There Are Brute Necessities

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ABSTRACT. A necessarily true sentence is 'brute' if it does not rigidly refer to any objects, and if it cannot be reduced to a logical truth. The question of whether there are brute necessities is an extremely natural one. Cian Dorr has recently argued for sweeping metaphysical claims on the basis of the principle that there are no brute necessities: he initially argued that there are no non-symmetric relations, and later that there are no abstract objects at all. I argue that there are nominalistically acceptable brute necessities, and that Dorr's arguments thus fail. My argument is an application of Gödel's first incompleteness theorem.

Roughly speaking, a necessarily true sentence is 'brute' if it does not rigidly refer to any objects, and if also it cannot be reduced to a logical truth. Cian Dorr has recently argued for sweeping metaphysical claims on the basis of the principle that there are no 'brute' necessities: he initially argued that there are no non-symmetric relations,[1] and later that there are no abstract objects at all.[2] For example, in the course of the latter argument, Dorr writes as follows (here a 'real definition' is a 'metaphysical analysis', e.g. of water as H_2O):

> …we have stumbled on an argument in favour of nominalism… only the nominalist… can adequately respect the idea that there are no brute necessities.
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> I think this is quite a powerful argument. To anyone not antecedently convinced of the falsity of nominalism, the idea of a metaphysically necessary truth whose necessity does not flow from real definitions plus logic should seem quite strange. A notion of necessity that allowed for such necessary truths would seem uncomfortably like nothing more than an extra-strong variety of nomological necessity. But when something strikes us as impossible… we don't just think of it as ruled out by a "law of metaphysics": we feel that in some important sense, the idea *just makes no sense at all*.[3]

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[1] C. Dorr, 'Non-Symmetric Relations', in D. W. Zimmerman (ed.), *Oxford Studies in Metaphysics*, vol. 1 (Oxford: Clarendon Press, 2004), pp 155–92.
[2] C. Dorr, 'There Are No Abstract Objects', in T. Sider, J. Hawthorne and D. W. Zimmerman (eds), *Contemporary Debates in Metaphysics* (Oxford: Blackwell Publishing, 2008), pp 32–63.
[3] Dorr, 'There Are No Abstract Objects', p. 53.

The problematic brute necessities include the claim that every relation has a converse, claims about the instantiation relation, and standard mathematical axioms.[4]

As the above quote suggests, the question of whether there are brute necessities seems to be an extremely natural one—quite independently of the issue of which abstract objects there are. Brute necessities appear to resist anything like the sorts of explanations given for the necessity of the usual examples of non-brute necessities, such as logical truths, 'Water is H_2O' or 'Socrates is human'. One can explain the necessity of such non-brute necessities by appealing to logic, metaphysical analyses or the notion of an essence: no such explanations will be possible for brute necessities. So it is very natural to ask whether there are any brute necessities, and, if so, whether the only such necessities involve abstract objects. These questions are also closely related to those of Wittgenstein's *Tractatus*, where it is argued that every proposition can be analysed using only logically independent atomic propositions,[5] and that the only necessary truths are those that turn into logical truths upon analysis.[6]

In this paper I give an argument for there being brute necessities: brute necessities that involve no mention of abstract objects; brute necessities that are nominalistically acceptable. This shows that even when one restricts attention to concrete objects, there is more to metaphysical modality than logic plus Kripke's a posteriori necessities (e.g. 'Water is H_2O', 'Socrates is not Plato' etc.). My argument also shows that Dorr is wrong to think that a nominalist can reject brute necessities: and wrong more generally to base arguments on the principle that there are no brute necessities. The argument I give is an application of Gödel's first incompleteness theorem (which I do not assume knowledge of).

The basic idea is to use if-then sentences about concrete objects arranged in natural number-like structures. For any sentence α of the language of arithmetic, there will be a corresponding sentence α^* about concrete objects that says essentially: any natural number-like structure of concrete objects will be α-like structured. And whenever α is a true sentence of arithmetic, α^* will be a necessary truth (a nominalist should interpret my set-up talk about numbers and sets however they interpret the talk of mathematicians). However I will show that it is a consequence of Gödel's theorem that there is no way to define 'natural number-

[4] Dorr, 'Non-Symmetric Relations', pp 161–67; 'There Are No Abstract Objects', pp 52, 51.
[5] L. Wittgenstein, *Tractatus Logico-Philosophicus*, trans. D. F. Pears and B. F. McGuinness (London: Routledge, 1921/1961), 4.21–4.221.
[6] Wittgenstein, *Tractatus*, 6.37–6.3751.

like' so as to make every necessarily true α* come out as a logical truth. The upshot is that some of these α*s will be (nominalistically acceptable) brute necessities.

In §1 I define 'brute necessity' and explain why the notion is a very natural one. I then exhibit the brute necessities and argue that they are what I say they are. In §2 I consider possible responses. The focus of §2 is on strengthenings of first-order logic that Gödel's theorem does not apply to (Gödel's theorem only applies to logics whose truths could in principle be output by a computer programme). I consider second-order logic, logics with infinitely long sentences and logic with a finitely-many quantifier. The issue is not whether these logics should really count as logics. Nor is the main issue whether they are nominalistically acceptable. Rather the main issue is whether these extensions of first-order logic yield correct metaphysical analyses of the predicates used in §1. I argue that they do not. These arguments involve somewhat subtle (although I think quite clear) judgements about mathematical notions. Because of this it is perhaps worth noting that one way of taking the argument of §1 is as demonstrating the following: general necessary truths can always be reduced to logic only if logic is substantially stronger than first-order logic (and this is so even if attention is restricted to nominalistically acceptable general necessary truths); that is, the rejection of brute necessities forces one to employ some substantial strengthening of first-order logic. This is already of interest: it is at the very least not obvious that the strengthened logics are nominalistically acceptable; and as one strengthens logic it becomes increasingly difficult to see a principled difference between the supposedly acceptable (brute) necessities and the supposedly unacceptable (non-brute) necessities.

1. The Brute Necessities

To define 'brute necessity' one needs the notion of a 'metaphysical analysis' of a predicate. This is a notion of analysis weaker than that of conceptual analysis. It is the notion of analysis at issue when one says things like 'To be made of water is to be composed of H_2O molecules' or 'For one thing to be hotter than another is for the former to have greater mean molecular kinetic energy than the latter'.[7] Intuitively, φ is a metaphysical analysis of ψ if what it really is to be a ψ is to be a φ. This is imprecise but sufficient for my purposes.

[7] See Dorr, 'Non-Symmetric Relations', pp 155–58.

I will say that a sentence is 'purely non-referential' if neither it nor any of its metaphysical analyses contain rigidly referring singular or plural terms (such as 'Socrates').

A sentence is a *brute necessity* if: (i) it is metaphysically necessary; (ii) it is purely non-referential; and (iii) it cannot be transformed into a logical truth via metaphysical analysis.

(ii) is required to rule out, for example: If Socrates exists then he is human. One might wonder: if one accepts that this sentence about Socrates is necessary without being metaphysically analysable into a logical truth, why should one be worried about brute necessities? But the necessity of the sentence about Socrates can be explained in a way very similar to that in which one would explain the necessity of 'Water contains hydrogen': by the 'new' or 'causal' theory of reference.[8] 'Socrates' and 'water' rigidly designate a certain object or sort of stuff, and the sentences in question are necessary because what it is to be that object or that stuff involves having the property ascribed (being human or containing hydrogen). No such explanation will be available for a brute necessity. So it is very natural to ask whether there are any such brute necessities and, if there are, whether they all concern abstract objects.[9]

I now move onto constructing the brute necessities. I want to construct nominalistically acceptable brute necessities. And I need some way of talking about natural number-like structures, but where these structures are not understood as they tend to be as abstract objects (e.g. ordered tuples of sets). There are various concrete surrogates that one could use instead: of course it need only be possible for these surrogates to exist, they need not actually exist. Here I use the following. Rather than considering numbers and sets, I consider pieces of string (including infinitely long pieces of string). I use $\in^*$ for the relation defined as follows: $x \in^* y$ iff x and y are pieces of string and y is tied to x. As my notation suggests, $\in^*$ is supposed to be a surrogate of the set-theoretic membership relation $\in$.

For these purposes rather than talking about both numbers and sets, I talk just about the set-theoretic 'numbers': 0 = Ø (the empty set), 1 = {Ø}, 2 = {Ø,{Ø}}, and so on. I use **N**

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[8] See for example S. A. Kripke, *Naming and Necessity*, revised edition (Oxford: Blackwell Publishers, 1972/1980).

[9] Dorr in fact uses 'brute necessity' in a variety of ways in the papers mentioned—and he correspondingly offers a variety of analogous arguments for his claims: that there are no non-symmetric relations and that there are no abstract objects. The argument in this paper goes through for each use; but for simplicity I stick to the central one. In any case, except for a use in 'Non-symmetric Relations' involving a priori knowledge, all alternatives are weaker than this central one.

for the set of these set-theoretic natural numbers. (I am not of course assuming that natural numbers really are sets, or anything like that.) One could do things simply in terms of (non–set-theoretic) natural numbers: however it is convenient to talk about sets because it is convenient to have a single object corresponding to the natural number structure.

This correspondence between $\in$ and $\in^*$ gives rise in the obvious way to a notion of isomorphism, defined as follows. Given a set x, the *transitive closure* of x, $tc(x)$, is the smallest set containing x's members, x's members' members, x's members' members' members, and so on. Analogously, given a piece of string y, the *transitive closure* of y, $tc(y)$, is the smallest set containing the pieces of string y is tied to, the pieces of string these pieces of string are tied to, and so on (so $tc(y)$ is the smallest set containing y's '$\in^*$-members', their '$\in^*$-members' etc.). A function f from a set z into another set w is a *bijection* if $f(a) \neq f(b)$ whenever $a \neq b$, and if for every $a \in w$ there is some $b \in z$ with $f(b) = a$. A set x and a piece of string y are *isomorphic* if there is a bijection from $\mathrm{tc}(x)$ to $\mathrm{tc}(y)$ such that for any $a,b \in \mathrm{tc}(x)$, $a \in b$ iff $f(a) \in^* f(b)$. So for example: Ø is isomorphic to any piece of string that is not tied to anything; {Ø} is isomorphic to any piece of string that is tied to exactly one piece of string, where this latter piece of string is not tied to anything; and so on.

This notion of isomorphism then yields a correspondence between predicates: given any predicate of sets φ, let φ^* be a predicate that applies to a piece of string just is case it is isomorphic to a set satisfying φ. In fact there are various importantly different correspondences one might have in mind here: the one that I will make use of is as follows. φ^* will be a predicate that ascribes certain '$\in^*$-structural' properties to pieces of string (properties concerning which 'members' a piece of string has, which 'members' they have and so on). It is not a predicate that says of pieces of string that they stand in certain relations to sets. I have talked about sets in introducing these φ^*s, but the predicates I have in mind are concerned solely with string and not with sets. If one doubts that there really are predicates of string as required just imagine what would have happened if mathematicians had always talked about possible pieces of string rather than sets (I am ignoring complications involving impure sets).

Let N be a 1-place predicate (atomic or otherwise) that applies exactly to the triple (taken to be a set) $\langle \mathbf{N},+,\times\rangle$.[10] N^* is then a predicate that applies exactly to pieces of string isomorphic to this set $\langle \mathbf{N},+,\times\rangle$.

For any given sentence α of the language of arithmetic (the first-order language with =, + and ×), let φ_α be a predicate of the language of set theory (the first-order language with = and ∈) that applies exactly to triples $\langle x,y,z\rangle$ satisfying α: triples $\langle x,y,z\rangle$ such that α is true when the domain is x, + is interpreted by y and × is interpreted by z (so y and z would be functions: sets of ordered pairs). One then has for each such α a predicate of pieces of string φ_α^* that applies exactly to pieces of string isomorphic to sets that interpret α so as to make it true.

Now for each α of the language of arithmetic, let S_α be: $\forall x(N^*(x) \rightarrow \varphi_\alpha^*(x))$. S_α says that any natural number-like structured piece of string will also be an α satisfying-like structured piece of string (to put it somewhat loosely—S_α is of course not actually about abstract objects at all). And if α is a true sentence of the language of arithmetic—if α is satisfied by $\langle \mathbf{N},+,\times\rangle$—then S_α will be necessarily true: this can be seen as follows. Suppose that α is a true sentence of arithmetic. Given any natural number-like structured piece of string, it will be determined by this piece of string's ∈*-structural properties that it is also α satisfying-like structured: determined in a way that is precisely analogous to the way in which the ∈-structural properties of $\langle \mathbf{N},+,\times\rangle$ determine that $\langle \mathbf{N},+,\times\rangle$ satisfies α. Thus for any true α, S_α will be necessary (in any finite world, every S_α will be vacuously true because no piece of string will be natural number-like structured).

I will argue that for at least some true α, S_α will be a brute necessity. I have already shown that if α is true, S_α will satisfy (i) of the definition of brute necessities: it will be metaphysically necessary. It also seems clear that any S_α satisfies (ii) of this definition: it is purely non-referential. I mentioned sets in introducing the predicates N^* and the φ_α^*s. But these predicates apply to pieces of string solely in virtue of their ∈*-structural properties, not in virtue of their relations to sets.

[10] There are various ways in which one can identify ordered tuples with sets: for example one can identify $\langle x,y\rangle$ with $\{\{x\},\{x,y\}\}$ and $\langle x_1,\ldots,x_{n+1}\rangle$ with $\langle\langle x_1,\ldots,x_n\rangle, x_{n+1}\rangle$.

What about (iii)—why could it not be that for every true α, S_α is metaphysically analysable into a logical truth? It is an easy consequence of Gödel's first incompleteness theorem that for some true α, $\forall x(N(x) \to \varphi_\alpha(x))$ will not be a logical truth (I am assuming the consistency of $\exists xN(x)$ and for the time being taking logic to be first-order logic—possible extensions of first-order logic are considered in §2(a–b)).[11] This point can be informally seen as follows. Gödel's theorem entails that no computer programme could generate exactly the truths of arithmetic. But now suppose that for each true α, $\forall x(N(x) \to \varphi_\alpha(x))$ is a logical truth. Then $\forall x(N(x) \to \varphi_\alpha(x))$ would be a logical truth just in case α is a truth of arithmetic: since if α is false then $\forall x(N(x) \to \varphi_\alpha(x))$ will be a set-theoretic falsehood and so clearly not a logical truth. But now any formal system of first-order logic is in effect a computer programme that generates exactly the logical truths. And under our hypothesis this would then clearly give rise to a programme generating exactly the truths of arithmetic: i.e. the programme that outputs α exactly when the former programme outputs $\forall x(N(x) \to \varphi_\alpha(x))$. So for at least some true α, $\forall x(N(x) \to \varphi_\alpha(x))$ will not be a logical truth.

And Gödel's result is extremely robust: it does not depend in any way on the details of N, the φ_αs or the language of set theory—it would apply to any sort of set theory in which one could interpret arithmetic. So there can be no hope of any sort of analysis (metaphysical or otherwise) of N and the φ_αs that would transform each instance of $\forall x(N(x) \to \varphi_\alpha(x))$ for true α into a logical truth.

But now suppose that there are metaphysical analyses of N^* and the φ_α^*s that turn each S_α for true α into a logical truth—as would be required to avoid the conclusion that some such S_αs are brute necessities. All that is relevant to whether or not a piece of string satisfies any of these predicates are its $\in^*$-structural properties: which 'members' it has (i.e. which pieces of string stand in the $\in^*$ relation to it), which 'members' they have and so on. So these $\in^*$-structural properties are similarly going to be all that are relevant to whether or not a piece of string satisfies the metaphysical analyses of these predicates. That is, the metaphysical analyses of these predicates will be concerned with nothing other than such $\in^*$-structural properties of pieces of string. But now given the correspondence between $\in^*$ and $\in$, these metaphysical analyses of N^* and the φ_α^*s will correspond to predicates of sets:

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[11] For Gödel's theorem see for example R. M. Smullyan, *Gödel's Incompleteness Theorems* (Oxford: Oxford University Press, 1992).

predicates concerning analogous $\in$-structural properties of sets. And these new set-theoretic predicates would of course give analyses of N and the φ_αs that would turn each $\forall x(N(x) \rightarrow \varphi_\alpha(x))$ for true α into a logical truth. But there can be no such analyses of N and the φ_αs: by Gödel's theorem, as we saw above. Thus there cannot be the required metaphysical analyses of N^* and the φ_α^*s. So some of the S_αs must be brute necessities. And since these brute necessities are about nothing other than which pieces of string are tied to which, they are clearly nominalistically acceptable, as desired.

2. Responses

(a) Second-Order Logic

In second-order logic there are basically two sorts of quantification: in addition to the first-order quantifiers ranging over the objects in the domain, there are also the second-order quantifiers that range over something like properties or concepts or pluralities of these objects. Formally the second-order quantifiers can usually be thought of as ranging over subsets of the domain. However the idea tends to be that they should officially be thought of as ranging over entities that are not really objects at all, as it were: or indeed that they should not strictly be thought of as 'ranging' over any new entities whatsoever—the idea being that second-order quantification is simply another way of talking about the first-order domain, not a way of talking about a distinct new lot of entities.

Gödel's theorem does not apply if one takes logic to include second-order logic, in the following sense. There is a sentence ζ of the second-order language of arithmetic that is satisfied exactly by interpretations that have precisely the structure of the natural numbers. Thus for any sentence of the second-order language of arithmetic α, $\zeta \rightarrow \alpha$ is a second-order logical truth just in case α is true of the natural numbers. ζ basically says: for any (second-order entity) X, if X is such that 0 is in X and whenever y is in X so is y+1, then everything is in X. The result is that ζ will only be satisfied by interpretations in which every object can be reached by starting with 0 and iterating the operation +1 a finite number of times (i.e. the things that play the role of 0 and +1 in the particular interpretation; there is no first-order sentence that can guarantee this: this is what leads to limitative results such as Gödel's theorem). Of course, there still cannot be a computer programme that generates exactly the arithmetical truths: but this time it is because there is no computer programme that generates all the second-order logical truths.

Analogous to ζ there is a predicate N_2 of the second-order language of set theory that is such that for each true α, $\forall x(N_2(x) \rightarrow \varphi_\alpha(x))$ is a second-order logical truth: and of course the 'string-theoretic' sentence $\forall x(N_2{}^*(x) \rightarrow \varphi_\alpha{}^*(x))$ is similarly a second-order logical truth. So it might seem natural to appeal to second-order logic to try to escape the conclusion of §1: that there are nominalistically acceptable brute necessities.

Such an appeal faces two serious problems, however. The first concerns a nominalist's use of this response. The problem is just that a nominalism that appeals to higher-order quantification would appear to at the very least lose much of its interest. Of course the nominalist will claim that the second-order 'entities' (properties, concepts, pluralities or whatever) are not really 'objects' at all, and so are not problematic in whichever way abstract objects are supposed to be. They will perhaps claim that given a commitment to the objects in the first-order domain, second-order quantification involves no further commitment to anything: the second-order entities (or whatever) come for free. But it is difficult to see what is really supposed to distinguish such a 'nominalism' from an anti-nominalism according to which there are abstract objects—but according to which they are completely different sorts of entities to concrete objects, entities which come for free given a commitment to an initial domain of concrete objects. Is our grasp of the natural language term 'object' really supposed to be sufficient to ground a fundamental distinction between these two positions?

In presenting the second problem with appeals to second-order logic, and for similar problems in (b), I talk simply about the predicate η which I take to apply to the natural numbers (as opposed to pieces of string or set-theoretic numbers): for these purposes nominalist worries can be bracketed, and once they are metaphysical analyses of η raise issues precisely analogous to those of metaphysical analyses of N^*—but that are easier to present. I also assume for convenience that the second-order quantifiers range over pluralities. A metaphysical analysis of η using second-order logic will be basically along the lines of ζ mentioned above (although one must of course specify in some way which particular natural number-like structured objects are the numbers: e.g. that they are those that do no have any other properties, to put it loosely; but this issue can be ignored here).

The second-order analysis of η may be extensionally correct, but it is not a correct metaphysical analysis—it does not tell us what it really is to be an η—for the following reason. A metaphysical analysis of η should not mention the plurality of prime numbers, say: what it is for something to be a natural number has nothing to do with this particular plurality—and similarly for much more complicated and unnatural pluralities of numbers. But a second-order metaphysical analysis of η says that what it is for something to be a natural number is for such pluralities (and others) to have certain properties: thus such analyses cannot be correct.

Here is another way to see this point (this time in terms of string): for a piece of string to be an N^* is for there to be $\aleph_0$ pieces of string that are related to this piece of string in a certain way, and that are themselves $\in^*$-structured in a certain way; it is not for the piece to string to stand in certain relations to $2^{\aleph_0}$ non-pieces of string ($\aleph_0$ is the number of natural numbers, $2^{\aleph_0}$ is the larger number of real numbers; whether the non-pieces of string are genuine objects or not is irrelevant here).

(b) Other Extensions of Logic

Extending first-order logic by allowing infinitely long sentences leads to a failure of Gödel's theorem similar to that with second-order logic: there is an infinitary sentence θ that is satisfied exactly by interpretations with the structure of the natural numbers, and $\theta \rightarrow \alpha$ is an infinitary logical truth just in case α is true of the natural numbers. θ basically says: for any x, either $x = 0$ or $x = 0+1$ or $x = 0+1+1$ or…. The infinitary metaphysical analysis of η would involve a similar infinite disjunction. This infinitary analysis of η may be extensionally correct, but it will be incorrect as a metaphysical analysis for reasons similar to those in the second-order case (i.e. for reasons similar to the second problem with appeals to second-order logic). This can be seen most clearly by analogy. It would obviously be wrong to give the following as a metaphysical analysis of 'x is a prime number': either $x = 2$ or $x = 3$ or $x = 5$ or $x = 7$ or…. This is clearly a bad metaphysical analysis since what it is to be a prime number is to have some particular property in virtue of which 2, 3, 5 etc. are all prime numbers—i.e. the property of having no divisor other than 1 and itself—and a metaphysical analysis of 'x is a prime number' must capture this. An infinitary disjunctive metaphysical analysis of η is wrong for just the same sort of reason: a metaphysical analysis of η must capture what 0, 1, 2 etc. all have in common (that they all result from starting with 0 and finitely iterating +1, say).

Another way to extend logic so as to make Gödel's theorem fail is to count as logical something like 'finitely many': e.g. a quantifier $\exists_{<\infty}$ where $\exists_{<\infty}x\varphi$ means there are less than infinitely many φs (possibly none). Again there will be a sentence ξ satisfied exactly by interpretations with the structure of the natural numbers, and with $\xi \rightarrow \alpha$ a logical truth (in this extended sense) just in case α is true of the natural numbers. This time the basic idea is that ξ says: every x has finitely many y with $y < x$; and the metaphysical analysis of η in this extended logic would take a similar approach. This time it is not clear that the envisaged metaphysical analysis of η must fail. That is, perhaps η should be analysed in terms of the notion of finiteness: maybe what it is to be a natural number is to be something that can be arrived at by starting with 0 and iterating +1 a finite number of times—i.e. maybe one can make sense of iterating +1 an infinite number of times, and so one may need to rule-out such infinite numbers if one is to give a metaphysical analysis of η. However the problem for the opponent of brute necessities is that even if this is so, it depends on something particular about the operation +1: that it makes sense to think of it being iterated an infinite number of times so as to arrive at things that cannot be reached in a finite number of iterations (from 0). And all the defender of brute necessities need do is find some operation that does not have this property, and use an analogue of N^* in terms of this operation to produce brute necessities that cannot be analysed in terms of $\exists_{<\infty}$. For example instead of pieces of string and N^* one could consider people and a predicate N' that applies to a person if they have exactly one child, and if also they are such that every descendent of theirs has exactly one child. If N' applies to p then there will be an isomorphism between the natural numbers and a structure consisting of p and p's descendents. One will thus be able to produce necessary truths using N' just as one could with N^*. However neither 'x is a descendent of y' nor N' should be metaphysically analysed in terms of finiteness or $\exists_{<\infty}$. The descendents of y are the children of y, the children of the children of y, the children of the children of the children of y, and so on. Of course x is a descendent of y only if there is a finite family tree leading from y to x. But this fact relating descendenthood to finiteness is surely a consequence of what it is to be a descendent—not simply part of what is to be a descendent—and thus should not be part of a metaphysical analysis of 'is a descendent of': just as having the smallest prime number of hydrogen atoms is a consequence of what it is to be a water molecule, not something that should be built into a metaphysical analysis of 'water'. So extending logic by adding $\exists_{<\infty}$ will not ultimately help the opponent of brute necessities.

The only extension option left for the opponent of brute necessities would appear to be to extend logic in an (as far as I know) hitherto unconsidered way by adding 'structural' expressions: e.g. an expression $\varphi(d,f,x)$ that applies to an object x iff x belongs to the structure that is just like that of the natural numbers but with d in place of 0 and f in place of +1. This extension of first-order logic might give better metaphysical analyses of η and N^* (i.e. that are immune to the sort of objections raised for previous analyses). But—needless to say—the problem is that these structural expressions do not appear to be genuinely logical in any recognizable sense: they are a far cry from general principles of reasoning or 'laws of thought', for example. And the danger is of course that it will become increasingly hard to see a principled distinction between the supposedly acceptable necessary truths (the logical or 'structural' ones) and the unacceptable ones (the 'brute' ones). For example the necessities that Dorr claims to be problematic need not be existentially committing, and can have something of a 'structural' ring to them: for example he argues that the anti-nominalist is problematically committed to the (brute) necessity of the negation of 'The world consists of just 17 entities, $a_1,\ldots, a_{17}$ such that a_1 instantiates a_2, a_2 instantiates $a_3\ldots$ and a_{17} instantiates a_1'.[12] So this structural expansion of first-order logic does not seem to offer a way out for the opponent of brute necessities.

*(c) Rejecting N**

An alternative response to my argument might simply be to deny that there can be a predicate N^* as required. In particular one might try to claim that no predicate can apply only to those pieces of string that have a certain unique $\in^*$-structure. Such a claim would be analogous to claims to the effect that we do not have a concept of natural number (say) that is sufficiently determinate to pick out a unique structure of numbers. However any such claim that would block my argument would have to be an extremely strong one. Consider again claims about

[12] Dorr, 'There Are No Abstract Objects', p. 52.

our concept of natural number: it is one thing to claim that we do not pick out a unique structure; it is quite another to claim that we pick out only structures that satisfy a certain (first-order) axiomatization Φ of arithmetic—to the extent that we do not even rule out those that satisfy $\neg Con_{\Phi}$ (the arithmetically coded version of the claim that Φ is inconsistent; Φ does not prove Con_{Φ} by Gödel's second incompleteness theorem, so there are models of Φ that satisfy $\neg Con_{\Phi}$). Even the set of true sentences of arithmetic has models of the 'wrong' structure, but not of course models that satisfy $\neg Con_{\Phi}$ (since Con_{Φ} will be a true sentence of arithmetic): so the latter really is a stronger claim. A rejection of *N** would have to be analogous to the latter, stronger sort of claim to block my argument.

I conclude that a nominalist, like anyone else, must accept that there are brute necessities. So even when one restricts attention to concrete objects: there is more to metaphysical modality than logic plus Kripke's a posteriori necessities.[13]

[13] [Acknowledgements.]