Take a Ride on a Time Machine*

John Earman, Christopher Smeenk, and Christian Wüthrich

1. Introduction

Early in his career Arthur Fine made contributions to the philosophy of space and time. But he was so thoroughly seduced by the *verdammt* quantum that he never returned to this field. Although our project on the twin topics of time travel and time machines cannot take any specific directions from Arthur's pronouncements on these topics–for there are none–we have been guided by his views on the relation between science and the philosophy of science. In the "Afterword" to *The Shaky Game* (1986, 1996) Arthur comments on how his Natural Ontological Attitude (NOA) impacts the agenda of philosophy. To pursue NOA

> means to situate humanistic concerns about the sciences within the context of ongoing scientific concerns, to reach out with our questions and interests to scientist's questions and interests–and to pursue inquiry as a common endeavor ... For me it has frequently meant learning new science in order to pursue standard topics on the philosophical agenda. (1996, 174-175)

The central message here can be divorced from the controversial aspects of NOA: in a nutshell, Arthur is telling us to see philosophy of science as continuous with science. In his PSA Presidential Address, "Interpreting Science" (1988), Arthur spells out what this means in concrete terms: it means that philosophers of science should seek to "engage with active science in a more or less equal basis" (p. 9), that they should see themselves as "engaged cooperatively with scientists" (p. 10). This is a high standard which few of us can meet with the flair exhibited in Arthur's own work. But the failure to strive to conform to this standard makes for bad philosophy of science, as is attested all too clearly by the philosophical literature on time travel and time machines.[1]

The bulk of this literature ignores Arthur's advice altogether, and much of the rest pays lip service–e.g. by citing Gödel's cosmological model–and then turns inward to pursue philosophical gymnastics on the "grandfather paradox" and other paradoxes of time travel. The reader is left with the impression of a game played in the pages of philosophy journals, a game whose rules are unspecified save for the imperative to strive for ever more clever examples and counterexamples. Our interest in the topics of time travel and time machines was aroused by the fact that they are actively discussed in some of the leading physics journals and at some of the major physics conferences.[2] In line with Arthur's advice, we want to make common cause with the physicists in using time travel and time machines to probe the foundations of various theories of modern physics and in doing so to illuminate the concept of time as it is embodied in these theories. Whether or not Arthur agrees with the conclusions of our investigation, we hope he will approve of the manner in which it is conducted.

The central issue to be discussed here can be simply, if crudely, stated: Is it possible to build and operate a time machine, a device that produces closed timelike curves (CTCs)? [3] Our goals are, first, to specify the spacetime structure needed to implement a time machine and, second, to assess attempted no-go results against time machines. Section 2 reviews the status of CTCs in the light of the paradoxes of time travel. Our position is that these paradoxes do not threaten either the conceptual coherency or the physical possibility of time travel. And we want to emphasize that the objections to time machines to be discussed here are independent of the paradoxes of time travel since they arise out of the physics of what transpires *before* the CTCs appear. Section 3 surveys attempts to give necessary and/or sufficient conditions for time machines. We reject various key conditions that appear frequently in the physics literature, and we show why it is so difficult to specify an interesting sense in which a device operating in a relativistic spacetime can be said to be responsible for the development of CTCs. In the light of these melancholy conclusions we review in Sections 4-7 purported no-go results for time machines that come from classical GTR, semi-classical quantum gravity, quantum field theory on curved spacetime, and Euclidean quantum gravity. Our conclusions are presented in Section 8.

2. Review of time travel

Many things can be meant by the question "Is time travel possible?" For present purposes we restrict ourselves to the issue of whether a spacetime structure containing CTCs is physically possible. This issue can be given two senses, corresponding to a weak and a strong sense of physical possibility. Time travel will be said to be *physically possible in the weak sense* just in case the laws L (of the actual world) are such that CTCs are compatible with L, or, in possible worlds talk, just in case there is a possible world in which there are CTCs and in which L are true. Time travel will be said to be *physically possible in the strong sense* just in case the laws L (of the actual world) are such that there is a possible world in which there are CTCs and in which L are not only true but are laws. If the laws L are the laws of GTR, then time travel is physically possible in the weak sense, for there are many known solutions to Einstein's field equations (EFE) which satisfy various energy conditions,[4] thought to be necessary for physically realistic matter-energy sources, and which contain CTCs. The issue of physical possibility in the strong sense is more delicate and controversial because it turns on one's conception of laws of nature. If, for example, one wants an empiricist conception of laws on which the laws supervene on the occurrent regularities, then the weak sense of physical possibility does not entail the strong sense since lawhood is not necessarily transmitted down the chain of nomological accessiblity. Consider, for sake of illustration, David Lewis's (1973) "best systems analysis" of laws,[5] and suppose that the best system for the actual world is axiomatized by the postulates of GTR. There is no guarantee that these postulates will be among the axioms or theorems of the best system for a possible world that contains CTCs and that satisfies these postulates. Nevertheless, we would be very surprised if there were not *some* possible worlds that correspond to solutions to EFE with CTCs and that return the laws of GTR when the best systems analysis is applied to them. On other analyses of laws, such as Carroll's (1994), the present issue doesn't arise since the weak sense of physical possibility entails the strong sense. The price to be paid for obtaining this entailment is the failure of laws to be Humean supervenient, a price we think is too high; but that is a topic for another occasion.[6]

If it is so obvious that time travel is physically possible in the weak sense and (probably) also in the strong sense, what is one to make of the discussions in the philosophical literature on the paradoxes of time travel, which are often taken to threaten the conceptual coherency of time travel? We view these paradoxes as a (not very perspicuous) way of calling attention to a phenomenon long known to physicists; namely, in the presence of CTCs, consistency constraints can arise for initial data. In order to explain the point let us begin with a familiar situation in Minkowski spacetime. Choose a *time slice* Σ (i.e. a spacelike hypersurface) which may be either local or global (in the latter case Σ has no edges). One hopes and expects that the specification of the relevant initial data on Σ for some field ψ will determine a unique solution of the field equations for ψ, at least for the *domain of dependence* $D(\Sigma)$ of sigma. ($D(\Sigma)$ is the union of the *future domain of dependence* $D^+(\Sigma)$ of Σ and the *past domain of dependence* $D^-(\Sigma)$ of Σ, which are defined respectively as the set of all spacetime points p such that every timelike curve which passes through p and which has no past (respectively, future) endpoint meets Σ in one point. Σ is said to be a *Cauchy surface* if $D(\Sigma)$ is the entire spacetime.) This hope will be frustrated if singularities in ψ can develop from regular initial data. For a massless scalar field obeying the Klein-Gordon equation and for a source-free electromagnetic field obeying Maxwell's equations, global existence and uniqueness results do hold for Minkowski spacetime. In what follows we implicitly restrict ourselves to such fundamental fields that have this nice behavior in Minkowski spacetime.

Now consider a general relativistic spacetime M, g_{ab}. Treat it as a fixed background on which a test field ψ propagates. Again choose a time slice Σ and specify on Σ the relevant initial data for the field ψ. Even if M, g_{ab} is Minkowski spacetime this initial data may not be freely specifiable since, in general, the relativistic field equations governing ψ imply constraints on the initial data. Call these the *constraint laws.* (As an example, the source free Maxwell equations for electromagnetism imply the vanishing of the divergence of the electric and magnetic fields, which places constraints on initial data.) But if M, g_{ab} contains CTCs it may not be the case that every initial data set for ψ satisfying the constraint laws gives rise to a globally consistent solution on all of M, g_{ab} of the field equations for ψ; and this failure can manifest itself even if, as is the case in all of the examples we will consider, the equations of motion are of a purely local character.[7] The additional constraints, over and above the normal constraint laws, that are needed to guarantee the existence of a global solution are called the *consistency constraints*. A trivial but instructive illustration is obtained by rolling up two-dimensional Minkowski spacetime along the time axis (see Fig. 1). Consider the matter field for point mass particles. Suppose that the equations of motion specify that the particles move along inertial trajectories except when they collide, in which case they undergo an elastic bounce. Take the initial data on a global time slice Σ to specify that a single particle is present. The consistency constraints for such data are very severe, for unless the initial velocity of the particle is chosen so that the inertial trajectory it picks out smoothly joins to itself after a loop around the cylinder, the initial specification will be contradicted.[8]

Both the philosophical and physics literature contain assertions to the effect that there are no genuine paradoxes of time travel. If this is taken to mean that the grandfather paradox (Kurt travels into his past, kills his grandfather before granddad sires Kurt's father, thus preventing Kurt from making the journey) and its ilk do not demonstrate that time travel is logically or conceptually absurd, then we agree wholeheartedly. For, to repeat, such paradoxes are but crude illustrations of the presence of consistency constraints in spacetimes with CTCs; and the presence of such constraints do not show CTCs are physically impossible in either the strong or weak sense. But sometimes the assertion that there are no genuine paradoxes of time travel is taken to mean that non-trivial consistency constraints are absent (see, for example, Krasnikov 1997a). In unqualified terms, such assertions are patently false, as illustrated not only by the above artificial example but by many other examples of self-interacting and non-self-interacting fields on a variety of general relativistic spacetimes with CTCs. There is, however, a qualified sense in which this assertion points to an interesting truth.

Define the *chronology violating region* V of a spacetime M, g_{ab} to be the set of all points

$p \in M$ such that there is a CTC through p. A *vicious* spacetime M, g_{ab} is one such that $V = M$; a *virtuous* spacetime is one such that $V = \varnothing$.[9] Between the vicious and the virtuous, V never consists of merely isolated points, for if $V \neq \varnothing$, then the interior of V is non-empty. Now if the slice Σ intersects V then one expects that consistency constraints arise. (In some vicious spacetimes, such as Gödel spacetime, there are no global time slices, although, of course, there will be local slices.) However, if $\Sigma \cap V = \varnothing$, then it may happen that no consistency constraints arise for Σ, and this can be so even though $J^{+}(\Sigma) \cap V \neq \varnothing$. (Here $J^{+}(X)$ denotes the *causal future* of X, i.e. the set of all $p \in M$ such that there is a future directed causal curve from X to p. The *causal past* $J^{-}(X)$ of X is defined analogously.) Of course, since Σ cannot be a Cauchy surface even if it is a global slice, global determinism cannot be expected to hold in that the initial data for fields on Σ may not in general determine a unique future development. But it is interesting and, perhaps, surprising that every initial data set satisfying the constraint laws on Σ can give rise to at least one globally consistent development.[10] Indeed, we do not know of any counterexample to the conjecture that for any relativistic spacetime M, g_{ab} admitting a global time slice Σ and for any fundamental field ψ, there are no consistency constraints on ψ for Σ if $\Sigma \cap V = \varnothing$; and a result of Krasnikov (2002), which will be discussed below, can perhaps be adapted to prove this conjecture for a broad range of cases. Anticipating the discussion in the following section, our characterization of time machines implies that there is a time slice such that $\Sigma \cap V = \varnothing$. Thus, there is a clear sense in which the so-called paradoxes of time travel do not affect time machines. The objection to such devices must come from other considerations.

Our implicit moral here is that the hand wringing in the philosophical literature over the paradoxes of time travel hasn't advanced the subject very far; the interesting action is in the details of the physics. There remains, however, a perplexing philosophical issue: What is the status of the consistency constraints? If they have the status of additional laws, over and above the basic laws of motion for the fields at issue, then we have a satisfying explanation of why the constrained away data cannot arise--they are forbidden by the laws. If not, we are left with an unsatisfying *reductio* explanation--they cannot arise because if they did a consistent global solution would not be possible. We will not attempt to resolve this issue since it would sidetrack us into the vexed question of what is a law of nature. But we do want to make one comment that leads to an unsettling perspective on the notion of consistency constraint.

As understood above, consistency constraints are consequences of the laws of motion plus the spacetime structure. Since the latter is treated as a fixed background, is it not then fair to say that, as consequences of the basic laws, the consistency constraints are themselves laws? But, of course, one of the lessons of GTR is that spacetime is not fixed and that its structure covaries with the matter-energy content. Treating a field as a test field on a fixed spacetime can be justified only as an approximation which is useful for some purposes. For other purposes it may be misleading. So let us attempt to remove the approximation. On Σ put some initial data for the ψ field which obey the constraint laws but which would be forbidden by the consistency constraints if the spacetime were treated as a fixed background; then remove the idealization of a fixed background spacetime and let this data evolve according to the coupled Einstein-ψ field equations. There will be a unique (upto diffeomorphism) development for which Σ is a Cauchy surface. There are then two possibilities. (a) This development may be maximal simpliciter,[11] which means CTCs do not arise from the initial data. (b) This development may not be maximal simpliciter, in which case CTCs may be present in some or all of its extensions. But, of course, by construction every extension, including ones with CTCs, is consistent. Either way, the consistency constraints have vanished. No doubt, some way can be found to reintroduce consistency constraints using counterfactual discourse. But our guess is that the truth values of such counterfactuals will be so heavily dependent on contextual factors that the notion of consistency constraints is rendered uninteresting. But if consistency constraints fade away, so too do the "paradoxes of time travel." Is this then the way to a final dissolution of the paradoxes? The answer to this question does not affect the discussion to follow since the no-go results for time machines do not use or mention the paradoxes of time travel.

3. Operating a time machine

That time travel is physically possible--that there are physically possible worlds (in either the weak or strong sense) with CTCs--does not necessarily mean that it is physically possible to operate a time machine. We now want to try to get a grip on what the operation of a time machine would mean in terms of relativistic spacetime structure, over and above the existence of CTCs. In the first place, if the spacetime M, g_{ab} is to accommodate a time machine we want to be able to choose a global slice Σ, corresponding to a time before the time machine goes into operation, such that no funny causal structure exists at "time" Σ; for otherwise we would have no need of a time machine to produce it. One way to make this precise is to require that Σ is a *partial Cauchy surface*, i.e. no future directed timelike curve intersects Σ more than once (although, of course, there will be inextendible causal curves that never intersect Σ unless it is a Cauchy surface). If a global time slice Σ exists, then the requirement that Σ is a partial Cauchy surface can always be met by passing, if necessary, to a covering spacetime. Next we want to set the data on Σ so that something that can reasonably be called time machine goes into operation. And whatever the details, the time machine is supposed to be confined to a finite region of space and to operate for a finite amount of time. Putting together these desiderata, we are led to the requirement that there is some compact spacetime region $K \subset D^+(\Sigma)$, which we will call the *time machine region*, such that as a result of the happenings in K, CTCs evolve. So, at a minimum, we want the chronology violating region V to satisfy $V \subset J^+(K)$ (see Fig. 2).

This is very sketchy, but even without filling in details two difficulties should be apparent. The first is that no matter how this scenario is filled in, it will not yield the kind of time travel beloved by science fiction writers. In order to rule out that CTCs already existed before the time machine was set into motion we required that Σ be a partial Cauchy surface. This rules out time travel to the past of the initial time Σ since no future directed causal curve departing from the future side of Σ can enter $J^-(\Sigma)$ (otherwise Σ would not be a partial Cauchy surface). Time travel is thus confined to the future of this instant. We see no way around this conundrum as long as time machines are to be implemented in orthodox relativistic spacetimes.

A second difficulty lies in making sense of the notion that CTCs are the "result of" the operation of the time machine. This cannot be taken in the sense of causal determinism since V always lies outside $D^+(\Sigma)$. Nor does it help to point to our condition that $V \subset J^+(K)$. That condition only guarantees that every point in the time travel region can be causally influenced by the operation of the time machine in K, and it does not rule out the possibility that the region V is influenced, perhaps decisively, by influences not emanating from K. It would seem desirable, then, to require also that $J^-(V) \cap \Sigma$ has compact closure. This guarantees that any influences on the time travel region emanating from Σ must originate from a finite portion thereof, which is certainly what we want since the conditions on Σ specify the operation of the time machine. The trouble is that the condition in question does not assure that there aren't other influences, perhaps decisive, on V which originate from outside $J^-(V) \cap \Sigma$ or even from outside of all of Σ. The worry is illustrated by the simple but contrived example of Fig. 3. The missing point represents a singularity from which causal influences, not determined by the conditions on Σ, can emerge and propagate to effect V, even though $J^-(V) \cap \Sigma$ has compact closure. This trivial singularity can be made irremovable, without changing the causal structure pictured in Fig. 3, by replacing the metric g_{ab} by $\Phi^2 g_{ab}$ where Φ is a scalar field that goes to zero as the missing point is approached. Alternatively, if Φ is chosen to blow up as the missing point is approached, the singularity can be made to disappear (at least by the criteria that identify the presence of a singularity by geodesic incompleteness or the like.) But in that case V is subject to influences coming from "infinity."

Hawking (1992a) presumably had in mind these sorts of difficulties when he imposed the requirement that the *future Cauchy horizon* $H^+(\Sigma)$ of Σ be *compactly generated* (see also Hawking 1992b). $H^+(\Sigma)$ is the future boundary of $D^+(\Sigma)$. (More precisely, $H^+(\Sigma)$ is defined as $D^+(\Sigma) - I^-(D^+(\Sigma))$, where $I^-(X)$ denotes the chronological past of X, i.e. the set of all points p such that there is a non-trivial past directed timelike curve from X to p.) This surface, which separates the part of spacetime that is causally determined by conditions on Σ from the part that is not, is necessarily a null surface, and its generators are null geodesics. If these generators, when traced far enough into the past, fall into and remain in a compact set, then the Cauchy horizon is said to be compactly generated. This precludes the possibility of the generators emerging from a curvature singularity or coming from infinity and, thus, rules out the example of Fig. 3, whether or not it is doctored by the addition of a conformal factor; for no neighborhood of the missing point has compact closure, and thus every past directed generator of $H^+(\Sigma)$ will eventually emerge from any compact set.[12]

It is not clear at the outset whether or not Hawking's condition is either necessary or sufficient for the development of CTCs to be *due*--in a sufficiently strong sense--to the time machine. One obvious and seemingly attractive way to obtain such an assurance is to require that every suitable extension of $D^+(\Sigma)$ contains CTCs (call this the *Potency Condition* for the time machine). This remains a schema rather than a definite condition until "suitable extension" is defined. At a minimum the extension must be smooth and maximal (i.e. not further smoothly extendible). The more additional restrictions that are put on the allowed extensions, the weaker the potency. There is little hope, however, that the strongest form of the Potency Condition is ever realized. To see the reason for the pessimism, consider the case of Misner spacetime (see Fig. 4a), a two-dimensional spacetime which captures some of the causal features of Taub-NUT spacetime, itself a homogeneous vacuum solution to EFE. Misner spacetime is topologically $S^1 \times \mathbb{R}$. In the lower, or Taub region, the surfaces of homogeneity, such as Σ, are spacelike. But because the light cones "tip over," the surfaces of homogeneity eventually become more and more lightlike, until eventually one reaches $H^+(\Sigma)$, which is not only compactly generated but compact. (In the case of four-dimensional Taub-NUT spacetime, $H^+(\Sigma)$ is non-compact but is compactly generated.) If the Potency Condition is going to be satisfied in a strong form, Misner spacetime would seem to be an ideal candidate. But not every smooth maximal extension of the Taub region $D^+(\Sigma)$ contains CTCs; nor does every such extension have a compact or compactly generated $H^+(\Sigma)$. A smooth maximal extension sans CTCs and sans compactly generated $H^+(\Sigma)$ can be constructed by cutting out vertical strips as shown in Fig. 4b and slapping on appropriate conformal factors that preserve the causal structure and assure maximality by "blowing up" as the cut-out regions are approached. Requiring the satisfaction of EFE and energy conditions will presumably rule out all such constructions. But a CTC free maximal extension can still be constructed by the alternative method of taking the universal covering spacetime of the spacetime in Fig. 4a.[13] To rule out such artifices we require that the extension be *hole free*. A spacetime M, g_{ab} is said to be hole free just in case for any spacelike $\Sigma \subset M$ (not necessarily a global time slice) there is no isometric imbedding i: $D^+(\Sigma) \to M'$ into a spacetime M', g'_{ab} such that $i(D^+(\Sigma))$ is a proper subset of $D^+(i(\Sigma))$. That it is not question begging to require hole freeness in the present context follows from two considerations. First, we are seeking, so to speak, the next best thing to the notion that the operation of the time machine causally determines that CTCs emerge. And, second, causal determinism itself has no prayer of being satisfied unless hole freeness is required (see Earman 1995, Sec. 3.8). In sum, our proposed version of the Potency Condition is that every extension of $D^+(\Sigma)$ contains CTCs if it is smooth, maximal, hole free, and satisfies EFE and energy conditions.

Taking the Potency Condition as the key to seeing a time machine at work might be criticized on the grounds that it mixes two issues: What is a time machine? vs. Is a time machine physically possible? But if we are right these two issues cannot be separated.

To see the bite of the Potency Condition, consider a proposal of Krasnikov (1998a) for defining relativistic time machines.

<u>Definition</u>. Let M, g_{ab} be an inextendible acausal spacetime. $L_M \subset M$ is said to be a *Krasnikov time machine* iff (i) the chronology violating region V ($\neq \varnothing$) is a subset of L_M, and

(ii) $M - J^+(L_M)$ is isometric to $M' - J^+(L_{M'})$, where M', g'_{ab} is a spacetime that does not contain CTCs and $L_{M'} \subset M'$ is compact.

Krasnikov (1998a) exhibits a spacetime which contains a time machine in his sense, which is singularity free, and which satisfies the weak energy condition (the significance of this condition is discussed in the following section). The construction proceeds in two steps. First, construct a Politzer spacetime (see Fig. 5a) by deleting the points p_1-p_4 from Minkowski spacetime and then gluing together the strips as shown. Because of the missing points the spacetime is singular in the sense of geodesic incompleteness.[14] This is taken care of in the second step. The introduction of an appropriate conformal factor leaves the causal structure the same but makes the spacetime geodesically complete (and, indeed, b-complete[15]) without violating the weak energy condition. The Def. is satisfied if L_M is chosen to be the causal future of the shaded rectangle shown in Fig. 5a and $L_{M'}$ is chosen to be the shaded region of Minkowski spacetime shown in Fig. 5b. But this reveals the defect of the definition: there is no separation between the operation of the supposed time machine and the CTCs it is supposed to produce. To overcome this defect one could modify the Def. to require that $V \subset J^+(L_M)$ and that L_M, like $L_{M'}$, has compact closure. The modified definition is satisfied if L_M is chosen to be the shaded rectangle in Fig. 5a rather than its causal future. But now another glaring defect emerges: there is no reasonable sense in which this new L_M is a time machine. Up to "time" Σ', which is after the alleged time machine has operated, the Politzer spacetime and the corresponding portion of Minkowski spacetime (Fig. 5b) are isometric. So there is no reasonable sense in which the CTCs in Politzer spacetime are "due to" the conditions in the time machine region L_M--or for that matter to the entirety of the conditions prior to Σ', which may be chosen to lie as close to the chronology violating region V as desired. For no interesting form of the Potency Condition is fulfilled in this case since there are extensions beyond Σ'--e.g. to Minkowski spacetime--which do not contain CTCs but which satisfy every condition that could be reasonably demanded of a suitable extension.

If the Potency Condition is accepted as the key to seeing a time machine at work, then one could seek to motivate Hawking's condition of compactly generated future Cauchy horizons by proving a form of the conjecture that, in some broad and interesting class of spacetimes, when $H^+(\Sigma)$ is compactly generated, the Potency Condition is realized for $D^+(\Sigma)$. The seeming plausibility of such a conjecture follows from the fact that strong causality is violated on the horizon if it is compactly generated.[16] (*Strong causality* is violated at a point p if, intuitively, there are almost closed causal curves near p; more precisely, there is a neighborhood of p such that every subneighborhood has the property that some causal curve intersects it more than once.) This violation can be taken as an indication that the seeds of acausality have been planted in $D^+(\Sigma)$ and are ready to bloom in the form of CTCs in any non-artificially constructed maximal extension.

At first blush the suggested approach to identifying the operation of a time machine by means of the Potency Condition is threatened by a result of Krasnikov (2002) showing that every time oriented spacetime M, g_{ab} has a maximal time oriented spacetime such that any CTC in the extension lies to the chronological past of the image of M in the extension. Furthermore, the construction allows local conditions on the metric to be carried over to the maximal extension. But at second glance it is not at all obvious that Krasnikov's construction guarantees that his maximal extension will violate our preferred form of the Potency Condition. And unless the guarantee is made good, his result does not contradict the intuition that in, say, the case of Misner spacetime, the "tipping over" of the light cones in the lower region means that CTCs will develop unless interdicted by means that violate one of the conditions in the Potency Condition—e.g. inserting barriers consisting of cut-out regions of spacetime and then preventing these missing regions from being restored in a maximal extension by taking covering spaces (which can mean that the extension is not hole free) or by slapping on a conformal factor that "blows up" as the missing regions are approached (which can mean that the maximal extension violated energy conditions or EFE).

We conjecture that our preferred version of the Potency Condition is not doomed to failure by the Krasnikov construction and similar constructions. If this conjecture is false, then those who want to see time machines at work in general relativistic spacetimes appear to have two options. First, the list of conditions that pick out a "suitable" extension could be strengthened--and, thus, Potency Condition weakened--so that it can be fulfilled in some general relativistic spacetime models. Second, the Potency Condition could be abandoned as the key to seeing a time machine at work in favor of the idea that a device deserves to be called a time machine if its operation raises the chances of the appearance of CTCs. As for the first option, we confess that we do not see any non ad hoc implementation. As for the second option, the implementation would start by defining a measure on the class of relevant solutions to EFE and then proceed to proving that within the subclass of solutions in which the would-be time machine operates the measure of the set extensions containing CTCs is larger than in the subclass where the machine does not operate. No doubt this can be done. But it is of little avail unless the measure can be justified as reflecting objective chances. For classical GTR it is hard to see how such a justification can be given without the help of a metaphysical extravagance that is tacked onto the theory, e.g. the Creator chooses which extension to actualize by throwing into the solutions to EFE a cosmic dart whose pattern of hits follows the contemplated measure.

We proceed in the hope that our conjecture is correct. But we also proceed with the realization that it may be false, in which case we doubt that that there is any coherent and interesting content to the notion of a time machine operating in a classical spacetime of GTR.

4. No-go results for time machines in classical GTR

In view of the fact that "time machine" does not present a definite target in classical GTR, it might seem that it is hopeless to try to prove impossibility or no-go results for time machines in this setting. However, the program of producing no-go results can be pursued by proving what can be labeled *chronology protection theorems* that take the form: "If no CTCs are present to begin with, then CTCs cannot emerge without violating conditions (CP)," where the chronology protection conditions (CP) have to be justified either on the grounds that they are necessary for seeing a time machine in operation or else that they are conditions that must be satisfied in any physically reasonable model of GTR. (Hawking's (1992) original version of the *chronology protection conjecture* stated that: "The laws of physics prevent the appearance of closed timelike curves.") We suggest that much of the rather sizable and growing physics literature on time machines be interpreted in exactly this fashion. In this section we review no-go results proved within classical GTR. In sections 5, 6, and 7 we take up results that rely respectively on semi-classical quantum gravity, quantum field theory on curved spacetime, and Euclidean quantum gravity. But before embarking on this odyssey we remark that from the perspective of the

Potency Condition analysis of time machines, chronology protection theorems are overkill—if no suitable extensions contain CTCs then necessarily some suitable extensions don't contain CTCs, but establishing the latter without establishing the former is sufficient to ground time machines.

We first review some no-go results due to Stephen Hawking. If Σ is a partial Cauchy surface and $H^+(\Sigma)$ is compact, then Σ must be compact (Hawking 1992a and Chrusciel and Isenberg 1993). This is a purely geometrical result.[17] Thus, whether or not EFE or energy conditions hold, if Σ is non-compact (i.e. the universe is spatially open), then $H^+(\Sigma)$ is non-compact. But can $H^+(\Sigma)$ nevertheless be compactly generated when Σ is non-compact? Hawking (1992a) showed that the answer is no under plausible assumptions. Specifically, he proved that if Σ is non-compact and if EFE and the weak energy condition (WEC) are satisfied, then $H^+(\Sigma)$ cannot be compactly generated.

Recall that the WEC requires that the stress-energy tensor T_{ab} of the source fields satisfies $T_{ab}V^aV^b \geq 0$ for all timelike vectors V^a. In the case of perfect fluid with energy density μ and pressure p, this requirement is satisfied if $\mu \geq 0$ and $\mu + p \geq 0$, which one normally expects to hold. However, Vollick (1997) has shown how the WEC can be violated using a charged dust interacting with a scalar field. Although the dust and the scalar field separately have positive energy densities, the interaction part of the stress-energy tensor can have negative energy densities. Using this fact, Vollick shows that it is possible to build a sphere of charged dust with negative energy density throughout the interior.[18]

But apart from doubts about the WEC, the crucial issue is whether the condition of compactly generated Cauchy horizons can be justified as a necessary condition for the operation of a time machine since it certainly cannot be justified as a condition that any physically reasonable model of GTR must fulfill. By our lights the justification would have to proceed by showing that when $H^+(\Sigma)$ is not compactly generated a strong and interesting form of the Potency Condition necessarily fails. This seems implausible, for there is no evident reason why, in the generic case of a four-dimensional inhomogeneous spacetime, it cannot happen that the behavior of the metric in one region of $D^+(\Sigma)$ implies that all smooth, maximal, and hole free extensions that satisfy EFE and energy conditions contain CTCs (because, for example, the light cones are "tipping over" in an appropriate way in the said region of $D^+(\Sigma)$), while overall the behavior of the metric implies that $H^+(\Sigma)$ is not compactly generated in some or all such extensions (because, for example, naked curvature singularities develop in other regions). Of course, this objection falls flat if our preferred version of the Potency Condition always fails. But in that case the issue of whether Hawking's condition of compactly generated Cauchy horizons is necessary for the operation of a time machine is moot since it is then entirely unclear how to characterize a time machine.

Hawking's theorem does not apply when the partial Cauchy surface Σ is compact, for otherwise Taub-NUT spacetime and Misner spacetime would be counterexamples. To attempt to fill the gap, one can bring to bear Theorem 3 of Tipler (1977). The proof of this theorem can be adapted to show that if the WEC and EFE hold and if the *generic condition* holds somewhere on $H^+(\Sigma)$ ($\neq \varnothing$), then $H^+(\Sigma)$ is not compactly generated. Taub-NUT spacetime and Misner spacetime are not counterexamples because they violate the generic condition which requires that $V^aV^bV_{[c}R_{d]ab[e}V_{f]} \neq 0$, where R_{abcd} is the Riemann tensor and V^a is the tangent to a timelike or null geodesic. The generic condition entails that the geodesic feels a tidal force. The absence of such a force on all of $H^+(\Sigma)$ would signal that the spacetime is of a rather "special" character.[19]

But then no one thought that spacetimes implementing time machines would be generic.[20] One expects more from a no-go result than a statement of the form: a time machine can operate only under such-and-such and such special conditions.

Hawking (1992a) attempted to fill the gap in another way. When Σ is compact, $H^+(\Sigma)$ can be compactly generated. But if it is, both the convergence and shear of the null geodesic generators of $H^+(\Sigma)$ must vanish if the WEC holds. "This would mean," Hawking wrote, "that no matter or information, and in particular no observers, could cross the Cauchy horizon into the region of closed timelike curves" (p. 606). But this implication doesn't constitute an anti-time machine result *per se*; it says rather that the operator of the time machine cannot take advantage of his handiwork.

There are other results that could be reviewed, but we do not believe that they change the conclusion that the no-go results that have been achieved to date do not give confidence that chronology is protected by classical GTR.[21]

5. No-go results using semi-classical quantum gravity

The introduction of quantum considerations cuts in two directions with respect to proving chronology protection results: in one direction the quantum would appear to undermine the classical no-go results since quantum fields can, for example, violate the WEC assumed in many of these results;[22] but in the other direction the quantum introduces new mechanisms that may prevent the formation of CTCs. In this section we study one such mechanism which arises in the context of semi-classical quantum gravity (SCQG).

The ambitions of SCQG do not extend to quantizing the metric; rather the aim is to estimate how quantum fields affect the metric by computing the quantum expectation value $<\xi|T_{ab}|\xi>$ of the (renormalized) stress-energy tensor T_{ab} that arises from the quantum fields, and then by inserting this expression into EFEs in place of the classical stress-energy tensor in order to calculate the "backreaction" of the fields on the metric. Work by Frolov (1991) and Kim and Thorne (1991) suggested that for physically interesting states $|\xi>$ of linear quantum fields, $<\xi|T_{ab}|\xi>$ diverges as the Cauchy horizon $H^+(\Sigma)$ is approached in classical general relativistic spacetimes where CTCs occur to the future of $H^+(\Sigma)$. Hawking (1992a) argues that when the backreaction has an attractive gravitational effect, the divergence behavior would cause the formation of singularities along $H^+(\Sigma)$, cutting off future development of the spacetime, whereas when the backreaction has a repulsive gravitational effect "the spacetime will resist being warped so that closed timelike curves appear" (p. 610); either way, the mechanism of SCQG acts to prevent the formation of CTCs. Hawking's argument has not been made rigorous; indeed, it cannot be made rigorous in our present state of knowledge since, presumably, the details will depend on the presently non-existent quantum theory of gravity.

Hopes for the original version of this program faded when Krasnikov (1996) and Sushkov (1997) studied some toy models with CTCs and a partial Cauchy surface Σ and exhibited states $|\xi>$ for which $<\xi|T_{ab}|\xi>$ remains bounded as $H^+(\Sigma)$ is approached. For example, Sushkov (1997) proved that for an automorphic quantum field (characterized as a complex scalar field φ in an external electromagnetic field) in four-dimensional Misner space, carefully setting the automorphic parameter yields a field such that $<\xi|T_{ab}|\xi>$ is zero everywhere on the initially globally hyperbolic region $D^+(\Sigma)$.[23] This "Sushkov state" is not entirely well behaved: other field quantities such as $<\xi|\varphi^2|\xi>$ diverge as the Cauchy horizon is approached. If anything, these examples show that one can circumvent the heuristic arguments for the divergence of $<\xi|T_{ab}|\xi>$ by cleverly choosing the field and/or the spacetime.[24] However, a general problem with these results is that even if there is a well-defined limit of $<\xi|T_{ab}|\xi>$ as the horizon is approached, the value of $<\xi|T_{ab}|\xi>$ on the horizon may be singular. Despite the fact that for the "Sushkov state" the renormalized stress energy tensor vanishes on the initially globally hyperbolic region, it vanishes on the Cauchy horizon as well only if continuity holds (see Cramer and Kay 1996). A result due to Kay, Radzikowski, and Wald (1997) (hereafter, KRW) shows that continuity necessarily fails: $<\xi|T_{ab}|\xi>$ cannot be non-singular on all of $H^+(\Sigma)$ when $H^+(\Sigma)$ is compactly generated.[25] In any case, researchers gave up on the idea that the "blow up" of the expectation value of the stress-energy tensor of the quantum field at the chronology horizon would provide a mechanism for chronology protection.

Rather than focusing on the behavior of field properties as $H^+(\Sigma)$ is approached, KRW prove that the "point-splitting prescription" for evaluating $<\xi|T_{ab}|\xi>$ yields singular values in neighborhoods of the "base points" of $H^+(\Sigma)$. Difficulties in evaluating $<\xi|T_{ab}|\xi>$ stem from the fact that T_{ab} is not an element of the algebra of observables. Rather than enlarging the algebra, KRW adopt the point-splitting procedure and restrict the admissible states $|\xi>$ to accommodate this procedure (see Wald 1994, Chapter 4, and KRW 1997). Point-splitting is required since there is no natural way to define a product of distributions.[26] In order to evaluate φ^2 terms in the stress energy tensor, the product of two distributions is treated as the coincidence limit of a bidistribution defined over two points, e.g. $<\xi|\varphi(x)^2|\xi>:= \lim_{x\to x'}[<\xi|\varphi(x)\varphi(x')|\xi> - H(x, x')]$. $H(x, x')$ corresponds to "vacuum stress-energy" and it is subtracted to get rid of possible divergences. The quantity in brackets is a smooth, well-defined function of x and x' only if the two terms have the same short range singularity behavior. In Minkowski spacetime H(x, x') for the preferred vacuum state is singular if and only if the points (x, x') are null related. So the procedure yields a well-defined quantity $<\xi | \varphi(x)^2 | \xi>$ if we require that $<\xi | \varphi(x)\varphi(x') | \xi>$ has the same singular behavior as H(x, x').

Generalizing this approach to curved spacetimes leads to the requirement that physically admissible states satisfy the analogous Hadamard condition. Curved spacetimes lack a preferred global vacuum state, so to apply the point-splitting prescription we need to determine the properties of H(x, x'). As Wald (1994) shows, specifying four desirable properties for $<\xi | T_{ab} | \xi>$ leads to constraints on H(x, x')--roughly, the upshot is that H(x, x') is singular for null-related points but well-defined for other points (more precisely, it is "locally weakly Hadamard"). The Hadamard condition limits physically admissible states to those whose two-point functions $<\xi | \varphi(x)\varphi(x') | \xi>$ have the same singularity structure as H(x, x').

KRW's result shows that in a neighborhood of a base point of a compactly generated Cauchy horizon there is no way to consistently define a Hadamard state. The set of *base points* B is defined as the set of points x which are past terminal accumulation points for some null geodesic generator γ of $H^+(\Sigma)$ --intuitively, γ continually re-enters any given neighborhood of the point x.[27] (In the case of the Misner spacetime of the preceding section, every point of $H^+(\Sigma)$ is a base point.) KRW prove that

<u>Lemma</u>: (a) When $H^+(\Sigma)$ is compactly generated, $B \neq \emptyset$. (b) For any globally hyperbolic neighborhood U of a base point, there are points $y, z \in U \cap D^+(\Sigma)$ such that y and z are connected by a null geodesic in M, g_{ab} but cannot be connected by a null curve lying entirely in U.

(The reader should verify property (b) for the case of Misner.) Because of the clash between the local and global senses of null-related points and because the singularity structure of a Hadamard state depends on which points are null related, the point splitting procedure fails to yield a well-defined value for $<\xi | T_{ab} | \xi>$ at a base point.[28]

It is expected that analogous results hold for cases where $H^+(\Sigma)$ is not compactly generated but serves as a chronology horizon in the sense that it separates the initially globally hyperbolic region $D^+(\Sigma)$ from a region where there are CTCs (see Cramer and Kay 1996 and KRW 1997). The collection of such results promises to serve as a basis for a general chronology protection theorem to the effect that the laws of physics prevent the formation of CTCs in a spacetime initially free of causal anomalies, whether or not the formation can be attributed to a time machine. But exactly how is this promise to be realized? The original idea was that because the expectation value of the stress-energy tensor "blows up" as the chronology horizon is approached, backreaction effects shut down the incipient formation of CTCs. That idea, to repeat, has been shot down by a number of counterexamples. The new approach, while much more rigorous, is explanatorily less satisfying in that it doesn't provide a mechanism for preventing the formation of CTCs; rather it proceeds by trying to show that, under such-and-such conditions, the formation of CTCs is inconsistent with the applicability of SCQG, which requires that expectation value of the stress-energy tensor be well-defined on $H^+(\Sigma)$. As is often the case with *reductio* arguments, this *reductio* leaves the beholder searching for the explanatory force.

Visser (1997, 2002) has argued that the situation is even worse because the chronology horizon lies beyond the "reliability horizon," which marks the limit beyond which SCQG cannot be reliably applied. The "unreliable region" Ω consists of points connected to themselves by spacelike geodesics shorter than the Planck length (ca. 10^{-35} m), and the reliability horizon is the boundary of $J^+(\Omega)$. The existence of these closed Planck scale loops spell disaster for SCQG because the mode sum for a quantum field defined over a region with such periodic spatial identifications will include momentum terms on the order of the Planck energy. These energies and associated metric fluctuations are great enough that we should not trust SCQG to give an accurate rendering of backreaction effects.

6. No-go results from quantum field theory on curved spacetime

If SCQG is a kind of first order approximation to full quantum gravity, then quantum field theory (QFT) on curved spacetime is a 0^{th} order approximation: there is no ambition to compute backreaction effects; rather, a spacetime of classical GTR is treated as a fixed background on which one attempts to do QFT. There is a general recipe for cooking up a natural algebra of observables $\mathbf{A}(M, g_{ab})$ for a scalar quantum field defined over a globally hyperbolic spacetime M, g_{ab} (see Wald 1994). The result of this recipe is a C*-algebra generated by the elements representing "smeared" quantum fields φ(f), $\varphi(f)^*$ and the identity, where $f \in C^\infty{}_o(M)$ (i.e. smooth functions with compact support on M).[29] This procedure for constructing a QFT on curved spacetime breaks down for non-globally hyperbolic spacetimes, and one can wonder whether it is even possible to construct a decent QFT when the spacetime is so causally ill-behaved as to contain CTCs and time machines. This opens the door for another approach to proving chronology protection theorems; namely, state and defend as minimally necessary a condition that a spacetime must satisfy in order to admit a decent QFT; then show that if a spacetime is initially free of CTCs, it can develop CTCs only at the price of violating said minimally necessary condition.

Kay (1992) has argued that a requirement he dubs *F-quantum compatibility* is just such a minimally necessary condition. To explain it, note first that even if a spacetime M, g_{ab} fails to be globally hyperbolic, it is still possible to choose for any $x \in M$ a globally hyperbolic neighborhood $U \subset M$, i.e. U, $g_{ab}|_U$ considered as a spacetime in itself is globally hyperbolic. One can then proceed per usual to construct the intrinsic algebra **A**(U, $g_{ab}|_U$) for this spacetime. But another possible choice of an algebra associated with U is the algebra **A**(M, g_{ab}; U) induced on U by the global algebra **A**(M, g_{ab}).[30] M, g_{ab} is said to satisfy *F-locality* with respect to a given **A**(M, g_{ab}) just in case for every $x \in M$ there is a globally hyperbolic neighborhood such that the algebras **A**(U, $g_{ab}|_U$) and **A**(M, g_{ab}; U) are isomorphic. And M, g_{ab} is said to be *F-quantum compatible* just in case it admits a global algebra **A**(M, g_{ab}) with respect to which it is F-local.

In order to relate F-locality to chronology protection we need to be more specific about how the algebras are constructed. Here we restrict attention to a scalar quantum field φ satisfying the Klein-Gordon equation $(\Box_g - m^2)\varphi = 0$, where $\Box_g$ is the Laplace-Beltrami operator with respect to the metric g_{ab}. When M, g_{ab} is globally hyperbolic, **A**(M, g_{ab}) is taken to be the C*-algebra of smeared quantum fields satisfying the following conditions (KRW 1997):

1. $\varphi(f) = \varphi(f)^*$
2. $\varphi(\lambda_1 f_1 + \lambda_2 f_2) = \lambda_1\, \varphi(f_1) + \lambda_2\, \varphi(f_2)$
3. $\varphi((\Box_g - m^2)f) = 0$
4. $[\varphi(f_1), \varphi(f_2)] = i\Delta(f_1, f_2)$

where Δ is the advanced-minus-retarded fundamental solution to the Klein-Gordon equation. The existence and uniqueness of Δ is guaranteed for globally hyperbolic spacetimes. This guarantee fails when the spacetime is not globally hyperbolic. Nevertheless we can construct for any globally hyperbolic neighborhood U the C*-algebra of smeared fields such that 4. holds for $f_1, f_2 \in C^{\infty}{}_o(U)$ with Δ replaced by $\Delta|_U$, the advanced-minus-retarded fundamental solution for U, $g_{ab}|_U$.

Now consider a spacetime where $H^+(\Sigma)$ is compactly generated. We know from the Lemma of the previous section that the set $B \subset H^+(\Sigma)$ of base points is non-empty. Let U be any globally hyperbolic neighborhood of a base point. Then the restrictions of **A**(U, $g_{ab}|_U$) and of **A**($D^+(\Sigma)$, $g_{ab}|_{D+(\Sigma)}$) to $U \cap D^+(\Sigma)$ cannot coincide, as follows from the Lemma quoted in the preceding section. It is a basic property of Δ that it vanishes for spacelike related points whereas it is singular for null related points. Thus, $\Delta|_U$ and $\Delta|_{D+(\Sigma)}$ cannot coincide when U is a globally hyperbolic neighborhood of a base point. This shows that the usual field algebra defined above for initially globally hyperbolic region $D^+(\Sigma)$ cannot be extended in any way so as to satisfy F-locality on a compactly generated $H^+(\Sigma)$. Under some extra mild technical assumptions KRW (1997) show additionally that there is no field algebra at all that satisfies F-locality on $H^+(\Sigma)$ when it is compactly generated.

In assessing the significance of the KRW theorems for chronology protection, it is worth noting that F-locality is not incompatible with time travel *per se*: there are spacetimes that contain CTCs but are F-quantum compatible--the spacetime of Fig. 1 being just such a case (see Kay 1992 and Fewster and Higuchi 1996). But F-locality is incompatible with the development of CTCs by means of a process that produces compactly generated $H^+(\Sigma)$ or, more generally, a $H^+(\Sigma)$ containing base points with the property (b) of the Lemma. This is a chronology protection result, albeit of limited scope. If the condition on $H^+(\Sigma)$ on were a necessary condition for the operation of a time machine–as Hawking initially thought–then the demonstrated incompatibility would be a completely general no-go result for time machines in SCQG. But as we have seen there is reason to doubt Hawking's *ansatz*.

Apart from issues of scope, the main worry about the effectiveness of the demonstrated incompatibility as a chronology protection theorem revolves around the status of F-locality. Most directly, one can worry whether F-locality constitutes a *sine qua non* for QFT. Krasnikov (1998b) has argued that a modified locality condition is perfectly compatible with the existence of a compactly generated Cauchy horizon. If this argument is correct, then the constraint on spacetime structure illustrated by the KRW results merely indicate that the F-locality condition includes what Krasnikov calls an "arbitrary requirement" responsible for the non quantum compatibility of spacetimes with compactly generated Cauchy horizons. As it stands, however, Krasnikov's modified F-locality condition is not an adequate replacement for F-locality since he focuses on defining a field algebra over the union of globally hyperbolic subsets of a manifold rather than over the entire manifold.[31] This leaves out precisely those parts of the manifold where F-locality is a substantial requirement! Friedman (1997) has also suggested a weaker version of a locality principle: instead of requiring that there is a global algebra which satisfies the F-locality condition, he considers a set of local C*-algebras with suitable overlap conditions. However, Friedman's considerations are limited to strongly causal spacetimes, and it is not clear that extending his approach to spacetimes with Cauchy horizons will lead to a condition weaker than F-locality. Although this is a subject for further research, as of yet there is no suitable replacement for F-locality which avoids the KRW result.

The results on the incompatibility of F-locality and the development of CTCs does not furnish us with a mechanism that prevents the formation of CTCs. Presumably that mechanism will have to come from the full theory of quantum gravity of which QFT on curved spacetime is only a 0-th order approximation. Assume for sake of argument that quantum gravity has a semi-classical limit in which QFT on curved spacetime emerges as the interaction between the quantized matter fields and the metric is turned off. Why should one think that this limit will always obey F-locality? Kay writes: “One also expects that, if full quantum gravity satisfies a set of local laws, then some remnant of this locality would survive at the semiclassical level and it seems worthwhile to anticipate what this might be” (1992, 171-172). In the absence of even a dim view of the final theory of quantum gravity it is not clear why the antecedent should be realized, or why, if it is realized, the semi-classical limit of quantum gravity will conform to F-locality. Still, there is an interesting implication to be drawn from trying to suppose that F-locality fails. Suppose that we live in a world that can be described by QFT on a curved spacetime of GTR, and suppose further that this is a time machine world whose operation involves the existence of base points in $H^+(\Sigma)$. Then in principle we would be able to detect the fact that we live in such a world by observing that commutativity fails, $[\varphi(f_1), \varphi(f_2)] \neq 0$, even when the supports of f_1 and f_2 are relatively spacelike when viewed from within the local spacetime regions we inhabit (see Kay 1997).

7. No-go results from Euclidean Quantum Gravity

Cassidy and Hawking (1998) have argued that Euclidean methods provide a “probabilistic” no-go result. The ambitions of the Euclidean approach extend beyond those of either QFT on curved spacetimes or the semi-classical approach: techniques borrowed from the path integral formulation of QFT are used to study "full quantum gravity" effects, ranging from the initial singularity to black hole evaporation.[32] In QFT path integrals are used to calculate the amplitude for a transition from one field configuration to another. These are often mathematically more well-behaved in Euclidean space (obtained by replacing t with –*i*t in global inertial coordinates) than in Lorentz-signature spacetime, so field theorists frequently use Euclidean path integrals and then analytically continue the relevant functions back to Minkowski space to find S-matrix elements and the like. The Euclidean approach focuses on path integrals like those from QFT, such as that for gravity coupled to a scalar field φ:[33]

$$Z(M) = \int d\mu(g_{ab})\, d(\varphi)\, \exp(-S_E[g_{ab},\varphi]).$$

The analog of the "$t \rightarrow it$" trick in curved space is to replace a Lorentzian metric with a Riemannian metric; the path integral extends over all the space of Riemannian metrics defined over the manifold M. In *some* cases it is possible to analytically continue the Riemannian metric back into a Lorentzian one to calculate physical quantities.

There are a number of difficulties in interpreting the quantity Z(M). The tricks for handling the measure over field configurations in perturbative calculations in QFT do not immediately extend to this case, and there is no general definition for the measures $d\mu(g_{ab})$ and $d(\varphi)$. Furthermore, it is not clear whether Z(M) can be interpreted as a "probability amplitude" by analogy with QFT. Following the familiar Born rule, suppose we take $\int_O |\Psi(h_{ij})|^2\, d\mu(h_{ij})$ to be the probability that the three-geometry of the universe falls within a range of values "O". This "naïve interpretation" faces a number of obstacles.[34] What does it mean to say that the universe has a given three-geometry on a surface Σ, without specifying a time when this geometry obtains? In addition, there is no guarantee that the dynamical evolution of Ψ has properties that would make it possible to interpret these probabilities as being somehow related to a curve through the space of allowed metrics. The Hartle-Hawking "no boundary" proposal interprets these probabilities as follows: define the ground state wave function(al) of the universe $\Psi(h_{ij}, \varphi_0, \Sigma)$ as a sum of Z(M) ranging over compact Euclidean manifolds that have a unique boundary Σ, with induced metric h_{ij} and field configuration φ_0. The probability calculated using this ground state is interpreted as the probability for the Universe (with an initial spatial geometry Σ, h_{ij}) to "appear from nothing" (Hartle and Hawking 1983).

The Euclidean approach is a promising way to study spacetimes with CTCs, since some of these spacetimes have well-behaved Euclidean analytic continuations and the approach applies to non-globally hyperbolic spacetimes. Cassidy and Hawking (1998) give a no-go result based on applying the Euclidean approach to a fixed background spacetime. In particular, they study scalar fields with finite temperature on background spacetimes constructed as analytic continuations of periodic Euclidean spaces; i.e., flat Euclidean space with points identified under discrete isometries. These spaces are equivalent to an Einstein universe rotating around some fixed axis; as the rotation rate increases, at a critical value points sufficiently far from the axis rotate faster than c. In the original space, the critical value of the boost parameter corresponds to the formation of CTCs. The path integral for the scalar field diverges to negative infinity at the critical value. At first blush, this result seems to indicate that acausal spacetimes would be incredibly probable according to the no boundary proposal. But an analogy with thermodynamics suggests a conflicting interpretation of this divergence: Hawking and Cassidy calculate the entropy of the scalar field, and interpret the divergence behavior of their result as implying that the density of quantum states tends to zero. In short, the quantum states that would be associated with a chronology violation simply are not available.

Results along these lines could provide a broad basis for chronology protection. They extend more broadly than previous results, since they would outlaw CTCs and not just time machines. But we have two reasons for thinking that the case is not closed. The first relates to the vexed issue of interpreting path integrals and associated probabilities in this setting. We mentioned some of the difficulties associated with probabilities in the Euclidean program above. Cassidy and Hawking's (1998) introduction of thermodynamic considerations muddies the waters further: Can these considerations be consistently combined with assignments of probabilities based on the no-boundary proposal, or do they come into play only in case of divergences? Second, does every acausal spacetime have a Euclidean section? If the answer is no, as we conjecture, the Euclidean techniques do not apply across the board. At best this approach would give a plausibility argument that quantum effects prevent CTCs in spacetimes outside its domain of applicability.

8. Conclusion

The physics literature on time machines has evolved in the absence of any precise delineation of the spacetime structure that would characterize the operation of these devices. This seemingly scandalous situation is partly explained by the combination of the facts that most of the physics literature is aimed at producing no-go results and that most of the contributors to the literature have accepted Hawking's dictum that compactly generated Cauchy horizons are a necessary feature of the operation of time machines. Since this dictum is questionable,[35] so are the effectiveness of the no-go results based on it. But, to put a better face on the endeavor, what has become known as the time machine literature in physics can be construed as an investigation of the prospects of achieving no-go results showing that, under physically reasonable conditions, CTCs cannot develop in spacetimes initially free of these pathologies. Such chronology protection theorems, if sufficiently general, would also constitute no-go results for time machines, however the vagaries associated with such devices are settled. Our review indicates that this sought after generality has not been achieved. At the same time, the pursuit of chronology protection results has proved to be a fruitful way to probe the foundations of classical GTR and the interface between general relativity and quantum field theory.

We have attempted to obey Arthur Fine's injunction to strive to make philosophy of physics a partnership with physics while recognizing that in this particular instance the partnership is necessarily an unequal one since the mathematical physicists have to do the heavy lifting. But it also seems clear that a little more cooperation with philosophers of science in attending to the analysis of what it takes to be a time machine could have led to some helpful clarifications in the physics literature.

Finally, at the risk of repetition, we want to underscore two points that have not penetrated very far into philosophical consciousness. The first is that the physics literature on time machines engages a set of issues that are wholly distinct from those involved in the so-called paradoxes of time travel. More attention to the former by philosophers could reinvigorate what has become a hackneyed discussion. The second point is more contentious because it relies on the Potency Condition analysis of time machines: chronology protection theorems are sufficient but not necessary to showing that a time machine cannot operate; so even if the loopholes in the presently existing protection theorems cannot be plugged, there remains another strategy for grounding time machines.

Notes

* We are grateful to Robert Geroch, Tim Maudlin, John Norton, Steve Savitt, and Robert Wald for helpful suggestions. Special thanks are due to the participants of the Finefest for stimulating discussion.

1. For an extensive, but by no means complete, list of references to philosophical literature on time travel up to 1995, see Earman (1995a, Ch. 6). Arntzenius and Maudlin (2000) is also an excellent review article, containing more up-to-date references.

2. The journals include *The Physical Review*, *Classical and Quantum Gravity*, and *Communications in Mathematical Physics*. Visser (2002) estimates that there are now over 200 articles on time travel and time machines in the physics literature. A session of the Eighth Marcel Grossmann meeting on general relativity was devoted to time machines; see Piran (1999, 564-610). Physicists have also been turning out semi-popular books on time travel and time machines; see, for example, Davies (2002), Gott (2001), Hawking et al. (2002), and Nahin (1999).

3. A timelike curve represents the world line of some possible observer. We deal throughout with spacetimes that are *temporally orientable*, so that there is a globally consistent past/future distinction. We assume that one of the orientations has been singled out, by whatever means, as giving the future direction. By a CTC we then mean a future directed timelike curve that intersects itself. Such a curve represents the world line of an observer who travels always in the future direction, only to arrive back at the very spatiotemporal location from which the journey commenced.

4. Two of these conditions, the *weak* and *strong energy conditions*, will be discussed in the

coming sections.

5. Roughly, this view holds that the laws (of the actual world) are given by the axioms or theorems of the best deductive system. A deductive system is a deductively closed axiomatizable set of sentences. The best such system is the one that achieves the best compromise between strength and simplicity.

6. See Earman and Roberts (2002) for a defense of Humean supervenience of laws.

7. There are many different meanings of "locality," but in the present case the equations of motion can be as local as can be possibly desired, e.g. they may be in the form of hyperbolic partial differential equations for local field quantities.

8. If Σ is chosen to be a local rather than a global time slice of the spacetime of Fig. 1, the associated consistency constraints aren't so severe. However, if further identifications are made along the space axis in this spacetime so as to produce a torus, then the only solutions to the homogeneous relativistic wave for a scalar field ψ are ψ = constant. Thus the observation of any variation in ψ in the smallest regions of our world would tell us that we do not inhabit such a spacetime.

9. The spacetime of Fig. 1 is vicious. A more interesting example of a vicious spacetime is Gödel spacetime, which is topologically $\mathbb{R}^4$; in this case the CTCs cannot be removed by passing to a covering space, as they can in the case of Fig. 1.

10. For examples, see Friedman and Morris (1991) and Friedman et al. (1991).

11. A spacetime is maximal if it cannot be (smoothly) embedded as a proper subset of another spacetime. By "smooth" we will mean C^∞.

12. Kay, Radzikowski, and Wald (1997) have given another motivation for the requirement of

compactly generated Cauchy horizonss. The idea is that if one starts with globally hyperbolic spacetime--a spacetime which admits a Cauchy surface--and smoothly deforms the metric in a compact region (which we may think of as the region of the operation of the time machine) in such a way that CTCs are admitted, then Cauchy horizons are compactly generated. Proof sketch (courtesy of Robert Wald): Let Σ be a Cauchy surface for the initially globally hyperbolic region and let K be a compact region in $J^+(\Sigma)$ where the metric is deformed. In the undeformed region choose a point p such that K is contained in $J^-(p)$. Let C be the intersection of $J^-(p)$ with $J^+(K)$, where the pasts and futures are taken in the undeformed metric. Then C is compact. It is then "obvious" that in the deformed spacetime, any generator of the Cauchy horizon when followed into the past must enter and remain within C. We have two worries about drawing morals from this result. First, why can't the deformation of the originally compact region K turn it into a non-compact region because a singularity develops in the region as a result of the deformation? To say that such a deformation is not smooth threatens to beg the question. But waving this qualm one can also fault the holding fixed what is outside K. True, the operation of the time machine itself is to be confined to a compact region, but as a result of this operation a non-compact region can the changed by influences emanating from its operation.

13. We are indebted to Robert Geroch for this example, as well as for the suggestion to use the condition of hole freeness (see below), which was first introduced in Geroch (1977).

14. A spacetime is *geodesically complete* just in case every geodesic can be extended to arbitrarily great values of one of its affine parameters.

15. b-completeness is stronger than geodesic completeness. For a definition of b-completeness see Hawking and Ellis (1973), p. 259.

16. This follows from the result that the generators of $H^+(\Sigma)$ for a partial Cauchy surface Σ are past endless. If $H^+(\Sigma)$ is compactly generated, we thus have a past endless causal curve totally imprisoned in a compact set, and thus by Prop. 6.4.7 of Hawking and Ellis (1973, p. 195) strong causality fails in this set.

17. Assuming that the spacetime is time orientable, there always exists a non-vanishing timelike vector field. Using the integral curves of this vector field, map the points of $H^+(\Sigma)$ into the interior $D^+(\Sigma)$. This mapping is one-to-one and onto. Thus, if $H^+(\Sigma)$ is compact, so is the resulting surface. In addition, the resulting surface is spacelike and diffeomorphic to Σ.

18. Vollick shows that for a sphere of charged particles coupled to a scalar field, $T_{44} < 0$ in the interior of a sphere provided that $(\alpha/m)^2 \mu_o R^2 > 2$, where α is the coupling constant between matter and the scalar field, μ_o is the rest mass density, and R is the radius of the sphere. Although this violation of the weak energy condition is achieved without recourse to zero-point energies and other subtleties of QFT, the scalar field must be very strong coupled to matter--$(\alpha/m)^2 > G$--over large distances. We take the existence of such a scalar field to be physically implausible.

19. There are other results which confirm the fact that a time machine spacetime must be "special." For example, Isenberg and Montcrief (1985) considered analytic vacuum and electrovacuum solutions to EFE which contain a compact null hypersurface ruled by closed null geodesics. They showed that such spacetimes must be special in the sense that they contain a Killing symmetry.

20. The situation differs from Penrose's cosmic censorship conjecture, which was watered down with a clause requiring "generic" initial conditions in light of a counterexample studied by numerical relativists.

21. For a review of some earlier no-go results within classical GTR, see Earman (1995b). Two results we do not discuss in the text deserve brief mention. Maeda et al. (1998) prove that there is no maximal spacetime containing a non-empty chronology violating set V if a number of conditions are satisfied. The idea seems to be to piggy-back a no-go result for time machines on Penrose's cosmic censorship conjecture, which posits that EFEs and energy conditions forbid the formation of naked singularities under physically plausible conditions. Maeda et al.'s theorem is self-consciously styled as a no-go result for time machines, but as such it is aimed mainly at the finitely vicious spacetimes that arise from the sorts of identifications made in the construction of Politzer spacetime. We argued in the preceding section that Politzer spacetime is not a plausible candidate for a time machine spacetime, so this theorem does not recommend itself as a general chronology protection theorem. Another recent no-go result operates against the possibility of producing CTCs by means of traversable wormholes: Hochberg and Visser (1997, 1999) establish the violation of energy conditions at or near the throats of traversable wormholes.

22. However, Ford and Roman (1996) argue that "quantum inequalities" place limitations on the violation of the weak energy condition and that in turn these limitations constrain the geometry of traversable wormholes that might allow CTCs.

23. An automorphic field at a point x satisfies the following condition: $\varphi(Lx) = \exp(2\pi i\alpha)\varphi(x)$, where L is an element of the spacetime isometry group and $0 \leq \alpha \leq 1/2$ is the automorphic parameter.

24. Hiscock (2000) has rejected these and other purported counterexamples to Hawking's chronology protection conjecture on the grounds that they either rely on special properties of the quantum fields or else they depend on "fine tuning" the metric parameters. In a similar vein,

purported counterexamples to Roger Penrose's cosmic censorship hypothesis have been rejected on the grounds that they are highly non-generic.

25. Cramer and Kay (1996) generalize the KRW result slightly to cover Sushkov's example; $H^+(\Sigma)$ is not compactly generated in four-dimensional Misner spacetime considered by Sushkov, but the KRW result applies to any manifold which is the product of an n - dimensional spacetime with a compactly generated Cauchy horizon and a 4 - n dimensional Riemannian manifold.

26. The stress-energy tensor will generally involve quadratic terms; for a Klein-Gordon field, for example, $T_{ab} = \nabla_a\varphi\nabla_b\varphi - 1/2g_{ab}(\nabla_c\nabla^c\varphi + m^2\,\varphi^2)$.

27. More precisely, a *past terminal accumulation point* of a curve γ: I $\rightarrow$ M is defined as a point $x \in M$ such that for every open neighborhood N(x) of x and every $s_o \in I$, $\exists s(s \in I: s > s_o \& \gamma(s) \in N(x))$, where we parametrize γ so that it is past directed. Since the null geodesic generators of $H^+(\Sigma)$ are past inextendible, for every base point there is a null curve which intersects any given neighborhood of that point more than once. Thus, strong causality fails at each base point.

28. The no-go result also relies on the propagation of singularities theorem which says, very roughly, that singularities in a bi-solution to the Klein-Gordon field equation for near by points on a null geodesic are propagated along the entire geodesic.

29. The smeared field is defined by $\varphi(f) := \int_M \varphi(x)f(x)\eta$, where η is the volume element for the spacetime i.e. the square root of the determinant of the metric.

30. **A**(M, g_{ab}; U) is the C^*-completion of the set of all **A**(M, g_{ab})(O) for which the closure of the open set O is contained in U, and **A**(M, g_{ab})(O) is the subalgebra of **A**(M, g_{ab}) generated by the smeared fields on M with $f \in C^\infty_o(O)$.

31. Krasnikov distinguishes globally hyperbolic subsets from intrinsically globally hyperbolic subsets: a subset $N \subset M$ of a spacetime M, g_{ab} is globally hyperbolic iff i) $\forall p,q \in N$, $(J^{+}(p) \cap J^{-}(q)) \subset N$ and ii) strong causality holds in N, whereas N is intrinsically globally hyperbolic iff $N, g_{ab}|_N$ is a globally hyperbolic spacetime. The difference between the two lies in the importance of M - N: strong causality fails in N if a CTC (or almost-CTC) loops through M - N and returns to N, but such a set will still be intrinsically globally hyperbolic. The union of globally hyperbolic subsets excludes the chronology violating regions, whereas the union of intrinsically globally hyperbolic subsets can cover a chronology violating spacetime.

32. Our comments here about the Euclidean program will be brief; for a careful and thorough assessment of the thorny interpretational issues, see Isham and Butterfield (1999). For a collection of papers introducing the technical apparatus of the Euclidean approach, see Gibbons and Hawking (1993).

33. The Euclidean action for general relativity coupled to a scalar field is given by

$$S_E = -1/16\pi \int_M R \sqrt{g}\, d^4x + 1/8\pi \int_{\partial M} K \sqrt{h}\, d^3x - \int L(\varphi) \sqrt{g}\, d^4x$$

where $h := \det(h_{ij})$, h_{ij} being the spatial metric of the boundary of M, written ∂M; K is the trace of the extrinsic curvature of ∂M; and $L(\varphi)$ is the Lagrangian of the scalar field.

34. The term is borrowed from Unruh and Wald (1989), who also discuss the difficulties with two other less naïve attempts to introduce probabilities.

35. We are not entirely alone; see Krasnikov (1997b) and Ori (1993).

References

Arntzenius, F. and Maudlin, Tim. 2000. Time Travel and Modern Physics. *Stanford Encyclopedia of Philosophy*. URL http://plato.stanford.edu.

Carroll, J. W. 1994. *Laws of Nature.* New York: Cambridge University Press.

Cassidy, M. J. and Hawking, S. W. 1998. Models for Chronology Selection. *Physical Review D* 57: 2372-2380.

Chrusciel, P. T. and Isenberg, J. 1993. Compact Cauchy Horizons and Cauchy Surfaces. In *Directions in General Relativity*, Vol. 2, edited by B. L. Hu and T. A. Jacobson, 97-107. Cambridge: Cambridge University Press.

Cramer, C. R. and Kay, B. S. 1996. Stress-energy must be singular on the Misner space horizon even for automorphic field. *Classical and Quantum Gravity* 13: L143-L149.

Davies, P. 2002. *How to Build a Time Machine*. New York: Viking Penguin.

Earman, J. 1995a. *Bangs, Crunches*, *Whimpers, and Shrieks: Singularities and Acausalities in Relativistic Spacetimes*. New York: Oxford University Press.

------. 1995b. Outlawing Time Machines. *Erkenntnis* 42: 125-139.

Earman, J. and Roberts, J. 2002. Contact with the Nomic. Pre-print.

Fewster, C. J. and Higuchi, A. 1996. Quantum field theory on certain non-globally hyperbolic spacetimes. *Classical and Quantum Gravity* 13: 51-61.

Fine, A. 1986. *The Shaky Game. Einstein Realism, and the Quantum Theory*. Chicago: University of Chicago Press. 2nd ed. 1996.

Fine, A. 1989. Interpreting Science. *PSA 1988*, Vol. 2, 3-11.

Ford, L. H. and Roman, T. A. 1996. Quantum field theory constrains traversable wormhole geometries. *Physical Review D* 53: 5496-5507.

Friedman, J. L. 1997. Field Theory on Spacetimes that Are Not Globally Hyperbolic. *Fields Institute Communications* 15: 43-57.

Friedman, J. L. and Morris, M. S. 1991. The Cauchy Problem on Spacetimes with Closed Timelike Curves. *Annals of the New York Academy of Sciences* 631: 171-181.

Friedman, J. L., Morris, M. S., Novikov, I. D., Echeverria, F., Klinkhammer, G., and Yurtsever, U. 1991. Cauchy problem in spacetimes with closed timelike curves. *Physical Review D* 42: 1915-1930.

Frolov, V. P. 1991. Vacuum polarization in a locally static multiply connected spacetime and time machine problems. *Physical Review D* 43: 3878-3894.

Geroch, R. P. 1977 . Prediction in General Relativity. In *Foundations of Spacetime Theories, Minnesota Studies in the Philosophy of Science*, Vol. 8, edited by J. Earman, C. Glymour, and J. Stachel, 81-93. Minneapolis: University of Minnesota Press.

Gibbons, G. W. and Hawking, S.W. (1993). *Euclidean Quantum Gravity*. Singapore: World Scientific.

Gott, R. 2001. *Time Travel in Einstein's Universe*. New York: Houghton Mifflin.

Hartle, J. and Hawking, S. W. 1983. Wave function of the universe. *Physical Review D* 28: 2960-2975.

Hawking, S. W. 1992a. Chronology protection conjecture. *Physical Review D* 46: 603- 611.

---------. 1992b. The chronology protection conjecture. In *The Sixth Marcel Grossmann Meeting.* edited by H. Sato and T. Nakamura, 3-13. World Scientific.

Hawking, S. W. and Ellis, G. F. R. (1973). *The Large Scale Structure of Spacetime*.

Cambridge: Cambridge University Press.

Hawking, S. W., Thorne, K. S., Novikov, I., Ferris, T., and Lightman, A. 2002. *The Future of Spacetime*. New York: W. W. Norton.

Hiscock, H. A. 2000. Quantized Fields and Chronology Protection. gr-qc/0009061 v2.

Hochberg, D. and Visser, M. 1997. Geometric structure of the generic traversible wormhole throat. *Physical Review D* 56: 4745-4755.

------. 1999. General Dynamic Wormholes and Violation of the Null Energy Condition. gr-qc/9901020.

Isenberg, J. and Moncrief, V. 1985. Symmetries of cosmological Cauchy horizons with exceptional orbits. *Journal of Mathematical Physics* 26: 1024-1027.

Isham, C. and Butterfield, J. 1999. On the emergence of time in quantum gravity. In *The Arguments of Time,* edited by J. Butterfield, 111-168. Oxford: Oxford University Press.

Kay, B. S. 1992. The Principle of Locality and Quantum Field Theory on (Non-Globally Hyperbolic) Curved Spacetimes. *Reviews of Mathematical Physics*, Special Issue: 167- 195.

------. 1997. Quantum Fields in Curved Spacetime: Non Globally Hyperbolicity and Locality. gr-qc/9704075.

Kay, B. S., Radzikowski, M. J. and Wald, R. M. 1997. Quantum Field Theory on Spacetimes with Compactly Generated Cauchy Horizons. *Communications in Mathematical Physics* 183: 533-556.

Kim, S.-W. and Thorne, K. S. 1991. Do vacuum fluctuations prevent the creation of closed timelike curves? *Physical Review D* 43: 3929-3957.

Krasnikov, S. V. 1996. Quantum stability of the time machine. *Physical Review D* 54: 7322-

7327.

------. 1997a. Causality and paradoxes. *Physical Review D* 55: 3427-3430.

------. 1997b. Time machines with non-compactly generated Cauchy horizons and "handy singularities," gr-qc/9711040.

------. 1998a. A singularity-free WEC-respecting time machine. *Classical and Quantum Gravity* 15: 997-1003.

------. 1998b. Quantum field theory and time machines. gr-qc/9802008.

------. 2002. No time machines in classical general relativity. *Classical and Quantum Gravity 19*: 4109-4129.

Lewis, D. K. 1973. *Counterfactuals.* Cambridge, MA: Harvard University Press.

Maeda, K, Ishibashi, A. and Narita, M. 1998. Chronology protection and non-naked singularities. *Classical and Quantum Gravity* 15: 1637-1651.

Nahin, P. J. 1999. *Time Machines*, 2nd ed. New York: AIP Press.

Ori, A. 1993. Must Time-machine Construction Violate the Weak Energy Condition? *Physical Review Letters* 71: 2517-2520.

Piran, T. (ed.), 1999. *The Eighth Marcel Grossmann Meeting*. Singapore: World Scientific.

Sushkov, S. S. 1997. Chronology protection and quantized fields: complex automorphic scalar field in Misner space. *Classical and Quantum Gravity* 14: 523-533.

Tipler, F. J. 1977. Singularities and Causality Violations. *Annals of Physics* 108: 1-36.

Unruh, W. and Wald, R. 1989. Time and the interpretation of canonical quantum gravity. *Physical Review D* 40: 2598-614.

Visser, M. 1997. The reliability horizon for semi-classical quantum gravity: metric fluctuations are often more important than back-reaction. *Physics Letters B* 115: 8-14.

----------- . 2002. The quantum physics of chronology protection. gr-qc/0204022. To appear in *The Future of Theoretical Physics and Cosmology*. Cambridge: Cambridge University Press.

Vollick, D. N. 1997. "How to produce exotic matter using classical fields," *Physical Review D* 56: 4720-4723.

Wald, R. M. 1994. *Quantum Field Theory in Curved Spacetime and Blackhole Thermodynamics*. Chicago: University of Chicago Press.

Figure captions

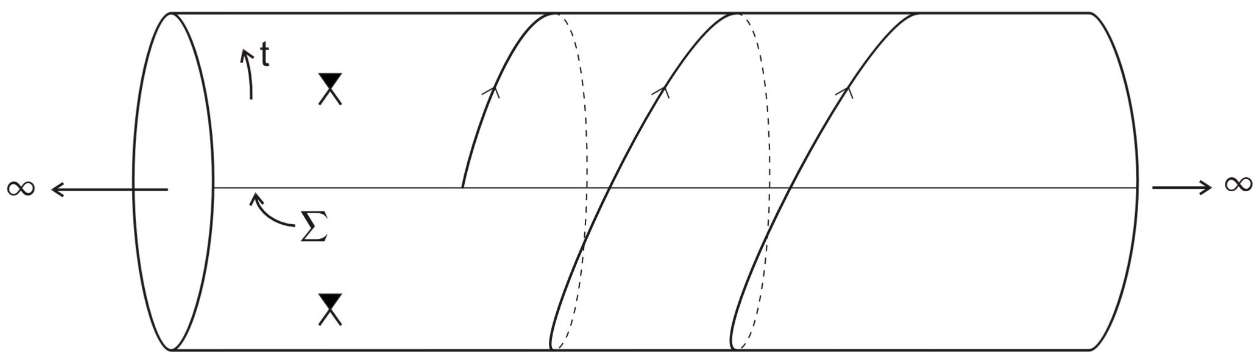

Figure 1

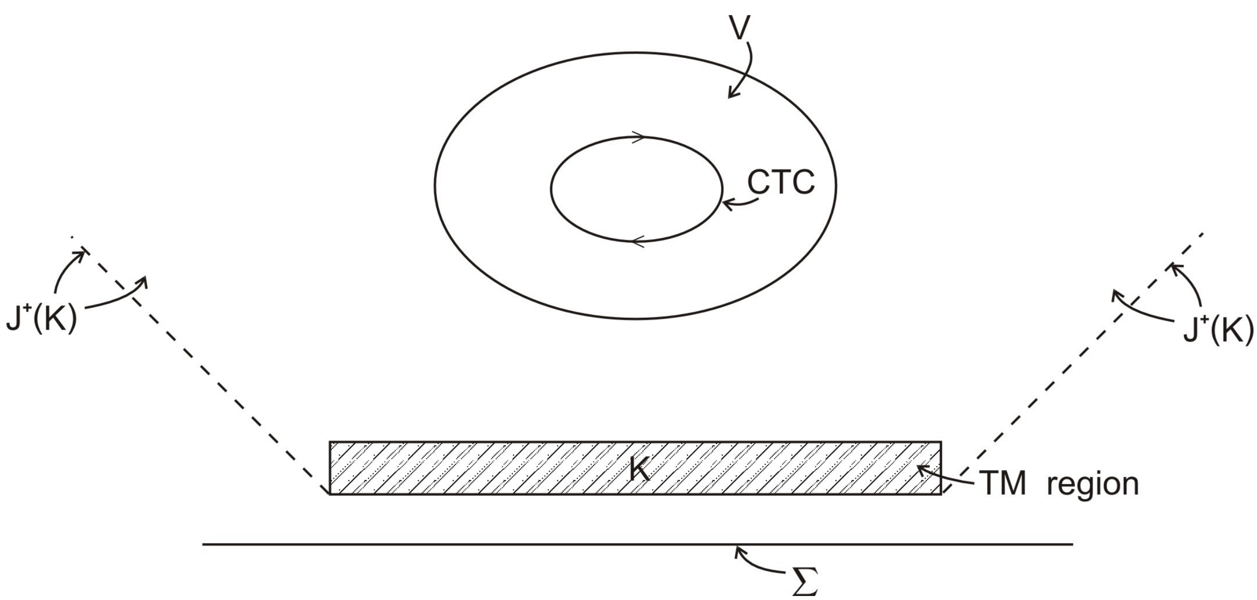

Figure 2

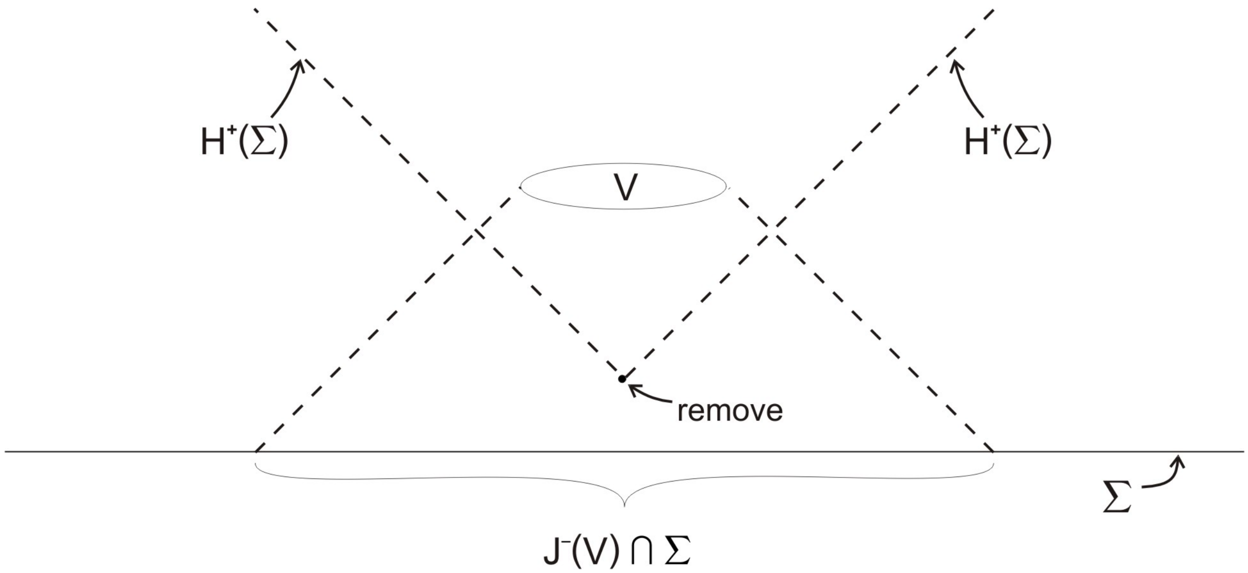

Figure 3

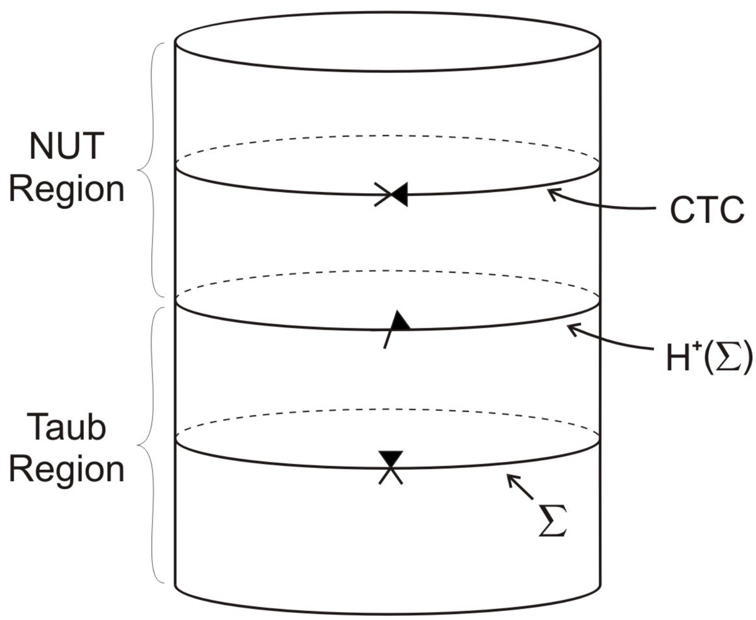

Figure 4a

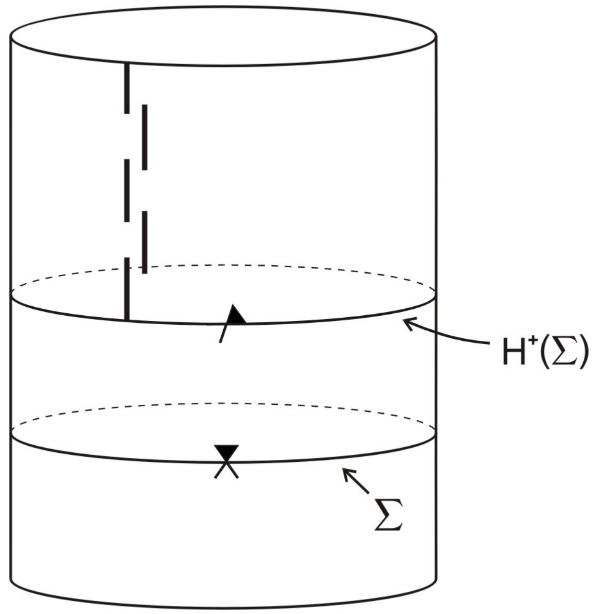

Figure 4b

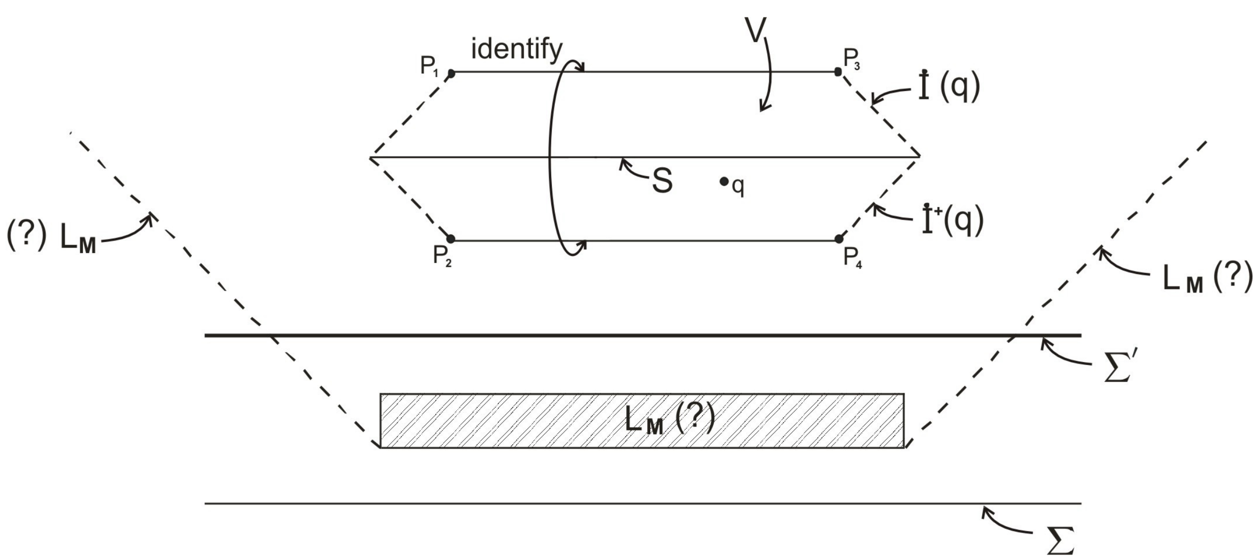

Figure 5a

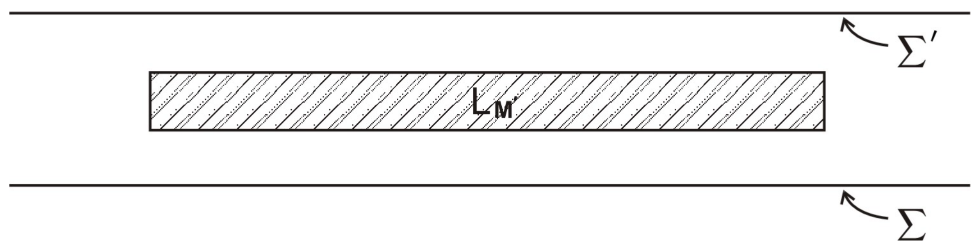

Figure 5b