

Worlds are Pluralities

Isaac Wilhelm

(penultimate version; please cite final version in Australasian Journal of Philosophy)

Keywords: possible worlds, pluralities, cardinality, linguistic ersatzism.

Abstract

I propose an account of possible worlds. According to the account, possible worlds are pluralities of sentences in an extremely large language. This account avoids a problem, relating to the total number of possible worlds, that other accounts face. And it has several additional benefits.

1 Introduction

There are many accounts of possible worlds. According to some, possible worlds are sets of sentences in a particular language. According to others, possible worlds are states of affairs.

According to still others, possible worlds are sets of propositions. And according to others still, possible worlds are concrete wholes.

These accounts face a variety of different problems relating to cardinalities (Forrest and Armstrong, 1984; Nolan, 1996; Sider, 2002; Hawthorne and Uzquiano, 2011). In this paper, I raise a related but distinct cardinality problem for these accounts: they imply that there is an upper bound on the number of possible worlds. As I show, for many different reasons, accounts of possible worlds should not imply any bounds like that.

To deal with the problem, I propose an alternative account of possible worlds. Roughly put, according to the account, possible worlds are pluralities of sentences in an extremely large language. Because the language is so large, and because pluralities can be bigger than any given set, this account does not imply an upper bound on how many worlds there are.

In Section 2, I present my account of possible worlds. In Section 3, I describe the particular cardinality problem that so many accounts face, and I show that my account avoids it. In Section 4, I list some other benefits of my account.

2 The Plural Account of Possible Worlds

In what follows, I formulate my account of possible worlds. To start, I review the notion of a plurality. Then I describe the sorts of languages which will be relevant here. Finally, I present the account.

A plurality is, basically, a bunch of entities. My books form a plurality, for example, as do the justices of the U.S. Supreme Court. And pluralities have members: Sotomayor is a member of the U.S. Supreme Court, for instance.

Pluralities are different from sets.¹ Pluralities obey the principles of plural logic, rather than the principles of standard set theory. And the plural membership relation is distinct from the set membership relation. To be clear about that distinction, in what follows, I refer to the plural membership relation as the ‘p-member’ relation. So I will say that s is a p-member of w just in case w is a plurality of entities, one of which is s .²

For another example of the difference between pluralities and sets, take the ordinal numbers. According to standard set theory, there is no set containing all and only the ordinal numbers as members. But according to plural logic, there is a plurality whose p-members are all and only the ordinals. Similarly, according to standard set theory, there is no set of all sets. But there is a plurality of all sets, according to plural logic.³

A related difference, between pluralities and sets, will be especially relevant here: pluralities, unlike sets, can get arbitrarily large.⁴ Standard set theory implies that for each set A , there is a cardinal number μ such that the number of members of A is at most μ . Plural logic,

¹ Similarly, as Boolos argues, pluralities are not entities [1984]. A plurality is not a new piece of ontology, that is, over and above the entities which it contains. Think of pluralities as the referents of plural terms.

² Many different plural languages—corresponding to many different logical systems—could be used to formulate the plural account of possible worlds. To keep things simple, I will not explore those differences here. My preferred approach to plurals is the approach in [Linnebo 2003; 2013].

³ All of this is compatible with the view—to which I subscribe—that in many cases, both the set containing some items and the plurality of those items exist. Just as there is a set of all natural numbers, for instance, there is a plurality of all natural numbers too.

⁴ Another important difference is that the unrestricted comprehension principle for pluralities, unlike the unrestricted comprehension principle for sets, avoids Russell’s paradox. The reason, in rough outline, is this: in plural logic, the formal regimentation of the sentence “there is a plurality consisting of all and only those entities which are not p-members of themselves”—this is the sentence whose formal analog in set theory generates the paradox—is ungrammatical. So the unrestricted comprehension principle for pluralities does not imply the formal regimentation of that sentence.

however, does not imply anything like that: there are pluralities which have more p-members than any cardinal number: the plurality of all sets, for instance.

Now for the languages on which I will focus. For any language L, an ‘L-sentence’ is a sentence formulated using the vocabulary that L features. Say that a language L is ‘absolutely infinite’ just in case the number of L-sentences is larger than any cardinal number. So absolutely infinite languages have massive vocabularies. In particular, the vocabulary of such a language is so large that it can be used to formulate more sentences than any set contains. These languages are too big to ‘fit into’ a set.

For example, take the Lagadonian language, in which each actual world object is a name for itself [Lewis 1986: 145]. Lagadonian is absolutely infinite. To see why, just note that the sets are more numerous than any cardinal number. In Lagadonian, each of those sets is a name for itself. So there are more Lagadonian names than cardinals. And so there are more Lagadonian sentences than cardinals too.⁵

It is possible to define absolutely infinite languages using only, or primarily, the resources of plural logic.⁶ The definition, in rough outline, is this: a language L is absolutely infinite just in case there is a special sort of plural function from (i) the plurality consisting of L-sentences, to

⁵ To see why, just consider all sentences of the form ‘Set x is self-identical,’ where x is a set which names itself. Since the number of such x is greater than every cardinal, the number of such sentences is greater than every cardinal too.

⁶ In fact, the resources of plural logic can also be used to define something that corresponds to the standard set-theoretic definitions of cardinalities. The former definition, which follows a construction similar to the one given in footnote 7, could also be interpreted as a way of quantifying size. So ultimately, there are two formal constructions that could be taken to quantify how big various collections are: one based on pluralities, and one based on sets. In this paper, I assume that only the set-theoretic construction provides a genuine, joint-carving account of what size is; other constructions, based on pluralities, do not. This assumption might not be necessary, for my purposes here. But it also might be; it is hard to tell. My worry—which may or may not be well-founded—is that without this assumption, my account might face a variant of the problem that I raise in Section 3. Given this assumption, however, the risk of that seems low: since the formal construction based on pluralities does not capture what size really is, according to the assumption in question, no version of the problem in Section 3 arises for the account of possible worlds to come.

(ii) the plurality consisting of all the cardinal numbers. This function can be defined using the vocabulary of plural logic.⁷

Now for the account of possible worlds. By way of preparation, say that a plurality x of L -sentences is 'maximal' just in case for every L -sentence s , either s or its negation is a p -member of x . And say that a plurality x of L -sentences is 'consistent' just in case there is no way to derive a contradiction from the p -members of x . Then the account of possible worlds is as follows.

The Plural Account

Let L be an absolutely infinite language. Then w is a possible world if and only if w is a maximal, consistent plurality of L -sentences.

In other words, possible worlds are bunches of sentences in a huge language.

The language L , in the Plural Account, may be any absolutely infinite language which can be used to draw all modal distinctions. That is, like other accounts that construct possible worlds out of sentences in some language, the Plural Account is more-or-less neutral on the language L in question. So long as a given absolutely infinite language can be used to draw all the modal distinctions needed, that language can be used in the Plural Account.

Lagadonian is one such language. As mentioned above, in Lagadonian, each object in the actual world is also a constant symbol which denotes itself. Furthermore, each property and relation in the actual world is also a predicate which expresses itself. In addition, Lagadonian

⁷ Here is how. Say that x is a 'pair plurality' just in case x is a plurality such that each p -member of x is a pair. The 'domain' of a pair plurality x is the plurality $\text{dom}(x)$ such that u is a p -member of $\text{dom}(x)$ if and only if for some v , $\langle u, v \rangle$ is a p -member of x (here and in what follows, ' u ' and ' v ' are singular terms while ' x ' is a plural term). The 'range' of a pair plurality x is the plurality $\text{ran}(x)$ such that v is a p -member of $\text{ran}(x)$ if and only if for some u , $\langle u, v \rangle$ is a p -member of x . A plurality x is 'functional' just in case x is a pair plurality such that for all p -members $\langle u, v \rangle$ and $\langle u', v' \rangle$ of x , v is v' . A functional plurality x is 'one-to-one' just in case for all p -members $\langle u, v \rangle$ and $\langle u', v' \rangle$ of x , u is u' . Finally, the language L is 'absolutely infinite' just in case there exists a functional plurality x such that $\text{dom}(x)$ is the plurality of all L -sentences and $\text{ran}(x)$ is the plurality of all cardinal numbers.

includes a bunch of quantifiers, sentential connectives, variables, and so on. So this absolutely infinite language—Lagadonian—can be used to draw a huge variety of modal distinctions.⁸

Strictly speaking, the Plural Account conflicts with standard views of propositions, modal operators, and other such things. Propositions, for instance, are often taken to be sets of possible worlds. And tautologies are often identified with the set of all possible worlds whatsoever. But given the Plural Account, there is no set like that: there are too many possible worlds—too many maximal, consistent pluralities of L-sentences—to form a set. Similarly, the truth conditions for modal operators are often formulated in terms of quantification over the set of all possible worlds. But again, given the Plural Account, there is no set like that.

To resolve this conflict, between the Plural Account and these other views, posit what are called ‘super-pluralities’. A super-plurality is, basically, a bunch of pluralities. Just as the p-membership relation obtains between entities and the pluralities to which they belong, another membership relation—call it the p’-membership relation—obtains between pluralities and the super-pluralities to which they belong. And importantly, super-pluralities are different from pluralities: whereas super-pluralities contain pluralities as p’-members, for instance, pluralities do not contain other pluralities as either p’-members or p-members.

There are many reasons to think that super-pluralities exist. They can be used to formulate an attractive account of groups [Wilhelm 2022].⁹ They can be used to provide a coherent semantics for certain possible languages [Rayo 2006]. And as some have argued [Linnebo and Nicolas 2008], English seems to contain super-plural terms; and so some natural languages seem committed to the existence of super-pluralities.

⁸ Note that Lagadonian may also include higher-order vocabulary. And Lagadonian allows for alien properties: for on my preferred view, properties can exist even if nothing instantiates them.

⁹ Superpluralities also facilitate the combination of (i) the Plural Account of worlds, and (ii) the account of groups in [Wilhelm 2020].

Super-pluralities can be used to resolve the conflict between the Plural Account and standard views of propositions, modal operators, and other such things. For instance, simply take propositions to be super-pluralities—rather than sets—of possible worlds. So tautologies may be identified with the super-plurality that contains all possible worlds whatsoever. Similarly, formulate the truth conditions for modal operators in terms of quantification over the super-plurality of all possible worlds. And do likewise for other views which invoke sets of possible worlds: reformulate those views so that they invoke super-pluralities of possible worlds instead.¹⁰

3 The Upper Bound Problem

In this section, I show that the Plural Account avoids a problem—based on cardinalities—which arises for many other accounts of possible worlds. In Section 3.1, I present an extremely attractive principle concerning how many worlds there are. In Section 3.2, I present the problem: put roughly, the problem is that many accounts of possible worlds do not validate the attractive principle. In Section 3.3, I show that the Plural Account validates the principle, and so avoids the problem.

3.1 Abundance

¹⁰ Likewise for philosophical theories that are formulated in terms of relations between (i) possible worlds, and (ii) entities like sentences, propositions, objects, agents, and so on. Such relations are usually assumed to obtain between entities only: they relate one entity—namely, a possible world—to another. But the corresponding philosophical theories can be reformulated so that they posit relations that obtain between pluralities and entities instead. The resulting relations can then obtain between (i) possible worlds, understood in accord with the Plural Account, and (ii) entities like sentences, propositions, objects, agents, and so on.

There is a very attractive principle which accounts of possible worlds should validate. Here it is.

Abundance

For each cardinal number μ , there are more than μ possible worlds.

In other words, the possible worlds are more numerous than any cardinal number.

For at least four reasons, Abundance is worth endorsing. First, it seems extremely intuitive. Pre-theoretically, there is little reason to think that the number of distinct metaphysical possibilities is bounded from above.

Second, it is costly to deny Abundance, because the negation of Abundance imposes a pretty arbitrary upper bound on the number of possible worlds. For if Abundance is false, then some cardinal number μ places an upper bound on how many worlds there are. But any particular choice of μ seems arbitrary. Why think that there are only μ different ways that the world could be? Why think that μ is an upper bound, rather than some other cardinal μ' ? Absent any clear answers, those who deny Abundance are committed to an unattractively arbitrary bound on the number of distinct possibilities.

Third, Abundance follows from two principles, both of which seem to be well worth endorsing.

Spacetime

For each cardinal number μ , it is possible that spacetime has cardinality μ .

Occupation

For each world w , and for each set S of spacetime points in w , it is possible that all and only the points in S are occupied.¹¹

¹¹ Principles akin to these are discussed in various places throughout the literature; for instance, see [Hawthorne and Uzquiano 2011; Bricker 2020].

To derive Abundance from Spacetime and Occupation, let μ be any cardinal number. By Spacetime, there exists a possible world w whose spacetime has cardinality μ . For each set S of spacetime points in w , Occupation implies that there exists a possible world w_S in which all and only the elements of S are occupied. So there are at least 2^μ possible worlds:¹² one for each w_S . By Cantor's theorem, 2^μ is greater than μ . It follows that there are more than μ possible worlds.

Both Spacetime and Occupation are extremely intuitive. Regarding Spacetime: it seems quite plausible that for any cardinal, it is at least metaphysically possible for spacetime to contain that many points. And it seems quite implausible to suppose that for some cardinal, it is metaphysically impossible for spacetime to contain that many points or more. Regarding Occupation: it seems quite plausible that for any set of spacetime points, it is at least metaphysically possible that all and only those points are occupied. And it seems quite implausible to suppose that for some set of spacetime points, it is metaphysically impossible for those points to be all and only the occupied ones.

Furthermore, the negation of either Spacetime or Occupation has problematic implications for empirical science; so it is better to endorse both Spacetime and Occupation, ultimately, than to endorse either of their negations. To see why, let us focus on Spacetime.¹³ Note that both Spacetime and its negation are metaphysical theses: they make claims about what is, or is not, metaphysically possible. But in addition, both Spacetime and its negation have implications for fundamental physics. In particular, the negation of Spacetime limits the range of spacetimes which our physical theories can posit: if Spacetime is false, then as a purely metaphysical matter, physics cannot appeal to spacetimes of arbitrarily large size. So faced with the choice between Spacetime and its negation, we should choose the former. For in general, we should prefer

¹² 2^μ is the cardinality of the power set of μ .

¹³ Analogous considerations suggest that it is better to endorse Occupation than to endorse its negation.

metaphysical theses which impose as few restrictions as possible on empirical theorizing.¹⁴ The negation of Spacetime imposes significant restrictions on empirical science: it suggests, for instance, that for purely metaphysical reasons, our world's spacetime cannot be thus-and-so large. Spacetime, in contrast, does not impose any restrictions like that. And that is a reason to prefer Spacetime over its negation.

Fourth, Abundance follows from another plausible principle. The principle is based on some complicated topological definitions, but for now, rough descriptions of those definitions will suffice. A 'path' in a set S of spacetime points is a continuous trajectory through S . A set S of spacetime points is 'path-connected' just in case for any two points in S , there is a path in S from one point to the other. And for any cardinal number μ , a set S of spacetime points contains exactly ' μ isolated components' just in case there is a partition of S into exactly μ subsets such that (i) each subset is path-connected, but (ii) for any two of those subsets, there is no path in S that connects a point in one subset to a point in the other. With that as background, here is the principle which implies Abundance.

Isolation

For each cardinal number μ , it is possible that the set of spacetime points contains exactly μ isolated components.

In other words, for any cardinal, it is possible for spacetime to contain that many disjoint spatiotemporal chunks.

¹⁴ One might object that empirical science does not need cardinalities higher than sets of reals. I agree that this may be true for empirical science in the actual world. But even if this is actually true, it is possibly false: it seems at least metaphysically possible, in other words, that empirical science will posit spacetimes of higher cardinalities. And while Spacetime allows for that, its negation does not.

Here is a sketch of the derivation of Abundance from Isolation. Roughly put, Isolation implies that every cardinal μ corresponds to a world which contains exactly that many isolated components. So there are more possible worlds than any given cardinal. Hence Abundance.

3.2 The Upper Bound Problem

Most accounts of possible worlds, that have been discussed in the literature, contradict Abundance. For they imply that there is an upper bound on how many worlds there are. Let us see why.

To start, consider linguistic ersatzism: for some specific language L , a possible world is a set of L -sentences. Most versions of linguistic ersatzism adopt a language L which has the following two features. First, the vocabulary of L forms a set, and that set has a particular—possibly infinite—cardinality. That is, the number of symbols in L is some particular cardinal number κ_L . Second, the sentences of L all have length bounded above by some particular—possibly infinite—cardinal. That is, for some cardinal number λ_L , every sentence of L has length at most λ_L . These features are not, of course, unusual or special. Nearly all formal languages have them.¹⁵

These two features imply a bound on how many sets of L -sentences there are. To see why, simply note that each L -sentence is a sequence of at most λ_L symbols, and the number of such

¹⁵ Infinitary languages are sometimes written in the form $L_{\chi\lambda}$, where (i) χ and λ are cardinals, (ii) χ bounds the number of formulas which can be conjoined ‘at once’, and (iii) λ bounds the number of variables which can be quantified over ‘at once’ [Dickmann 1975]. In this paper, I do not use that notation. My arguments apply to any language which has the two features mentioned above, regardless of whether that language is infinitary in the sense just described.

symbols is at most κ_L . So the number of L-sentences is at most $\kappa_L \cdot \lambda_L$.¹⁶ Therefore, there are at most $2^{\kappa_L \cdot \lambda_L}$ sets of L-sentences; in what follows, let μ_L be the cardinal number $2^{\kappa_L \cdot \lambda_L}$.

So Abundance is false. For given any language L that satisfies the two conditions listed above, the number of sets of L-sentences is at most μ_L . According to the corresponding version of linguistic ersatzism, every possible world is a set of L-sentences. So there are at most μ_L possible worlds.¹⁷

This derivation focused on a particular class of linguistic ersatz views: those which take possible worlds to be sets of sentences [Roper 1982]. But a similar upper bound can be derived for other versions of linguistic ersatzism. For instance, a similar upper bound can be derived for versions of linguistic ersatzism which posit an ‘ersatz pluriverse’ of possible worlds [Nolan 2002; Sider 2002].

¹⁶ If both κ_L and λ_L are finite, then $\kappa_L \cdot \lambda_L$ is whichever finite number is equal to their product. If either κ_L or λ_L is infinite, then by the axiom of choice, $\kappa_L \cdot \lambda_L$ is whichever of κ_L or λ_L is largest.

¹⁷ In order to avoid this conclusion, one might adopt a potentialist conception of sets (thanks to an anonymous reviewer for pointing this out). Roughly put, according to the potentialist conception, sets are constructed in an iterative hierarchy: at each stage in the construction, new sets can be constructed using the sets which were constructed at previous stages. So there is no completed totality of all sets; rather, at each stage in the construction, many sets exist potentially, in that they would exist if they were constructed, but they do not actually exist at the stage in question. Now consider a Lagadonian language—call it ‘Potentialized Lagadonian’—which is relativized to stages: in particular, the constants of this language include all the sets which exist at the stage in question, but no sets which merely potentially exist. Arguably, a version of linguistic ersatzism which takes possible worlds to be maximal, consistent sets of Potentialized Lagadonian sentences—call this view ‘potentialized ersatzism’—validates both Spacetime and Occupation. Potentialized ersatzism may validate Spacetime because at each stage, potentialized ersatzism implies that possibly, more sets exist; and for each cardinal number μ , those merely possible sets could be used to construct a possible world out of Potentialized Lagadonian sentences whose spacetime has cardinality μ . And potentialized ersatzism may validate Occupation for similar reasons. Whether or not potentialized ersatzism really does validate Spacetime and Occupation, of course, depends on the details of the potentialist conception of sets in question; and it is beyond the scope of this paper to fully explore that here. Perhaps there is a way of formulating potentialized ersatzism which validates both Spacetime and Occupation, and which also satisfies several other relevant desiderata. But very briefly, it is worth pointing out two issues that may arise in the attempt to formulate an attractive version of potentialized ersatzism. First, because the potentialist conception of sets is often formalized using modal notions [Linnebo 2013], potentialized ersatzism might be viciously circular: the truth conditions for the modal operators used to state its potentialist conception of sets might presuppose the very worlds that potentialized ersatzism is used to analyze. Second, if possible worlds are identified with maximal, consistent sets of sentences in Potentialized Lagadonian, then each possible world may always be merely ‘potentially’ completed: for a set of sentences may be maximal and consistent—and so be a possible world, according to this version of linguistic ersatzism—at one stage, but non-maximal at later stages, as it will not contain sentences featuring the later stages’ sets as constants.

All this shows that many versions of linguistic ersatzism face a problem; call it the ‘upper bound problem’. The problem is that they contradict Abundance, since they imply an upper bound on the number of distinct possibilities. And for the sorts of reasons mentioned in Section 3.1, that is problematic. First, Abundance is intuitive; so by contradicting Abundance, these accounts contradict an intuitively plausible principle. Second, if Abundance is false, then there is an unattractively arbitrary bound on how many worlds there are; so by contradicting Abundance, these accounts commit to a certain amount of arbitrariness. Third, Abundance follows from the independently plausible principles Spacetime and Occupancy; so by contradicting Abundance, these accounts contradict the conjunction of those two attractive principles. Fourth, Abundance follows from the independently plausible principle Isolation; so by contradicting Abundance, these accounts contradict Isolation too.

To avoid the upper bound problem, linguistic ersatzers might abandon sets for classes. They might adopt a language whose vocabulary forms a proper class: for instance, Lagadonian. Then linguistic ersatzers might identify worlds with sets of sentences in that language.

But this leads to problems. For instance, linguistic ersatzers typically adopt a simple account of truth at a world: for every sentence s and every world w , s is true at w if and only if s is a member of w . In order for this account to have certain desirable features—for instance, the feature that if s is not true at w , then the negation of s is true at w —linguistic ersatzers often adopt a ‘maximality assumption’: for every world w and every sentence s , either s or its negation is a member of w . If worlds are sets of sentences in a language L whose vocabulary forms a proper class, however, then the maximality assumption is false. The L -sentences form a proper class, and so every maximal collection of L -sentences is a proper class too. As a result, this account of worlds and truth does not have the desirable features.

Alternatively, linguistic ersatzers might identify worlds with classes of L-sentences instead. For by doing so, linguistic ersatzers can hold onto the maximality assumption. As a result, this account of worlds and truth has the desirable features.

This account, however, might face a class-theoretic variant of the upper bound problem. Whether or not it does, though, ultimately depends on whether certain class-theoretic axioms hold. For given certain standard axioms of class theory, it can be shown that there is no proper class of all the classes that exist: in fact, it can be shown that proper classes get bigger and bigger without bound.¹⁸ And that, along with some reasonable assumptions about the sizes of languages, implies an upper bound on the possible size of spacetime.¹⁹

Even setting these issues aside, this approach has other shortcomings. For instance, it commits linguistic ersatzers to the existence of proper classes. And for a variety of reasons, it may be good to remain neutral on that [Roy 1995].

The upper bound problem is not unique to linguistic ersatzism. Consider combinatorialism: a possible world is a molecular, possible state of affairs [Armstrong 1986]. Suppose that the number of possible states of affairs is bounded above by some cardinal κ . A molecular possible state of affairs is a conjunction of possible states of affairs. Suppose that the lengths of these conjunctions are bounded above by some cardinal λ . Then there are no more than $\kappa \cdot \lambda$ possible worlds. So this version of combinatorialism, like the different versions of linguistic ersatzism, implies an upper bound on how many worlds there are. And so it faces the upper bound problem.

¹⁸ For more discussion of assumptions concerning the sizes of classes, and how those assumptions generate the same sorts of issues concerning size that standard set theory generates, see [Uzquiano 2015; Bricker, 2020].

¹⁹ This provides a reason to favor the Plural Account over linguistic ersatz accounts that appeal to classes. For arguably, while the axioms of class theory can be used to generate a version of the upper bound problem for linguistic ersatz accounts of the latter sort, the axioms of plural logic cannot be used to generate a version of the upper bound problem for the Plural Account.

Similarly, take propositionalism [Adams 1974]: a possible world is a maximal, consistent set of propositions. A set of propositions S is maximal just in case for every pair of contradictory propositions, exactly one element of that pair is in S . Let \mathcal{P} be the collection of all propositions that exist. As a simple argument shows, \mathcal{P} is a set rather than a proper class.²⁰ Therefore, \mathcal{P} has a particular cardinality λ . It follows that the power set of \mathcal{P} has cardinality 2^λ . So there are at most 2^λ maximal, consistent sets of propositions. In other words, Adams' propositionalism implies an upper bound on the number of possible worlds. And so it faces the upper bound problem.

Finally, take modal realism [Lewis 1986]: a possible world is a collection of spatiotemporally connected, existent things. In formulating modal realism, Lewis basically assumes that there is an upper bound on the number of possible worlds. And Lewis is happy to make that assumption, since Lewis is happy to assume that the possible worlds form a set [1986: 104]. So Lewis's version of modal realism faces the upper bound problem.²¹

This shows that the upper bound problem has a lesson for all accounts of possible worlds. In order to avoid various unattractive implications, those accounts must be careful about the way

²⁰ The argument is as follows. Let S be any maximal, consistent set of propositions; note that given propositionalism, and given the existence of at least one possible world, such a set exists. Let S' be the propositions in \mathcal{P} which are not in S . Since S is maximal and consistent, the following holds: for every proposition in \mathcal{P} , either that proposition is in S or the negation of that proposition is in S (but not both). Therefore, there is a bijection between S and S' . It follows that S' is a set. Since \mathcal{P} is the union of S and S' , \mathcal{P} is a set too.

²¹ Lewis would not describe the upper bound problem as a problem, of course. For as mentioned above, Lewis more-or-less straightforwardly assumes an upper bound on how many worlds exist. There are, basically, two reasons why. First, without this assumption, Lewis's recombination principle would imply Spacetime: for any cardinal μ , possibly, spacetime has cardinality μ . But it seems fishy, to Lewis, that a recombination principle about how spacetime might be occupied would have consequences for the possible size of spacetime itself [1986: 89]. Second, it provides Lewis with a way around an argument due to Forrest and Armstrong [1984]. As Forrest and Armstrong show, if the possible worlds are few enough to form a set, then Lewis must make that assumption: otherwise, Lewis's recombination principle is inconsistent. But neither of these reasons—for making the assumption—strike me as compelling. It makes sense, to me, that a principle about how spacetime might be occupied would have consequences for the possible size of spacetime. And there is a more plausible way of avoiding the argument due to Forrest and Armstrong: deny that the possible worlds are few enough to form a set.

in which they construct possibilia. If those constructions are subject to certain sorts of bounds, then the corresponding accounts will imply unattractive limits on how many worlds there are.²²

3.3 A Solution

The Plural Account avoids the upper bound problem. For the Plural Account implies that for any cardinal number μ , there are more than μ possible worlds. In other words, the Plural Account implies Abundance.

To see why, let L be an absolutely infinite language. It follows that for every cardinal number μ , there are more than μ maximal pluralities of L -sentences. In order to show that more than μ of those maximal pluralities are consistent, at least one other assumption—from a suite of different assumptions—must be made. The available assumptions are all extremely technical, so to keep things simple, I will not present them here.²³ But they are compatible with pretty much any absolutely infinite language that one might think to use in the Plural Account. They are compatible with Lagadonian, for instance.

Sider [2002: 307] says that all accounts of possible worlds run into trouble with cardinalities. And indeed, a great many do. As discussed in Section 3.2, the upper bound problem arises for standard versions of linguistic ersatzism, combinatorialism, propositionalism, and modal realism.

²² The upper bound problem is somewhat similar to, yet distinct from, other cardinality problems in the literature. It differs from Lewis's original objection to linguistic ersatzism, which assumed that linguistic ersatzism constructs worlds using sentences in a countable language [1973: 90]. The upper bound problem also differs from other objections that have been raised to Lewis's theory of possible worlds [Forrest and Armstrong 1984; Nolan 1996; Hawthorne and Uzquiano 2011]: while many of those objections depend on principles of recombination or transworld fusions, the upper bound problem does not.

²³ The assumptions depend, in fairly sensitive ways, on the details of how consistency is defined. See [Dickmann 1975] for discussion of the notion of consistency in the context of massive languages.

Because it drops sets for pluralities, however, the Plural Account avoids this problem. It does not face the sort of cardinality trouble that so many other accounts face. And that is a significant point in favor of the Plural Account.

4 Additional Benefits of the Plural Account

There are many other reasons to like the Plural Account. In this section, I discuss three of them.

First, unlike many of the versions of linguistic ersatzism discussed in Section 3.2, the Plural Account is compatible with a standard maximality assumption; indeed, it is formulated in terms of an assumption like that. That is, the Plural Account is compatible with the assumption that for every world w and every L -sentence s , either s or its negation is a p -member of w .

Second, and relatedly, the Plural Account is compatible with a particularly simple account of truth at a world: sentence s is true at world w if and only if s is a p -member of w . This account of truth at worlds, when combined with the Plural Account, has all the standard desirable features. For instance, it implies that if s is not true at w , then the negation of s is true at w .

Third, unlike the version of linguistic ersatzism advocated by Roy [1995], the Plural Account does not imply that there exist numerically distinct worlds at which exactly the same sentences are true.²⁴ According to the Plural Account, if worlds w and w' contain exactly the

²⁴ Roy argues that worlds are sets of sentences in a language whose vocabulary forms a proper class [1995: 220–225]. Though there is much to like about Roy's account, it implies that for every world w , there are proper-class-sized many worlds w' such that (i) exactly the same sentences are true at w and w' , and (ii) w and w' are distinct. In other words, Roy's account implies the existence of many 'redundant' worlds: the world-building language 'overgenerates' how many worlds are needed, in order to represent all the possibilities that there are.

same sentences, then—by the axiom of extensionality for pluralities— w and w' are identical. So given the above account of truth, it follows that if worlds w and w' are distinct, then some sentence is true at one of those worlds but not at the other.

5 Conclusion

When proposing accounts of possible worlds, philosophers have focused too much on sets. The lesson of the upper bound problem is that philosophers should focus on pluralities instead. For pluralities do useful philosophical work: they can be used to formulate the Plural Account, and in so doing, to avoid the upper bound problem.²⁵ The Plural Account, moreover, has additional benefits: it is consistent with a standard maximality assumption, it supports a simple account of truth at worlds, and it does not imply that some pairs of numerically distinct worlds verify exactly the same sentences. So the Plural Account is worth taking seriously.

Acknowledgements

Thanks to Ethan Brauer, Phillip Bricker, Melissa Fusco, Martin Glazier, Eugene Ho, Christopher Menzel, Dee Payton, Alexandru Radulescu, Jonathan Schaffer, Ted Sider, Yip Sze Kay, two anonymous referees, and the audience at the 2021 Central APA, for much helpful feedback and discussion.

²⁵ The basic idea of the Plural Account—drop sets for pluralities—can be extended to other accounts of possible worlds. Plural versions of combinatorialism, propositionalism, and modal realism, can be formulated using pluralities too.

References

- Adams, R. 1974. Theories of Actuality, *Noûs* 8/3: 211–231.
- Armstrong, D. M. 1986. The Nature of Possibility, *Canadian Journal of Philosophy* 16/4: 575–594.
- Boolos, G. 1984. To Be is to be a Value of a Variable, *The Journal of Philosophy* 81/8: 430–449.
- Bricker, P. 2020. *Modal Matters*. New York, NY: Oxford University Press.
- Dickmann, M. A. 1975. *Large Infinitary Languages*. New York, NY: North-Holland.
- Forrest, P., and D. M. Armstrong 1984. An Argument Against David Lewis' Theory of Possible Worlds, *Australasian Journal of Philosophy* 62/2: 164–168.
- Hawthorne, J., and G. Uzquiano 2011. How Many Angels Can Dance on the Point of a Needle? *Mind* 120/477: 53–81.
- Lewis, D. 1973. *Counterfactuals*. Malden, MA: Blackwell.
- Lewis, D. 1986. *On the Plurality of Worlds*. Malden, MA: Blackwell.
- Linnebo, Ø. 2003. Plural Quantification Exposed, *Noûs* 37/1: 71–92.
- Linnebo, Ø. 2013. The Potential Hierarchy of Sets, *The Review of Symbolic Logic* 6/2: 205–228.
- Linnebo, Ø., and D. Nicolas 2008. Superplurals in English, *Analysis* 68/3: 186–197.
- Nolan, D. 1996. Recombination Unbound, *Philosophical Studies* 84/2&3: 239–262.
- Nolan, D. 2002. *Topics in the Philosophy of Possible Worlds*. New York, NY: London.
- Rayo, A. 2006. Beyond Plurals, in *Absolute Generality*, eds. A. Rayo and G. Uzquiano, New York, NY: Oxford University Press: 220–254.
- Roper, A. 1982. Towards an Eliminative Reduction of Possible Worlds, *The Philosophical Quarterly* 32/126: 45–59.
- Roy, T. 1995. In Defense of Linguistic Ersatzism, *Philosophical Studies* 80/3: 217–242.

Sider, T. 2002. The Ersatz Pluriverse, *The Journal of Philosophy* 99/6: 279–315.

Uzquiano, G. 2015. Recombination and Paradox, *Philosophers' Imprint* 15/19: 1–20.

Wilhelm, I. 2020. The Stage Theory of Groups, *Australasian Journal of Philosophy* 98/3: 661–674.

Wilhelm, I. 2022. Pluralities, counterparts, and groups, *Philosophical Studies* 1–21.

<https://doi.org/10.1007/s11098-021-01755-5>