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PART IV

Emergence

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Nonlinearity and metaphysical emergence

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Introduction

The nonlinearity of a composite system or 'entity', whereby certain of its features (including associated powers and behaviours) cannot be seen as linear or other broadly additive combinations of features of the system's composing entities, has been often taken to be a telling mark of metaphysical emergence, coupling a composite system's *dependence* on an underlying system of composing entities, with the composite system's having some degree of ontological *autonomy* from this underlying system.[1] But why think that nonlinearity is a mark of emergence, and moreover, of metaphysical rather than merely epistemological emergence? Are there diverse ways in which nonlinearity might enter into an account of properly metaphysical emergence? And what are the prospects for there actually being phenomena that are metaphysically emergent in any of these ways?

Here I will explore the mutual bearing of nonlinearity and metaphysical emergence, with an eye towards answering these and related questions. Given considerations of space I cannot hope to be comprehensive, but by focusing on a representative range of relevant accounts of emergence, and associated paradigm cases of nonlinearity, fairly robust results can, I think, be achieved.

I'll start, in Section 1, by considering how nonlinearity entered into the British Emergentist account of 'strong' metaphysical emergence, as involving the presence of new fundamental powers, forces/interactions, or laws; the emergence here is 'strong' in being of the sort that, were the base entities to be physical or physically acceptable, would falsify physicalism (or 'mechanism') were it to exist.[2] In particular, British Emergentists typically

[1] It is additionally typically supposed that the ontological autonomy at issue will bring some degree of causal autonomy in its wake: epiphenomenalist views are not to the point here.

[2] The question of how best to characterize the physical, hence physicalism, is controversial, but nothing in what follows will turn on the specific characterization of the base entities at issue. Most commonly, the physical entities are taken to be those treated more or less directly by (ideal, or at least approximately accurate) fundamental physics; physically acceptable entities are entities that are supposed to be, one way or another 'nothing over and above' the physical entities—e.g. by being lower-level aggregates of physical particulars standing in physical relations, or by being weakly emergent from such aggregates, as the case may be (see Wilson 2005).

supposed that non-linearity of composite features and associated powers or behaviours was a sufficient indication of strong metaphysical emergence of the features, hence systems, involved. As we'll see, this supposition (properly understood as involving a broad notion of additivity, involving not just intrinsic but also certain lower-level relational features of composing entities as 'summands') was plausible enough at the time. It was shown to be incorrect, however, by the discovery and creation of nonlinear complex systems (e.g. turbulent fluids, populations reproducing under conditions of finite resources, the Game of Life) clearly failing to involve new fundamental interactions or laws; indeed, awareness of such complex systems provides another reason, besides the advent of quantum mechanical explanations of chemical phenomena noted in McLaughlin 1992, for the 'fall' of British Emergentism. That said, it remains a live if somewhat outside possibility that some complex phenomena involve new fundamental interactions, and (drawing on Wilson 2002) I identify a better, scientifically supported criterion of the presence of a new fundamental interaction, in which a recognizable descendant of nonlinearity as indicative of strong metaphysical emergence is present in the form of seeming violations of conservation laws.

In Section 2 I consider a representative variety of accounts of 'weak' emergence of the sort supposed to be compatible with physicalism (the idea being that weakly emergent entities are physically acceptable if their base entities are), which typically take as their starting point one or other variety of nonlinear system. Some such accounts—e.g. Newman's (1996) account of features such as *being in the basin of a strange attractor* as identical, though inaccessibly so, to lower-level features—cannot be seen as characterizing emergence in other than merely epistemological terms. Others, including Bedau's (1997, 2002) account of features such as *being a glider gun* in the Game of Life as explanatorily or algorithmically 'incompressible', and Batterman's (1998) account of phase transitions as involving asymptotic universality, aim (or can be seen as aiming) to characterize emergence in genuinely metaphysical terms. As I argue, however, incompressibility and universality are compatible with the ontological reducibility of nonlinear systems and associated features; hence these criteria do not in themselves succeed in establishing that any nonlinear systems are even weakly emergent in a properly metaphysical sense.

In Section 3 I turn to an account of weak emergence that does successfully combine physical acceptability with ontological autonomy, based in the notion of *degrees of freedom* (DOF)—parameters needed to specify the states relevant to the law-governed properties and behaviour of a given entity or system. I first motivate and summarize the key features of the DOF-based account (presented in detail in Wilson 2010), according to which it suffices for a composite entity *E*'s being metaphysically weakly emergent from the system of its composing entities e_i that *E* has DOF some of which are eliminated relative to the comparatively unconstrained system of e_i. I then argue that various paradigmatic nonlinear systems are plausibly seen as having DOF some of which are eliminated relative to entities in their composing base system, hence as being weakly metaphysically emergent.

1 Nonlinearity and strong metaphysical emergence

1.1 Nonlinearity in the British Emergentist tradition

Mill (in 'On the Composition of Causes', 1843/1973) distinguished two types of effects of joint or composite causes. 'Homopathic' effects conform to 'the principle of composition of causes' in being (in some sense) mere sums of the effects of the component causes when acting in relative isolation, as when the weight of two massy objects on a scale is the scalar sum of their individual weights, or when the joint operation of two forces conforms to vector addition in bringing an object to the same place it would have ended up, had the forces operated sequentially. 'Heteropathic' effects, by way of contrast, violate the principle in *not* being mere sums in any clear sense. What is most crucial for Mill about this distinction is that the advent of heteropathic effects is taken to be indicative of the operation of new laws. Mill says:

> This difference between the case in which the joint effect of causes is the sum of their separate effects, and the case in which it is heterogeneous to them; between laws which work together without alteration, and laws which, when called upon to work together, cease and give place to others; is one of the fundamental distinctions in nature. (1843/1973: 408–9)

And he offers chemical compounds and living bodies as entities that are capable of producing heteropathic effects.

Mill did not use the term 'emergence' (evidently Lewes 1875 first did so), and his discussion appears to target a diachronic notion of emergence rather than the broadly synchronic emergence typically at issue in debates over whether some 'higher-level' goings-on are properly seen as emergent from some 'lower-level' goings-on. But given the reciprocal connection between powers and effects, it is straightforward to translate between the two approaches: to say that an effect of a feature of a composite entity is non-additive, relative to effects of features of the parts acting separately, is just to say that the higher-level feature has a power not had by the lower-level base features when in additive combination.[3] Mill himself moves seamlessly from talk of heteropathic effects to talk of new properties of and laws governing entities capable of causing such effects:

[3] A background issue here concerns whether lower-level features 'in additive combination' are to be understood as features of a relational entity, consisting in lower-level entities standing in lower-level relations (as well as Boolean and mereological combinations) or not; this question is related to the question of whether realization should be understood as a one-one or many-one relation (see Gillett 2002). *Contra* Gillett, I don't think anything of importance hangs on this issue, since it's clear that both the one-one and the many-one approaches to characterizing lower-level (i.e. physically acceptable) base features target the same phenomena. To wit: on the non-relational approach we consider the nature of the dependence (as involving either emergence or realization) of higher-level features on features of comparatively non-relational lower-level entities given certain allowable combinatorial principles, whereas on the relational approach we consider the nature of the dependence of higher-level features on features of a relational lower-level entity understood as 'put together' by the allowable combinatorial principles. In my view, it's more perspicuous to take the one-one approach, as reflecting or encoding the sorts of features of lower-level aggregates that all parties agree would not give rise to emergence. But to gloss over this issue I will typically speak of the composing base 'system', leaving open whether the system is comprised by many lower-level entities or a single lower-level relational entity.

> [W]here the principle of Composition of Causes [. . .] fails [. . .] the concurrence of causes is such as to determine a change in the properties of the body generally, and render it subject to new laws, more or less dissimilar to those to which it conformed in its previous state. (Mill 1843/1973: 435)

Both Mill's reference to 'new laws' and his taking such cases to contrast with 'the extensive and important class of phenomena commonly called mechanical' indicate that Mill's appeal to non-additivity of effects is aimed at identifying a criterion for a higher-level feature's having a new fundamental power, enabling it (or its possessing 'body') to override the usual composition laws in the production of certain effects. As McLaughlin (1992) notes, 'Mill holds that collocations of agents can possess fundamental force-giving properties' (65). Hence it is that Mill's conception is appropriately seen as a variety of 'strong' metaphysical emergence, of the sort that, were it to exist, would falsify physicalism ('mechanism').

Other British Emergentists followed Mill in characterizing strong metaphysical emergence in terms of nonlinearity, as involving violations of broadly additive composition laws, including Alexander (1920), who characterized emergent properties as having powers to produce heteropathic effects; Morgan (1923), who contrasted resultant with emergent features as being 'additive and subtractive only'; and Broad (1925), who offered scalar and vector addition as paradigms of the compositional principles whose violation was characteristic of emergence.[4]

As in Mill's case, these appeals to nonlinearity are best seen as attempts to provide substantive metaphysical criteria of the operation of new fundamental powers, forces, or laws that come into play only at certain complex levels of existence, as when Broad (1925) says, '[T]he law connecting the properties of silver-chloride with those of silver and of chlorine and with the structure of the compound is, so far as we know, an unique and ultimate law' (64–5). As such, McLaughlin accurately characterizes British Emergentism as:

> [. . .] the doctrine that there are fundamental powers to influence motion associated with types of structures of particles [. . .] In a framework of forces, the view implies that there are what we may call 'configurational forces': fundamental forces that can be exerted only by certain types of configurations of particles [. . .].[5] (1992: 52)

1.2 Is nonlinearity a mark of fundamentality?

Were the British Emergentists right, or at least reasonable, to take nonlinearity as a mark of emergent fundamentality? In answering this question we need first to address a clear

[4] Lewes's (1875) was an exception to this rule, in characterizing emergence as involving any failure of 'general mathematizability'. Lewes's characterization is faulty, however: since one can always construct a broadly mathematical function from base to higher-level features, such a characterization will rule out any features as being emergent.

[5] McLaughlin goes on to note that some British Emergentists were suspicious of forces per se, and to claim that the thesis could be preserved just as a thesis about new fundamental powers. For reasons that I'll get to in Section 1.3, I think the appeal to forces/interactions is essential to making out the position, but nothing in this subsection turns on this specific issue.

objection to the sufficiency claim, arising from taking the contrasting understanding of linearity to require that every feature (or associated power, etc.) of a composite entity be a linear combination of *intrinsic* features (powers, etc.) of its composing entities, or as it is sometimes put, a linear combination of features of its composing entities when these are 'in relative isolation'. Consider, for example, the shape of a molecule composed of some atoms.[6] The molecule's shape (and associated powers, etc.) is clearly not the product of any new fundamental interactions; but on the other hand this shape is clearly not a function (linear or otherwise) of *just* the shapes of the atoms when in relative isolation, but also of the bonding relations between the atoms holding these at some distance from each other.

British Emergentists were aware of this concern, and the most sophisticated and careful of them—that is, Broad—included pairwise and other relatively non-complex relations between the composing entities (or states of affairs consisting of lower-level entities standing in relatively non-complex lower-level relations) as among the physically acceptable 'summands' apt to be combined in broadly additive or mereological fashion, and against which a given claim of emergent nonlinearity was to be assessed. Hence it was that Broad characterized 'pure mechanism' as involving broadly additive deducibility of all higher-level features from features of lower-level entities 'either individually or in pairwise combination', and couched his official formulation of emergence in terms of failures of in-principle deducibility of composite features from features of composing entities both 'in isolation' and 'in other wholes'. The notion of linearity in these construals clearly adverts to lower-level relational as well as intrinsic features of composing entities, hence can accommodate the physical acceptability of, e.g., the shape of a molecule, as a broadly additive combination of shapes associated with atoms in pairwise or other relatively non-complex combination. Note that the broadly linear deducibility here can help itself to vector addition of forces exerted by atoms standing in bonding relations.

Given an appropriately sophisticated understanding of the notion of linearity at issue, then, it was quite reasonable for the British Emergentists to take nonlinearity to be a mark of emergent fundamentality. To start, various paradigm cases of non-emergent features of composite entities are simple scalar sums of features of their composing entities, as when the mass of a composite entity is the sum of the masses of its composing entities. More generally and more importantly, at the time it was common to suppose that effects ultimately involve the exertion of various fundamental forces, including gravity and electromagnetism, operating either singly or together. Moreover, as Mill's discussion makes especially clear, the combination of fundamental forces was taken to proceed in accord with linear composition laws—that is, by means of vector rather than scalar addition. As such, and again recalling the correspondence between powers and effects, linearity looked to provide a general handle on when the features of composite

[6] I assume here that molecules (or larger entities, such as tables) and atoms have fairly contained shapes.

entities did *not* involve or invoke any new fundamental powers, forces, or laws. Failures of features or behaviours of composite systems to be subject to linear analysis would thus have been reasonably interpreted as indicating that some additional fundamental force—a force not operative at lower, less complex levels of natural reality—was now on the scene.

Though reasonable at the time, the Emergentist supposition that nonlinearity is sufficient indication of strong metaphysical emergence is no longer plausible. For one thing, the picture of causal relations as constituted by the operation of additive combinations of fundamental push–pull forces is now seen as largely heuristic, or in any case not generalizable; it is fundamental interactions, involving particle exchanges, or yet more abstract accounts of the existence and evolution of natural phenomena, that provide the ultimate story as regards the 'go' of events, and to the extent that broadly Newtonian forces can be seen as real (as they arguably can be; see Wilson 2007), they are now assumed to be non-fundamental (as, e.g., constituted by fundamental interactions). For another, 20th-century investigations into a wide range of complex systems revealed not just that nonlinearity was rampant, but that in many of these cases the nonlinearity was generated simply as a result of complexity, without any need to posit additional fundamental interactions.

The recognition of many complex composite systems as genuinely nonlinear proceeded along several fronts. Even as early as the late 1880s, there were difficulties in seeing complex systems of the sort associated with turbulence in fluids and gases, and phase transitions in general, as linear. Attempts were made to explain away failures of linear prediction in these cases as due to noise or imprecision in measurement; but in a nice recapitulation of the move from a Ptolemaic to a Copernican system of astronomy, the anomalies and epicycles associated with the supposition of linearity eventually gave way to an understanding of chaotic complex systems as having genuinely nonlinear dynamics. This is not to say, of course, that failures in prediction were thereby (always) overcome; rather, such failures were alternatively explained as reflecting, most saliently, the highly sensitive dependence of the associated nonlinear functions on initial conditions (a.k.a. 'the butterfly effect').

That the predictive anomolies in some complex systems could not generally be put down just to noise or imprecision was confirmed by attention to natural and artificial nonlinear systems for which the relevant initial conditions could be specified with complete accuracy. Population growth, for example, is straightforwardly modelled by the nonlinear logistic map:

$$\mathrm{x}_n + 1 = a\mathrm{x}_n - a(\mathrm{x}_n)^{*}2$$

Here *a* is a parameter representing birth and death rates, and is different for different systems. The behaviour of a given system is strongly dependent on *a*. For most values of *a*, the system evolves to a fixed point; as *a* approaches 4, the system's behaviour becomes periodic, and subject to increasingly rapid bifurcation; for *a* = 4, the system's behaviour becomes chaotic, with very small differences in initial conditions x_i, associated with distant decimal places, eventually leading to wildly different trajectories. The discovery of natural nonlinear systems encouraged attention to nonlinear systems in general, and with the advent of computers in the latter half of the 20th century much attention focused on artificial complex systems such as cellular automata, where, as in Conway's 'Game of Life' (see Section 2.2 below), the stipulated dynamics are nonlinear.

What is the bearing of such genuinely nonlinear systems on the British Emergentist supposition that nonlinearity is sufficient for strong metaphysical emergence? Most obviously, the sufficiency claim is undermined, in that at least some cases of nonlinear complex systems—e.g. that associated with population growth or cellular automata—clearly do not involve any additional fundamental forces/interactions, etc. (in the case of automata, as a matter of explicit stipulation).

More generally, it is often taken for granted that strong metaphysical emergence, of the sort, again, whose existence would falsify physicalism (mechanism), is not at issue in any cases of nonlinear systems along lines of those mentioned above. Hence Bedau says:

> An innocent form of emergence—what I call 'weak emergence'—is now a commonplace in a thriving interdisciplinary nexus of scientific activity [...] that includes connectionist modeling, non-linear dynamics (popularly known as 'chaos' theory), and artificial life. (Bedau 1997: 375)

It is worth noting, however, that stated reasons for thinking that nonlinear systems *in general* are physically acceptable aren't good. Newman (1996) cites the fact that complex systems are 'strictly deterministic' in support; but nothing prevents emergent features from entering, both as regards their emergence and their subsequent evolution, into a deterministic nomological net.[7] Nor does the fact that features of nonlinear systems are 'derivable' from nonlinear equations and initial (or boundary) conditions establish physical acceptability, since—as the British Emergentist tradition makes explicit—unlike linear combinations, that nonlinear combinations of physically acceptable features are themselves physically acceptable is not obvious. Bedau (1997) claims that features of nonlinear systems are physically acceptable because 'structural'—that is: are features of a relational system consisting in composing entities standing in lower-level relations; but given that the features of such a relational entity do not consist solely in additive combinations of features of the parts, that such features are *merely* structural (as with, e.g., the shape of a molecule), in a sense that would entail physical acceptability, is again not obvious. One might aim to support the general claim via an argument by analogy, maintaining that insofar as various surprising features associated with nonlinearity (period doubling, extreme sensitivity to initial conditions) can be modelled in comparatively

[7] Cf. Broad's (1925) claim that emergent features are 'completely determined' by lower-level features, in that 'whenever you have a whole composed of these [...] elements in certain proportions and relations you have something with the [compound's] characteristic properties' (64).

simple and artificial systems for which no fundamental novelty is at issue, there is no reason to suppose that more complex natural nonlinear systems involve fundamental novelty, either. But this argument by analogy fails, for precisely what is at issue is whether, in the more complex natural cases, the nonlinearity at issue has a physically acceptable source.

Indeed, there is in-principle room for maintaining that strong metaphysical emergence is at issue in at least some cases of nonlinearity. Consider, for example, cases where the non-linear phenomena involves feedback between the micro-entities constituting the base, associated with strange attractors and other dynamic phenomena. As Silberstein and McGeever (1999) note, non-linearity might be taken to involve a kind of system-level holism:

> What is the causal story behind the dynamics of strange attractors, or behind dynamical autonomy? The answer, it seems to us, must be the non-linearity found in chaotic systems. [...] But why is non-linearity so central? [...] Non-linear relations may be an example of what Teller calls 'relational holism' [...]. (1999: 197)

Silberstein and McGeever go on to suggest that relational holism of this sort might reflect emergent features' possessing fundamentally new powers ('irreducible causal capacities'). It has also been suggested (or interpreted as being suggested) that the singularities standardly associated with thermodynamic phase transitions are indicative of strong emergence. Hence Callender and Menon (forthcoming) interpret Batterman's claim that 'thermodynamics is correct to characterize phase transitions as real physical discontinuities and it is correct to represent them mathematically as singularities' (Batterman 2005: 234) as signalling Batterman's commitment to phase transitions' being emergent along British Emergentist lines. Whether Teller and Batterman would agree that relational holism or thermodynamic singularities should be understood as involving new fundamental powers/interactions/laws is disputable (see in particular my upcoming discussion of Batterman's view). Still, for present purposes the crucial point is that one *could* maintain strong emergence as underlying some features associated with complex natural nonlinear systems.

Let's sum up the results so far.

First, whether or not all nonlinear systems are physically acceptable, in any case there's no doubt that *some* are. The general moral to be drawn from the identification of straightforwardly mechanistic and artificial nonlinear systems is that, contra Mill and the other British Emergentists, nonlinearity is not sufficient indication of fundamental higher-level powers/interactions/laws. This result represents a second salient reason for the fall of British Emergentism, besides the advent of quantum mechanical explanations of chemical phenomena noted in McLaughlin (1992).

Second, though many nonlinear systems clearly do not involve strong metaphysical emergence, stated reasons for generalizing this result to all nonlinear systems do not go through, and the general claim, as it stands, appears to be something of an article of faith. It would be nice if, given that nonlinearity *itself* can no longer be seen as criterial of strong metaphysical emergence, there was an alternative criterion which could distinguish physically acceptable from physically unacceptable cases of nonlinearity (assuming any of the latter exist). I turn now to identifying such a criterion, as entering into a somewhat refined account of strong metaphysical emergence.

1.3 Nonlinearity's descendant

In Wilson (2002) I offered an account of strong metaphysical emergence along British Emergentist lines, as explicitly (as opposed to implicitly, as on Broad's characterization in terms of 'in-principle failure of deducibility') involving a new fundamental force/interaction. As above, at the heart of the Emergentist position is that an emergent composite system has features bestowing new powers, grounded in new fundamental forces or interactions, to produce effects that the composing system can't enter into producing.[8] McLaughlin claimed (see note 3) that those suspicious of forces (and presumably also of interactions) could dispense with this aspect of the view, retaining only the appeal to new powers; but as I argued in my (2002), this claim is likely incorrect, for on a wide variety of accounts of which powers are bestowed by a given feature, the powers of features of a composite system are inherited by the features of the composing system, simply in virtue of the latter features' being nomologically necessary preconditions for the former. Without a more fine-grained way of assigning powers to features, the possibility of strong metaphysical emergence threatens to be ruled out of court—a bad result, since surely the truth of some version of physicalism is not that easy to establish. I considered various ways of getting around this problem—by appeal, e.g., to a distinction between 'direct' and 'indirect' bestowing of a power, or by appeal to different systems of laws—but concluded that the most straightforward and metaphysically illuminating solution to the problem proceeds by recognizing that powers are typically grounded in specific fundamental forces/interactions, and incorporating this dependence into the operative account of strong metaphysical emergence.

That powers are grounded in distinct fundamental forces/interactions is, I take it, scientifically uncontroversial. The power of being able to bond with an electron, in circumstances where one is in the vicinity of a free electron, is grounded in the electromagnetic (or electroweak) force/interaction, as opposed to the strong nuclear or gravitational forces/interactions. The power of being able to fall when dropped, in circumstances where one is poised above Earth's surface, is grounded in the gravitational force, as opposed to the other fundamental forces in operation. The power of being able to bond with other atomic nuclei in a stable configuration is grounded in the strong nuclear force/interaction, as opposed to the electromagnetic, weak, or gravitational forces/interactions. The power of being able to sit on a chair without falling through it is grounded (at least) in the gravitational and the electromagnetic forces/interactions. In

[8] The notion of 'power' here serves in the first instance as a metaphysically neutral way of registering what effects an entity may enter into causing, when in certain circumstances. Nothing in my use of this notion depends on controversial assumptions.

grounding the powers bestowed by properties, fundamental forces/interactions explain, organize, and unify vast ranges of natural phenomena. More could and eventually should be said about the details of the grounding relation at issue, but for present purposes it will suffice to note that we appear to be within our rights to speak of a feature's bestowing (or not bestowing) a power, *relative to* a given set of fundamental interactions.

But are there philosophical problems with positing fundamental forces or interactions—in particular, should we be suspicious of such entities on metaphysical grounds? One thing to say here is that, as above, Newtonian forces, understood as pushes and pulls having magnitudes and directions, have been replaced in fundamental theorizing by fundamental interactions, involving particle and energy exchanges; as such, one might maintain that Newtonian forces are no longer directly relevant to the characterization of strong emergence. That said, the primary reason for rejecting Newtonian forces—namely, that their posit is redundant, since the properties giving rise to the purported forces are sufficient unto the task of explaining causal relations—might also be supposed to apply to fundamental interactions. It is thus worth noting that ways of responding to this concern about forces (set out in Wilson 2007) also serve to vindicate the posit of fundamental interactions. Fundamental interactions are thus both scientific and metaphysical posits in good standing; hence it is reasonable to appeal to them in characterizing the strong emergence of broadly scientific entities.

My suggestion, then, is that strong emergence (and also its 'complement', realization) should be understood as a *relative* notion—relative, that is, to a specified set of fundamental interactions $\{F\}$. Let us assume, as per usual, that every feature of a composite entity E is (at least) nomologically necessitated by a feature of the system of its composing entities e_i.[9] Schematically expressed, the account is as follows:

Interaction-relative Strong Emergence: Feature P of composite entity E is strongly emergent from feature Q of the system of composing entities e_i, relative to $\{F\}$, if (i) Q nomologically necessitates P, and (ii) P is associated with at least one power that is *not* identical with any power of Q that is grounded only in fundamental interactions in $\{F\}$.[10]

The British Emergentist's account of strong emergence is then accommodated by taking the fundamental interactions in {F} to be the fundamental physical interactions—that is, those coming into play at or below the atomic level of organization.[11] And British Emergentism would then be the thesis (contrasting with any version of physicalism), that at least one feature of at least one composite entity is strongly emergent in having at least one power that is not grounded *only* in fundamental physical interactions.

[9] Recall that in my talk of a 'system' of composing entities I am remaining neutral between the one-one and one-many characterizations of the system; see note 1.

[10] This account can be extended to apply to cases of non-composite emergence (e.g. emergence of melded 'fusions' as discussed in Humphreys 1997).

[11] See Wilson 2005 for discussion of how to characterize the physical entities (including interactions) so as to avoid concerns stemming from 'Hempel's dilemma'.

As prefigured, *Interaction-relative Strong Emergence* makes room for there to be strong metaphysical emergence: even if, taking *all* fundamental interactions into account, features of the composing system inherit all the powers of any features they nomologically necessitate, it remains that composite features may be associated with powers that are relevantly 'new', in not being grounded only in the set of physical fundamental interactions.

Interaction-relative Strong Emergence also has three features relevant to investigating the mutual bearing of nonlinearity on strong emergence.

First, the account nicely accommodates the supposition that features of a composite entity that *can* be analysed as broadly additive combinations of physically acceptable features of the composing entities will *not* be strongly emergent. For whatever the precise account of how powers are grounded in fundamental interactions, in any case it is clear—to go back to Mill's original discussion—that every power of a feature that is a broadly additive combination of physically acceptable features will be grounded only in fundamental physical interactions, hence fail to be strongly emergent.

Second, the account suggests an alternative criterion for strong emergence, that not only survives the advent of physically acceptable nonlinear systems, but moreover provides the means of distinguishing, at least in principle, between cases of nonlinearity that *do* and cases that *don't* involve strong emergence. On the interaction-relative account, strong emergence explicitly involves the coming into play of a new fundamental interaction; hence in determining whether there is any such emergence we can adopt whatever scientific methods are available for tracking such fundamental novelty. Here it is especially useful to attend to the discovery of the weak nuclear interaction, since this interaction occurs at a lower-level but still configurational level. This discovery was motivated by its seeming to be that certain nuclear processes, involving alpha radiation in particular, violated well-entrenched conservation laws of mass-energy and momentum. Rather than reject the associated conservation laws, the books were balanced by positing an additional fundamental interaction. A similar strategy makes in-principle empirical room for testing whether or not the unusual features associated with complex natural nonlinear systems are due to configurational fundamental interactions, by comparing the values of relevant conserved quantities predicted by fundamental physical theory as attaching to composite entities, with the actually observed values of these quantities. If there's less (or more) energy coming out than going in, for example, we might well be inclined to conclude, following accepted scientific procedure and as per the strong emergentist thesis, that a new configurational force/interaction is in operation.

Third, in the appeal to apparent violations of conservation laws as a sufficient criterion of strong emergence, we have, it seems to me, a recognizable descendant of the British Emergentist appeal to apparent violations of linearity as such a criterion. For in both cases, the apparent violations serve to flag that the whole is more than the mere sum of its parts; and that additional fundamental entities must be posited, if the sum—of forces, of conserved quantities—is to come out right.

All this said, I take it that there is not much motivation for thinking that any complex natural nonlinear systems involve new fundamental interactions. Still, it is useful to observe, first, that strong metaphysical emergence understood in interaction-relative terms is conceptually viable, and second, that (unlike bare appeals to relational holism or representational mismatch, whose status as criterial for new fundamental interactions is controversial) such an account suggests a criterion for strong emergence upon which all parties are likely to agree, and which could, in principle, be tested for.

2 Nonlinearity and weak metaphysical emergence

I now want to turn to the role nonlinearity has played in a representative range of accounts of weak metaphysical emergence (of the sort, again, whose occurrence is supposed to be compatible with physicalism) intended to apply to three different kinds of nonlinear systems. As above, accounts of properly metaphysical emergence aim to combine the *dependence* of a composite system *E* on the system of its composing base entities *e_i* with *E*'s ontological (and also causal) autonomy from the composing *e_i*; and in cases of weak emergence the autonomy is supposed to be compatible with the composite system and its features' being physically acceptable. But as I'll argue, (a representative range of) accounts of weak emergence are compatible with the ontological (hence causal) reducibility of the nonlinear systems at issue, and hence establish, at best, that such systems are epistemologically or representationally emergent. I'll establish this, in turn, for Newman's account of emergence as a matter of epistemologically inaccessible identity between composite and base features, applied to properties such as *being in the basin of a strange attractor*; Bedau's account of weak emergence as involving algorithmic or explanatory incompressibility, applied to properties such as *being a glider gun* in the Game of Life; and Batterman's account of emergence as involving asymptotic universality, as applied to thermodynamic systems undergoing phase transitions.[12] I'll then present my (2010) account of weak metaphysical emergence as involving an *elimination in degrees of freedom* associated with the composite entity as compared to the unconstrained system of its composing base entities, and argue that at least some nonlinear systems are weakly emergent, by lights of the DOF-based account.

2.1 Newman's inaccessible-identity-based emergence

Newman's focus (1996) is on chaotic nonlinear systems, characteristically exhibiting (among other interesting features) extreme sensitivity to initial conditions (as per the 'butterfly effect'). Such systems are, he notes, associated with 'strange attractors':

[12] As we'll see, there's good reason to think that Batterman doesn't really have any sort of metaphysical account of emergence in mind. Still, asymptotic universality runs through his work and it is worth considering whether this (or the related notion of stability under permutation) could serve as a basis for weak metaphysical emergence. Another feature of the sort of systems Batterman discusses receives a thoroughly metaphysical treatment in my own account of weak emergence (Section 3).

> A strange attractor is a non-periodic trajectory in the state space that exhibits sensitive dependence on initial conditions. This means that as the state of a chaotic system evolves toward the attractor in its state space, it will never be in exactly the same state twice, and any two nearby points in the state space will diverge exponentially under the dynamical evolution of the system. (1996: 254)

Typically, for certain parameters of a given non-linear dynamical equation, a system governed by the equation will evolve towards a strange attractor A in its state space, no matter what its initial state; such a system is said to be 'in the basin' of strange attractor A. So, for example, in the case of the logistic map, the onset of chaos occurs for $a = 4$.

Chaotic nonlinear systems exhibit two epistemological characteristics. First, due to the sensitive dependence on initial conditions we are unable to predict the future trajectory of a chaotic non-linear system:

> [F]or any chaotic system, since the measurements that we can make of its state are less than perfect [. . .] the future states of that system are impossible to predict. [. . .] The kind of unpredictability introduced by chaotic systems is a kind of epistemic impossibility rather than a metaphysical impossibility. (Newman 1996: 254)

Second, difficulties in measurement also mean that it is epistemically 'impossible to determine exactly which chaotic system we are dealing with'. These characteristics enter into Newman's account of weak emergence, on which emergent features of composite systems are, while identical to some lower-level feature of the composing system, such that it is epistemically impossible to make the identification, as follows:

> A property designated by a predicate P in an ideal theory T is emergent [from T'] if and only if the following conditions are met:
>
> (i) T describes a class of systems which are structured aggregates of entities described by T'.
> (ii) T' is an ideal theory of those entities, and the entities described by T strongly supervene on those described by T'[13]
> (iii) Occurrences of the property designated by P are epistemically impossible to identify with occurrences of any property finitely describable in T'
> (iv) Each occurrence of the property designated by P is an occurrence of one of a set of properties PC, which is modelled by T'. Each member of PC is epistemically indistinguishable in T' from some other member of PC.

Newman argues that the property of *being in the basin of a strange attractor* is such an emergent property of nonlinear systems. Nonlinear chaotic systems (described by the relevant higher-level science T) are structured aggregates of lower-level physically acceptable entities (ultimately described by fundamental physical theory), as per (i);

[13] This clause is intended to characterize emergent dependence as physically acceptable, though as it happens there are good reasons to think that even the strongest varieties of supervenience do not guarantee transmission of physical acceptability; see, e.g., Horgan 1993, Wilson 2005.

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nonlinear chaotic systems, Newman assumes, strongly supervene on entities described in fundamental physical theory, as per (ii); moreover, every occurrence of a property of a chaotic nonlinear system is, Newman assumes, identical to the occurrence of a lower-level physically acceptable property, as per (iv); but given that occurrences of the higher-level property are so extremely sensitive to initial conditions, the precise values of which are epistemically inaccessible to us, we are never in position to specify the relevant identities, as per (iii).

Newman's account is straightforwardly epistemological, and indeed in supposing that every property of a nonlinear dynamic system is identical with some lower-level physically acceptable property, he is clear that he thinks the former are ontologically, hence causally, reducible to the latter. I mention Newman's account as representative of perhaps the most common approach to the weak emergence of nonlinear systems—namely, one on which the 'emergence' of such systems is cashed in terms of one or other epistemological feature commonly associated with such systems (see also Popper and Eccles 1977; Klee 1984; Rueger 2001). However illuminating such accounts may be as regards why we find the features and behaviour of such systems interesting, novel, or unpredictable, they go no distance towards establishing nonlinearity as a mark of weak *metaphysical* emergence. The accounts to be next considered aim (or can be seen as aiming) to do better.

2.2 Bedau's incompressibility-based emergence

Bedau's focus is on a feature of nonlinear systems shared by both chaotic and non-chaotic nonlinear systems; namely, that such systems typically fail to admit of analytic or 'closed' solutions. The absence of analytic or otherwise 'compressible' means of predicting the evolution of such systems means that the only way to find out what this behaviour will be is by 'going through the motions': set up the system, let it roll, and see what happens. It is this feature—namely, *algorithmic incompressibility*—that serves as the basis for Bedau's account of weak emergence, as follows:

> Where system S is composed of micro-level entities having associated micro-states, and where microdynamic D governs the time evolution of S's microstates:
>
> Macrostate P of S with microdynamic D is weakly emergent iff P can be derived from D and S's external conditions but only by simulation. (1997: 378)

Derivation of a system's macrostate 'by simulation' involves iterating the system's micro-dynamic, taking initial and any relevant external conditions as input. The broadly equivalent conception in Bedau's (2002) takes weak emergence to involve 'explanatory incompressibility', where there is no 'short-cut' explanation of certain features of a composite system. In being derivable by simulation from a micro-physical dynamic, associated macrostates are understood to be physically acceptable; as Bedau (1997) says, such systems indicate 'that emergence is consistent with reasonable forms of materialism' (376).

By way of illustration, Bedau focuses on Conway's Game of Life, an example of a non-chaotic nonlinear map. The game consists in a set of simple rules, applied simultaneously and repeatedly to every cell in a lattice of 'live' and 'dead' cells.[14] Here there is no problem of sensitivity to initial conditions, since these conditions consist just in the discrete 'seeding' of the lattice. Still, Bedau argues that the property of *being a glider gun* in the Game of Life is weakly emergent, in his sense. That this property (involving a gun-like shape that moves across the grid, emitting 'bullets') does not involve any strong emergence is clear, since for cellular automata the long-term behaviour of the system is completely determined by ('derived from') the lower-level 'rules' applying to cells in the grid. But that a given system will evolve in such a way as to generate a glider gun can typically not be predicted from knowledge of initial conditions (seeding) and these rules.

On the face of it Bedau's account, like Newman's, does not characterize a genuinely metaphysical account of weak emergence, an impression seemingly confirmed when Bedau says that 'weakly emergent phenomena are autonomous in the sense that they can be derived only in a certain non-trivial way' (Bedau 2002: 6). Indeed, Bedau is explicit that he takes weakly emergent features of composite systems to be both ontologically and causally reducible to features of their composing systems:

> [W]eakly emergent phenomena are ontologically dependent on and reducible to micro phenomena. (2002: 6)
>
> [T]he macro is ontologically and causally reducible to the micro in principle. (2008: 445)

Notwithstanding these reductive assumptions, Bedau maintains that the autonomy of weakly emergent entities on his account is not just epistemological, but also properly metaphysical. He offers two reasons for thinking this, but as I'll now argue, neither establishes the claim.

The first is that the incompressibility of an algorithm or explanation is an objective metaphysical (if broadly formal) fact:

> The modal terms in this definition are metaphysical, not epistemological. For P to be weakly emergent, what matters is that there is a derivation of P from D and S's external conditions and any such derivation is a simulation. [. . .] Underivability without simulation is a purely formal notion concerning the existence and nonexistence of certain kinds of derivations of macrostates from a system's underlying dynamic. (1997: 379)

But such facts about explanatory incompressibility, though objective and hence in some broad sense 'metaphysical', are not suited to ground the metaphysical autonomy of emergent entities. What is needed for such autonomy is not just some or other metaphysical

[14] The Game of Life takes place on a two-dimensional grid of cells, which may be either 'alive' or 'dead'. At each step in time, cells are updated as per the following rules: (1) any live cell with fewer than two live neighbours dies; (2) any live cell with two or three live neighbours stays alive; (3) any live cell with more than three live neighbours dies; (4) any dead cell with exactly three live neighbours becomes alive.

distinction between macro- and micro- goings-on, but moreover one which plausibly serves as a basis for rendering weakly emergent features ontologically autonomous from—that is, *distinct* from—the lower-level features upon which they depend.

The second reason Bedau gives is somewhat more promising; namely, that weakly emergent features typically enter into macro-level patterns and laws. As Bedau says:

> [T]here is a clear sense in which the behaviors of weak emergent phenomena are autonomous with respect to the underlying processes. The sciences of complexity are discovering simple, general macro-level patterns and laws involving weak emergent phenomena. [...] In general, we can formulate and investigate the basic principles of weak emergent phenomena only by empirically observing them at the macro-level. In this sense, then, weakly emergent phenomena have an autonomous life at the macro-level. (1997: 395)

As such, Bedau maintains, 'weak emergence is not just in the mind; it is real and objective in nature' (2008: 444). Attention to macro-level patterns sounds like a move in the right direction towards autonomy; but, two points. First, I don't see how Bedau can maintain that some weakly emergent goings-on are not 'merely epistemological' (Bedau 2008: 451) and rather reflect an 'autonomous and irreducible macro-level ontology', while also maintaining that all weakly emergent goings-on are metaphysically ontologically and causally reducible to the micro-level goings-on. Either the metaphysical autonomy or the metaphysical reducibility has to go.[15]

Second, it isn't enough to block the potential reducibility of composite features to features of the composing base system, merely to point to the fact that the composite features enter into nomological patterns that are in some sense more general than those into which the composing features enter. To see this, it is useful to recall a related dialectic in the physicalism debates. There, would-be non-reductive physicalists point to the fact that mental features are associated with functional roles (i.e. 'macro-level patterns') that can be multiply implemented, or realized, by lower-level physical features; but the reductionist's ready response is that the existence of functional or other comparatively general patterns can be accommodated, on their terms, by identifying a given mental feature with (to cite the most salient candidates) *a disjunction* of its first-order physical realizers or the second-order *existential* property of having one or other of its first-order physical realizers. The dialectic sometimes continues, with the non-reductive physicalist rejecting, e.g., the existence of disjunctive features on grounds that they are too gerrymandered, are not projectible, or otherwise suspect; but such considerations seem, from a reductionist perspective, either unprincipled or uncompelling. Similarly, in the case of *being a glider gun*, while something seems right about attending to the comparative generality of the patterns into which this feature enters, more needs to be said if reductionist accommodation of such macro-level patterns is to be blocked in a principled and compelling fashion.

[15] My impression is that the tension here reflects Bedau's assumption that if a higher-level system is to be physically acceptable, then it must be ultimately ontologically reducible to some lower-level system. This is incorrect, as I will discuss in Section 3.1.

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Bedau does not say more along these lines, however, and the upshot is that this strategy, like the previous, fails to establish that features of nonlinear systems are even weakly metaphysically emergent.

2.3 Batterman's asymptotic emergence

Batterman has in recent years written a great deal concerning the status as emergent or reducible of special science entities (see his 2002, 2005, and elsewhere); but perhaps because his discussions have tended to engage primarily with theoretical/representational and explanatory issues, there has remained unclarity as regards whether he takes emergence to be a metaphysical phenomenon, and if so, of what strength. As I'll discuss, there is good reason to think that Batterman does not have even a weak account of metaphysical emergence in mind (much less the strong account some attribute to him). Still, no discussion of emergence as relevant to nonlinear phenomena should neglect Batterman's work on emergence in asymptotic regions of the sort associated with phase transitions, since these share key features with chaotic nonlinear systems; and whatever Batterman's view of the matter, various features of systems in asymptotic regions are at least promising so far as indicating that weak metaphysical emergence is in place.

I'll start, then, by discussing some general features of Batterman's account of emergence as associated with asymptotic limits, and the specific application of his account to the case of systems undergoing phase transitions. I'll identify three related characteristics of such systems, each of which generally applies to chaotic nonlinear systems and might be thought promising as regards characterizing weak metaphysical emergence; namely, *eliminations in micro-level degrees of freedom (DOF)*, *asymptotic universality*, and *stability under permutation*. I'll say why it's fairly clear that Batterman does not see these features as tracking a genuinely metaphysical form of emergence. I'll then consider whether the latter two features, which are the primary running themes in Batterman's work (see his 1998, 2000, 2002, 2005), might nonetheless serve as sufficient criteria for weak emergence, and I'll argue that they cannot do so, since here again the having of these features is compatible with ontological reducibility. In the next section I turn to my preferred, explicitly metaphysical DOF-based account of weak emergence, which, I'll argue, is up to the task of guaranteeing ontological autonomy.

Let's summarize the basics of Batterman's account of asymptotic emergence.[16] An asymptote in mathematics is a limiting value of a function that is approached indefinitely closely, but never reached. So, for example, as $x \to 0$ the function $1/x$ goes to infinity; in this case (though not all, of course) the asymptote is associated with a discontinuity. Interestingly, many 'near-neighbour' scientific theories involve asymptotes: special relativity asymptotically approaches Newtonian mechanics in the limit as $v/c \to 0$, wave optics→ray optics as $1/\lambda$ approaches 0, (quantum mechanics) → Newtonian mechanics as Planck's constant→0, statistical mechanics→thermodynamics in the 'thermodynamic

[16] My discussion here leans heavily on Hooker's (2004) admirably clear summary of the view expressed in Batterman (2002).

limit', where particle number N and volume V→infinity. Now, in some of these cases—in particular, the latter three—the asymptotic limits at issue are associated with discontinuities in the regions near the asymptote. In such cases of 'singular' asymptotic limits, Batterman suggests, we have reason to take various objects or features associated with the asymptotic region (or associated theory) as emergent. In particular, to cut to the case which concerns us, Batterman suggests that various features of systems undergoing phase transitions, including those associated with certain critical exponents, are emergent features of such systems. Why so, and in what sense?

Batterman's most explicit stated considerations concern broadly explanatory factors. Again focusing on the case which concerns us, one explanatory concern reflects a kind of theoretical mismatch between the near-neighbour theories at issue, insofar as the discontinuities associated with taking the thermodynamic limit, and which are commonly supposed to be needed to accommodate the associated asymptotic phenomena, find no representational mirror in the analytic functions of statistical-mechanics.[17] Even if there were no problem with deriving specific instances of asymptotic features from the micro-theory, however, Batterman's second explanatory concern would remain; namely, that the characteristic *universality* of asymptotic phenomena cannot be properly explained by reference just to lower-level 'causal-mechanical' explanations. As above, the behaviour of systems undergoing phase transitions is characterized by a small set of dimensionless numbers called 'critical exponents'. As Batterman (1998) says:

> What is truly remarkable about these numbers is their universality [...] the critical behavior of systems whose components and interactions are radically different is virtually identical. Hence, such behavior must be largely independent of the details of the microstructures of the various systems. This is known in the literature as the 'universality of critical phenomena'. Surely one would like to account for this universality. (198)

Lower-level causal-mechanical explanations, even if available, cannot account for universality, for these 'will be infinitely various in detail and this will block any reconstruction of what is universal about them' (2004: 442).

By way of contrast, Batterman argues, various methods for modelling asymptotic phenomena—most notably, the Renormalization Group (RG) method—*do* provide an explanation of the universal features of systems undergoing phase transitions. The applicability here primarily reflects that near critical points, the systems at issue cease to have any characteristic length scale, and are 'self-similar' in that the laws governing the systems take the same form at all length scales. Accordingly, the RG method—which takes a system's governing laws (e.g. its Hamiltonian) and iteratively transforms these into laws having a similar form but (reflecting moves to increasingly 'larger' scales) fewer parameters—can be applied. In the limit, the resulting Hamiltonian describes the behaviour of a single 'block', corresponding to the macroscopic system. As Batterman (1998) puts it:

[17] See Callender and Menon (in progress) and Wilson (in progress) for discussion.

One introduces a transformation on this space that maps an initial physical Hamiltonian describing a real system to another Hamiltonian in the space. The transformation preserves, to some extent, the form of the original physical Hamiltonian so that when the interaction terms are properly adjusted (renormalized), the new renormalized Hamiltonian describes a system exhibiting the same or similar thermodynamical behavior. Most importantly, however, the transformation effects a reduction in the number of coupled components or degrees of freedom within the correlation length. Thus, the new renormalized Hamiltonian describes a system which presents a more tractable problem. It is to be hoped that by repeated application of this renormalization group transformation the problem becomes more and more tractable until one can solve the problem by relatively simple methods. In effect, the renormalization group transformation eliminates those degrees of freedom (those microscopic details) which are inessential or irrelevant for characterizing the system's dominant behavior at criticality. (1998: 200)

(I quote this and the next passage at length, in order to make explicit, for future purposes, Batterman's appeal to certain features potentially relevant to emergence.) In particular, application of the RG method in this case enables calculation of the critical exponents associated with phase transitions, and hence provides an explanation of the universal behaviour of systems near critical points:

[I]f the initial Hamiltonian describes a system at criticality, then each renormalized Hamiltonian must also be at criticality. The sequence of Hamiltonians thus generated defines a trajectory in the abstract space that, in the limit as the number of transformations goes to infinity, ends at a fixed point. The behavior of trajectories in the neighborhood of the fixed point can be determined by an analysis of the stability properties of the fixed point. This analysis also allows for the calculation of the critical exponents characterizing the critical behavior of the system. It turns out that different physical Hamiltonians can flow to the same fixed point. Thus, their critical behaviors are characterized by the same critical exponents. This is the essence of the explanation for the universality of critical behavior: Hamiltonians describing different physical systems fall into the basin of attraction of the same renormalization group fixed point. This means that if one were to alter, even quite considerably, some of the basic features of a system (say from those of a fluid F to a fluid F' composed of a different kind of molecule and a different interaction potential), the resulting system (F') will exhibit the same critical behavior. This stability under perturbation demonstrates that certain facts about the microconstituents of the systems are individually largely irrelevant for the systems' behaviors at criticality. (1998: 201)

Batterman's account of asymptotic emergence, especially as relevant to phase transitions, looks very promising, so far as vindicating the emergence of nonlinear systems is concerned. To start, there are three features associated with this account having at least *prima facie* promise for characterizing a form of weak metaphysical emergence, in particular:

(1) elimination of micro-level degrees of freedom (DOF);
(2) universality of certain features or behaviour;
(3) stability of certain behaviour under perturbation.

Moreover, these common features of systems undergoing phase transitions are more generally features of chaotic nonlinear systems, as Hooker (2004) implies:

> In every case of so-called 'critical phenomena', e.g. near the 'critical point' beyond which there is no vapour phase between liquid and gas, the asymptotic domain shows a universally self-similar spectrum of fluctuations. [...] This is indicative of chaos and occurs when behaviours are super-complexly, but still systematically, interrelated [...]. (2004: 440)

Indeed, the core similarities between critical phenomena in statistical mechanics and chaotic nonlinear phenomena, including period doubling and intermittency routes to chaos of the sort displayed by the logistic map, have led to an active area of investigation in which 'the logistic map is [...] becoming a prototypical system [...] for the assessment of the validity and understanding of the reasons for applicability of the nonextensive generalization of [...] Boltzmann-Gibbs statistical mechanics' (Mayoral and Robledo 2006: 339). If we can understand the features entering into Batterman's account of emergence in metaphysical terms, we would have a nice result as regards the bearing of nonlinearity and metaphysical emergence.

To be sure, there is good reason to believe that Batterman does *not* intend his discussion to be interpreted as offering either an account of or a case of metaphysical emergence. As above, his discussion is squarely focused on the question of what is required if the critical behaviours of the systems in question are to be explained, with the general idea being that, in the case of such systems, neither theoretical derivations nor causal–mechanical considerations can do the trick. Hence Morrison (2012) reads Batterman as offering an 'explanatory' account of emergence:

> I characterize Batterman's account [of emergence] as explanatory insofar as the main argument centers on how asymptotic methods (via the RG) allow us to explain features of universal phenomena that are not explainable using either intertheoretic reduction or traditional causal mechanical accounts [...]. (2012: 143)

I think Morrison's reading is correct,[18] but one might wonder if it is undermined by Batterman's seeming to suggest, especially in some passages in his (2002) book, that he sees metaphysical emergence as following from explanatory emergence, as when he says, 'It seems reasonable to consider these asymptotically emergent structures [as constituting] the ontology of an explanatory "theory"' (Batterman 2002: 96). For another

[18] In context, it is worth registering that Morrison (2012) makes this observation en route to offering her own account of metaphysical emergence, as inspired by cases of spontaneous symmetry breaking in superconducting fluids. I do not enter into the details of Morrison's account here, mainly because it is not completely clear to me whether Morrison's account is a variety of strong or of weak emergence, and a full investigation into this issue would take us too far afield. Morrison herself supposes that 'When one is dealing with emergence in physics, physicalism is not an issue. No one denies that condensed matter phenomena such as superconductivity, phenomena often described as emergent, [...] are physical in nature' (141–2). Still, on her account (which in certain respects follows Batterman's and my own in attending to the relative independence of weakly emergent phenomena from micro-physical details), 'the characteristics that define the superconducting state are not explained or predicted from those processes'—due, in particular, to the advent of spontaneous symmetry breaking and the introduction of a new order parameter—which to my mind opens the door to the possibility that the phenomena at issue are strongly, not weakly, emergent. This and other issues are explored in Wilson (forthcoming).

example, in discussing the asymptotic domain between wave and ray optics, Batterman first argues that rainbows cannot be explained without referring to 'caustics'—ray tangent curves associated with the higher-level, but not lower-level theory, then suggests that we are ontologically committed to such optical objects:

> [I]f I'm right and there is a genuine, distinct, third theory (catastrophe optics) of the asymptotic borderland between wave and ray theories—a theory that of necessity makes reference to both ray theoretic and wave theoretic structures in characterising its 'ontology'—then, since it is this ontology that we take to be emergent, those phenomena are not predictable from the wave theory. They are 'contained' in the wave theory but aren't predictable from it. (2002: 119)

These suggestive passages are misleading, however, since Batterman has explicitly denied that the emergence at issue here carries ontological weight:

> I do not believe that there is any new ontology in the asymptotic catastrophe optics. The wave theory has replaced the ray theory and there simply are no caustics (as characterized by the ray theory). Asymptotic analysis of the wave equation yields terms (think syntax here) which require for their interpretation (semantics) that we make reference to ray theoretic structures. In effect, it is the understanding/interpretation of these terms in the asymptotic expansions that requires 'appeal' or reference to structures that 'exist' only in the ray theory. (p.c. with Hooker, discussed in Hooker 2004: 448)

Hooker concludes that 'the status claimed for caustics based on essential reference is not after all that of ontological commitment but one of ineliminable semantic role, without ontological commitment' (2004: 448). Similar remarks would presumably go for the 'emergent' phenomena at issue in phase transitions or chaotic nonlinear systems, more generally. Strictly speaking, Batterman's account of asymptotic emergence is an account of theoretical/representational or epistemological, not metaphysical, emergence.

That said, nothing prevents the more metaphysically inclined from considering whether any of the aforementioned features—elimination in micro-level DOF, universality, and/or stability under perturbation—might serve as sufficient indicators of metaphysical emergence. With an eye to sticking somewhat closely to Batterman's work, we might look especially to the latter two features, since these features have been consistent themes in his work, which he has discussed as attaching to a variety of special science entities to which the RG approach and its associated strategy for eliminating DOF does not apply.[19]

Universality and stability under perturbations are really two sides of the same coin; as Batterman says, 'most broadly construed, universality concerns similarities in the behavior of diverse systems' (2000: 120). The suggestion that weak emergence might be a matter of universality or of stability under micro-level perturbations is common enough;

[19] Hence, for example, Batterman (2000) argues that certain considerations showing that universal features of systems undergoing phase transitions are emergent (in whatever sense) readily extend to pendulums and other special science entities, and he says that 'there is really no difference between the example of the pendulum and some of the more interesting cases' (121).

indeed, we have already seen a version of this claim in Bedau's suggestion that the fact that certain features of nonlinear automata (e.g. glider guns) enter into 'macro-level patterns' might support such features' being metaphysically autonomous. It is unsurprising, then, that the same concerns with Bedau's suggestion, which again echo the debate over the import of multiple realizability in the metaphysics of mind, also attach to attempts to locate metaphysical emergence in universal or stable features of composite entities; namely, that reductionists have various strategies for accommodating such features, and that the standard anti-reductivist responses (rejecting disjunctive properties, denying that these track genuine natural kinds, and the like) are not compelling. What is needed, if these features are to be seen as tracking the ontological autonomy (distinctness) of composite entities, is a better response to the usual reductivist strategies; but Batterman does not provide such a response—arguably because his concern is ultimately with whether an appropriately explanatory account of universal or multiply realized features is available, and not with whether such features are (or are not) ontologically reducible.[20]

Let's sum up the results of this section. I've considered a representative survey of accounts of weak, or physically acceptable, emergence intended to apply to nonlinear systems of one or the other variety, but in each case the occurrence of the feature at issue (epistemic indiscernibility, algorithmic or explanatory incompressibility, presence in a macro-pattern, universality or stability under perturbation) has turned out to be, at least for all that proponents of these accounts have shown, compatible with ontological reduction of the systems at issue. So far, then, it has not been established that any nonlinear systems are even weakly metaphysically emergent. I turn now to considering the somewhat better prospects, as regards both viability and potential application to nonlinear systems is concerned, of weak emergence understood as involving an *elimination in degrees of freedom* (DOF).

3 Nonlinearity and eliminations in DOF

In Wilson (2010) I proposed an account of weak emergence on which weakly emergent systems are both physically acceptable and ontologically autonomous from (that is, distinct from) the systems from which they emerge.[21] In this section I'll motivate and present this account, discuss how it accommodates both physical acceptability and autonomy, then argue that chaotic nonlinear systems satisfy the conditions of the account, hence are weakly metaphysically emergent.

[20] See especially the introductory remarks in his 2000: 115–16, where he expresses his desire to explore the nature of multiple realizability rather than enter into 'the reductionism debate'.

[21] Following contemporary concern with physicalism, I characterize the dependence condition in terms of physical acceptability, but the account more generally suffices to guarantee 'nothing over and aboveness' (in recently fashionable terms: 'grounding') of weakly emergent entities vis-à-vis whatever type of base entities might be at issue.

3.1 A DOF-based account of weak metaphysical emergence

My account is based in the notion of a degree of freedom (DOF). Call states upon which the law-governed (i.e. nomologically possible) properties and behaviour of an entity E functionally depend the 'characteristic states' of E. A DOF is then, roughly, a parameter in a minimal set needed to describe an entity as being in a characteristic state. Given an entity and characteristic state, the associated DOF are relativized to choice of coordinates, reflecting that different sets of parameters may be used to describe an entity as being in the state.[22] More precisely, the operative notion of DOF is as follows:

Degrees of Freedom (DOF): For an entity E, characteristic state S, and set of coordinates C, the associated DOF are parameters in a minimal set, expressed in coordinates C, needed to characterize E as being in S.

I'll sometimes speak for short of 'characterizing an entity', with state and coordinates assumed.

Some common characteristic states, and DOF needed to characterize certain systems as being in those states, are as follows:

The configuration state: tracks position. Specifying this state for a free point particle requires three parameters (e.g. x, y, and z; or r, ρ, and θ); hence a free point particle has 3 configuration DOF, and a system of N free point particles has $3N$ configuration DOF.

The kinematic state: tracks velocities (or momenta). Specifying this state for a free point particle requires six parameters: one for each configuration coordinate, and one for the velocity along that coordinate; hence a free point particle has 6 kinematic DOF, and a system of N free point particles has $6N$ kinematic DOF.

The dynamic state: tracks energies determining the motion. Specifying this state typically requires at least one dynamic DOF per configuration coordinate, tracking the kinetic energy associated with that coordinate; other dynamic DOF may track internal/external contributions to the potential energy.

Why attend to DOF in thinking about emergence? To start, as above, different systems, treated by different sciences, may be functionally dependent on the same characteristic state (e.g. the configuration state). Moreover, as above, the DOF needed to characterize intuitively different systems as being in these states typically vary. Following these observations, I take the main cash value of attention to DOF to lie in the fact that DOF track the *details* of a system's functional dependence on a given characteristic state in a more fine-grained way than the mere fact that the system is in the state does. Driving my account is the idea that the fine-grained details concerning functional dependence that are encoded in the DOF needed to characterize a given entity/system serve as a plausible ontological basis for the individuation of broadly scientific entities/systems.

[22] This relativization won't matter in what follows, since the relations between (sets of) DOF at issue in the ensuing account will be in place whatever the choice of coordinates.

I start by observing an important tripartite distinction (again, see Wilson 2010) relevant to such functional dependence, reflecting that the DOF needed to characterize an entity may be *reduced*, *restricted*, or *eliminated* in certain circumstances (typically associated with the imposition of certain constraints or more generally, the presence of certain energetic interactions), compared to those needed to characterize a (possibly distinct) entity, when such circumstances are not in place. To prefigure: eliminations in DOF, in particular, will enter into the upcoming account of weak metaphysical emergence. Let's get acquainted with these different relations and note an example of each.

First, constraints may *reduce* the DOF needed to characterize an entity as being in a given state. So, for example, a point particle constrained to move in a plane has 2 configuration DOF, rather than the 3 configuration DOF needed to characterize a free point particle. In cases where a given DOF is given a fixed value, the laws governing an entity so constrained are still functionally dependent on the (now constant) value of the DOF; hence such constraints do not eliminate the DOF, but rather reduce it to a constant value. By way of example: rigid bodies treated in classical mechanics have such a reduced set of DOF relative to the unconstrained system of their composing entities.

Second, constraints may also *restrict* DOF needed to characterize an entity. A point particle may be constrained, not to the plane, but to some region including and above the plane. Characterizing such a particle still requires 3 configuration DOF, but the values of one of these DOF will be restricted to only certain of the values needed to characterize the unconstrained particle. Cases of restriction in DOF are more like cases of reduction than elimination of DOF, in that, again, the entity's governing laws remain functionally dependent on specific values of the DOF. By way of example: molecules, whose bonds are like springs, have a restricted set of DOF relative to the unconstrained system of their composing entities.

Third, sometimes the imposition of constraints *eliminates* DOF. So, for example, N free point particles, having $3N$ configuration DOF, might come to compose an entity whose properties and behaviour can be characterized using fewer configuration DOF, not because certain of the DOF needed to characterize the unconstrained system are given a fixed value, but because the properties and behaviour of the composed entity are *functionally independent* of these DOF. By way of example: a spherical conductor of the sort treated in electrostatics has DOF that are eliminated relative to the system of its composing entities, since while the E-field due to free particles depends on all charged particles, the E-field due to the spherical conductor depends only on the charges of particles on its surface. Certain quantum DOF are also eliminated in the classical (macroscopic) limit. So, for example, spin is a DOF of quantum entities; entities of the sort treated by classical mechanics are ultimately composed of quantum entities; but the characteristic states of composed classical–mechanical entities do not functionally depend on the spins of their quantum components.[23]

[23] Recall that the characteristic states of an entity are those upon which its law-governed properties and behaviour functionally depend; for classical entities it is the laws of classical mechanics that are at issue. Hence notwithstanding that the values of quantum parameters may lead to macroscopic differences, it remains the case that quantum DOF such as spin are eliminated from those needed to characterize entities of the sort treated by classical mechanics.

Two features of the illustrative special science case studies are important for what follows. First is that the holding of the constraints relevant to reducing, restricting, or eliminating DOF occurs as a matter entirely of physical or physically acceptable processes. Such processes suffice to explain why sufficiently proximate atoms form certain atomic bonds, why atoms or molecules engage in the energetic interactions associated with SM ensembles, and so on. More generally, for each of the aforementioned special science entities E, the constraints on the e_i associated with their composing E are explicable using resources of the theory treating the e_i (or resources of some more fundamental theory, treating the constituents of the e_i). Call such constraints 'e_i-level constraints'. A second important feature of these special science entities E is that all of their properties and behaviour are completely determined by the properties and behaviour of their composing e_i, when these stand in the relations relevant to their composing E.[24] People disagree about the metaphysical ground for this determination, but all parties agree that the determination is in place.

These preliminaries in hand, the DOF-based account of weak metaphysical emergence is as follows:

DOF-based weak metaphysical emergence: An entity E is weakly metaphysically emergent from some entities e_i if:

(1) E is composed by the e_i, as a result of imposing some constraint(s) on the e_i.
(2) For some characteristic state S of E: at least one of the DOF required to characterize the system of unconstrained e_i as being in S is eliminated from the DOF required to characterize E as being in S.
(3) For every characteristic state S of E: every reduction, restriction, or elimination in the DOF needed to characterize E as being in S is associated with e_i-level constraints.
(4) The law-governed features of E are completely determined by the law-governed features of the e_i, when the e_i stand in the relations relevant to their composing E.

Systems that are emergent by lights of the above account are physically acceptable, given that the unconstrained system of composing entities is physically acceptable. In my (2010) I argue for this, but due to considerations of space, here I'll just observe that this result is to be expected, given that the constraints relevant to composing a weakly emergent entity E, as well as all of E's law-governed features, are physically acceptable.

I also argue in my (2010) that entities satisfying DOF-based weak metaphysical emergence plausibly satisfy *Non-reduction*. Since the possibility of ontologically irreducibility is, as above, the main sticking point so far as accounts of weak metaphysical emergence is concerned, it's important to present this for the reader's perusal; but in order to keep the presentation flowing, I've put

[24] Note that this feature does not in itself follow from E's being composed by e_i as a result of imposing e_i-level constraints: the British Emergentists agreed that the composition of base systems was a matter of lower-level processes, but maintained that some of the properties or behaviours of composite systems were not fully determined by lower-level processes.

my main argument for this claim in an appendix to this paper (in Section 4). In brief: the argument is an argument by cases, where I consider each of the lower-level entities that are candidates for being identical with a weakly emergent composite entity *E*; for each case I show that the candidate reductive base entity has *as many* DOF as the unconstrained system of *E*'s composing entities, and so (given that *E* is weakly emergent, and so has strictly *fewer* DOF than the unconstrained system) cannot, by Leibniz's Law, be identical with *E*. Unlike all other accounts of weak emergence on the market, a DOF-based account provides the basis for an *argument* that a weakly emergent entity is ontologically autonomous from—that is, distinct from—the system of composing entities upon which the weakly emergent entity depends.

3.2 DOF-based weak emergence and chaotic nonlinear systems

Let's return now to cases of nonlinear systems, and consider whether they can be understood as weakly metaphysically emergent. Given Batterman's work on the Renormalization Group (RG) method, and my results concerning the sufficiency of eliminations of DOF (along with certain other suppositions which are here in place, concerning e_i constraints and e_i determination), we can be comparatively brief.

As previously noted, the RG method applies to systems undergoing phase transitions, which are relevantly similar to and indeed can be understood as chaotic nonlinear systems; and that the RG applies to a given system provides as good indication as we are likely to get that the system has DOF that are not just reduced or restricted, but *eliminated* as compared to the unconstrained system of its composing entities (that is, as compared to the system of composing entities when not energetically interacting in the way associated with phase transitions, or more generally, with chaotic behaviour). We can thus argue as follows:

(1) Systems that can be modelled by the RG have eliminated DOF (Batterman 1998 and elsewhere).
(2) Chaotic nonlinear systems are modelled by the RG.
(3) Therefore, chaotic nonlinear systems have eliminated DOF.
(4) Systems with eliminated DOF are weakly metaphysically emergent (Wilson 2010).
(5) Therefore, chaotic nonlinear systems are weakly metaphysically emergent.

As confirmation of the fact that chaotic nonlinear systems have eliminated DOF, it is worth noting that one of the puzzles that Batterman raises for thermodynamic systems carries over to nonlinear chaotic systems, and is answered in just the same way. The puzzle he raises concerns how thermodynamic systems can be a viable object of study. Such systems—e.g. an isolated gas *E*—are composed of massively large numbers of particles or molecules e_i. Since the composite entity *E* in this case is (boundary restrictions aside) effectively unstructured, shouldn't it have the same DOF as the system of unconstrained e_i ? Supposing so, however, the success of SM is mysterious, since obviously we can't track $\sim 10^{26}$ DOF. As Batterman (1998) puts it:

> One wants to know why the method of equilibrium SM-the Gibbs' phase averaging method is so broadly applicable; why, that is, do systems governed by completely different forces and composed of completely different types of molecules succumb to the same method for the calculation of their equilibrium properties? (1998: 185)

The answer reflects that while the e_i are not bonded, they are interacting via exchanges of energy, and such interactions may not only restrict or reduce, but eliminate DOF, which is indicated by the fact that the renormalization group method is appropriately applied to such systems. This, then, is the answer to the puzzle: such systems are tractable since the modes of interaction of their composing entities result in their having DOF that are massively *eliminated* compared to the unconstrained system of composing entities. Again:

> [T]he renormalization group transformation eliminates those degrees of freedom (those microscopic details) which are inessential or irrelevant for characterizing the system's dominant behavior at criticality. (1998: 200)

A similar puzzle applies to chaotic nonlinear systems. Recall that chaotic nonlinear systems are characterized by their extreme sensitivity to initial conditions. If nonlinear systems are so sensitive and their resulting trajectories so 'chaotic', how is it that they can be, as they are, a viable object of scientific study? The answer, I take it, is effectively the same: the composing entities, though not bonded, are energetically interacting, in ways that, as application of the RG method reveals, massively eliminate the DOF needed to characterize the composite system. Here we have a solution to the puzzle, and more to the present point, a decisive, empirically supported case for taking the important class of chaotic nonlinear systems to be weakly metaphysically emergent.[25]

4 Appendix: the irreducibility of weakly emergent entities

The main objection to the claim that an entity can be both physically acceptable and metaphysically irreducible is the objection from theoretical deducibility, as follows.

To start, laws governing weakly metaphysically emergent entities are (I am granting) deducible consequences of the laws of the more fundamental theory. But in that case, the objector asks, why doesn't such deducibility indicate that weakly metaphysically emergent entities *E* are ontologically reducible to their composing *e_i*? Indeed, that theoretical deducibility entails ontological reducibility seems initially plausible, and has been commonly endorsed (Nagel 1961; Klee 1984).

[25] Might an equally good case be made for taking non-chaotic nonlinear systems to be weakly emergent, by lights of a DOF-based account? I am inclined to think that a case *can* be made, though perhaps not one as good, based in certain patterns of difference-making and counterfactual dependence (see Wilson (in progress)).

The concern can also be presented as a dilemma. As above, the constraints entering into the composition of our target special science entities are plausibly e_i-level constraints, and as indicated, this feature is crucial to establishing the physical acceptability of the target entities. But if the constraints are e_i-level, why can't the associated e_i-level relations enter, one way or another, into an ontological as well as a theoretical reduction of E to its composing e_i? This line of thought is, I think, at the heart of suspicions that there is no way for entities to be weakly metaphysically emergent.

To see how attention to DOF provides a way of avoiding the dilemma, let's first get clear on what would count as an ontological reduction of E to the e_i. Traditionally and reasonably, the candidate reducing entities include all the lower-level entities, properties and relations, plus any ontologically 'lightweight' constructions—lower-level relational, Boolean (conjunctive or disjunctive), or mereological—out of these. Hence if E is to be ontologically reducible to the e_i, then E must be identical to either:

(i) a system consisting of the jointly existing e_i;
(ii) a relational entity consisting of the e_i standing in e_i-level relations; or
(iii) a relational entity consisting in a Boolean or mereological combination of the entities at issue in (i) and (ii).

Since E is weakly metaphysically emergent, there is some state S such that characterizing E as being in this state requires fewer DOF than are required to characterize the unconstrained system of E's composing e_i as being in S. Hence a necessary condition on E's being identical with an entity of the type of (i)–(iii) is that the DOF required to characterize the candidate reducing entity as being in S are similarly eliminated, relative to the unconstrained system. I'll now argue that this condition isn't met for any of the entities of type (i)–(iii).

First, consider the e_i understood as (merely) jointly existing, as per (i). Such a system of e_i is not subject to any constraints; hence for any state, characterizing this system will require the same DOF as are required to characterize the system of unconstrained e_i.

So E is not identical to the system consisting of (merely) jointly existing e_i.

Second, consider the relational entity e_r consisting of the e_i standing in certain e_i-level relations, as per (ii). Though e_r realizes a constrained entity (namely, E), e_r is not itself appropriately seen as constrained—even throwing the holding of the constraints into the mix of e_i-level relations at issue in e_r. Why not? Because the laws governing entities (such as e_r) consisting of the e_i standing in e_i-level relations are, unlike the laws governing E, compatible with the constraints *not* being in place. Hence characterizing e_r as entering into these laws, hence as capable of evolving into states (of, perhaps other e_i level entities) where the constraints are not in place, requires all the DOF associated with the unconstrained system of e_i. So E is not identical to e_r.

Third, consider a Boolean (disjunctive or conjunctive) or mereological combination of entities of the previous varieties (or the closure of any such entity). To start, consider a relational entity consisting in a disjunctive combination of entities of the sort at issue in (i) or (ii). The occurrence of a disjunctive entity consists in one of its disjunct entity's occurring. Hence for any state, characterizing a disjunctive entity as being in that state will require all the DOF required to characterize any of its disjunct entities as being in that state. Such disjunct entities, being of type (i) or (ii), will require all the DOF required to characterize the system of unconstrained e_i, for any state. So characterizing a disjunctive relational entity will require all the DOF required to characterize the system of unconstrained e_i, for any state. The same goes for conjunctive entities. So *E* isn't identical to a disjunctive or conjunctive relational entity. Finally, consider a relational entity consisting in a mereological combination of entities of the sort at issue in (i) or (ii). Mereological wholes are identified with the mere joint holding of their parts; hence characterizing the whole will require all the DOF required to characterize each of the parts. Such parts, being of type (i) or (ii), will require all the DOF required to characterize the system of unconstrained e_i, for any state. So characterizing a mereological relational entity will require all the DOF required to characterize the system of unconstrained e_i, for any state. So *E* isn't identical to a mereological relational entity.

That exhausts the available candidates to which weakly metaphysically emergent entities might be ontologically reduced. In each case a weakly emergent entity has fewer DOF than the candidate reducing entity, so by Leibniz's Law the former is not identical to the latter. Attention to the metaphysical implications of the eliminations of DOF at issue in weak metaphysical emergence indicates that theoretical deducibility is compatible with ontological irreducibility. So physical acceptability is compatible with ontological autonomy, as weak metaphysical emergence supposes.