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Laws: An Invariance- Based Account

1. Introduction

This paper defends an account that links laws of nature to *invariance* (a.k.a *stability, robustness)*. Laws are generalizations[1] about repeatable relationships[2] that are invariant over variations in initial and other sorts of conditions, at least within an appropriate range of such variations —invariant in the sense that laws will or would continue to hold under such variations. Alternatively, laws are generalizations that exhibit a certain sort of *independence* from initial conditions, where initial conditions themselves are understood as meeting, at least ideally, certain sorts of (additional) independence requirements.

I have defended an invariance-based account of laws elsewhere (Woodward, 2003, 2013) and broadly similar accounts have been proposed by other writers, including Mitchell, 2000 and Lange, 2009, although their treatments differ in detail from mine. As nearly as I can tell, such accounts have been less influential than several alternatives, including Best Systems Analyses (Lewis, 1986) and accounts that either attempt to "ground" laws in metaphysically special entities[3] (e.g., relations between universals, dispositions possessed essentially) or to replace laws with claims about such entities (Mumford, 2004). I thus begin with some general remarks about why invariance- based accounts of laws deserve more attention.

1.1) They fit scientific practice well (or at least better than alternatives). Explicit discussion of laws, at least in physics, commonly links these to symmetry and invariance principles, as illustrated by the remarks of Eugene Wigner discussed in more detail in section **2**. It is natural to think of symmetry principles as special cases of a more general invariance-based conception of laws—see below. By contrast, in my view and as argued

[1] The word "law" is used in both philosophy and science to refer both to (i) generalizations, formulated mathematically or in some other representational format, as when one says that Maxwell's equations are laws of nature, and (ii) to whatever it is *in* nature that "corresponds" to these generalizations. For the most part, I adopt the former usage on the grounds that it is less likely to beg important questions about what the "objective correlates" of laws in sense (i) above are.

[2] Laws are often taken (not just by philosophers but by scientists—see the quotations from Wigner below) to describe "regularities". My talk of repeatable relationships is meant to avoid some of the problematic features of this view but little will turn on this in what follows.

[3] Sometimes called "non-Humean" entities or properties or "whatnots" (Lewis, 1986). Although I will sometimes use similar terminology, it is not entirely perspicuous. In my view, there is nothing problematic about such ordinary language claims as "salt has a disposition to dissolve in water"— problems only arise when this disposition is invested with various other features and accorded the explanatory role of "grounding" laws. Insofar as there is an objection to "non-Humean stuff", it ought to be directed at this latter use of disposition-talk.

by others (e.g., Lange, 2004), the idea that laws must satisfy symmetry/ invariance conditions does not emerge very naturally, if at all, from alternative accounts of laws—instead it has to be imposed, to the extent that it can, by hand.

1.2) Laws are often described by having a kind of necessity or inviolability. Neither we nor the rest of nature can "break" or avoid them, at least in regimes in which they hold. However, this notion of necessity, if intelligible at all, stands in need of explication, and alternatives to invariance- based accounts have not been very successful at this. Some (e.g. Bird, 2007) claim that the necessity of laws is a kind of metaphysical necessity, but, in addition to its obscurity, current statements of this idea do not fit well with how scientists reason about laws. Scientists seem quite willing to consider scenarios in which such alleged metaphysical necessities are violated—e.g., what the trajectories of the planets would be if gravity conformed to an inverse cube law[4]. "Metaphysical necessity" seems too strong a notion and too far removed from what might be established by empirical investigation to capture the kind of necessity possessed by laws.

Best Systems analyses have the opposite problem: they take laws to be regularities described by axioms or theorems in a deductive systemization involving a best balance of simplicity and strength imposed on a Humean Basis specified in terms of spatio-temporal relations and perfectly natural monadic properties, characterized non-modally. Although defenders can stipulate that what it is for a generalization to be "physically necessary" is just for it to be such an axiom or theorem, it is not obvious what this status has to do with notions of inviolability or necessity or, more generally, why one should expect the axioms and theorems of the BSA to be invariant in the sense described above. On the contrary, the BSA threatens to make laws too sensitive to (non-invariant with respect to) initial condition information[5].

By contrast, invariance-based accounts provide a naturalistic, scientifically respectable and non-mysterious treatment of what non-violability and physical necessity amount to: this just amounts to the claim that within the domain of invariance of a law there are no initial and background conditions that might be realized—nothing that might be done by nature or an experimenter—under which the law will fail to hold[6]. Inviolability, non-breakability and so on are thus just other names for invariance.

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[4] This point is emphasized by Lange, 2009.

[5] See Woodward 2003 and below for additional discussion. On the invariance-based account, laws are those generalizations that continue to hold under a range of different initial conditions. But sufficiently different initial conditions will generate significantly different Humean Bases, which may require different axioms for their systemization. Thus which generalizations are BSA-laws will depend on the actual distribution of initial conditions. This reflects the fact the BSA does not cleanly separate lawful and initial condition information in the way the invariance-based account does.

[6] As explained in more detail below, this is not just the triviality that laws hold when they hold; it rather involves the claim that laws would continue to hold if non-actual conditions (and non-actual processes leading to these conditions) within their domain of invariance were to be realized, where this domain can be given an independent specification—in physics this is often done by specifying length or energy scales in which the law holds.

Moreover, the invariance of a generalization is something for which we can get empirical evidence (Woodward, 2013 and Section 4 below).

1.3) Invariance-based accounts combine some of the most plausible elements in both BSA-type accounts, and more "metaphysical" accounts, while rejecting less attractive elements of both accounts. Invariance-based accounts agree with the BSA and disagree with the metaphysical accounts in holding that the postulation of special metaphysical entities is unhelpful. However, like metaphysical accounts, and unlike the BSA, invariance –based accounts do not aspire to provide a reduction of the modal features of laws to non-modal features.

Although the invariance-based account can be thought of, in the respects described above, as situated midway between the BSA and metaphysical accounts, it departs from both in a fundamental resect. Both the latter aspire to provide "truth-makers" or "metaphysical foundations" for laws, with this presumably consisting of something like the full Humean Basis in the case of the BSA, and the metaphysically special entities and relations in the case of non-Humean theories. As commentators have noted (and, often, complained[7]) the invariance-based account provides no such metaphysical story about truth-makers or foundations, either in the form of special entities or a reduction. For many philosophers, the absence of such a story in an account of laws is rather like Hamlet without the prince-- the most important element has been left out. Bird, for example, writes that although the invariance-based account makes an "important contribution to understanding the superstructure [of how we reason about laws] it does not tell us (nor is it intended to tell us) about the metaphysical underpinnings" (2007, p. 5). He also suggests that any one of the alternative accounts mentioned above might provide such underpinnings for talk of invariant relationships.

In response, I would emphasize that, first, there is a great deal of value in providing a descriptively accurate characterization of the "superstructure" of our thinking about laws and the role they play in scientific theorizing—this superstructure is far more complex and subtle than many philosophers suppose. Second, contrary to what Bird suggests, it is not true that all of various accounts of the metaphysical underpinnings of laws on offer can be made to fit with the invariance-based account. Instead, the invariance-based account captures features of scientific practice not readily captured by *any* of these alternative accounts.

A third and more fundamental consideration is whether, as Bird implies, the invariance-based account not only leaves something out (a story about metaphysical foundations) *but also leaves something out that can and needs to be provided*. Disciplines other than philosophy (including in particular the mathematics and the natural sciences) recognize the possibility of *ill-posed* problems—problems which as currently formulated rest on mistaken presuppositions or which assume constraints on what counts as a solution that are unsatisfiable or ill-motivated. Sometimes solving a problem requires rejecting one or more of these presuppositions/ constraints and/or reconfiguring the problem itself or rethinking what is required to solve it.

I'm inclined to think that the problem of identifying truth makers for laws in the sense of "truth-maker" that most philosophers have in mind is just such an ill-posed

[7] See, e.g., Bird, 2007, Psillos, 2004.

problem. A detailed argument for this claim is beyond the scope of this paper but I try to briefly suggest (Section 9) why I think it is true.

2. Wigner on Invariance and Independence.

With this as motivation, I turn to some remarks about laws, invariance and initial conditions taken from several of the essays in Eugene Wigner (1979). I quote extensively because I want to make detailed use of Wigner's ideas in elucidating the connection between lawfulness and invariance.

Wigner's point of departure is the contrast between laws and initial conditions He writes:

> The world is very complicated and it is clearly impossible for the human mind to understand it completely. Man has therefore devised an artifice which permits the complicated nature of the world to be blamed on something which is called accidental and thus permits him to abstract a domain in which simple laws can be found. The complications are called initial conditions; the domain of regularities, laws of nature. Un-natural as such a division of the world's structure may appear from a very detached point of view, and probable though it is that the possibility of such a division has its own limits, the underlying abstraction is probably one of the most fruitful ones the human mind has made. It has made the natural sciences possible.

He also stresses the role of invariance principles in the identification of laws:

> However, the possibility of isolating the relevant initial conditions would not in itself make possible the discovery of laws of nature. It is, rather, also essential that given the same essential initial conditions, the result will be the same no matter where and when we realize these. This principle can formulated, in the language of initial conditions, as the statement that the absolute position and the absolute time are never essential initial conditions is the first and perhaps the most important theorem of invariance in physics. If it were not for it, it might have been impossible for us to discover laws of nature….

Commenting further on the contrast between laws and initial conditions, Wigner introduces several additional ideas:

> The fact that initial conditions and laws of nature completely determine the behavior is … true in any causal theory. The surprising discovery of Newton's age is just the *clear separation* of laws of nature on the one hand and initial conditions on the other. The former are precise beyond anything reasonable; we know virtually nothing about the latter (my italics).

Wigner also suggests that if there were regularities or relations or constraints among initial conditions, this would suggest that our specification of the laws of nature was inadequate:

> how can we ascertain that we know all the laws of nature relevant to a set of phenomena? If we do not, we would determine unnecessarily many initial conditions in order to specify the behavior of the object. One way to ascertain this would be to prove that all the initial conditions can be chosen arbitrarily. (40)

> The minimal set of initial conditions not only does not permit any exact relation between its elements: on the contrary, there is reason to contend that these are, or at some time have been, as random as the externally imposed, gross constraints allow.

He explains what he has in mind by reference to Kepler's laws and Bode's law. These involve regularities in what we think of as initial conditions. Wigner comments:

> However, the existence of the regularities in the initial conditions is considered so unsatisfactory that it is felt necessary to show that the regularities are but a consequence of a situation in which there were no regularities.

As an illustration, he cites a hypothesis advanced by von Weizsacker according to which the "solar system consisted of a central star, with a gas in rotation but otherwise in random motion around it". Von Weizsacker attempts to deduce regularities like Bode's law from this hypothesis; thus, in Wigner's words,

> Attempt[ing] to show that the apparently organized nature of these initial conditions was preceded by a state in which the uncontrolled initial conditions were random.

He adds

> These are, on the whole, exceptional situations. In most cases, there is no reason to question the random nature of the noncontrolled, or nonspecified, initial conditions (41)

From these remarks, we may extract the following picture. In many although perhaps not all cases, science proceeds by (somehow) making a "cut" or "separation" between initial conditions and laws. Wigner does not describe procedures for doing this (nor will I) but he does tell us something about the features that each component (initial conditions versus laws) will possess once we have made the separation—thus describing in part what success in making the separation will consist in. First, the laws are associated with "regularities" that possess invariance properties. Part of what this means is that the laws possess certain symmetries in the sense of returning exactly the same solutions or predictions under certain kinds of transformations in the initial conditions—for example, the laws may tell us that otherwise similar systems which are spatially translated with respect each other will behave in the same way and similarly for other sorts of transformations. According to Wigner, some minimal degree of invariance of this sort is probably necessary for doing science at all.

Laws also possess additional invariance properties which we can think of as generalizing these symmetry-linked invariance conditions: the laws themselves (and the relationships described by the laws) should be stable or continue to hold as the initial conditions vary, at least for some range of such variations, even though the laws will yield different predictions as we combine them with different initial conditions. While the invariance/symmetry of, say, Newton's gravitational inverse square law under spatial translations implies that two masses separated by a fixed distance will exert the same force on each other if this system undergoes a spatial translation, this more general notion of invariance requires that the law itself -- $F= Gm_1m_2/r^2$ and the relationship it describes -- should continue to hold as the magnitudes of the masses and the distance between them varies, at least for some "appropriate" range of such variation. (See below for what this means). In other words, we should make the cut between laws and initial conditions in such a way that the laws are such that when conjoined with a range of different initial conditions, they continue to correctly describe the behavior of the systems to which they apply. Moreover, we should formulate individual laws in such a way that they are invariant not just over initial conditions in the sense of values taken by variables explicitly figuring in those laws (*m* and *r* in the case of the inverse square law) but under changes in other variables as well --- e.g., changes in the colors or shapes of the masses in particular gravitating systems or other sorts of "background" changes. In what follows I will use the notion of invariance under changes in "initial conditions" to include such additional changes as well.

To pick up on other aspects of Wigner's remarks and to make a connection with what will come later, one way of thinking about these invariance conditions is that they require a kind of *independence* of laws from initial conditions—the cut between laws and initial conditions should be made in such a way that they are "independent" of each other or as "separate" as possible. Of course, in this sort of context, "independence" does not mean statistical or probabilistic independence[8], at least in any literal sense: laws do not correspond to random variables characterized by joint probability distributions also involving initial conditions. Nor does it seem right to think of the independence in question as a kind of causal independence of any straightforward sort.

One possibility suggested both by Wigner's discussion and some other ideas derived from machine learning described briefly below (Section 6) is to think of the independence in question as a kind of *informational* independence: we want the split between laws and initial conditions to be chosen in such a way that these are as informationally independent as possible, in the sense that specific information about which initial conditions obtain for a system should not provide information (or at least specific, non-generic information) about the laws that describe that system and conversely. In Wigner's language there should be "no relation" (at least of a sort we are able to describe) between whatever is specifically true of the initial conditions and whatever is true of the laws and conversely. Of course this requirement needs to be understood in such a way that it is consistent with the same variables characterizing both the laws and initial conditions—otherwise we would not be able to apply the laws to the initial conditions. So in this respect there will be some connection between the laws and initial conditions. Rather the idea is that specific values taken by the initial conditions or

[8] Or at least there is no obvious warrant for such an interpretation. A Bayesian treatment in which it is assumed that there is a joint probability distribution over laws and initial conditions may be possible but I ignore this in what follows.

specific parameterizations of those conditions or specific relationships characterizing patterns in those conditions should not occur in the laws, if this can be avoided. (Again, see Section 6.) Speaking metaphorically, given the laws, we or nature should be able to independently and freely choose whatever initial conditions we wish to combine them with[9].

So far we have been talking about the "independence" of laws from initial conditions. Recall, however, that Wigner also describes what are in effect independence conditions applying to the initial conditions themselves—roughly, these "should be as random as the externally imposed gross constraints will allow" with the existence of regularities in initial conditions being considered "unsatisfactory" (1979, p. 41).

Wigner does not further explain what he means by "random" or by "the existence of a regularity in the initial conditions" but here statistical or quasi-statistical interpretations seem more appropriate: absent specific information to the contrary, there is a default in favor of assuming that initial conditions characterizing different systems or characterizing appropriately distinct elements of the same system (where these are taken at the "same time" -- that is, on a surface of simultaneity—see below) should be statistically independent of one another. Or, more weakly, one might take the idea to be that such initial conditions should at least be capable of varying independently of each other—there should be no lawful constraints among the initial conditions themselves. Moreover, one might expect that under the right circumstances this possibility of free variation should manifest itself in statistical independence or something close to it, as well as in the ability of experimenters to set up experiments at separated locations realizing "arbitrarily" different initial conditions[10]. If, contrary to this default, a regularity or non-independence is present among the initial conditions, one would expect that there is some further explanation of this, which traces the dependency or regularity back to earlier conditions satisfying the randomness requirement, as illustrated by the hypothesis about the formation of the solar system.

This suggestion about initial conditions is allied to assumptions that are sometimes described under such rubrics as the independence of incoming influences or the principle of the common cause, where the latter says that if X does not cause Y and Y does not cause X and they have no common cause Z, one should assume that X and Y are statistically independent. This provides a connection between this portion of Wigner's remarks and a commonly accepted, if sometimes controversial, methodological maxim.

Finally, to avoid misunderstanding, let me emphasize, that whether some proposed set of laws (and accompanying sets of claims about initial conditions) actually have the features of invariance and independence described above is always an empirical matter. It is nature and not any kind of a priori or purely formal analysis that settles whether or not some proposed law is or is not invariant under some specified set of changes in initial conditions.

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[9] In certain speculative multiverse cosmological models, this picture of different sets of laws and different sets of initial conditions being generated independently of one another in different "worlds" may be understood non-metaphorically.

[10] The notion of "free assignability" of initial conditions is sometimes used to express both this idea about the absence of relations among initial conditions and their independence from the laws.

3. Wigner Explicated Further

Let me next underscore some additional features of these "Wignerian" ideas. Note first that both initial conditions and laws are characterized via a set of interrelated constraints that must be satisfied together: we choose what to count as a law and what to regard as initial conditions in such a way that these constraints are simultaneously satisfied. In particular, the invariance possessed by laws is *not* understood as a matter of the law continuing to hold under just any arbitrary counterfactual stipulation but rather as invariance under a much more specific set of changes – changes in initial conditions, where these meet further requirements having to do with independent realizability. Note that the "independence" and the "realizability" component of this requirement themselves have a modal character[11].

This is one of several respects in which, in the account described above, invariance is understood in a non-reductive way—the (modally committed) notion of a change in an independently realizable initial condition enters into the characterization of invariance. This picture contrasts with one in which one somehow begins with access to information that is completely non-modal in character, and that can be characterized independently of "laws", identifies regularities from this information, and then promotes some of these to the status of laws on the basis of additional criteria such as those specified by the BSA.

Another feature of this invariance-based picture is that the initial conditions are regarded as subject to very different constraints than the laws—roughly, one tries to put as much structure or regularity as possible into the laws (in this sense one looks for "simple" laws), while choosing what counts as initial conditions in such a way that these are unstructured as possible (If one thinks of this also as simplicity constraint, it has different consequences than the simplicity constraint imposed on laws). One can think of this as a matter of choosing so as simultaneously satisfy different desiderata (order for laws, disorder for initial conditions) but this choice appears to have a different structure than the "trade-off" described in the BSA.

Also worth noting is Wigner's pragmatic attitude toward the "split" between laws and initial conditions. Although he describes it as "one of the most fruitful divisions that the human mind has made", he also says it is an "artifice" and perhaps "un-natural" from a "very detached perspective", adding "that the possibility of such a division has its own limits". In a footnote (p. 3) he suggests that "the artificial nature of the division of information into "initial conditions" and "laws of nature" is perhaps most evident in the realm of cosmology", adding that, "It is in fact impossible to adduce reasons against the assumption that the laws of nature would be different even in small domains if the universe had a radically different structure".

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[11] "Realizability" is a subtle notion that deserves more explication than I can give it here. At least in many cases, it means that the *occurrence* of the initial condition taken in itself is physically possible. However, in a number of cases, no such requirement of physical possibility is imposed on whatever process is modeled as leading to the initial condition in question—e.g., one may model the initial condition as imposed by a Dirac-delta function-like impulse.

In remarking that there may be physical situations or contexts in which the law/initial condition distinction breaks down or fails to apply, I take Wigner to be suggesting that our ability to make this distinction requires that nature behaves in a way that "supports" the distinction. The distinction thus tracks "objective" features in nature, but these features are not guaranteed to be present in all situations.

At the same time, however, Wigner also emphasizes that there is an element of "artifice" in the construction of theories embodying this distinction, with such theories being adopted because they are "fruitful" and not just because nature allows us to construct them. At the risk of putting words in Wigner's mouth, this suggests that there is a "functional" story[12] to be told about role of law/initial condition distinction in our thinking– functional in the sense that this structure or architecture is adopted because it is more fruitful to contributing to certain goals than alternative systems of representation that do not involve a separation of laws and initial conditions. (For more on what these goals might be, see Section 5 below.)

If one were to focus entirely on this second feature, having to do with the goal of finding theories with a certain structure, to the exclusion of the first, having to do with the features of nature that permit the construction of theories with this structure, one might be led to a view according to which the law/initial condition contrast is merely something that we "make up" and "project" onto nature. The fact that nature has to behave in certain ways for the attempt to describe it in accord with the law/initial condition contrast to be successful suggests instead that more is going on than mere projection—it points us in the direction of (or suggests that there must be something correct about) a more "realistic" picture of the status of laws. But I suggest that what is supported is (at best) a kind of tempered and restricted realism: we should think of the law/initial condition distinction as a distinction that works well for us, at least often and around here, and we should not neglect the fact that the distinction is employed because there are things we want to *do* with it—e.g., predict and explain. Like "cause"[13], "law" thus has a dual or janus-faced character; it functions to organize our thinking in a certain way (it has a "design" component) *and,* when applicable, tracks features of the world. To understand the notion of law one needs to keep both of these aspects in mind.

To bring out the significance of this last point, observe that is certainly possible to describe nature in ways that do *not* employ a distinction between independent laws and initial conditions. Following Wigner, I will take it that the (or a) major reason for not doing this is that the resulting descriptions are less fruitful or effective than those incorporating the law/initial condition split. Although I lack space to argue for this claim in detail, I take this (the possibility and fruitfulness of the distinction) to be a *discovery* that occurs, perhaps gradually, in the early modern period, with a key element involving rejection of pictures of nature in which, from a modern perspective, law and initial condition information are intermingled or conflated rather than distinguished and regarded as independent. Thinking, as scholastics and Aristotelians were wont to do, of nature in terms of active powers and tendencies inherent in bodies that lead them to behave as they do and focusing on what happens most frequently or "for the most part"

[12] For more on what is meant by a "functional" story in the context of understanding causation, see Woodward (2014).

[13] See Woodward, 2014.

involves just such an intermingling, which needed to be rejected for modern science to take the form that it did.[14]

A key element in this change was the recognition that the world of common sense and ordinary observation is disorderly in many respects and that this disorder can be regarded as the upshot of often hidden orderly constraints (the laws), interacting with disorderly initial conditions, with the former capable of being formulated independently of the latter, so that the disorder can, as it were, be segregated off into the initial conditions. Thinking of laws as in some way "external" to (or independent of) the particular objects they describe, however puzzling this may seem metaphysically, seems to have been a key part of this change in thinking.

Let me conclude this section by acknowledging what will be obvious to cognoscenti: the picture I have presented, when construed as a general account of laws in science (or even in physics) is (to put it charitably) hugely oversimplified. To begin with, although Wigner does not explicitly say this, his formulation seems to require (or at least to apply most naturally) to systems in which there is a well-posed "initial value" problem: a so-called Cauchy surface which allows for a complete specification at a time-slice of the initial conditions governing the system. (It is these conditions that are supposed to be capable of freely varying or independent of one another. Obviously initial conditions at different time slices will not be independent.) Wigner thinks of the fundamental laws (as many philosophers of science do) as dynamical laws expressed by hyperbolic differential equations specifying the evolution of these systems over time.

At present by no means all candidate laws take this form. Some candidate laws are not of evolutionary type and are not expressed by means of hyperbolic differential equations. Relatedly, in focusing, as Wigner does, on invariance under *initial* conditions, I have completely ignored the role played by *boundary* conditions, needed for the characterization of many systems and for the solution of the differential equations (the laws) applied to those systems. As emphasized by Wilson (1990), the equations governing the interior of a system may accept only certain boundary conditions and not others; Indeed, certain boundary conditions (and the physics they implicitly assume) may be *inconsistent* with the equations governing the interior. The boundary conditions themselves may embody additional modal information (e.g. information about what can be changed independently along the boundary and what cannot), so that again we have a situation in which not all such information is carried by the laws alone.

Obviously the simple characterization of invariance related notions attributed above to Wigner requires elaboration and qualification if it is to be extended to these more complicated situations. I will not try to do this here. One has to begin somewhere.

4. More on Invariance

[14] What happens most frequently reflects a kind of mixture of which initial conditions are realized in nature most often and the laws applying to these conditions. Thinking in terms of powers which are characterized in terms of their effects also seems to lead in many cases to a mixing or lawful and initial condition information, since the effects depend on which initial conditions are present.

Let me turn now to a closer look at the notion of invariance. One very natural construal is modal or counterfactual:

(4.1) generalization G is invariant if it *would* continue to hold *were* certain changes in initial conditions to hold.

(This is not the only possible construal. Section 6 will briefly consider a interpretation in terms of informational independence.) An obvious question is then how (4.1) should be understood. If we think in terms of the standard Lewisian possible worlds analysis, we seem led to construals like the following:

(4. 2) In the closest possible worlds in which alternative initial conditions Vi occur, G continues to hold

We then encounter the problem that the Lewsian similarity measure for closeness among possible worlds makes reference to the laws of nature, which include G. We thus seem to have a vicious circle: to evaluate (4.2) it looks as though we need to already know what the laws of the actual world are (and perhaps whether and to what extent such laws are "preserved" under various counterfactual suppositions). How then can we use a notion of invariance understood in terms of such counterfactuals to get a purchase on laws?

Thoughts along these lines are presumably part of what underlies the complaint that appealing to invariance to explicate laws of nature is (viciously) "circular". For example, Psillos (2004) argues that it is the laws that determine which variations in initial conditions are physically possible and hence which variations are appropriate for purposes of assessing invariance, an assessment which is also endorsed by Bird, 2007. Suppose, to use Psillos' example, we want to assess the invariance of generalization G. According to Psillos, it would not be appropriate to ask whether G is invariant under some variation that involves a particle moving faster than light since this is a physically impossible variation. But (Psillos then argues) this judgment requires that we have already identified what the laws are, thus landing us in a "circle". More generally, many philosophers hold that laws are among the "truth-makers" for counterfactuals, so that to the extent invariance is bound up with counterfactuals, we can't appeal to invariance considerations to elucidate the notion of law.

These complaints move much too quickly from the non-reductive character of an invariance-based account to the conclusion that it is epistemically or otherwise circular in a vicious way. First, let me repeat that what invariance requires is stability of a generalization under variation in (independently realizable) *initial conditions* and not under all possible counterfactual suppositions that philosophers may be willing to make. Thus for the purposes of assessing invariance we need not worry about the truth –value of such counterfactuals as "if the wires on this table (which are copper) had been insulators, "copper conducts electricity" would not be a "law". (cf. Lange, 2009, pp 38-

9). The antecedents of such counterfactuals do not correspond to assumptions about initial conditions, let alone independently realizable ones[15].

Second, our *epistemic access* to which initial conditions are physically possible, singly or in combination, obviously does not require that we already know what the laws of nature are. One can learn about what is physically possible both from observation of the variation in states and initial conditions that occur naturally and by active experimental manipulation. For example, one can determine empirically that there is no way of realizing initial conditions involving faster than light velocities or insulating copper wires. When such variations in conditions are actually realized one can often determine empirically whether some candidate generalization continues to hold under such variations. (This is what Boyle did, when he systematically manipulated the pressure of air, measured the associated volume and found a relatively invariant relationship between the two.) Moreover, to at least some extent inference to what would happen under unrealized variations can be justified as a matter of ordinary inductive extrapolation (or interpolation): seeing that the relation $F=-kX$ describing the relation between the extension of a particular kind of spring and the force it exerts continues to hold for various variations in X, one infers that it will hold for intermediate unrealized values of X and perhaps for small increases in X beyond those measured.

More generally, whether or not laws of nature are among the truth conditions for counterfactuals, it is not true that in all cases one must know what those laws are to evaluate counterfactuals. If I want to know whether it is true that if I were to drop this rock it would fall to the ground, I don't need to know the laws governing gravitation or freely falling bodies. Instead I can just drop the rock, taking care to avoid possible confounding factors and observe what happens—the world, rather than elaborate inferences involving possible worlds, gives us the answer. It is bizarre to suppose that I cannot assess counterfactual claims in cases like this or do not understand what they mean without having prior knowledge of laws of nature or a detailed treatment in terms of Lewisian semantics. A similar point often holds for the counterfactuals involved in assessments of invariance.

What about the idea that, nonetheless, laws are required to state "truth conditions" for counterfactuals and that this makes the invariance-based account damagingly circular? This complaint only has force if truth conditions for laws can be given in a form that makes no use of counterfactuals[16]. That is, the complaint assumes that either (i) laws-claims can be reduced to claims that do not presuppose counterfactual or other modal notions, as in the BSA or (ii) law clams can be elucidated by appealing to special entities like dispositions which are somehow "prior" to counterfactuals. I deny both (i) and (ii)—see Sections 7 and 8.

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[15] This is one reason why I would resist attempts to take a very general and undifferentiated notion of counterfactual dependence as primitive and then to use this to explicate the notion of law, as in Lange 2009. It is only certain kinds of counterfactuals that are relevant to assessments of invariance.

[16] Some will think that either laws must be "prior" to counterfactuals (and hence suitable for explicating them) or counterfactuals must be "prior" to laws. I see no reason to suppose that one of these alternatives "must" be true, independently of an exhibition of the explication in question. Perhaps neither is prior to the other.

5. Explanation

I suggested above that an important part of understanding the law/ initial condition distinction involves an appreciation of what the distinction is to be *used* for—what purpose(s) or function is served by the distinction. There are many interrelated candidates for such purposes. To begin with, if we can identify a generalization that holds across a range of variations in initial conditions, we can "export" it to different situations within this range and use it to make correct predictions about what will happen in those situations. More generally, satisfaction of the independence conditions of the sort described above will often lead to the elimination of redundancy and unnecessary complexity, as when we replace a number of different local generalizations each holding only in a limited domain with a single overarching generalization which is invariant across all those domains[17]. (Think of informational dependence or at least unexplained informational dependence as a kind of redundancy.) Relatedly, as suggested above, making a split between laws and initial conditions allows for the segregation of the disorderly part of what we observe in information about initial conditions, allowing for the formulation of orderly laws.

In addition to this, however, it is also natural to think of the law/initial condition distinction as functioning in the service of a certain regulative ideal for explanation and causal analysis. According to this ideal, described in more detail in Woodward, 2003, explanation and causal analysis often proceed by providing answers to a range of what-if things-had -been different questions-- that is, by showing how the behavior of system of kind S would change, under changes in the initial conditions characterizing S, given one or more generalizations applying to S that are invariant under these changes. In this way, we come to see how the behavior of S depends on these initial conditions and how these initial conditions "make a difference" for the behavior of S. The connection with my previous discussion should be obvious: realizing this ideal of explanation requires that we effect a split between laws and initial conditions, with the former invariant over changes in the latter[18].

Extending this further, one may also think of the independence conditions imposed on initial conditions as also having a natural motivation in terms of a related ideal of explanation. Explanations that make use of highly structured initial conditions or correlations among initial conditions or highly unusual initial conditions with no further account of where these come from are generally not regarded as fully satisfying—for the obvious reason that they seem to raise but leave unanswered the question of where that structure comes from (cf. my remarks on the principle of the common cause above). By contrast, explanations that appeal to "random" or relatively unstructured initial

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[17] That is, when a generalization holds only in a limited domain, we may think of this as a kind of failure of complete independence between the generalization and the initial conditions under which it holds.

[18] This suggests the following question for those who advocate replacing law-talk laws with something else such as disposition-talk: Can such replacements accomplish the same goals as law-talk and its accompanying law/initial condition split? I'm inclined to think not, but this question seems to go unaddressed in the dispositions literature.

conditions do not seem to raise corresponding questions with quite the same urgency—random, uncorrelated and unstructured initial conditions are a natural stopping place in explanations[19].

A common complaint directed against "regularity" accounts of laws, including the BSA, is that if laws are mere summaries of what happens, it is hard to see how they can figure in explanations. The evaluation of this complaint is a complicated matter, but it is worth noting that the invariance-based account is not subject to it. On that account, laws do *not* just record regularities—instead they encode information about invariance/independence properties and relations. These fit naturally into an account of explanation like that described above that does not take explanation to consist merely in a demonstration that various explananda "instantiate" regularities.

6. Invariance and Independence Extended

I have advocated a common framework for characterizing both the relationship between laws and initial conditions and initial conditions themselves that appeals to notions of "independence". In this section I want to briefly describe several ways in which this common element might be extended and developed.

6. 1) Independence Relations among Variables. Laws often describe relationships between the values taken by one variable (often but not always placed on the left hand side of an equation) and several other "independent" variables (placed on the right hand side). Even though the equation may not explicitly say this, it is often presumed that these right hand side variables can vary in value independently of each other. For example, in connection the Newtonian inverse square law, $F= Gm_1 \, m_2/r^2$, it is usually assumed that, as far as this equation taken in itself goes, the distance r between the two masses m_1 and m_2 can change independently of the values taken by those masses and similarly the masses can change in magnitude independently of each other. In the Lorentz force law, $F= qvxB$, the charge q of the moving body can vary independently of the intensity of the magnetic field, and of its velocity. This independence feature is closely linked to ideas about the possibility of independent variation in initial conditions mentioned in section 2 above. Note also this is a feature that is not captured by the standard (6.1) "all *A*s are *B*s" or (6.2) "all *A*s and *C*s are *B*s" representations of laws (or, for that matter, the claim that laws represent or are "regularities"). In (6.1) there is only one rhs variable and in (6.2), there is no representation of whether *A* and *C* can vary independently. This is another illustration of the complex and variegated independence information carried by laws which is missed in many philosophical treatments.

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[19] None of this should be read as an endorsement of what is sometimes called "inference to the best explanation". Instead my picture is this: we look for theories and models that exhibit the kind of structure described above because we value explanation as a goal. But the fact that such theories/ models would if true provide good explanations is not automatically a reason for taking them to be true. For that we require independent evidence (independent of judgments of explanatory power) for taking the theories/ models to be true.

6.2. Independence Relations among Laws. Marc Lange (e.g. 2009), among others, has drawn attention to the way in which scientists often reason about laws in ways that appear to embody assumptions about the independence of individual *laws* from one another[20]. For example, an investigator may ask what the motion of a body would be if it were subject to a gravitational force obeying an inverse cube rather than inverse square law. Such investigations seem to assume that the laws of motion describing how this body responds to forces (e.g. $F=ma$) are independent of the specific law describing the force itself, so that one can coherently assume a force law different from the actual one, while also assuming the laws of motion remain as they actually are, and use these together to calculate the resulting trajectory[21]. Newton used reasoning of this sort to argue that the evidence supported his inverse square law over alternative gravitational force laws with different exponents. Again, independence in this context might naturally be understood as involving a kind of informational independence, in the sense that alternative gravitational force laws are not inconsistent with the laws of motion (that is, do not imply alternative laws of motion) .

6.3. Independence Assumptions in Causal Inference. Although our topic is laws rather than causal claims, there are very interesting applications from the machine learning literature (e.g. Janzing, 2012) illustrating the power of the independence/invariance based ideas described above, particularly in connection with determining the direction of causation from statistical information. These provide concrete illustrations of what it might mean for initial conditions to fail to be informationally independent of one another and of the "laws/causal generalizations" governing a system. Suppose first that X and Y are random variables whose values we observe. Suppose also (i) Y can be written as a function of X plus an additive error term U that is probabilistically independent of X: $Y= f(X) + U$ with X _|_ U (where _|_ means probabilistic independence) . Then it can be shown that if the distribution is non-Gaussian, there is no such additive error model from Y to X — that is no model in which (ii) X can be written as $X= g(Y)+ V$ with Y _|_ V. When applied to empirical data in which the correct causal direction is independently known, the assumption that the correct causal direction is the one in which the cause is independent of the error leads to correct results a majority of the time. One can think of this inference procedure as making use of default assumptions about the independence of incoming influences or initial conditions of the sort described in section 2. The reliability of the results in this very different context illustrates both the power and generality of such assumptions.

Remarkably even in cases in which there are just two variables—X and Y — which are deterministically related via an invertible function (i.e., there is no error term

[20] This is my formulation. Lange may not agree with this way of putting things

[21] If I have understood him correctly, Lange gives this reasoning a very strong modal interpretation: he takes it to involve commitment to the idea that (i) nature is such that if the inverse square law had not held, Newton's laws of motion still would have held. It seems to me that a weaker interpretation (advocated above) will often suffice to make sense of such reasoning: there is no inconsistency or incoherence in combining the laws of motion with an alternative to the inverse square law. This seems to require a kind of informational independence between the laws, but perhaps not the idea that there is some structure in nature that ensures the truth of the counterfactual (i).

U), it is also possible to determine causal direction by making use of a kind of analog of the idea that laws should be (informationally) independent of initial conditions described as described in Section 2. Very roughly, the idea is that if the causal direction runs as $X \rightarrow Y$, then we should expect that the function f describing this relationship to be informationally independent of the description of the (marginal) distribution of X (which corresponds in this context to an initial condition) —independent in the sense that knowing this distribution will provide no non-generic information about the functional relationship between X and Y and vice-versa. By contrast, it is possible to show that for "most" functional relations g when (X—> Y) is the correct direction, writing X as function g of Y ($X= g(Y)$)will result in a g that is not informationally independent of the distribution of Y. The relevant notion of independence can be understood in terms of algorithmic information theory or in terms of the absence of terms in f that are finely tuned to the distribution of X and the presence of corresponding terms tuned to the distribution of Y in g . Again the procedure is relatively reliable as an empirical matter. This illustrates how looking for representations in which lawful (and causal) relationships are independent of or invariant across initial conditions, with independence understood as informational independence is a fruitful one, which is at work both in contexts in which physical laws are deployed and in the causal inference contexts described immediately above.

This suggests a further speculation: perhaps an alternative to explicating the notion of invariance (and "law" and "cause") in terms of counterfactuals is to employ instead some appropriately behaved notion of informational independence --perhaps at bottom these are alternative ways of getting at the same thing. According to this conception, one looks for a formulation of laws and initial conditions such that the initial conditions are (informationally) independent of one another, the laws are independent of the initial conditions, and the laws are independent of one another. Perhaps such a framework, if it could be developed, might be appealing to those who are uncomfortable with the use of counterfactuals in explicating the notion of invariance

7. Comparison with Alternatives: BSA

I turn next to a discussion of the relationship between the invariance-based account and alternatives, beginning in this section with the BSA. I have criticized this account elsewhere (e.g., 2013). Rather than repeating those criticisms, I want to proceed (somewhat) more constructively. From the perspective of the invariance-based account, the key question is whether the resources that are employed in the BSA—the information in the Humean basis, and the notion of a trade-off between simplicity and strength, can be used to (reductively) capture notions like invariance. To adopt assumptions most favorable to the BSA, suppose we had a clear account of the various notions (simplicity, strength and a best-trade off between these) that go into the BSA and that in our universe there is a single best systemization. What will be the relationship between the axioms and theorems picked out in the BSA framework as "laws" and those claims picked out as laws in the invariance-based account? One possibility is that the two accounts largely agree: the axioms/theorems of the BSA turn out to largely coincide with those generalizations that are invariant under variations in initial conditions when initial conditions meet the independence constraints described above. This would be a happy outcome; presumably the plausibility of each account would be increased by this sort of agreement. Moreover, it would also suggest that the invariance-based account might be reinterpreted so as to meet reductivist requirements, assuming that the BSA does.

The other possible outcome is that the two accounts diverge in their judgments of lawfulness, which presumably would mean that the resources of the BSA don't capture invariance- based notions. While I would not claim that this would show the BSA is mistaken, I believe it would put some pressure on that account, given that the invariance-based account appears to substantial roots in scientific practice. If, in the case of divergence, BSA advocates claim that it is the invariance based account that is misguided, what would be basis for such a claim?

Without trying to settle the issue of whether the two accounts agree in their assessments of lawfulness, let me note several points. The first is that, conceptually, the notion of a generalization being an axiom or theorem in a best balanced systemization by no means obviously coincides with notion of a generalization being invariant in the manner described above. Prima-facie, it looks as though a generalization might be "simple" in some relevant sense (or figure as an axiom in some simple systemization) and such that one could derive a lot from it (thus strong in some sense) and yet be relatively non-invariant[22]. Perhaps some cosmological generalizations have this character—assumptions about the homogeneity or isotropy of the universe on a large scale might be part of a simple systemization and might enable one to derive a lot when conjoined with other candidate laws but might nonetheless be non-invariant (would have failed to hold if initial conditions in the early universe had been different.) To the extent this is so, the BSA will fail to capture invariance-based notions.

8. Comparison: Special Entities

Although the invariance-based account appears to diverge from the BSA in the ways described, it agrees with the BSA in rejecting treatments that appeal to metaphysically special "modal" or "non-Humean" entities (or properties or relations). From an invariance-based perspective, what is problematic about such treatments is not their invocation of modality per se (after all, invariance is naturally understood as a modal notion) but rather their "reification" or "ontologizing" of modal claims—their treatment of modal claims as (or as made true by) *existence* claims about special entities, with such existence claims having some kind of non-trivial explanatory or "grounding" role for modal claims.

I see such accounts as subject to the following dilemma. Suppose, on the one hand, the accounts have no additional physical consequences for what laws are like beyond what is suggested by the invariance-based account. In this case, one worries that the accounts are mere redescriptions of invariance claims in a metaphysical vocabulary. For example, if it is claimed that laws are made true by relations of necessitation between

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[22] This outcome (simple and strong generalizations failing to coincide with invariant ones) seems particularly likely if the relevant notion of simplicity is understood, as it often is in discussions of the BSA, purely syntactically—a disjunct is less simple than either of its disjuncts etc. By contrast, whether a generalization is invariant is not something that can be identified through its syntax.

universals, then, assuming that the invariance account is correct as far as it goes, such necessitation relations will need to endow laws with various invariance properties—from the truth of N(F,G) in present circumstances it somehow has to follow that, e.g., All Fs would continue to be Gs if initial and background conditions were to be different in various ways and so on. If this connection with invariance exhausts the physical/empirical content of what N(F,G) involves, then it looks as though we have a mere redescription of these invariance features in the more metaphysical language of necessitation. Similarly for the invocation of dispositions as truth-makers: one can build it into one's conception of a disposition that for aspirin to have a disposition to relieve headaches, aspirin must manifest that disposition (under appropriate triggering conditions) under a range of different circumstances, but this sounds very close to building invariance-linked requirements into the notion of a disposition. In this case there does not seem enough distance between the two for the attribution of the disposition to "explain" the invariance-claim.

The other possibility is that the metaphysical entity accounts do have additional physical implications that go beyond what is claimed by the invariance account, either by excluding possibilities that the invariance-based account permits or by adding further physical consequences to that account. Either result seems unwelcome since these additional consequences seemed to be reached on non-empirical grounds. As an illustration of the first alternative, many defenders and critics of disposition-based accounts agree that such accounts have difficulties capturing certain kinds of laws or features of laws -- for example, symmetry principles and conservation laws. Apparently the problem is that dispositions are most naturally ascribed to objects, and it does not seem natural to regard, e.g., the conservation of charge as the manifestation of a disposition attributable to any particular object. This leads one leading advocate (Bird, 2007) to the conclusion that the disposition-based account results in the judgment that symmetry and conservation principles are "pseudo- laws" that may turn out to be "eliminated" as "being features of our form of representation rather than features of the world requiring to be accommodated within our metaphysics" (p. 214) This judgment/ prognostication is certainly at odds with the deep significance attributed to such principles by most physicists and looks like a clear case in which restrictions on the content of science are being motivated by appeals to metaphysical considerations. By contrast, an invariance-based account provides a natural and straightforward treatment according to which conservation and symmetry principles are regarded as genuine laws.

9. Conclusion.

The upshot of my discussion so far may seem unsettling. I have rejected both reductivist accounts of laws and accounts that appeal to both special metaphysical entities. Putting aside accepting laws as "primitive"[23], the conventional wisdom is that these accounts exhaust the possible alternative positions. In this concluding section I want to briefly sketch another possibility. This will also constitute a further response to the complaint the invariance-based account fails to provide "metaphysical underpinnings".

[23] As in Maudlin 2007. I regret that I do not have space to discuss this alternative.

Consider, as an analogy, claims about chances when attributed to macroscopic gambling devices, such as roulette wheels, which we can treat as accurately described by deterministic laws, as in

(9.1) The chance of red on the next spin is 0.5.

Although such devices are accurately described by deterministic laws, they generate stable relative frequencies of outcomes. These are frequencies that the croupier cannot alter by manipulations that are accessible to her. Moreover, those who have access only to macroscopic information cannot do any better in prediction than by making use of information about such frequencies.

A considerable physical/mathematical literature, going back to (at least) Poincare on "the method of arbitrary functions" explains these features. Very roughly, one can show that

(9.2) For a range of different possible dynamics and a large range of possible distributions of initial conditions meeting very generic constraints including the presence of certain symmetries (constraints that a macroscopic croupier will be unable to violate) , stable frequencies for device outcomes result.

In effect one shows that such frequencies are invariant under manipulations that the croupier is able to perform and under many other changes in the state of the wheel or even its dynamics—in this sense explaining or elucidating why we see stable frequencies. We can thus think of (8.2) as describing those features "in the world" that "support" the ascription of chances to the systems in question and, in some perfectly good sense, "explain" its chancey behavior. Note, however, this does not require that there be some discrete, isolable entity or property with a mysterious ontological status corresponding to "chance" that provides the "metaphysical underpinnings" for the chancey behavior of such systems. The explanation for such behavior instead just involves the diffuse, distributed physical considerations described above. Moreover, if we want to understand the behavior of such systems it would not be fruitful to attempt to provide a "reduction" of chance to something else – e.g., relative frequencies (or some measure of "fit" to relative frequencies) plus additional factors like "simplicity" Such a reduction, even if it could be provided, would tell us nothing about *why* such systems exhibit the behavior they do. Again, the explanation for this behavior lies in facts about the relative insensitivity of the behavior of the systems in question to the details of the laws characterizing their dynamical behavior and details of the particular initial conditions induced by the croupier on successive spins. Neither reductivist treatments of chance nor the introduction of chance into our fundamental ontology gives us any insight into such facts. Note that the problem with the latter approach is not that it is *false* that the device exhibits chance- behavior; it is rather that ontologizing "chance" wrongly suggests that there is a special kind of discrete thing or property that (metaphysically) explains this chance- behavior, when the actual explanation has to do with the diffuse considerations described above.

Finally, note that if one is puzzled about chance and its ascription to gambling devices, much more is relevant than besides the physical story provided above.

Considerations having to do with chance also play distinctive architectural or design roles in our reasoning and decision-making—these are matters studied in statistics and decision-theory. Understanding chance is a matter of understanding *both* these architectural considerations and the various ways in which chance ascriptions can have physical underpinnings/explanations.

Although the analogy is inexact, I suggest that there are a number of parallels with the notion of law. At least in the case of non-fundamental laws, we don't have to take their invariance properties as brute or primitive. Instead, there is an important *scientific* project of explaining why various relationships we find in nature are invariant to the extent that they are. The goal of such explanations is to explain why certain kinds of variations in the values of certain variables do not matter for why the relationships hold—why the relationships are stable across variations in those variables. Similarly to the explanations of the behavior of gambling devices, such explanations typically take the form of showing that provided systems subject to such laws satisfy certain very generic constraints, further variations in other variables will not affect whether they exhibit certain stable patterns of behavior. Details vary across cases (and in many cases we do not presently know what is responsible for the invariant relationships we see) but examples include explanations in terms of statistical mechanics for why variations in molecular details that are consistent with certain macroscopic constraints do not matter for whether various laws of thermodynamics hold, explanations of various aspects of critical point behavior in terms of the renormalization group, and demonstrations in particle physics in the form of "decoupling theorems" show that, as long as generic constraints are met, the detailed structure of presently unknown high energy theories is irrelevant to laws at lower energy scales.

Several points about such explanations deserve emphasis. Note first that the explananda are not (or not just) generalizations specifying that certain regularities hold. Instead the explananda are facts about the invariance of various relationships. This involves a different kind of explanation than a mere derivation showing that some generalization is true—instead we are looking for an explanation of the generalization's stability. As with chance, such explanations will appeal to complex and distributed considerations and to generic constraints rather than reductions or special entities and properties. Indeed, it is hard to see how any of the latter could explain facts about invariance any more than "chance" is the name of something that explains the behavior of roulette wheels. My suggestion is that insofar as explanations of (or underpinnings for) invariance in terms of what is in nature are possible, what we should be looking for are explanations that have the character described above. I concede that this strategy does not help us to understand the invariance of laws, if any, that are ultimate or fundamental in the sense of having no further explanation, but at this point in the development of science it is not clear what would provide illumination about this. Note also that, on this view, just as the case with "chance" what is wrong with the postulation of special entities to serve as metaphysical grounds or truth-makers for laws of nature is the reification and illusion of explanation this involves. It is not that it is false that aspirin has the power to relieve headaches, rather the point is the ascription of this power to aspirin does not explain its behavior[24].

Finally, let me add that just as in the case of chance, there is more to be said about laws than what in nature "underpins" them. An important part of the project of understanding the notion of law involves understanding the role this notion plays in our reasoning and decision-making, what sorts of evidence is appropriate for establishing law-claims, how this notion connects to other scientifically important notions and so on. These fall into the general category of architectural/design considerations. They connect with the methodological role played by laws. A "metaphysics" of laws will not tell us about these matters. I have tried to provide some preliminary suggestions above.

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[24] There is a tendency in philosophical discussion to describe views that deny that there are special metaphysical entities that serve as truth-makers for laws as "anti-realist". This strikes me as tendentious. If realism about laws is just the thesis that claims about laws can be true or false, my view of laws is a "realist" one, even though it rejects "special entity" accounts.

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