Attomes," for example, Newton contends that objections to the indefinite divisibility of body apply equally in mathematics. But they have absurd consequences in the mathematical case. Therefore, these objections are unpersuasive in the case of bodies as well (Janiak 2000, 213). Leibniz, throughout his career, regards matter as actually infinitely divided (1875–90, II:77; II:268; Rutherford 1990, 544–49). Yet in early works, he presents infinite division as compatible with atomism (IV:15–26; IV:228–29).

[36] Aristotle's *Physics* defines the continuous as that which can be endlessly divided into further divisibles (1985, 200b19; 232b25). While apparently allowing some completed actual infinities, Aristotle does not countenance actual infinite collections of parts of continua (White 1992, 111–12n54; 156–61; Coope 2005, 80–81). Broadly Aristotelian approaches to continuity were still authoritative in the eighteenth century. Outside of relatively isolated mathematical works, continuity lacked a clear, positive definition; it was conceived as a mere lack of division or determination (De Risi 2021). See for example Wolff's definition of continuity (2001/1730, §554).

These continuity claims have straightforward consequences for actual material things. They entail that atomism is false and that change in matter is continuous. Du Châtelet's optical theory also relies on continuity assumptions (Gessell 2019, 872–73). I disagree, then, with Reichenberger's (2016, 163) suggestion that these continuity principles are mere regulative guidelines for inquiry, in a Kantian sense, that lack objective purport.

At several points, Du Châtelet appeals to the principle of sufficient reason to support these claims about continuity (1742, 32ff.; 137–40; 151–52). But it is unclear how, on its own, the principle of sufficient reason could establish them. Atoms as ultimate parts might seem to provide a conclusive, unqualified explanation for the composite bodies they compose. A material continuum, meanwhile, courts an unsatisfying infinite explanatory regress, one that might not satisfy the principle of sufficient reason.

Here Leibniz helps himself to the premise that if atoms existed, then God would have arbitrarily decided, without sufficient reason, that a certain fundamental mereological level for matter obtains rather than some other possibility (Leibniz & Clarke 2000, 28). But it is unclear why a parallel problem does not arise for levels of non-mereological metaphysical dependence. What is God's sufficient reason for making monads the fundamental created beings, with no level below them? In any case, Du Châtelet is cautious about reading off conclusions about divine choice from observed facts in nature. She does not present this Leibnizian premise in her arguments for material continuity.

Attempts to prove the continuity of change merely from the principle of sufficient reason also risk begging the question. Supposing that change did occur in discrete stages, why couldn't each stage find its sufficient reason in the stage prior to it? Atomists about change could argue that these discrete stages are more intelligible and explanatory than Leibniz's alternative. Stages provide ultimate units of change, and hence an unconditioned explanatory bedrock, as opposed to the explanatory regress suggested by continuous change.

This is where Du Châtelet's views on the relationship between mathematics and physics come in. These continuity theses, on her account, follow not just from the principle of sufficient reason, but require the additional principle that things are in spatiotemporal nature as they are in mathematics. More specifically, she appeals to the idea that if a thesis of sufficient generality holds in geometry, it therefore also holds of matter. Geometrical continuity is a sufficiently general thesis, in this sense. On this basis, Du Châtelet can use the following argument schema:

(1) A continuity thesis *C* holds in geometry.
(2) If *C* holds in geometry, then it holds for physical matter.
(3) Therefore, *C* holds for physical matter.

This schema is then applied both to the continuity of matter itself, and to the continuity of change.[37]

To begin with the continuity of matter, consider one of Du Châtelet arguments against atomism. Any alleged atom takes up some space, thus has spatial parts and can in principle be further divided, even if this is "physically" impossible (1742, 139). This argument seems to presuppose premises elaborated in Du Châtelet's account of matter. Extension is an essential property of matter, and in turn, extension logically implies indefinite divisibility (160). Therefore, matter is essentially indefinitely divisible.

But why does extension logically imply indefinite divisibility? Here she is relying on the assumption, from geometry, that extension is always potentially divisible into further parts. We can compare the argument from Keill discussed in section 2: since in geometry extension is provably infinitely divisible, matter must be infinitely divisible as well. Du

[37] Leibniz's arguments for continuity sometimes use the premise that "nature…observes the same [rule]" as does geometry (1875–90, IV:375–76; IV:568–69). Du Châtelet probably did not know these texts, however.

Châtelet thus accepts arguments from mathematical truths to physical truths that Voltaire and Wolff rejected.

This geometrical assumption makes it easier to see the principle of sufficient reason's relevance to the question of matter's continuity. For if matter is composed of atoms, then these atoms ground the size and shape of whatever they mereologically compose. But the atoms themselves are extended, and therefore, by the premise from geometry, must have some size and shape. Their size and shape cannot be grounded in further parts, on pain of contradicting the hypothesis that they are atoms. But what other grounds for their size and shape could there be? Some atomists fall back on "the will of the creator" to "give a reason for the extension of the atom" (1742, 140). Du Châtelet rejects this solution: God is volitionally responsible for the existence of matter, but not for its essence or possibility (73–74). The demand for a reason arises, she holds, even for possible or conceivable atomic parts of matter, so long as we assume that matter is essentially spatial and so corresponds to geometry. So she concludes that there are no material atoms: the size and shape of any part of matter is (partly) grounded in the size and shape of its parts.

Du Châtelet uses similar reasoning to defend the continuity of material change. An equivalent to the continuity of change holds in geometry, on her view. For example, the deformation of a parabola must proceed continuously (Du Châtelet 1742, 36–37). She also discusses inflections (*points de rebroussement*) in third-degree algebraic curves. A "concave" curve, for example, becomes "convex" by "infinitely small degrees" (33). By the "same principle," the quantity of motion of a given body can be continuously decreased until it is at rest (1742, 37).

Like some earlier authors, she considers geometrical curves as abstracted from the motions of bodies.[38] A continuous geometrical curve is one traced by (idealized) continuous motion. The continuity of geometrical curves, then, can be seen as governed by the same principle as the continuity of motion. So although continuous motions are metaphysically prior to geometrical curves, one can deduce properties of motions from properties of curves.

Given these arguments from geometry, it is easier to see why for Du Châtelet, failures of continuity in the material world violate the principle of sufficient reason. If the argument from geometry is successful, then matter and change are continuous in a classical, Aristotelian sense. If one attempts to explain matter or change in terms of an arbitrary number of discrete parts or stages, some intervening parts or stages will always be missed by an explanatory account.

Before concluding, let's consider a passage that has been taken to support a quite different reading of Du Châtelet on the relationship between physics and mathematics. "The divisibility of extension to infinity," she writes, "is at the same time a geometrical truth and a physical error," citing Keill as confused on this point (1742, 194). At first glance, this might seem to contradict her doctrine of the continuity of matter, as well as the correspondence between geometry and nature (cf. Carson 2004, 169–70). This passage, however, needs to be read in its context, which is an attempt to resolve ancient paradoxes about continuity.

> It is in confusing geometrical extension and physical extension, and in assuming that physical extension is always composed of extended parts to infinity, that the Ancients [sc. Zeno and Melissus] formed these so very false and specious arguments against the possibility of motion. (1742, 191–92)

Du Châtelet also accuses Keill of reasoning that, because a grain of sand has an infinite number of parts, these parts could "fill the entire universe" (194). This is a variant of Zeno's second paradox of plurality, which is standardly read as arguing that because a line is

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[38] Descartes identified "the subject-matter of pure mathematics" with "corporeal nature" or "material things" (1984, 49; 264; 55). Newton arguably took lines and circles to be, in the first instance, physical motions (Guicciardini 2004).

divisible into an infinite number of parts and each part has some size, the line must, absurdly, have infinite size (Vlastos 1971).

Despite her remarks about confusing geometrical and physical extension, however, Du Châtelet is not objecting to the continuity of matter and change as such in these cases. Nor is she ruling out inferences from general geometrical principles to conclusions about material nature. For "physical error" she diagnoses here is not the continuity of matter, but rather that Zeno and Keill present matter as *actually* "composed of" an infinite number of parts, such that these parts are prior to the whole. This is also a point of disagreement with Wolff, who regarded simple substances as prior to and collectively composing spatial wholes, even though each simple substance "does not…fill a space" (1720, §602). Du Châtelet, by contrast, upholds the infinite *potential* divisibility of matter. In this sense, physical extension is not divisible "to infinity." But it is not actually divided into atoms, either.[39]

What does Du Châtelet mean by the claim that the actual infinity of extended parts is a "geometrical truth"? Euclid defines points as partless (1908, I.153). Presumably, she does not want to give this up. Given her dictum that things are in nature as they are in geometry, if she accepts infinite collections of actual geometrical points, then it might seem that she must accept infinite collections of actual physical points, despite her official potentialism about parts of matter.

But it is only the "ideal division" of geometrical objects that can go to infinity, because we can conceive of geometrical points (1742, 194). The points in question are merely ideal "possible parts" (1742, 190). She supports this point by first appealing to the familiar process of dividing a line into parts. We can "mentally proceed to infinity" by conceiving of this process as extending indefinitely: there is no principled reason this "ideal division" cannot continue (193). Next, we reflect on this *capacity* to divide continuous geometrical objects *ad indefinitum* or "to infinity," and finally conceive of the logically "possible" ultimate result of such boundless division (193; 109). But given our finite cognitive capacities, such an "infinite" result is not in our power (193).

With geometrical points, it seems we have reached a limitation on Du Châtelet's dictum that things are in nature as they are in geometry. Perhaps this limitation rests on an epistemological distinction between different kinds of mathematical object. Figures and lines can be given to us, at least approximately, in perception. Geometrical points cannot be perceptually given in this way: they are merely conceivable.

More precisely, Du Châtelet's dictum that things are in nature as they are in geometry is plausibly restricted to geometrical objects with determinate magnitude. A physical object such as a watch has a "fixed number" of determinate parts, each with a measure, and these parts are largely mind-independent, even if they are in principle further divisible (1742, 191). By contrast, the number of points in a line is "absolutely indeterminate": it is nonsensical to speak of the determinate length of these points, even though the line has a determinate length (190; cf. Rohault 1723, I.32–33). A line should be treated not as made up not of an infinity of points but as divisible into *some* "non-finite" but unknown "quantity of parts" (193). Geometrical points, as merely conceivable, lack determinate magnitude or number. This circumvents the difficult question of whether points are Archimedean or non-Archimedean magnitudes.

[39] An influence here may be Jacques Rohault's introductory work on physics, which Du Châtelet owned (Du Châtelet, 2018, I.367). Near the beginning of this work, Rohault offers a proof that matter cannot have simple parts, but he also rules out an actual infinity of material points. Instead, "matter is indefinitely divisible" and has no definite number of actual parts (Rohault 1723, I.32).

To historically situate Du Châtelet's position on the continuity of matter, I want to conclude by juxtaposing it with Kant's taxonomy of traditional accounts of the part–whole structure of the world in the Second Antinomy.[40]

The Antithesis of the Second Antinomy denies that there are any simples in the created world. Du Châtelet is by contrast committed to simple or at least non-composite substances, namely the non-spatiotemporal souls and elements making up the fundamental level of the created world. Therefore, she would not accept the Antithesis.

Nor would Du Châtelet endorse the Thesis:

> Every composite substance in the world consists of [*besteht aus*] simple parts, and nothing exists anywhere except the simple or what is composed [*zusammengesetzt*] of simples. (Kant 1998, A434/B462)

Du Châtelet take there to be some composite substances, although these are merely phenomenal substances. But she denies that matter is mereologically composed of simples. Matter is indefinitely divisible into further material parts. Simple substances—or more precisely, *non-composite* substances—partly ground matter. But this is not a standard mereological composition relation, since these non-composite substances are non-spatial and cannot compose spatial wholes. This is particularly clear from Du Châtelet's discussions of souls.

Given these properties of matter and Du Châtelet's commitment to the existence of material things, it follows that (i) not every composite substance in the world consists of simple parts and (ii) something exists somewhere that is neither simple nor composed of simples. In turn, (i) and (ii) negate the two clauses of the Thesis of Kant's Second Antinomy. Du Châtelet is not committed, then, to either of the apparent contradictory propositions making up the Second Antinomy.

Kant apparently resolves the Antinomy by claiming that both the Thesis and the Antithesis are false (1998, A485/B513). The truth, on his view, is that the "division" of matter into parts can be continued "*in infinitum*," but because of our finitude cannot actually reach a complete division into "infinitely many parts" (A523–4/B551–2; A515–17/B543–45; Marschall 2019). This solution resembles Du Châtelet's potentialist approach to the division of matter.

## 5 Conclusion

I began by examining the positions of some of Du Châtelet's influential contemporaries on the application of mathematics to physics. Maupertuis, Voltaire, and Wolff take relatively conservative positions on this issue, often leaving it unclear what role is to be played by the mathematics in post-Newtonian physics. Du Châtelet's position, I then sought to show, contrasts vividly with theirs. Even though mathematical objects are abstracted from the physical world, they are partly grounded in magnitudes that are in material things. This allows for relations of approximation between mathematical objects and physical things. And although Du Châtelet takes these objects to be fictions, a closer look at her account reveals that they are public, and under a number of objective and non-voluntary constraints which do not apply to literary fictions. I argued that these commitments underwrite Du Châtelet's claim that the same thing happens in nature in geometry. And I showed that Du Châtelet, in turn, uses the parallel between nature and geometry to establish two important metaphysical theses, namely the continuity of matter and change.[41]

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[40] As for the continuity of change, McNulty (2019) argues that Kant considers it an "a priori…truth of metaphysics," linked to a demand for intelligible explanation, though also grounded in the "geometric" continuity of space and time (1595; 1600; 1605–7). McNulty's reading suggests similarities between Du Châtelet and Kant on the continuity of change.

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