Eating Goldstone bosons in a phase transition: A critical review of Lyre's analysis of the Higgs mechanism

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Abstract

In this note, I briefly review Lyre's (2008) analysis and interpretation of the Higgs mechanism. Contrary to Lyre, I maintain that, on the proper understanding of the term, the Higgs mechanism refers to a physical process in the course of which gauge bosons acquire a mass. Since also Lyre's worries about imaginary masses can be dismissed, a realistic interpretation of the Higgs mechanism seems viable.

Contents

1 Scalar electrodynamics

For the present purposes of reviewing Lyre's analysis[1] concerning the ontology of the Higgs mechanism, I will not go into the details of the complete model of spontaneous symmetry breaking of the electroweak interaction. The simpler model of the electrodynamics of charged spinless particles also exhibits the relevant features. I, like Lyre for the main part of his analysis, will therefore restrict myself to this model. For the following exposition of this model, I use the lecture notes by Wiese (2004) as my basis.

[1] Lyre 2008.

The simplest version of the model is one in which the particles are free. The model contains a complex scalar field $\Phi = \Phi_1 + i\Phi_2$, $\Phi_1, \Phi_2 \in R$ and the dynamics of this field and its quanta are described by the Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial_\mu \Phi^* \partial^\mu \Phi - V(\Phi), \tag{1}$$

where

$$V(\Phi) = \frac{m^2}{2}|\Phi|^2. \tag{2}$$

From the Lagrangian of this most simple of systems we can derive the corresponding Euler-Lagrange equations

$$\partial_\mu \frac{\delta \mathcal{L}}{\delta(\partial_\mu \Phi_i)} - \frac{\delta \mathcal{L}}{\delta \Phi_i} = 0 \qquad i = 1, 2, \tag{3}$$

which coincide, in this case, with the familiar Klein–Gordon equations for two free, spinless, charged fields with quanta of mass m:

$$\partial_\mu \partial^\mu \Phi_i + m^2 \Phi_i = 0. \tag{4}$$

In the next complex version of the model, the scalar particles interact directly among themselves. The interaction is described by a power of 4 in the field's absolute value which is added to the potential V such that it now reads

$$V(\Phi) = \frac{m^2}{2}|\Phi|^2 + \frac{\lambda}{4!}|\Phi|^4, \tag{5}$$

where λ parametrizes the strength of the interaction and the factor 4! is introduced for convenience in the perturbative solution of the dynamical equations of the system.

For such models to be interpretable physically, the potential V must be bounded from below. Otherwise the energy spectrum would also not be bounded from below and, accordingly, there would be no ground state of the system, which clearly cannot be the case for any real system. For the purely quadratic potential of equation (2) there is thus no other choice than $m^2 > 0$. For the potential describing the self-interaction of the field Φ (see equation (5)), however, $m^2 < 0$ is possible also.

2 Spontaneous breakdown of a global symmetry

For $m^2 > 0$, the model is globally symmetric with respect to $U(1)$ transformations $\Phi' = e^{iq\phi}\Phi$, where $\phi \in R$ is the parameter of the transformation and the factor q is introduced for more convenient identification of the charge of the particles. With $m^2 > 0$ and the potential of equation (5), the Lagrangian of equation (1) describes a system of particles of *approximately* the mass m. The mass of the particles is in this version of the model not exactly equal to m because of the interactions among the particles. This reminds us that the coefficient of the quadratic term in the Lagrangian is equal to (half) the mass of the particles only as long as the potential is approximately quadratic (like equation (2) or, more generally, like the potential of a harmonic oscillator). Only then do the Euler-Lagrange equations approximately coincide with the Klein–Gordon equation, on which our identification of the coefficient with the mass of the particle was based.

For $m^2 < 0$, the global $U(1)$ symmetry of the model is *spontaneously broken.* This means that the symmetry of the Lagrangian is still intact but the field configuration which leads to a minimal value of V is not invariant under the $U(1)$ transformations any longer. Before, in the case of $m^2 > 0$, the field configuration with minimal V was simply $\Phi = 0$. Now, with $m^2 < 0$, there is no unique configuration which minimizes V. A whole class of field configurations,

$$\Phi = \sqrt{-\frac{6m^2}{\lambda}}e^{i\chi}, \tag{6}$$

yield a minimum value for V. χ is the real parameter which characterizes a particular member of the class. For convenience, I will abbreviate $\sqrt{-\frac{6m^2}{\lambda}}$ by v such that the minimal configurations read $ve^{i\chi}$.

In order to estimate the masses of the quanta of the interacting fields, we have to be able to equate approximately the actual potential of equation (5) by the potential of equation (2), which is more readily interpretable in terms of a Klein–Gordon equation as discussed before. Therefore, we have to perform a series expansion of the Lagrangian around the point of minimal value of V. We obtain this expansion by substituting $v + \sigma + i\pi$ for Φ, where σ and π are two real fields of which we only consider infinitesimal excitations. In terms of the newly introduced σ and π fields the Lagrangian takes the form

$$\mathcal{L} = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\pi\partial^\mu\pi - \frac{1}{2}(-2m^2)\sigma^2 + \ldots, \tag{7}$$

where I left out higher order terms in the fields. This form of the Lagrangian, valid for small absolute values of σ and π, allows us to read-off the approximate masses of the quanta of this system of self-interacting fields: zero for the quanta of the π field, $\sqrt{-2m^2}$ for the quanta of the σ field.[2] In the complete electroweak model, the quanta of the π field would correspond to the Goldstone bosons and the quanta of the σ field to the Higgs boson.

We now also see that Lyre's worries, his "third observation",[3] about the imaginary mass that would result from the identification of the coefficient of $|\Phi|^2$ as the square of the particles' mass is unjustified. The identification can only be made if the potential V is approximately quadratic.

[2]Remember $m^2 < 0$. Therefore, $-2m^2 > 0$ and $\sqrt{-2m^2}$ positive and real.
[3]Lyre 2008, p. 126.

3 Spontaneous breakdown of a local symmetry

For reasons not to be discussed here, one prefers models which exhibit even a local symmetry, instead of a merely global one. In order to promote the global $U(1)$ symmetry, discussed above, to a *local* symmetry, one has to introduce a *gauge* field and a covariant derivative. The gauge field will eventually describe an interaction between the fields whereas, in the case of the global symmetry, the interaction between the particles was direct and immediate.

The Lagrangian that describes a locally symmetric model of spinless charged particles which interact through a gauge field is

$$\mathcal{L} = \frac{1}{2}(D_\mu\Phi)^* D^\mu\Phi - V(\Phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \tag{8}$$

where $D_\mu = \partial_\mu - iqA_\mu$ is the covariant derivative, A_μ the gauge field, q the strength of the coupling of the scalar field to the gauge field (in other words: the charge of the scalar field) and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ the field strength tensor associated with the gauge field. The combination $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ describes the kinetic energy of the gauge field. For the purposes of our simplified model of the electroweak interactions, A_μ is the electromagnetic field and its quanta the photons. $V(\Phi)$ reads, as in the case of global symmetry, $\frac{m^2}{2}|\Phi|^2 + \frac{\lambda}{4!}|\Phi|^4$.

As in the case of global symmetry, the Lagrangian describes either the symmetric phase (if $m^2 > 0$) or the broken phase (if $m^2 < 0$). In the symmetric phase, the field configuration which minimizes V is again just $\Phi = 0$. The masses are approximately given by the coefficients of the quadratic terms in the Lagrangian. There are two fields, Φ_1 and Φ_2, which both have quanta of mass m^2. Because there is no quadratic term of the gauge field, its quanta (the photons) are massless.

In the broken phase, i. e. when $m^2 < 0$, we have to do again the series expansion around one of the field configurations which minimize V, i. e. around $ve^{i\chi}$ for some $\chi \in R$, see equation (6). Apart from higher order terms, we obtain, as was the case with the spontaneous breakdown of the global symmetry, a quadratic term in σ. Its coefficient gives us the approximate mass of the quanta of the sigma field, $\sqrt{-2m^2}$, see equation (7). The terms of the Lagrangian involving derivatives, however, does now not only yield, a derivative of σ and π like in equation (8) but, apart from higher order terms, also a term which is quadratic in the gauge field, $\frac{1}{2}q^2v^2A_\mu A^\mu$, and a term which is a product of the gauge field and the derivative of the π field, $\frac{1}{\sqrt{2}}qvA_\mu\partial^\mu\pi$. The former is readily interpreted as a term describing a photon of mass qv. The latter term does not lend itself to such an interpretation nor is there any quadratic term in π that would do so.

Therefore, to determine whether there are other quanta, massive or not, apart from the quanta of the σ and the gauge field A_μ, we have to bring the Lagrangian in a more appropriate though equivalent form. This is achieved by taking advantage of the principle which states that physical states which are related by a gauge symmetry transformation are empirically indistinguishable.

	global	local
symmetric	$m_\sigma = m_\pi = m$	$m_\pi = m_\sigma = m$, $m_\gamma = 0$
		$\downarrow *$
broken	$m_\sigma = \sqrt{2}\lvert m\rvert$, $m_\pi = 0$	$m_\sigma = \sqrt{2}\lvert m\rvert$, $m_\gamma = qv$

Table 1: Approximate masses of the quanta which exist in the symmetric or broken model of interacting spinless charged particles, in the case of global or local symmetry. "m_γ" denotes the masses of the quanta of the gauge field, the photons. The transition, denoted by the starred arrow, from the symmetric to the broken phase, in the case of the local symmetry, is the Higgs mechanism.

This is the case for the members of the class of field configurations which minimize V, see equation (6). They are now related by the *gauge* transformation $\Phi(x)' = e^{iq\phi(x)}\Phi(x)$. Before, they were only related by the *global* symmetry transformation $\Phi' = e^{iq\phi}\Phi$, where the parameter ϕ does not depend on the space-time coordinate x.

Because of this freedom to choose the gauge, the real field, σ, suffices to describe the small excitations of the complex field Φ around the configuration which minimizes V. There is, actually, no π field needed at all. In slightly more technical terms, this means that we can choose a *unitary* gauge in which the Φ, appropriate for the series expansion around the minimal configuration, reads just $\Phi = v + \sigma$. Apart from higher order terms, there now only remain readily interpretable terms in the Lagrangian:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}(-2m^2)\sigma^2 + \frac{1}{2}q^2v^2A_\mu A^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \dots \tag{9}$$

In this form, we see that the Lagrangian describes massive quanta of the σ field and massive quanta of the gauge field. This is indicated by the quadratic terms in these fields; the other terms describe the kinetic energy of the fields. Contrary to the case of the spontaneous breakdown of the global symmetry, we see that here, in the case of local symmetry, we have a massive photon but no Goldstone boson (the quanta of the π field). Compared to the locally symmetric phase, the difference is that we have a massive, instead of massless, photon and no quanta of the π field at all, see table 1.

4 The Higgs mechanism

A closer inspection of table 1 reveals that the number of physical degrees of freedom is unaffected by the spontaneous breakdown of the symmetry, either local or global. In the case of the global symmetry, the number of degrees of freedom is two, before and after the spontaneous breakdown of the symmetry, because each scalar particle has one degree of freedom, irrespective of its mass. In the case of local symmetry, it might seem, at first sight, that one degree of freedom is somehow lost in the course of the spontaneous breakdown of the symmetry, because in the symmetric phase there is a quantum of the π field while in the broken phase there is none. However, the number of degrees of freedom of the gauge field depends on whether it is massive or not. In the symmetric phase, the photons are massless and thus have two physical degrees of freedom only. In the broken phase, the photon has a mass and thus has three physical degrees of freedom.

The *Higgs mechanism* is the transition from the symmetric to the broken phase in the case of a local symmetry, see table 1.[4] This is the transition from a state in which there are two massive scalar fields, σ and π, and a massless gauge field, A_μ, to a state in which there is only one scalar field, σ, with massive quanta, and a massive gauge field. The Goldstone boson, the massless quanta of π, which appears in the broken phase of a global symmetry, does not appear in the broken phase of a local symmetry. In a metaphorical manner of speaking, one therefore often says that the Goldstone boson, which would appear if the symmetry were global, is "eaten" by the photon which thus becomes massive.[5]

At the same time, this metaphor of eating might be responsible for the confusion behind Lyre's claim that the Higgs mechanism is nothing but a reshuffling of degrees of freedom and as such cannot possibly refer to a physical process. Such a claim can only be maintained if one means by "Higgs mechanism" the transition from the system described by the Lagrangian of equation (8) to the Lagrangian of equation (9). However, this is clearly not a transition between two physically distinct systems, as Lyre correctly points out, but a mere transition from one description of the system to another equivalent description. One might be tempted to apply the eating metaphor to this transition, too, because in the first description the π field appears in one of the terms of the Lagrangian while in the second description it does not. However, it is the same physical system, without π quanta, that is described in both cases. The only difference between the two cases is that one form of the description (equation (9)) clearly shows that, in fact, there are no π quanta, while the other form of the description (equation (8)) is less directly interpretable.

The Higgs mechanism has, therefore, the same ontological status like any other mechanism of spontaneous symmetry breaking, which we observe, for instance, in ferromagnets or superconductors. Lyre's analysis concerns the transition between two equivalent descriptions of the same physical system which should and, in fact, usually is not called the Higgs mechanism. The proper

[4]For some purposes, this statement may be over-simplified. The relation $m_\gamma = qv$ (see table 1) shows how the mass of the gauge boson depends on the strength of the coupling q of the scalar field to the gauge field. The second row of table 1 shows how, in the broken phase, the introduction of a gauge field and the requirement of a *local* symmetry, instead of only a global one, leads to the disappearance of the (massless) Goldstone boson π. These observations are emphasized in Higgs (1964) and Anderson (1963), for instance. Accordingly, in a more complete characterization, the Higgs mechanism should be seen in the combination of the two processes of coupling the scalar field to the gauge field (going from left to right in the second row of table 1) *and* the transition from the symmetric to the broken phase of a local symmetry (going from top to bottom in the second column of table 1).

[5]To my knowledge, the metaphor goes back to Coleman (1985, p. 123).

understanding of the term is that of a transition from a symmetric phase of a physical system to an asymmetric (or broken) phase. In the course of this transition one type of massive charged spinless particle disappears and the gauge field, the quanta of which are massless in the symmetric phase, becomes massive. According to most, or at least some, of today's cosmological theories,[6] such a process has happened during the cooling of the early universe and is, as such, as real as it can get.

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[6]See, e. g., Linde 1990, p. 17.