

# Nicholas JJ Smith: *Vagueness and Degrees of Truth*

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## 1 Overview

What is it for ‘baldness’ to be vague? Nick Smith answers: it is for ‘ $x$  is bald’ and ‘ $y$  is bald’ to be very similar in respect of truth, whenever  $x$  and  $y$  are very similar in hair coverage and distribution. More generally, ‘ $F$ ’ being vague is constituted by the following: that whenever two things are very similar in  $F$ -relevant respects, predicating  $F$  of those things gives results very similar in respect of truth. Smith argues that from this fundamental characterisation we can derive and explain the familiar surface characteristics of vagueness: borderlineness, fuzzy boundaries and soriticality (Smith, 2005, 2008, ch.3). (Henceforth all unsupplemented references will be to the latter work.)

What theory of vagueness makes sense of this characterization? Smith answers: a fuzzy one, allowing for degrees of truth—truth values with numerical structure that allows us to make sense of two predicates having ‘very close’ truth values.

Aren’t degree theories of vagueness subject to horrible objections? Here are three prominent criticisms, and Smith’s responses:

**Degree theories are objectionably revisionary:** they undermine the foundations of well-supported, classically-based, theories.

**Degree theories iterate a bad idea:** in response to worries about associating a single boundary with a vague predicate, they recommend we draw infinitely many boundaries.

**Degree theories require doublethink:** while recommending we give up classicism, their advocates use a classical metatheory.

*On Revisionism.* Smith shows how to save the letter of classical logic within a degree theory (aiming to remove some of the sting of revisionism) and shows how to adjust extant classically-based theories; for example, he adapts a Stalnakerian account of assertion to allow for degreed speech acts (§§5.2-3).

*On Iteration.* Whether the transition is from truth to falsity or from  $> 0.5$  truth to  $\leq 0.5$  truth, intuitively there is no fact of the matter about the *location* of boundaries for vague predicates. Smith’s response is to distinguish explanatory tasks. Degrees of truth explain *vagueness*, but vagueness does not exhaust the unsettledness we find in natural language; and the unsettledness in the location of boundaries isn’t a manifestation of vagueness per se. He argues that

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whatever style of model theory one adopts for vague language, one can add that there is no unique intended interpretation of natural language—there is instead a cloud of ‘acceptable interpretations’. Members of the cloud differ in where they locate the boundaries—so ‘no fact of the matter’ intuitions are accommodated. Once boundaries *per se* are seen as unproblematic, we are freed up to consider what kind of semantics best accounts for the *remaining* features of vague language. Smith argues for the smoothly varying transitions of degree theory against the arbitrary, jolting boundaries of other views (§6.10).

*On Doublethink.* A classical metatheory for degree theory requires doublethink only if the language in which it is framed is vague. But we’ve seen that predicates may be ‘cloudy’ (take different extensions on different acceptable interpretations) without being vague. Smith argues that ‘is an acceptable interpretation’ isn’t vague—though it may well itself be cloudy (§6.2).

The book is divided into three sections. The first surveys the space of theories of vagueness. The second argues for the definition of vagueness in terms of closeness or similarity, as in the first paragraph of this review—and argues that only degree theory can underwrite this. The third then develops and defends a degree theoretic approach to vagueness.

It is an excellent book: clearly written and packed full of interesting ideas and arguments. Researchers in the area must read it. Together with (Williamson, 1994) and (Keefe, 2000), it completes a triad of book-length treatments of the most prominent positive model-theoretic approaches to vagueness, and as such should become a cornerstone of the literature. The technical material is self-contained and well-presented, and the book would suit a graduate or an advanced undergraduate class not afraid of the odd bit of elementary mathematical formalism.

## 2 Arguments

Smith’s treatment of vagueness revolves around model theory. He endorses what he calls *model theoretic literalism*: models give a ‘literal (although not necessarily complete) description of the relationship between that language and the world’ (p.45). This he relates to *semantic realism*, the view that: ‘there are genuine semantic properties and relations: in particular, relations of reference holding between sub-sentential expressions . . . and parts of the world (objects, properties, . . . , relations)’ (p.46). (Semantic realism is contrasted with a Davidsonian instrumentalism and a ‘no semantics’ view attributed to Putnam (p.47, §2.7 respectively). Field (2003, 2008), Schiffer (2003) and Wright (2003) can be added to the list of prominent rejectors of semantic realism in the recent vagueness literature.)

The remainder of this review will evaluate three themes of Smith’s book: the relation between semantic realism and model theory; the role of model theory in distinguishing types of vagueness; and the role of logic in the system.

### *Semantic Realism*

Semantic realism is for Smith an ‘unargued presupposition’. Model theoretic literalism is meant to follow immediately. If our best theory of semantic properties involved commitment to intended models, this inference would be reasonable. But the issue is not so straightforward. Consider a textbook natural language semantics—as found e.g. in (Heim & Kratzer, 1998). There is no talk of models, whether acceptable, intended or otherwise (though you’ll find relativization to worlds, times and the like). A semantic realist might take this theory at face value, incurring no explicit commitment to models.

Perhaps there is tacit commitment here? With a bit of twisting, you can reintroduce talk of models—rewriting the axioms to say what conditions an intended model must meet. Instead of

simply stating that ‘ $F$ ’ denotes  $P$ , one might say that  $M$  is intended only if it maps ‘ $F$ ’ to  $P$ . It is a substantive question whether such rewriting is possible. A semantic axiom for a quantifier could stipulate an unrestricted interpretation. But the analogous rewritten condition requires that the model’s domain contain absolutely everything. For Cantorian reasons, no domain meets that condition (cf. *inter alia* Rayo & Uzquiano, 2006; Field, 2008).

Textbook semantics has wider significance for Smith’s project. It is our best take on what the metatheory for natural language looks like. This impacts on Smith’s strategy for responding to charges of doublethink. If all metatheory required was the identification of a class of mathematical constructs (models) and the use of a single theoretical predicate, ‘acceptable interpretation’, then a non-vague (albeit cloudy) metatheory might be a goer. But the metatheory we find in the textbooks is far richer than this, involving the *detailed axiomatic specification* of the intended model—using ordinary, vague vocabulary. If this metatheory relies on classical patterns of reasoning, then the doublethink charge will recur.

### *Model Theory and Worldly/Semantic vagueness*

One of the consistent themes of the book is a distinction drawn on the basis of model-theoretic literalism: between vagueness ‘in the world’ and vagueness ‘in language’. Smith says: if one’s model theory contains fuzzy properties as the referents of predicates (properties that things instantiate to degrees, rather than in an all or nothing manner), then that means that there must be fuzzy *parts of the world*, hence worldly vagueness. By contrast, if you get by with classical properties within the models, but posit a cloud of acceptable models of natural language rather than a unique intended interpretation, then vagueness is confined to language.

In what sense are properties ‘parts of the world’? Smith is happy to identify properties with set-theoretic constructs—functions from objects to numbers in  $[0, 1]$ , for example. Anybody who takes impure set theory at face value already has these entities around; but surely the existence of a function from colour patches to the reals isn’t enough by itself to demonstrate worldly vagueness. The distinctive claim of the degree theorist is that these common-ground entities play a certain functional role, being appropriate semantic values for our language. But their playing this role could be a reflection of our language use, with little to do with the constitution of the world independently of our activity. I’m no neutral observer on this point (cf. Williams, 2008; Barnes & Williams, 2011) but I admit that I don’t see what is gained by categorizing such vagueness as ‘worldly’ (though see Rosen & Smith, 2004). Caveat: if the ‘sparse’ properties that ‘carve nature at its joints’ (Armstrong, 1978; Lewis, 1983) were fuzzy in Smith’s sense, then there would be an intuitive sense in which the world itself was vague; but this claim about sparse properties can’t be read off model theory for natural language.

Again, this has wider significance. Smith makes the heterodox claim that the model theory involved in the paradigmatic semantic theory of vagueness—the supervaluationism of (Fine, 1975)—in fact commits adherents to worldly vagueness (p.96). The distinction between (worldly) supervaluationism and a position Smith calls ‘plurivaluationism’ leans heavily on whether technology doing essentially the same job is included within a model, or relates models to language. Unless the motivation for tying a substantive worldly/semantic distinction to these formal features can be reinforced, I suggest we think of these two ways of framing model theory as stylistic variants.

I’ve just argued that Smith’s taxonomy carves too finely. I think it’s also too coarse-grained. We can use some of Smith’s own very interesting development of degree theory to illustrate this. Notice that classical truth values—truth and falsity—are usually treated as *cognitively loaded*: truth is the aim of belief, and falsity the aim of disbelief. In terms of degrees of belief, the Godlike credence to have in a truth is 1 (full confidence), and the Godlike credence to have in a falsity is 0 (full rejection). Smith’s degrees of truth are cognitively loaded in a similar

fashion—the Godlike credence to have in a sentence that is  $k$ -true is  $k$ . But here’s an alternative cognitive loading: the Godlike state to be in when  $S$  is true, is to having conflicting partial dispositions to judge—to judge that  $S$   $k$  per cent of the time, and to judge that  $\neg S$  the rest of the time. *Prima facie*, this is a radically different conception of the role that vague sentences play in our lives, and deserves to be treated as a distinct theory, despite the identical model-theoretic underpinnings.

### Logic

One of the distinctive features of Smith’s degree-theory is the novel characterisation of consequence offered. A sentence with degree of truth  $> 0.5$  is called *inference-grade*. A sentence with degree of truth  $\geq 0.5$  is called *assertion-grade*.  $C$  follows from  $A$  when every model that makes  $A$  inference-grade makes  $C$  at least assertion-grade. This pulls a classical rabbit out of a fuzzy hat. For example, Smith endorses standard max/min and reversal accounts of disjunction, conjunction and negation, respectively. It follows that the law of excluded middle is never worse than 0.5, so it is always assertion-grade, hence tautologous. An explicit contradiction is never better than 0.5, so it is never inference-grade, so it implies anything whatsoever.

(Aside: Smith offers a proof that his definition extensionally coincides with the classical consequence relation (p.222, fn.17). However, suppose we had a sentence that takes the value 0.5 in every model (a logical constant for the 0.5 truth value). Then this would be never inference grade (so a contradiction) but always assertion-grade (so a tautology). It would follow from everything, but everything would follow from it. With this constant in the language the consequence relation better be non-transitive! So Smith’s stated qualification—that the proof only shows his logic to coincide with the classical one *in a standard first-order language*—is vital.)

Smith’s characterisation of consequence is ingenious. But it’ll only lead to a ‘non-revisionary’ account of logic in spirit as well as letter if the relation picked out *plays the logic-role*—if it does the job that makes logical consequence an interesting thing to theorise about in the first place (compare p.223). For example, one central part of our interest in logic is that it imposes *normative constraints on our belief state*. We should believe tautologies and shouldn’t believe inconsistent things.

Does Smith’s logic play the normative logic role, by his own lights? Smith argues that rational degrees of belief in the fuzzy setting should be expectations of truth values. Smith’s main case for this is that it fits what he takes to be the intuitive data about our attitudes to vague predications—though he also cites a dutch book argument by Peter Milne. (Joyce’s non-pragmatic argument for probabilism could equally be adapted to this setting. For defences of generalized probabilism in non-classical settings see Paris (2001); De Finetti (1974); Joyce (1998); Williams (2011b, manuscript); Smith (forthcoming).) The upshot of this characterization of degree of belief is that if we know that  $A$  is half-true, we should ideally be 0.5 confident in  $A \vee \neg A$  and 0.5 confident in  $A \wedge \neg A$ , matching their known truth values. But Smith’s logic would seem to advise otherwise: since the former is a tautology, it recommends full confidence; and since the latter is an inconsistency, it recommends utter rejection. Smith can avoid internal contradiction, of course—but only by draining the logic of normative significance.

The logic-role exists, and there is something that fills it. It is implicit in the following characterization of rational credences in a fuzzy setting; a belief-state  $b$  is an expectation of (fuzzy) truth values iff it satisfies the following (cf. Gerla, 2000; Di Nola *et al.*, 1999; Paris, 2001; Mundici, 2006; Williams, manuscript):

$$\begin{aligned}
 (\mathcal{P}1) \quad & \vdash A \quad \Rightarrow \quad b(A) = 1 \\
 & A \vdash \quad \Rightarrow \quad b(A) = 0 \\
 (\mathcal{P}2) \quad & A \vdash B \quad \Rightarrow \quad b(A) \leq b(B) \\
 (\mathcal{P}3) \quad & b(A \wedge B) + b(A \vee B) = b(A) + b(B)
 \end{aligned}$$

This states, in logical terms, localized constraints on degrees of belief. Smith’s expected truth value characterization gives a complete *global* characterization of belief states as expectations of truth value, but Smith (§5.3, (2010)) only discusses local characterizations in certain special cases. (Caveats: (a) the second axiom above is in fact a consequence of the others, but in other non-classical settings it plays an essential role, so its inclusion is merited in the characterization of a general logic-role. (b) the quoted result is language-specific, concerning propositional logic, and the presence of a Łukasiewicz conditional in the language is important.)

For the reasons already covered, for this result to hold,  $\vdash$  can’t pick out classical consequence. The appropriate choice—as emerged in personal communication with Smith—is a ‘no drop’ logic, where  $B$  follows from  $A$  iff there is no model where  $B$  is less true than  $A$ .

### 3 Evaluation

Smith makes a strong case that degree theory is philosophically stable and theoretically attractive. An ongoing task for friends of this account will be to continue the work of reconstructing extant classically based theories involving vague vocabulary (and I urge that portions of the metatheory of degree theoretic semantics itself remains on the to-do list, alongside applied physics, biology, and the like.) But suppose such reconstruction can be carried out. The final question is: should we be convinced that truth-functional degree-theory is a correct model-theory for vague natural language? I don’t think so. The theory has strong and surprising predictions, for example that some explicit contradictions deserve intermediate credence. Despite Smith’s entreaties (§5.5), I’m still in the camp who find these predictions implausible.

One of the striking things to emerge from Smith’s book is how much one can do with degrees of truth applied to basic sentences, and how little turns on what the degree-theorist says about the compositional inheritance of truth values. To those who find the compositional predictions of Smith’s theory unpalatable, I recommend supervalueational degrees of truth (cf. Lewis, 1970; Kamp, 1975), discussed in the book several times (e.g. p.88ff). To a first approximation, this gives a non-revisionary logic that genuinely plays the logic-role (only to a first approximation though! See (Williams, 2011a)). Characteristically, Smith offers interesting arguments against this position. The most powerful make heavy use of his particular views on semantic realism and model-theoretic literalism. As indicated earlier, there is more wriggle room on these fronts than Smith allows. As I see it, this only increases the interest of the book: it means that Smith’s many interesting positive proposals are more modular, and less tied to the specifics of his favoured model theory, than he contends.

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