The pathology of validity

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Abstract   Stephen Read has presented an argument for the inconsistency of the concept of validity. We extend Read's results and show that this inconsistency is but one half of a larger problem. Like the concept of truth, validity is infected with what we call *semantic pathology*, a condition that actually gives rise to two symptoms: inconsistency and indeterminacy. After sketching the basic ideas behind semantic pathology and explaining how it manifests both symptoms in the concept of truth, we present cases that establish the indeterminacy of validity and that link this indeterminacy with the concept's inconsistency. Our conclusion is that an adequate treatment of the semantic pathology thus revealed must deal with both of its symptoms. Further, it must extend to the occurrences of this condition elsewhere: in the concept of truth, in the other central semantic notions, and even in certain philosophical concepts outside semantics.

Keywords   Validity · Paradox · Truth · Inconsistency · Indeterminacy · Semantic pathology

1 Introduction

Stephen Read, extending an example from Pseudo-Scotus, presents and evaluates an argument for the inconsistency of validity. According to Read (2001, p. 193), what

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the paradox of validity shows is not just a problem with the "classical" account of validity assumed by Pseudo-Scotus; rather, what it shows is that our very *concept* of validity (and, thus, the language we use to express it) is inconsistent—at least *prima facie*. Moreover, Read maintains that such inconsistency takes off from considerations similar enough to variants of the liar paradox (presented by sentences like "This sentence is false") to suggest that a solution to one should extend to the other. His claim is that worries about the inconsistency of validity (if such worries exist) are as worrisome as those about the liar, but we can resolve the former if (and, perhaps, only if) we can resolve the latter.[1]

We believe that Read is right—that the concept of validity, like certain other familiar concepts, is at least *prima facie* inconsistent. Regardless of whether this is ultimately correct, our claim here is simply that if it is, inconsistency is but one half of the issue. If we have reason to take the concept of validity to be inconsistent, then we have just as much reason to take it to be indeterminate. Here, validity again bears comparison to truth, since there appears to be an indeterminate analogue for each liar-like case that *prima facie* exhibits the latter's inconsistency.[2] If, as Read maintains, liar-cases and the paradox of validity reveal that these *semantic* concepts are *prima facie* inconsistent, then the truth-teller (presented by sentences like "This sentence is true") and its extensions, along with (what we call) *the validity-teller* show that truth and validity are also indeterminate. Moreover, for both concepts we can link the inconsistency and the indeterminacy via a kind of unified, disjunctive case—Buridan-inspired pairs of sentences that we call *open pairs*.[3] The features of inconsistency and indeterminacy thus appear to be two symptoms of a single underlying phenomenon, a condition of these concepts that we, following others, call *semantic pathology*.[4]

If a solution to the paradox of validity should take off from one for the liar, then symmetry demands that a solution to the indeterminate case for validity (to be discussed below) should stem from one for the truth-teller. Further, the connection between these symptoms that the open-pair cases for truth and validity reveal suggests that one solution should ramify about all of these cases. Moreover, as we indicate below (Sect. 4), the dual symptoms of inconsistency and indeterminacy arise in concepts other than truth and validity, in a way similar to how they arise in these two cases. There are thus a number of concepts that are properly characterised as semantically pathological if truth and validity are. This strongly suggests that we need a single solution that applies to all of these semantically pathological concepts.

Although we believe that the open pair presents a difficulty that is perhaps more challenging than either the liar or the truth-teller, our goal here is to show something different. What we will establish is that the open pairs for validity share their features with the open pairs for truth—just as the paradox of validity and the validity-teller do with the liar and the truth-teller, respectively. We begin with the paradox of validity.

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[1] Read (1979, 2001) does not argue that variants of the liar and the paradox of validity require a *unified solution*, although his thesis suggests it. Rather, Read (personal correspondence) maintains that there is a structural similarity between the paradoxes, which suggests, but does not entail (at least without further premises), the requirement of a unified solution to both.

[2] See Woodbridge and Armour-Garb (2007) and the top of the appendix here.

[3] See Armour-Garb and Woodbridge (2006) and Woodbridge and Armour-Garb (2005, 2007).

[4] On semantic pathology, see Herzberger (1970), Kripke (1975), Grover (1977), and Gupta and Belnap (1993).

2 Read and the paradox of validity[5]

Per Read (1979, 2001), the classical account of validity holds that an argument is valid if, and only if, it is impossible for its premises to be true while its conclusion is false. While he takes this account to yield paradox, Read wishes to show that validity's paradoxicality is independent of the classical account. To this end, he considers a different set of conditions for validity—one sufficient and one necessary—that he claims are true of validity on any account:[6]

(1)   That a conclusion follows from the premises is a sufficient condition of validity; and

(2)   That the premises cannot be true while the conclusion is false is a necessary condition of validity.

Both (1) and (2) are, Read claims, "unquestionable facts about validity" (2001, p. 187). However, he acknowledges that (1) is fairly vacuous, and that even the addition of (2) to (1) does not uniquely characterise the concept of validity.

For purposes of generality, we will modify Read's conditions in what we assume is an unobjectionable way. To start, (2) differs slightly from the more usual (equivalent) formulation of the necessary condition for validity, namely,

(2′)   That the conclusion cannot be untrue while all the premises are true is a necessary condition of validity.

However, since the conditions under consideration are supposed to apply to validity on any account, they should also apply in the context of three (or higher) valued logics where "true" and "false" are not the only possible statuses. In general, validity (like entailment) has to do with preservation of designated value. Thus, we should generalize (2′) to

(2*)   That the conclusion cannot have an undesignated value while all the premises have a designated value is a necessary condition of validity.

In what follows, we will assume that (1) and (2*) are, respectively, sufficient and necessary for validity. Before moving on, we will also stipulate the following narrow sense of the status of *being not valid*: Something, α, is not valid if, and only if, (i) α is *validity-apt*—i.e., it is a member of a kind of things which are properly ascribed validity—and (ii) it is not the case that α is valid. With this restriction in place, the following equivalence holds, for an argument, α: α is invalid if, and only if, α is not valid.[7]

If, relying only on (1) and (2*), we can establish both the inconsistency and the indeterminacy of validity, then the pathology of validity will also emerge independently of the classical account.

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[5] For what follows, we rely primarily on Read (2001), which is an expansion of Read (1979).

[6] Read (2001, p. 183). The conditions are independent of the classical account in that they do not entail the classical account, although the classical account clearly entails these conditions.

[7] We assume that all arguments are validity-apt and thus are either valid or invalid (i.e., not valid). Your watch, on the other hand, cannot properly be described as valid (lovely though it may be), but neither can it properly be described as not valid (as we use the expression here), since it is not validity-apt.

2.1 Read and the inconsistency of validity

With (1) and (2*) set out, consider argument (I), which appears to entail the inconsistency of validity,

(I) $$\frac{1 = 1}{\therefore \text{ Argument (I) is not valid}}$$

This is a variant of Pseudo-Scotus' paradox of validity, which takes off from the assumed necessarily true premise that God exists, rather than that $1 = 1$. If (I) is valid, then, by (2*), it must have a true (i.e., designated) conclusion, since its premise is true. Because the conclusion "says" that argument (1) is not valid, it follows that (I) is not valid. So, if this argument is valid, it is not valid. Accordingly, (I) is not valid. But in establishing as much, we have relied on the truth of the premise; i.e., we have *deduced* that (I) is not valid, from the truth that $1 = 1$. But this is to show that the conclusion of (I) follows from the premise of (I), which, given Read's (1), shows that (I) is valid after all. As a result, and via some obvious fiddling, we conclude that (I) is both valid and not valid. Thus, (I) purports to establish the inconsistency of validity, and without relying on the classical account.

As a first step in establishing the pathology of validity, consider the validity-teller, the indeterminate analogue of the paradox of validity:

(II) $$\frac{1 = 1}{\therefore \text{ Argument (II) is valid}}$$

Relying only on (1) and (2*), we can show that (II) could be valid or it could be invalid, which, recall, is to say that it is *not valid*. Starting with the latter, then, a question: What would it take for (II) to be not valid? Since the premise is true (designated), by (2*), having an undesignated conclusion will suffice. The conclusion is undesignated if it is false, and, given what the conclusion "says", it is false if the argument is not valid.[8] Flipping this around, it is clear that, given (1) and (2*), together with the fact that $1 = 1$, the supposition that (II) is not valid is sufficient to derive that it is invalid.

Suppose instead that (II) is valid. By (2*), the assumed validity of (II), together with the truth of its premise, yields that its conclusion is true, which, given what the conclusion "says", entails that (II) is valid. Thus, (II)'s validity follows from the assumption that it is valid, just as its invalidity follows from the assumption that it is not valid. We conclude, therefore, that (II) reveals the *indeterminacy* of validity, as we can consistently ascribe this argument either of the available validity-statuses, with no obvious reason (*prima facie)* to pick one status over the other.[9]

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[8] By "false", here, we mean *false only*, so as to make the derivation dialetheically kosher. See Priest (1979).

[9] The result is indeterminacy, not inconsistency, because one should not conclude from the fact that (II) could be valid and it could be not valid, that it could be valid and not valid. We can see the illicitness of this inference by examining its application in an argument regarding the truth-teller considered by an anonymous author in a late 12th century example of the medieval tract *De insolubilibus*. In that argument, the fact that calling the truth-teller true and calling it false both count as "responding well" is supposed to yield that it is both true and false, and thus is inconsistent. However, this does not follow, since the fact that both ascriptions are legitimate does not mean that both are legitimate *together*. Each is legitimate on the assumption that the other is not—in other words, the truth-teller yields indeterminacy. Thanks to Read for pointing out this source on the truth-teller (in De Rijk (1966)).

3 The paradox of validity and semantic pathology

Read maintains that (I) yields the paradoxicality of validity, which makes this concept of a piece with other (familiar) semantically paradoxical concepts. We agree, but maintain that such concepts are more properly classed as semantically *pathological*. What shows them to be so is the fact that we can construct cases that exhibit a telltale resistance to semantic characterisation, arising either in the form of an inconsistency, which *over-populates* the extension and anti-extension of a given predicate, or in the form of an indeterminacy, which *under-populates* the extension and anti-extension of that predicate. This *dual-symptom* nature of semantic pathology requires both a general diagnosis and, we maintain, a general treatment.

In this section, we set out semantic pathology and put forward an example that makes it clear. After doing so, we revisit the paradox of validity, showing it to be a special case of semantic pathology, which will both verify Read's claim and extend it, since it will class the *pathology* of validity as of a piece with that of other familiar concepts.

3.1 Semantic pathology explained

A central feature of pathological semantic discourse is a sort of *semantic malfunctioning*, traditionally classed with either inconsistency or indeterminacy. In order to see this, consider a *liar sentence*,

(L) (L) is false,

which yields inconsistency, if we attempt to characterise it semantically (either as true or as false). Given standard, "classical" logical and semantic assumptions, what (L) suggests is that our naïve semantic concepts exhibit *semantic pathology*, allowing for the construction of cases that *resist semantic characterisation* in virtue of the ordinary operation of the semantic predicate employed—specifically, in virtue of the principles that govern its use.[10] While resistance to semantic characterisation is a *mark* of semantic pathology, as we use the term, "semantic pathology" also indicates the *source* of this resistance, as opposed to just marking its presence.

Now, while variants of the liar appear to exhibit contradiction-inducing semantic pathology, other pathological cases, like the *truth-teller sentence*,

(K) (K) is true,

appear to manifest indeterminacy. We can ascribe either truth or falsity to (K), with (logical) impunity, though, *prima facie*, there seems to be nothing that favours (or could favour) ascribing one truth-value over the other. So, (K) too appears to resist semantic characterisation. Here, as with a liar sentence, the theorist identifies (K) as *prima facie* semantically pathological, on the basis of failed attempts at semantically characterizing it. But she now recognises the semantic pathology of the concept of truth as giving rise to both inconsistency and indeterminacy.

This, of course, is all familiar. What is perhaps not familiar is that, while (L) and (K) each separately exhibit one of semantic pathology's two symptoms, the disjunction of these symptoms arises in versions of (what we call) *the open pair*. This sort of case comes in both a basic symmetric version,

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[10] In the cases of "is true" and "is false" we have in mind the instances of the standard schemata: (T) "p" is true iff p, (F) "p" is false iff ~p.

(M) (N) is false
(N) (M) is false,

and a (strengthened) asymmetric variant,

(O) (P) is not true
(P) (P) is not true → (O) is not true.

As is clear, if we insist on ascribing matching truth-values to the members of the respective pairs, (M)/(N) and (O)/(P), the result is inconsistency. If, instead, we demand consistency, we can maintain this by making divergent truth-value ascriptions to the members of the respective pairs. But while this avoids inconsistency, it yields indeterminacy, as there are two equally good ways to make divergent ascriptions and nothing favouring one over the other. Thus, the variants of the open pair resist semantic characterisation by manifesting one or the other of semantic pathology's symptoms.

The important difference between the two cases just presented is that, while they manifest the two symptoms under the same conditions, the second undermines the *grounds* for certain treatments one might offer for the first. To begin, replacing "false" with "not true" thwarts (at least a simple) appeal to truth-value gaps as a solution to (M)/(N). More significantly, the second pair is asymmetric, and this thwarts an attempt to trade in the indeterminacy of the open pair for inconsistency alone via an appeal to the symmetry of (M) and (N)—each "saying" of the other what the other "says" of it. Symmetry suggests they should always have the same truth-value, and so only be inconsistent. But (O) and (P) are not symmetric, so the indeterminacy re-emerges. More generally, as we have shown elsewhere,[11] variants of the open pair appear to admit of *revenge problems*, given any proposed resolution of their pathology, whatever mode of semantic characterisation—gappy, glutty, etc.—one cares to endorse. Thus, the open-pair cases present genuine semantic pathology, as they unavoidably result in one or another of its symptoms, thereby showing our concept of truth to be *semantically unworkable*.

With the forgoing in mind, we return to the paradox of validity, which appears to exhibit the inconsistency of validity. If we can construct a dual-symptom case for this concept, we will show that validity, like truth, is actually semantically pathological. Such a case would consist of a pair of arguments that yield inconsistency, if we ascribe them the same *validity-status* (i.e., if we say either that both are valid or that both are not valid), and indeterminacy, if we demand a consistent validity-status ascription. The latter results from the fact that the only way to satisfy this demand is to make a divergent ascription (i.e., say that one is valid and the other is invalid), but there are two ways of doing this and no reason for picking one over the other. In fact, such a case is constructible, as we show in the next section.

3.2 Dual-symptom pathology

To connect the two symptoms of semantic pathology exhibited separately by arguments (I) and (II), consider the following pair of arguments, which we call *the validity open pair*:

$$\text{(III)} \quad \frac{1 = 1}{\therefore \text{(IV) is not valid}}$$

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[11] Armour-Garb and Woodbridge (2006), and Woodbridge and Armour-Garb (2005, 2007).

(IV) $\dfrac{1 = 1}{\therefore \text{(III) is not valid}}$

We can establish the pathological features of (III) and (IV), by assuming just conditions (1) and (2*). Inconsistency follows, if we ascribe (III) and (IV) the same validity-status, i.e., if we say that both are valid or that both are invalid. Suppose that both arguments are valid. Then, given that $1 = 1$, by (2*), both conclusions must be true (designated), in which case both arguments are not valid, i.e., invalid, yielding inconsistency. Thus, if the arguments are valid, they are invalid. Since, trivially, it is also the case that both arguments are invalid if both are invalid, it follows that if these arguments have the same validity-status, they are both invalid. This result yields that (III) is not valid and that (IV) is not valid. Recalling that we relied on the truth of the arguments' premises, in order to derive this, it follows that we can derive the conclusion of each argument, given its premise. By (1), this means that both arguments are also valid, if they have the same validity-status, again yielding inconsistency.

We can maintain consistency, if we ascribe (III) and (IV) divergent validity-statuses. For example, if we ascribe validity to (III) then, by (2*), it cannot have a designated premise and an undesignated conclusion. Since the premise is (necessarily) true then, if (III) is valid, the conclusion must likewise be true. And the conclusion is true, so long as (IV) is not valid. What would it take for (IV) to be not valid? Given the truth of its premise, by (2*), having an undesignated conclusion will suffice. What would make the conclusion undesignated? It would be undesignated, if it were false, and it is false if it is not the case that (III) is not valid—i.e., if (III) is valid, which is what we assumed. It is therefore perfectly consistent to maintain that (III) is valid and that (IV) is invalid. *Mutatis mutandis* for the ascription of validity to (IV), which comports with the invalidity of (III). Accordingly, while we *can* consistently make either divergent ascription, there is no obvious reason to pick one over the other, thereby yielding the indeterminacy of validity.[12]

Perhaps, in line with Buridan, and recognising that (III) and (IV) are symmetric, we should adhere to the demand: "treat likes alike", which thereby yields only inconsistency. If this were so, then what Read and others might conclude is that there is a problem with our concept of validity, akin to that plaguing other of our central semantic concepts, but this problem emerges in (I) already, independently of (III)/(IV). We might then conclude that, while perhaps interesting, this pair case is just a variant of the paradoxical case, which we must resolve anyway.

To be sure, if considerations of symmetry were compelling, (III)/(IV) would just amount to a variant of (I). But, even granting such considerations initially, they quickly become irrelevant, for, just as the basic open pair for truth extends to a (strengthened) "symmetry-breaking" case, the one for validity does too, which shows that an appeal to symmetry is not enough to resolve the dual-symptom condition of the validity open pair.

The revenge problem for an appeal to symmetry in the case of validity is (what we call) *the asymmetric validity open pair*,

(V) $\dfrac{1 = 1}{\therefore \text{(VI) is not valid or (V) is not valid}}$

(VI) $\dfrac{1 = 1}{\therefore \text{(V) is not valid or ((VI) is not valid and (V) is not valid)}}$

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[12] Similarly, we can assume that (III) is not valid, thereby yielding the validity of (IV), and *mutatis mutandis* in the other direction. We leave this as an exercise for the interested reader.

Arguments (V) and (VI) have the same features as (III) and (IV). If we take both to be valid, then, given the truth of the premises, by (2*), both must have true conclusions. Both conclusions are true so long as either (V) is not valid or both (V) and (VI) are not valid. Thus, if both are valid, it follows that at least (V) is also invalid—a contradiction. Since, trivially, (V) is invalid if both arguments are invalid, it follows that (V) is not valid if these arguments have the same validity-status. Given an assumption of matching statuses, it then follows that (VI) is not valid as well. In any case, from (V) being not valid it follows both that either (VI) is not valid or (V) is not valid and that either (V) is not valid or both (VI) is not valid and (V) is not valid. That is, the conclusions of both (V) and (VI) follow. Since we relied on the truth of these arguments' premises to arrive at this result—specifically, in deriving that (V) is invalid—this shows that assuming these arguments have the same validity-status, the conclusion of each follows, given that argument's premise. By (1), this means that if these arguments have the same validity-status, they are both valid. Any matching validity-status assignment, therefore, yields inconsistency.

Consistency can be maintained, if we ascribe (V) and (VI) divergent validity-statuses. Suppose that (V) is valid. If (V) is valid then the truth of its premise and (2*) yields that it must have a true conclusion. Given the assumption of (V)'s validity, this yields that (VI) is invalid. Again, (VI) having an undesignated conclusion suffices to make it invalid. The falsity of the conclusion suffices for this, and the conclusion of (VI) will be false, if (V) is valid. Thus, if we ascribe validity to (V), the invalidity of (VI) follows.

Suppose now that (VI) is valid. If so, then the truth of its premise and (2*) yields, via the truth of its conclusion and some (obvious) fiddling, that (V) is not valid. What would it take for (V) to be not valid? Again, having an undesignated (e.g., false) conclusion would suffice. Given the assumption of (VI)'s validity, the first disjunct of (V)'s conclusion is false, making the status of the whole conclusion turn on the truth-value of the second disjunct. If this disjunct is true, then the conclusion is true, but given what the disjunct "says", it follows that (V) is not valid nonetheless. If the second disjunct is false, then, given the falsity of the first disjunct, the conclusion as a whole will be false, which is sufficient for the invalidity of (V).[13] Thus, if we ascribe validity to (VI), the invalidity of (V) follows. So, either divergent assignment works fine, but, again, there is no reason to pick one over the other, thereby yielding indeterminacy. Furthermore, since (V) and (VI) are not symmetric,[14] an appeal to symmetry

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[13] The falsity of the second disjunct appears to yield that (V) is valid, but it then follows that (V) is invalid. Since (V) is invalid if it is valid and (trivially) if it is not valid, it follows that (V) is invalid. So, the invalidity of (V) also follows if the second disjunct is false, as well as if it is true.

[14] The conclusion of (V) "says" that (VI) is not valid or (V) is not valid, while the conclusion of (VI) "says" that maybe both (V) and (VI) are not valid, but (V) is not valid, in any case. The latter conclusion is logically equivalent to "(V) is not valid", but the former is not logically equivalent to "(VI) is not valid". To be sure, it follows from this last sentence, but it is easy to verify that they do not have the same truth-tables, which we take to be a necessary condition for the two sentences to "say" the same thing. Thus, the conclusion of (V) "says" something different from the sentence that "says" of (VI) what the conclusion of (VI) "says" of (V), even assuming that logical equivalence is sufficient for "saying" the same thing (which we doubt in any case). The upshot is that the conclusions of the two arguments do not "say" the same thing of each other. So, unlike (III) and (IV), (V) and (VI) are not symmetric.

does not eliminate this indeterminacy and reduce (V)/(VI) to a case of inconsistency alone.

Accordingly, given the acceptance of conditions (1) and (2*), (I), (II), and the pairs, (III)/(IV) and (V)/(VI), show that, as in the case of truth, if validity is inconsistent, then it is semantically pathological in the broader sense we have explained here. Our concept of validity allows for cases that at least appear to resist semantic characterisation through manifesting not only inconsistency, but also indeterminacy, and both forms of resistance together in a single case. This last possibility indicates that, if genuine, these two forms of resistance are but two symptoms of a single underlying condition, which *prima facie* calls for a single treatment of validity's pathological features.[15]

4 Concluding remarks

Read (2001, p. 193) ends his discussion of the paradox of validity by classing it with other, familiar, semantic paradoxes, linked in virtue of their self-reference, direct or otherwise. We agree with the grouping but maintain that what the open-pair cases for truth and validity show is that inconsistency and indeterminacy are dual symptoms of semantic pathology. Resolving this (more fundamental) problem demands a resolution of both of its symptoms. More generally, we take variants of the open-pair to function as a sort of a revenge problem, one that thwarts attempts to resolve the standard cases, given their continued resistance to semantic characterisation. Moreover, this appears to be so, whether one attempts to extend the range of possible semantic characterisations, by appealing to gaps, gluts or both gaps and gluts.

In closing, we maintain that the triplet of paradox, indeterminacy, and a pair that binds them indicates that truth and validity are semantically pathological concepts. If our conclusions here are correct then there is then a further question about the scope of the problem we have identified. Is it limited to these two concepts, or does it infect others as well? In fact, we believe that truth and validity are not alone here, for we can generate open pair variants for myriad concepts, which, given the cases, appear to manifest both inconsistency and indeterminacy, and in precisely the way that truth and validity do. Our general hypothesis is that this indicates, at least *prima facie*, that semantic pathology infects a surprisingly broad range of notions, including: reference, predicate-satisfaction, knowledge, rationality, and others.[16] We see this as underwriting a demand for a general solution to the semantic pathology exhibited by many of our central philosophical (including several putatively *non*-semantic) concepts, though we will have to table this larger claim for another time.[17,18]

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[15] We would like to thank two anonymous referees for urging us to clarify the section on the asymmetric validity open pair.

[16] We lay out a broad spectrum of cases in Woodbridge and Armour-Garb (2007), some more tentatively than others. See the appendix below for a table summarizing the cases we think are at least worth considering.

[17] See Armour-Garb and Woodbridge (2007).

[18] We would like to thank Stephen Read for comments on an earlier draft.

Appendix: semantic pathology—the cases

Inconsistency

- (L) (L) is false

- (L+) (L+) is not true

- (1) (2) is not true
  (2) (1) is true

- (S_n) For all $k > n$, (S_k) is not true

- (C) (C) is true $\rightarrow \perp$

- Set (A):
  (3) The Earth is round.
  (4) The Moon is cheese.
  (5) Set (A) contains an odd number of true claims

Indeterminacy

- (K) (K) is true

- (K+) (K+) is not false

- (1′) (2′) is true
  (2′) (1′) is true

- (S_n) (S_{n+1}) is true

- (C′) (C′) is false $\rightarrow \perp$

- Set (A′):
  (3′) The Earth is round.
  (4′) The Moon is cheese.
  (5′) Set (A′) contains an even number of true claims

Inconsistency (Non-Alethic)

- "The least number not denotable in less than 18 syllables"

- "... is heterological" or "... does not satisfy itself"

- (I) $\dfrac{1 = 1}{\therefore \ \text{Argument (I) is not valid}}$

- (G) (G) is not known.

- (A) (A) is not assertible.

- (E) (E) is not rationally endorsable.

- (O) Do not obey (O)!

Indeterminacy (Non-Alethic)

- "The greatest number denotable in less than 18 syllables"

- "... is autological" or "... does satisfy itself"

- (II) $\dfrac{1 = 1}{\therefore \ \text{Argument (II) is valid}}$

- (G′) (G′) is known.

- (A′) (A′) is assertible.

- (E′) (E′) is rationally endorsable.

- (O′) Obey (O′)!

Dual-Symptom Cases

The Open Pair:

- (6) (7) is false
  (7) (6) is false

The Strengthened Open Pair:

- (8) (9) is not true
  (9) (8) is not true

The Curry Open Pair:

- (C1) (C2) is true $\rightarrow \perp$
  (C2) (C1) is true $\rightarrow \perp$

The Asymmetric Open Pair:

- (10) (11) is false
  (11) (11) is false $\rightarrow$ (10) is false

The Strengthened Asymmetric Open Pair:

- (12) (13) is not true
  (13) (13) is not true $\rightarrow$ (12) is not true

The Asymmetric Curry Open Pair:

- (C3) (C4) is true $\rightarrow \perp$
  (C4) [(C4) is true $\rightarrow \perp$] $\rightarrow$ [(C3) is true $\rightarrow \perp$]

Dual-Symptom Cases (Non-Alethic)

The Reference Open Pair:

- (N1) The thing(s) not referred to by the term labeled "(N2)"

  (N2) The thing(s) not referred to by the term labeled "(N1)"

The Satisfaction Open Pair:

- (P1) ... does not satisfy the predicate labeled "(P2)"

  (P2) ... does not satisfy the predicate labeled "(P1)"

The Validity Open Pair:

- (III) $\dfrac{1=1}{\therefore \ \text{(IV) is not valid}}$

  (IV) $\dfrac{1=1}{\therefore \ \text{(III) is not valid}}$

The Knowledge Open Pair:

- (G1) (G2) is not known.
  (G2) (G1) is not known.

The Assertibility Open Pair:

- (A1) (A2) is not assertible.
  (A2) (A1) is not assertible.

The Rationality Open Pair:

- (E1) (E2) is not rationally endorsable.
  (E2) (E1) is not rationally endorsable.

The Obligation Open Pair:

- (O1) Do not obey (O2)!
  (O2) Do not obey (O1)!

The Asymmetric Reference Open Pair:

- (N3) The thing(s) not referred to by the term labeled "(N4)"

  (N4) The thing(s) either referred to by both (N3) and (N4) or not referred to by (N3)

The Asymmetric Satisfaction Open Pair:

- (P3) ... does not satisfy the predicate labeled "(P4)"

  (P4) ... satisfies both (P3) and (P4) or does not satisfy (P3)

The Asymmetric Validity Open Pair:

- (V) $\dfrac{1=1}{\therefore \ \text{(VI) is not valid or (V) is not valid}}$

  (VI) $\dfrac{1=1}{\therefore \ \text{(V) is not valid or ((VI) is not valid and (V) is not valid)}}$

The Asymmetric Knowledge Open Pair:

- (G3) (G4) is not known.
  (G4) (G4) is not known → (G3) is not known.

The Asymmetric Assertibility Open Pair:

- (A3) (A4) is not assertible.
  (A4) (A4) is not assertible → (A3) is not assertible.

The Asymmetric Rationality Open Pair:

- (E3) (E4) is not rationally endorsable.
  (E4) (E4) is not rationally endorsable → (E3) is not rationally endorsable.

The Asymmetric Obligation Open Pair:

- (O3) Do not obey (O4)!
  (O4) Obey (O4) or do not obey (O3)!

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