Spectral Distribution of Photons Admitting Ambiguous Statistics

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By allowing photons to exhibit both Bose and Fermi statistics with predetermined probabilities, we calculate the correction to the thermal energy spectrum of photons when the probability of fermion states is small. Such a distribution may apply to a localized distribution of blackbody radiation where photons may exist in thermal equilibrium with photinos or to a condensed configuration of neutrons as in a neutron star where a small fraction of the neutrons admit to Bose statistics.

P.A.C.S. - 05.20 Statistical Mechanics

Introduction

For students of quantum theory and statistical mechanics alike it might be safe to say that of all the principles of these two subjects the "exclusion principle" and the spin-statistics connection represent the most ad-hoc and most poorly understood features of these theoretical frameworks [1], [2]. Geroch *et al.* [3] have pointed out that the exclusion principle might not be a principle that can be derived from the properties of space-time but is a result of topological properties of spin space. Whatever the generic origin of the spin statistics connection, we must admit that so many features of atomic, nuclear and particle physics depend on the exclusion principle [4]. In this regard, atomic stability, nuclear structure, the hadron spectrum and the very existence of the color degree of freedom are a direct result of the Pauli principle. Actually the first attempt to generalize Bose and Fermi statistics goes back to Gentile [5] who considered a statistics that allows up to k particles in a single quantum state. Other generalizations involve deformations of the commutation relations

$$a_j a_j^+ - q a_j^+ a_i = \delta_{ij}, \quad q \neq 1$$

a^+ = creation operative, a = annihilation operative) [6], [7], [8]. Concrete reasons to study non-conventional statistics emerge from studies of the 2 + 1 dimensional quantized Hall effect [9] along with the studies of (2 + 1) dimensional anayons [10]. Motivated by these studies Haldane [11] and Wu [12] have discussed a generalized statistics that interpolates between Bose and Fermi statistics. Generalizations of Bose and Fermi statistics generally also involve generalizations of the particle wave function permutation symmetry and may involve an arbitrary phase shift due to the permutation of any two particles [13]. In two previous notes we have applied the generalized Haldane statistics to anomalous photons [14] and anomalous fermions [15] and have calculated corrections to the black body spectrum and the properties of a free electron gas (specific heat, heat capacity) induced by the Haldane statistics. In what follows we consider a new generalization of Fermi and Bose statistics proposed by Medvedev [16], in this approach identical particles have the properties that there is a probability that each particle admits Bose statistics (P_b) and a probability that it admits Fermi statistics (P_f). After discussing the modified distribution resulting from this generalization we apply the distribution to a distribution of photons. We estimate the value of P_b and P_f to be in accord with the known C.M.B. anistropy and point out that the presence of unbroken supersymmetry [17] may make this a real possibility when a localized distribution of black body radiation contains photons in equilibrium with photinos.

2. Photons Admitting Bose and Fermi Statistics

We begin by considering N particles to be broken up into subgroups of size N_J ($\Sigma N_J = N$), for each realization (N_J) we have k bosonic states and $N_J - k$ fermion states, we then add up the number of ways of realizing these N_J particles ($k = 0, 1 \ldots k = N_j$), this gives

$$\omega_j = \sum_{k=0}^{N_j} \binom{N_j}{k} \omega_{b,j}(k)\, \omega_{f,j}(N_j - k)\, P_b^k P_f^{N_j - k} \tag{2.1}$$

here

$$\omega_{b,j} = \frac{(g_j + k - 1)!}{(g_j - 1)!k!}, \quad \omega_{f,j}(N_j - k) = \frac{g!}{(g_j - N_j + k)!(N_j - k)!}$$

Also P_b = probability of bosonic state, P_f = probability of fermionic state, $\binom{N_j}{k} = \frac{N_j!}{(N_j - k)!k!}$ = number of ways of arranging k bosons and $N_j - k$ fermions. For the total number of ways of realizing the system we have

$$\omega = \underset{j}{\pi}\, \omega_j \tag{2.2}$$

(here j = index specifying all the particle realizations with different g_j, and energy ε_j.)

When the entropy is calculated from Eq. (2.2) as $S = k\ln_e\omega$ and varied with respect to N_j along with the constraints

$$\delta \sum N_j = 0 \tag{2.3}$$

$$\delta \sum N_j \varepsilon_j = 0 \tag{2.4}$$

we obtain

$$\frac{N_j}{g_j} = \frac{(P_f + P_b) e^{\frac{\varepsilon_j - \mu}{\tau}}}{\left(e^{\frac{\varepsilon_j - \mu}{\tau}} + P_f\right)\left(e^{\frac{\varepsilon_j - \mu}{\tau}} - P_b\right)} X \left[1 + \sqrt{\frac{(P_f - P_b)^2 \left(e^{\frac{\varepsilon_j - \mu}{\tau}} + P_f\right)\left(e^{\frac{\varepsilon_j - \mu}{\tau}} - P_b\right)}{(P_f + P_b)^2 e^{\frac{\varepsilon_j - \mu}{\tau}} \left(e^{\frac{\varepsilon_j - \mu}{\tau}} + P_f - P_b\right)}}\right] \tag{2.5}$$

(Here the Lagrange multipliers for Eq. (2.3) and Eq. (2.4) respectively are $\mu/\tau, -1/\tau$; μ = chemical potential, $\tau = kT$ = normalized temperature, T = absolute temperature, k = Boltzmann constant, ε_j = energy of particles in level g_j). Eq. (2.5) was derived by Medvedev [16] using Stirling's formula for the factorials and Stirling's approximation for the sum in Eq. (2.1).

We now apply Eq. (2.5) to photons and photinos before supersymmetry is broken, here $\varepsilon_j = h\nu$ and we assume $P_b = 1 - \varepsilon$, $P_f = \varepsilon$ (ε small). We may also choose $\mu = 0$ [18], since the particles have zero rest mass prior to supersymmetry breaking.

Eq. (2.5) can be written as

$$\frac{N_j}{g_j}=\frac{e^{\frac{h\nu}{\tau}}}{\left(e^{\frac{k\nu}{\tau}}+\varepsilon\right)\left(e^{\frac{k\nu}{\tau}}-1+\varepsilon\right)}\left[1+\sqrt{\frac{(1-4\varepsilon)\left(e^{\frac{h\nu}{\tau}}+\varepsilon\right)\left(e^{\frac{h\nu}{\tau}}--1+\varepsilon\right)}{e^{\frac{h\nu}{\tau}}\left(e^{\frac{h\nu}{\tau}}-1+2\varepsilon\right)}}\right] \tag{2.6}$$

When Eq. (2.6) is expanded to order ε (no higher terms than ε) we obtain

$$\left(X=e^{\frac{h\nu}{\tau}}\right)$$

$$\frac{N_j}{g_j}=\frac{1}{X-1}-\frac{\varepsilon}{(X-1)}\left(\frac{1}{X}+\frac{1}{X-1}\right)+\frac{2\varepsilon^{1/2}}{X-1}\left(1+\frac{1}{4(X-1)}-\frac{1}{4X}\right)^{1/2} \tag{2.7}$$

In Eq. (2.7) if $\varepsilon = 0$, we recover the usual formula for pure bosons

$$\frac{N_j}{g_j}=\frac{1}{e^{\frac{h\nu}{\tau}}-1} \tag{2.8}$$

To estimate ε, we use the constraints from the C.M.B. anisotropy [19], [20], thus

$$\frac{\delta U}{U}\simeq 10^{-5}\simeq\frac{\delta N_j}{N_j}\approx\frac{\frac{2\varepsilon^{1/2}}{X-1}}{\frac{1}{X-1}}\simeq 2\varepsilon^{1/2} \tag{2.9}$$

Here $\varepsilon^{1/2} \gg \varepsilon$ and $e^{\frac{h\nu}{\tau}}-1>1$. Thus $\varepsilon \simeq 10^{-10}$ which is an extremely small value of ε for the probability of fermion states mixing with bosonic photon states. For $h\nu > \tau'$, Eq. (2.7) gives

$$\frac{N_j}{g_j}\simeq e^{\frac{h\nu}{\tau}}-2\varepsilon e^{-\frac{2h\nu}{\tau}}+2\varepsilon^{1/2}e^{-\frac{h\nu}{\tau}} \tag{2.10}$$

if ε depends on ν in a quadratic fashion, $\varepsilon \simeq C^2\nu^2$ (C = constant), then

$$\frac{N_j}{g_j}\simeq e^{-\frac{h\nu}{\tau}}-2C^2\nu^2 e^{-\frac{2h\nu}{\tau}}+2C\nu e^{-\frac{2h\nu}{\tau}} \tag{2.11}$$

The energy distribution per unit volume for Eq. (2.11) would be

$$\mathrm{d}U(\nu)=\left(e^{-\frac{h\nu}{\tau}}-2C^2\nu^2 e^{-\frac{2h\nu}{\tau}}+2C\nu e^{-\frac{h\nu}{\tau}}\right)\frac{8\pi h\nu^3}{C^3}\mathrm{d}\nu \tag{2.12}$$

Thus for high frequencies ($h'\nu \gg \tau$) we would get corrections to the thermal distribution energy that would involve higher powers of the frequency. Actually $\varepsilon = C^2\nu^2$ is just one possible phenomenological choice, and deviations from the usual high frequency spectrum $\mathrm{d}U(\nu)=e^{-\frac{h\nu}{\tau}}\frac{8\pi h\nu^3}{C^3}\mathrm{d}\nu$ could be used to constrain $\varepsilon(\nu)$.

Conclusion

The above discussion has demonstrated that for very small values of the asymmetry parameter between bosonic photons and fermionic photons ($P_f = \varepsilon = 10^{-10}$) the above theory can be made compatible with constraints imposed by the C.M.B. Since supersymmetry is most likely unbroken at high temperatures we could expect that the above discussion would be more apt to apply to configurations of black body radiation at high temperatures such as in the atmosphere of a hot star or at the center of a newly formed galaxy [21]. Another place to look for the corrections induced by Eq. (2.12) would be in the high frequency thermal spectrum that accompanies γ-ray burst phenomena. The problem here is to separate out what is thermal background and what is not. Other applications of the above statistics might be found in applications to the Bose-Einstein condensation [22] and to the problem involving the bound configuration of particles and their super-symmetric particles in the presence of a central super-heavy nucleus [23].

It is of interest here to ponit out that both neutron stars and white dwarfs possess critical masses above which they become unstable because the degeneracy pressure cannot support the graviational force that acts in a central direction within the star. The actual critical masses are $M_{W.D.} = 1.39\ M_0$, [24] $M_N = .7\ M_0$ [25] for a white dwarf and neutron star respectively. Present maximum limits on the mass of both of these types of stars have experimental tolerances that may very well accomodate slightly higher values of the critical mass that would be a manifestation of the breakdown of pure Fermi statistics through an increase in the degeneracy pressures supporting a more massive star [26]. Also present theoretical work on "neutrino balls" [27], (which are spheres of massive neutrinos supported by a degeneracy pressure) can predict a maximum critical mass which would be increased if the pure Fermi statistics is violated in the manner discussed in this paper. The experimental discovery of "neutrino balls" in an astrophysical setting would provide us with an experimental probe to the violation of Fermi statistics and a test for the presence of ambiguous statistics. Lastly, deep inelastic scattering of e^- off of nucleons is capable of testing for violations of Bose and Fermi statistics for the "sea quark" and gluons in a nucleon wherein the distribution of each of these species (seq quarks and gluons) determines the structure function which in turn determines the scattering cross sections [28]. Present limits on deviations from Bose and Fermi statistics for these phenomenon are small but signatures indicating deviations from Bose and Fermi statistics for "seq quarks" and gluons might very well appear in these experiments when more precise values of the experimental cross-sections are found.

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