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Research Article

Coordinating Pricing and Advertising Decisions for Supply Chain under Consignment Contract in the Dynamic Setting

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In this paper, via the differential game method, the problems of the pricing and advertising decision are investigated by considering the effect of number of the platform users on demand. In addition, a novel contract is developed to coordinate the supply chain. Firstly, the optimal strategies of the pricing and advertising are given in the decentralized and centralized scenarios by applying the differential game theory. Also, the comparison analysis concerning on the optimal strategies is proposed in two decision scenarios. It is shown that the centralized scenario could lead to the higher advertising effort of each member and a lower retail price. Next, we construct the state-dependent contract with hope to coordinate the supply chain and then improve the performance of the supply chain. Finally, a numerical example is provided to illustrate the impacts of the price-elasticity index of demand and the effectiveness of the number of retailer's platform users onto the feasible region of the corresponding contract.

1. Introduction

Over the past decades, the consignment contract with revenue sharing has attracted an increasing research attention due to its wide applications in the Internet commerce, the mobile application industry, and so on [1–4]. For example, as the largest international online e-commerce company, the Amazon invites other sellers to sell their products through "Amazon platform," where a consignment contract with revenue sharing is proposed to the sellers. For this contract, the Amazon charges the commission according to certain percentage of the selling price when the products are sold [1]. Note that the Amazon platform has high quantity of the users; hence, the sellers could expand the market demand of product by selling the product through the Amazon platform, which generally increases certain expense to the sellers. Therefore, the sellers should decide the retail price on the consignment platform. In addition, the Amazon needs to increase the number of the platform users and the advertising is an effective way commonly adopted. It should be noted that the investigation concerning on the numbers of the platform users in a dynamic setting is more general [5]. Therefore, in this paper, we make one of attempts to address the following several questions in a dynamic setting: (1) how to design the parameter of the consignment contract with the participating sellers? (2) What are the optimal pricing and advertising strategies in both centralized and decentralized scenarios? (3) How to design a contract to coordinate the decentralized supply chain via the evolution of the number of the platform users?

In recent years, much effort has been made to address the issues related to comparison between the wholesale price contract and consignment contract [2, 6, 7]. In addition, the performance analysis and coordination have been discussed for supply chain under consignment contract [8–11] and dynamic systems under certain indices [12–14]. For example, in [15], the effect from uncontrollable factors and controllable factors onto the contract choice of supply chain member has been discussed. As mentioned in [16], compared to the wholesale price contract, both the retailer and the manufacturer prefer to the consignment contract when the dominant retailer's sales effort is very effective. The optimal pricing and inventory decisions have been examined for supply chains under two regimes (i.e., the vendor-managed consignment inventory and the retailer-managed consignment inventory), where it has been illustrated that the performance of supply chain is higher in the vendor-managed consignment inventory scenario [8]. Subsequently, based on the results in [8], the customer return policy has been introduced in [17] and the impact from the customer return policy onto the performance of channel has been revealed, where it has shown that the vendor-managed consignment inventory still is beneficial to the performance of supply chain irrespective of the customer returns. In [18], the impact from risk attitudes of channel members onto the supply chain performance has been discussed. For the issue of contract coordination, new method has been provided in [19] on how to use the containing side payment terms to coordinate the supply chain with price-dependent demand. When the supply chain with uncertainty and stochastic demand, the mechanist of vendor-managed consignment inventory coupled with a production cost subsidy can achieve the coordination [20]. In [21], the impact from shelf space onto demand has been examined and two-part tariff contract has been designed to coordinate the supply chain system under consignment contract with revenue sharing contract.

On the other hand, the dynamic setting should be considered to face the various practical requirements [22–26] and enhance the channel efficiency under a dynamic setting as in [27]. So far, few results can be available for the advertising problem of the supply chain under consignment contract. However, it is worth mentioning that the advertising is an important factor effect demand of product and increasing attention has been paid to the advertising strategy [28–30], especially to the dynamic advertising strategy in differential games [31, 32]. For example, assume that the market demand at the same time was affected by the retail price and goodwill; the problems of pricing and advertising have been discussed in [33] for franchise system with a single franchisor and two independent franchisees. In [34], a differential game model has been used to analyze the advertising competition in a dual channel supply chain via the differential game theory. For the issue of contract coordination in a dynamic setting, an optimal two-part tariff has been characterized in [35] to replicate the vertically integrated performance both in static and dynamic games. Furthermore, in [36], the problem of strategic transfer pricing has been studied for the supply chain with quality level and advertising-dependent goodwill. However, it should be mentioned that the supply chain coordination is achieved by the committed dynamic transfer price contract. These results have focus mainly on the pricing and advertising strategies for traditional supply chain. In our study, the problems of pricing and advertising strategies of the two channel members as a Stackelberg differential game are addressed for supply chain under consignment contract, where the retailer is powerful and acts as the leader and the manufacturer as the follower. Moreover, it should be noted that the traditional contract commonly fails to realize the coordination in the dynamic setting, which constitutes another research motivation of this paper, where a new state-dependent contract is designed for addressed problem.

Motivated by the above discussions, the operation decision problem is investigated for a supply chain under consignment contract, where dynamics of the numbers of the retailer's platform users is considered. Here, we use the differential equation to model the evolution of the number of the retailer's platform users and depict the effect from number of the retailer's platform users onto the demand function. By applying the differential game theory, the optimal pricing and advertising strategies are derived under the centralized and decentralized cases, and a state-dependent contract is designed to coordinate the decentralized supply chain. Finally, a numerical example is given to show the change of contract feasible domain with the effectiveness of the store-assistance service level and other parameters and verifies the feasibility of coordination contract. The contributions of this paper can be summarized as follows: (1) compared with the existing results concerning on solely price-dependent demand models, this paper includes the numbers of the retailer's platform users as a demand accelerator; (2) the equilibrium strategies of supply chain under consignment contract are given by considering the dynamic of the number of the retailer's platform users; and (3) the state-dependent contract is designed for the supply chain under consignment contract, which can coordinate the decentralized supply chain in the dynamic setting.

2. The Problem Formulation

In this paper, we consider a monopolistic supply chain consisting of a retailer and a manufacturer, in which the retailer's sales platform has a higher number of the platform users (the name recognition) and the manufacturer produces one product and sells it through the retailer's sales platform under the consignment contract with revenue sharing. In addition, we assume that the retailer is not selling other competing products. Based on the above assumption, we aim to propose new pricing and advertising strategies under consignment contract in the dynamic setting. Specifically, the manufacturer produces a constant unit cost c_{M}, and the retailer incurs a unit cost c_{R} for selling the product. The total unit cost for the channel is $c = c_{\mathrm{M}} + c_{\mathrm{R}}$, where the share of the channel cost that is incurred a manufacturer is $\alpha = c_{\mathrm{M}}/c$. In addition, the demand of product $S(t)$ is affected not only by the retail price $p(t)$ but also by the number of the retailer's platform users $G(t)$, where the number of the retailer's platform users can expand the base market size of product. Thus, the demand function can be expressed as follows:

$$S(t) = (a + \gamma G(t))p(t)^{-b}, \tag{1}$$

where $a > 0$ is the base market size, $b > 1$ is the price-elasticity index of demand, and $\gamma > 0$ depicts the effect from the number of the retailer's platform users onto the market size.

It should be mentioned that the retailer should face one important problem on how to keep old consumers and develop new consumers. Hence, the retailer often actively takes the advertising investment in order to improve the number of the retailer's platform users. On the other hand, the number of the retailer's platform users has the decay property due to the consumers' forgetting the competition between similar platforms and so on. Thus, the number of the retailer's platform users is enhanced by the advertising campaigns and experiences the decay in a dynamic environment. Therefore, we use the following differential equation to describe the evolution of the number of the retailer's platform users $G(t)$:

$$\dot{G}(t) = U_{\mathrm{A}}(t) - \delta G(t), \quad G(0) = G_0, \tag{2}$$

where $U_{\mathrm{A}}(t)$ is the advertising effort at time t by the retailer in order to increase the number of the retailer's platform users, and the parameter δ represents the decay rate. The advertising cost functions are the convex and increasing with their own advertising effort, i.e., $C(U_{\mathrm{A}}(t)) = 1/2(U_{\mathrm{A}}^2(t))$.

The retailer offers the manufacturer a consignment contract with revenue sharing, which stipulates that the retailer keeps ϕ share (percent) of the revenue for per unit of the product sold. Given an infinite time horizon and a common discount rate ρ, the objective functions of the retailer, the manufacturer, and the supply chain are, respectively, expressed by

$$J_{\mathrm{R}} = \int_0^{+\infty} e^{-\rho t}\left[(\phi p(t) - c_{\mathrm{R}})S(t) - \frac{1}{2}U_{\mathrm{A}}^2(t)\right]dt, \tag{3}$$

$$J_{\mathrm{M}} = \int_0^{+\infty} e^{-\rho t}[((1-\phi)p(t) - c_{\mathrm{M}})S(t)]dt, \tag{4}$$

$$J_{\mathrm{C}} = \int_0^{+\infty} e^{-\rho t}\left[(p(t) - c_{\mathrm{M}} - c_{\mathrm{R}})S(t) - \frac{1}{2}U_{\mathrm{A}}^2(t)\right]dt. \tag{5}$$

Remark 1. In this paper, the infinite planning horizon is considered, that is, $t \in [0, +\infty)$. Although many studies focus on the analysis of supply chain within the short-term planning horizons [30, 37], an infinite horizon is often used and useful in the marketing problems in order to discuss the long-run impacts of the marketing dynamics as in [36, 38]. In fact, the advertising is a key factor of improving the number of the platform users in the present study, and the main reason of utilization of an infinite horizon is to examine the long-run nature of advertising strategy on *the numbers of the users of the platform.*

3. The Optimal Strategies under the Decentralized Decision

Under the decentralized scenario, the manufacturer and the retailer maximize their own objective functions, respectively. In the sequel, let the retailer be the Stackelberg leader and the manufacturer be the follower. The sequence of events is given as follows: the retailer first declares the revenue share for per unit sold φ and advertising effort $U_{\mathrm{A}}(t)$, then the manufacturer decides the retail price of the product based on the retailer's decision.

Next, the optimal strategies will be presented for supply chain under the decentralized scenario.

Theorem 1. *Under the decentralized scenario, the optimal strategies of supply chain members are given by*

$$\begin{aligned}
\phi^* &= \frac{\alpha + (b-1)(1-\alpha)}{(\alpha + b - 1)}, \\
p^{\mathrm{D}^*} &= \frac{bc(\alpha + b - 1)}{(b-1)^2}, \\
U_{\mathrm{A}}^{\mathrm{D}^*} &= \frac{\gamma (b-1)^{2(b-1)}}{(\rho+\delta)(\alpha+b-1)^{(b-1)} c^{(b-1)} b^b}.
\end{aligned} \tag{6}$$

Furthermore, the optimal trajectory of the number of the platform users is

$$G^{\mathrm{D}}(t) = \left(G_0 - G_{\mathrm{SS}}^{\mathrm{D}}\right)e^{-\delta t} + G_{\mathrm{SS}}^{\mathrm{D}}, \tag{7}$$

where $G_{SS}^{\mathrm{D}} = \gamma(b-1)^{(2b-1)}/(\delta(\rho+\delta)(\alpha+b-1)^{(b-1)}c^{(b-1)}b^b)$.

Proof 1. By applying the backward induction method, we first derive the manufacturer's retail price of the product $p(t)$, when both the retailer's revenue share for per unit sold ϕ and the advertising effort $U_{\mathrm{A}}(t)$ are given. The optimization problem of manufacturer is

$$\begin{aligned}
\max_{p>0} \quad & J_{\mathrm{M}} = \int_0^{+\infty} e^{-\rho t}[((1-\phi)p(t) - c_{\mathrm{M}})S(t)]dt \\
\text{s.t.} \quad & \dot{G}(t) = U_{\mathrm{A}}(t) - \delta G(t), \quad G(0) = G_0.
\end{aligned} \tag{8}$$

Subsequently, the Hamilton function is adopted for the established optimization model, where the optimal retail price of the product can be obtained. Hence, based on the differential game theory, we construct the following current-value Hamiltonian function for the manufacturer:

$$\begin{aligned}
H_{\mathrm{M}} &= ((1-\phi)p(t) - c_{\mathrm{M}})S(t) + q(t)(U_{\mathrm{A}}(t) - \delta G(t)) \\
&= ((1-\phi)p(t) - \alpha c)p(t)^{-b}(a + \gamma G(t)) \\
&\quad + q(t)(U_{\mathrm{A}}(t) - \delta G(t)),
\end{aligned} \tag{9}$$

where $q(t)$ is the costate variable to represent the shadow price associated with the state variable $G(t)$. The practical meaning of the parameter $q(t)$ refers to the case that the optimal strategies will lead to the increasing of the objective function at the rate of $q(t)$ along a small increment of the number of the platform $G(t)$ at time t. The first-order condition of the manufacturer's profit with respect to the retail price $p(t)$ is written as

$$\frac{\partial H_{\mathrm{M}}}{\partial p} = (a + \gamma G(t))\left[-(1-\phi)(b-1)p^{-b} + b\alpha c p^{-(b+1)}\right] = 0. \tag{10}$$

Next, it follows from (10) that

$$p = \frac{\alpha bc}{(1-\phi)(b-1)}. \tag{11}$$

The retailer's optimization problem is

$$\max_{\phi>0,U_{\rm A}>0} \quad J_{\rm R}=\int_0^{+\infty}e^{-\rho t}\left[(\phi p(t)-c_{\rm R})S(t)-\frac{1}{2}U_{\rm A}^2(t)\right]dt$$
$$\text{s.t.}\quad \dot{G}(t)=U_{\rm A}(t)-\delta G(t),\qquad G(0)=G_0. \tag{12}$$

And the current-value Hamiltonian function for the retailer is given by

$$\begin{aligned}H_{\rm R}&=(\phi p(t)-(1-\alpha)c)S(t)-\frac{1}{2}U_{\rm A}^2(t)+\mu(t)(U_{\rm A}(t)-\delta G(t))\\&=(\phi p(t)-(1-\alpha)c)p(t)^{-b}(a+\gamma G(t))-\frac{1}{2}U_{\rm A}^2(t)\\&\quad+\mu(t)(U_{\rm A}(t)-\delta G(t)),\end{aligned} \tag{13}$$

where $\mu(t)$ is a costate variable. Substituting the retail price p into (13) yields

$$\begin{aligned}H_{\rm R}=&\frac{(b-1)^{b-1}(a+\gamma G(t))}{(\alpha bc)^b}\\&\cdot\left[\alpha bc\varphi(1-\phi)^{b-1}-(b-1)(1-\alpha)c(1-\phi)^b\right]\\&-\frac{1}{2}U_{\rm A}^2(t)+\mu(t)(U_{\rm A}(t)-\delta G(t)).\end{aligned} \tag{14}$$

By applying the necessary conditions of the maximum principle, we obtain

$$\frac{\partial H_{\rm R}}{\partial\phi}=0, \tag{15}$$

$$\frac{\partial H_{\rm R}}{\partial U_{\rm A}}=0, \tag{16}$$

$$\frac{\partial H_{\rm R}}{\partial G}=\rho\mu-\dot{\mu}. \tag{17}$$

Together with (15) and (16), one has

$$\begin{aligned}\phi&=\frac{\alpha+(b-1)(1-\alpha)}{(\alpha+b-1)},\\U_{\rm A}&=\mu.\end{aligned} \tag{18}$$

Therefore, $p=bc(\alpha+b-1)/(b-1)^2$. From (17), we have

$$\dot{\mu}=\rho\mu-\frac{\partial H_{\rm R}}{\partial G}=(\rho+\delta)\mu-\frac{\gamma(b-1)^{2(b-1)}}{(\alpha+b-1)^{b-1}c^{b-1}b^b}. \tag{19}$$

By solving the above differential equation, one has

$$\mu=c_1e^{(\rho+\delta)t}+\frac{\gamma(b-1)^{2(b-1)}}{(\rho+\delta)(\alpha+b-1)^{b-1}c^{b-1}b^b}, \tag{20}$$

where c_1 is a constant. Thus, the optimal advertising effort of retailer is

$$U_{\rm A}=\mu=c_1e^{(\rho+\delta)t}+\frac{\gamma(b-1)^{2(b-1)}}{(\rho+\delta)(\alpha+b-1)^{b-1}c^{b-1}b^b}. \tag{21}$$

Once $c_1\neq 0$, we know that the retailer's advertising effort $U_{\rm A}$ in (21) will tend to be infinite when $t\to\infty$, which does not satisfy the reality. Therefore, we have $c_1=0$. Based on (21), the optimal advertising effort of the retailer under the decentralized decision is given as follows:

$$U_{\rm A}=\frac{\gamma(b-1)^{2(b-1)}}{(\rho+\delta)(\alpha+b-1)^{b-1}c^{b-1}b^b}. \tag{22}$$

Finally, by using (2), the optimal trajectory of the number of the platform users is given by

$$G^{\rm D}(t)=\left(G_0-G_{\rm SS}^{\rm D}\right)e^{-\delta t}+G_{\rm SS}^{\rm D}, \tag{23}$$

where $G_{\rm SS}^{\rm D}=\gamma(b-1)^{(2b-1)}/(\delta(\rho+\delta)(\alpha+b-1)^{b-1}c^{(b-1)}b^b)$.

Subsequently, substituting p, u, and v into (3) and (4), respectively, the profits of the retailer and manufacturer are expressed as follows:

$$\begin{aligned}J_{\rm R}^{\rm D^*}=&\frac{a(b-1)^{2(b-1)}}{\rho(\alpha+b-1)^{b-1}c^{b-1}b^b}+\frac{\gamma(b-1)^{2(b-1)}G_0}{(\rho+\delta)(\alpha+b-1)^{b-1}c^{b-1}b^b}\\&+\frac{\gamma^2(b-1)^{4(b-1)}}{2\rho(\rho+\delta)^2(\alpha+b-1)^{2(b-1)}c^{2(b-1)}b^{2b}},\\J_{\rm M}^{\rm D^*}=&\frac{a\alpha(b-1)^{2b-1}}{\rho(\alpha+b-1)^bc^{b-1}b^b}+\frac{\gamma\alpha(b-1)^{2b-1}G_0}{(\rho+\delta)(\alpha+b-1)^bc^{b-1}b^b}\\&+\frac{\alpha\gamma^2(b-1)^{4b-3}}{\rho(\rho+\delta)^2(\alpha+b-1)^{2b-1}c^{2(b-1)}b^{2b}},\\J_{\rm M+R}^{\rm D^*}=&\frac{a((2b-1)\alpha+(b-1)(1-\alpha))(b-1)^{2(b-1)}}{\rho(\alpha+b-1)^bc^{b-1}b^b}\\&+\frac{\gamma((2b-1)\alpha+(b-1)(1-\alpha))(b-1)^{2(b-1)}G_0}{(\rho+\delta)(\alpha+b-1)^bc^{b-1}b^b}\\&+\frac{((3b-2)\alpha+(b-1)(1-\alpha))\gamma^2(b-1)^{4(b-1)}}{\rho(\rho+\delta)^2(\alpha+b-1)^{2b-1}c^{2(b-1)}b^{2b}}.\end{aligned} \tag{24}$$

Proposition 1. *Under the decentralized channel structure, the following results can be obtained for the channel members:*

(1) $(\partial U_{\rm A}^{\rm D*}/\partial\gamma)>0$, $(\partial\phi^*/\partial b)<0$, *and* $(\partial p^{\rm D*}/\partial b)<0$. *Moreover,* $(\partial U_{\rm A}^{\rm D*}/\partial b)<0$ *if* $c>((b-1)^2/(b(\alpha+b-1)e^{-\alpha/\alpha+b-1}))$

(2) $(\partial J_{\mathrm{R}}^{\mathrm{D}^*}/\partial \gamma) > 0$, $(\partial J_{\mathrm{M}}^{\mathrm{D}^*}/\partial \gamma) > 0$, *and* $(\partial J_{\mathrm{R}}^{\mathrm{D}*}/\partial b) < 0$. *Moreover,* $(\partial J_{\mathrm{M}}^{\mathrm{D}*}/\partial b) < 0$ *if* $\ln((b-1)^2/bc(\alpha+b-1)) > ((-bJ_{\mathrm{M}}^{\mathrm{D}^*} - (b-1)N)/(J_{\mathrm{M}}^{\mathrm{D}^*} + N))$

where $N = \alpha\gamma^2(b-1)^{4b-3}/\rho(\rho+\delta)^2(\alpha+b-1)^{2b-1}c^{2(b-1)}b^{2b}$.

Proposition 1 characterizes the following managerial implications. Firstly, the retailer will invest more in advertising when the impact of the retailer's platform users on the market demand becomes higher (i.e., a higher γ). In the same time, the retailer will invest less in advertising as the price-elasticity index becomes higher (i.e., a higher b) if the total unit cost for the channel is higher. Also, when the impact from the price onto the market demand becomes higher, the manufacturer will decrease the retail price. Simultaneously, the retailer will set a lower revenue share as a response. Secondly, the profits of the manufacturer and the retailer are increasing functions with respect to the impact of the retailer's platform users on the market demand, which mean that both the manufacturer and the retailer will get more revenues when the impact of the retailer's platform users on the market demand becomes higher. Moreover, the retailer will get more as the price-elasticity index becomes higher, but the manufacturer will get more only when the total unit cost for the channel has satisfied certain condition.

4. The Optimal Strategies in the Centralized System

In this section, when discussing the centralized scenario, the retailer and the manufacturer are integrated to as a whole system. The major objective is to maximize the supply chain's profit by setting the retail price p and advertising effort U_{A}, where the revenue share φ is not involved in decision-makings. This scenario serves as a benchmark since it implements the first best outcomes.

Theorem 2. *In the centralized scenario, the optimal strategies of supply chain are given by*

$$\begin{aligned} p^{\mathrm{C}*} &= \frac{bc}{b-1}, \\ U_{\mathrm{A}}^{\mathrm{C}*} &= \frac{\gamma(b-1)^{b-1}}{(\rho+\delta)c^{b-1}b^b}. \end{aligned} \tag{25}$$

Furthermore, the optimal trajectory of the number of the platform users is

$$G^{\mathrm{C}}(t) = \left(G_0 - G_{SS}^{\mathrm{C}}\right)e^{-\delta t} + G_{SS}^{\mathrm{C}}, \tag{26}$$

where $G_{SS}^{\mathrm{C}} = \gamma(b-1)^{b-1}/(\delta(\rho+\delta)c^{b-1}b^b)$.

Proof 2. The optimization problem of supply chain members is

$$\begin{aligned} \max_{p>0,U_{\mathrm{A}}>0} \quad & J_{\mathrm{C}} = \int_0^{+\infty} e^{-\rho t}\left[(p(t) - c_{\mathrm{R}} - c_{\mathrm{M}})S(t) - \frac{1}{2}U_{\mathrm{A}}^2(t)\right]dt \\ \text{s.t.} \quad & \dot{G}(t) = U_{\mathrm{A}}(t) - \delta G(t), \quad G(0) = G_0. \end{aligned} \tag{27}$$

The current-value Hamiltonian function is given by

$$\begin{aligned} H_{\mathrm{C}} &= (p(t) - c_{\mathrm{R}} - c_{\mathrm{M}})S(t) - \frac{1}{2}U_{\mathrm{A}}^2(t) + \lambda(t)(U_{\mathrm{A}}(t) - \delta G(t)) \\ &= (p(t) - c)p(t)^{-b}(a + \gamma G(t)) - \frac{1}{2}U_{\mathrm{A}}^2(t) \\ &\quad + \lambda(t)(U_{\mathrm{A}}(t) - \delta G(t)), \end{aligned} \tag{28}$$

where $\lambda(t)$ is a costate variable. By applying the necessary conditions of maximum principle, we obtain

$$\begin{aligned} \frac{\partial H_{\mathrm{C}}}{\partial p} &= 0, \\ \frac{\partial H_{\mathrm{C}}}{\partial U_{\mathrm{A}}} &= 0, \\ \frac{\partial H_{\mathrm{C}}}{\partial G} &= \rho\lambda - \dot{\lambda}. \end{aligned} \tag{29}$$

Similar to Proof 1 of Theorem 1, the optimal retail price and advertising effort of supply chain are given by

$$\begin{aligned} p &= \frac{bc}{b-1}, \\ U_{\mathrm{A}} &= \frac{\gamma(b-1)^{b-1}}{(\rho+\delta)c^{b-1}b^b}. \end{aligned} \tag{30}$$

Substituting U_{A} into (2) and solving the differential equation, one has

$$G^{\mathrm{C}}(t) = \left(G_0 - G_{SS}^{\mathrm{C}}\right)e^{-\delta t} + G_{SS}^{\mathrm{C}}, \tag{31}$$

where $G_{SS}^{\mathrm{C}} = \gamma(b-1)^{b-1}/(\delta(\rho+\delta)c^{b-1}b^b)$.

Subsequently, the profit of supply chain is given as follows:

$$J_{\mathrm{C}}^* = \frac{a(b-1)^{b-1}}{\rho c^{b-1}b^b} + \frac{\gamma(b-1)^{b-1}G_0}{(\rho+\delta)c^{b-1}b^b} + \frac{\gamma^2(b-1)^{2(b-1)}}{2\rho(\rho+\delta)^2c^{2(b-1)}b^{2b}}. \tag{32}$$

Remark 2. Similar to Theorem 1, Theorem 2 suggests that all the optimal strategies are constant over time. Maintaining the constant price and advertising effort decisions is easy to implement from a managerial perspective. In addition, the efficacy coefficient of retailer's platform users on the demand γ has a positive impact on the advertising effort while the price does not depend on γ. Meanwhile, the price-elasticity index b has a positive impact on the retail price while the advertising effort does not depend on b. Moreover, the channel cost, decay rate, and discount rate have negative impacts on the advertising effort. These observations are in line with literature [36].

Proposition 2. *Compared with the optimal strategies and profit values in the decentralized and centralized scenarios, we have*

$$\begin{aligned} & p^{\mathrm{D}^*} > p^{\mathrm{C}^*}, \\ & U_{\mathrm{A}}^{\mathrm{D}*} < U_{\mathrm{A}}^{\mathrm{C}*}, \\ & J_{\mathrm{C}}^* > J_{\mathrm{M+R}}^{\mathrm{D}^*}\left(J_{\mathrm{R}}^{\mathrm{D}*} + J_{\mathrm{M}}^{\mathrm{D}*}\right). \end{aligned} \tag{33}$$

Proposition 2 shows that the retail price is higher and the advertising effort is lower in the decentralized scenario compared to those in the centralized scenario. On the other hand, the channel's profit is higher when the two members are integrated to as a whole system, which highlights the superiority of the centralized scenario. Furthermore, this proposition suggests that a constant consignment contract in a dynamic game cannot coordinate the decentralized supply chain. This raises two questions, i.e., whether there exists a dynamic consignment contract in such a way that the decentralized supply chain can be coordinated, and both members can become better under the dynamic consignment contract compared to the decentralized scenario, which will be shown later.

5. Coordination Contract

Motivated by the coordination method in [38], a state-dependent contract (ϕ, k, ψ) is provided to coordinate the supply chain and enhance the profits of the decentralized supply chain member. The main idea is to introduce the slotting allowance $f(G)$ (transfer from the manufacturer to the retailer) [21, 39], which linearly depends on the number of the retailer's platform users $G(t)$ in this paper. Here, the contract provisions are structured as follows. Firstly, the retailer announces a constant percentage allocation of sales revenue for per unit sold ϕ and a linear state-dependent slotting fees $f(G)$, i.e., $f(G) = k + \psi G(t)$, where k and ψ are constants. Secondly, the manufacturer decides the retail price by maximizing its own profit, while the retailer controls the advertising effort by maximizing its own profit. It is worth noting that the contract parameters φ and ψ are used to coordinate the supply chain, and the parameter k is used to adjust the profit distribution between supply chain members.

Theorem 3. *The supply chain can be coordinated if the contract (ϕ, k, ψ) satisfies $\phi = 1 - \alpha$ and $\psi = (\alpha\gamma(b-1)^{b-1})/(c^{b-1}b^b)$.*

Proof 3. Under the state-dependent contract (ϕ, k, ψ), the optimization problem of manufacturer is

$$\begin{aligned} \max_{p>0} \quad & J_{\mathrm{M}}^{\mathrm{S}} = \int_0^{+\infty} e^{-\rho t}[((1-\phi)p(t) - \alpha c)S(t) - k - \psi G(t)]dt \\ \text{s.t.} \quad & \dot{G}(t) = U_{\mathrm{A}}(t) - \delta G(t), \quad G(0) = G_0. \end{aligned} \tag{34}$$

The current-value Hamiltonian for the manufacturer is given as follows:

$$\begin{aligned} H_{\mathrm{M}}^{\mathrm{S}} = {} & ((1-\phi)p(t) - \alpha c)p(t)^{-b}(a + \gamma G(t)) - k \\ & - \psi G(t) + q^{S}(t)(U_{\mathrm{A}}(t) - \delta G(t)), \end{aligned} \tag{35}$$

where $q^{S}(t)$ is the costate variable. The first-order condition of the manufacturer's profit with respect to the retail price $p(t)$ is written as

$$\frac{\partial H_{\mathrm{M}}^{\mathrm{S}}}{\partial p} = (a + \gamma G(t))\left[-(1-\phi)(b-1)p^{-b} + b\alpha c p^{-(b+1)}\right] = 0. \tag{36}$$

From (36), we have

$$p = \frac{\alpha bc}{(1-\phi)(b-1)}. \tag{37}$$

Next, the retailer's optimization problem is

$$\begin{aligned} \max_{U_{\mathrm{A}}>0} \quad & J_{\mathrm{R}}^{\mathrm{S}} = \int_0^{+\infty} e^{-\rho t}\left[(\phi p(t) - (1-\alpha)c)S(t) + k + \psi G(t) - \frac{1}{2}U_{\mathrm{A}}^2(t)\right]dt \\ \text{s.t.} \quad & \dot{G}(t) = U_{\mathrm{A}}(t) - \delta G(t), \quad G(0) = G_0. \end{aligned} \tag{38}$$

And the current-value Hamiltonian function for the retailer is given by

$$\begin{aligned} H_{\mathrm{R}}^{\mathrm{S}} = {} & (\phi p(t) - (1-\alpha)c)p(t)^{-b}(a + \gamma G(t)) + k + \psi G(t) \\ & - \frac{1}{2}U_{\mathrm{A}}^2(t) + \mu^{\mathrm{S}}(t)(U_{\mathrm{A}}(t) - \delta G(t)), \end{aligned} \tag{39}$$

where $\mu^{\mathrm{S}}(t)$ is a costate variable. By employing the necessary conditions of the maximum principle, we obtain

$$\frac{\partial H_{\mathrm{R}}^{\mathrm{S}}}{\partial U_{\mathrm{A}}} = 0, \tag{40}$$

$$\frac{\partial H_{\mathrm{R}}^{\mathrm{S}}}{\partial G} = \rho\mu - \dot{\mu}. \tag{41}$$

Next, it follows from (40) that

$$U_{\mathrm{A}} = \mu^{\mathrm{S}}. \tag{42}$$

In addition, from (41), we get

$$\begin{aligned}\dot{\mu}^{\mathrm{S}} &= \rho\mu^{\mathrm{S}} - \frac{\partial H_{\mathrm{R}}^{\mathrm{S}}}{\partial G} \\ &= (\rho+\delta)\mu^{\mathrm{S}} - \left(\gamma(\phi p(t) - (1-\alpha)c)p(t)^{-b} + \psi\right).\end{aligned} \tag{43}$$

By solving the differential equation in (43), one has

$$U_{\mathrm{A}} = \mu^{\mathrm{S}} = c_2 e^{(\rho+\delta)t} + \left(\gamma(\phi p(t) - (1-\alpha)c)p(t)^{-b} + \psi\right). \tag{44}$$

Similar to Proof 1 of Theorem 1, the store-assistance service investment is

$$U_{\mathrm{A}} = \gamma(\phi p(t) - (1-\alpha)c)p(t)^{-b} + \psi. \tag{45}$$

Jointly solving (37) and (45), the optimal strategies of supply chain under the state-dependent contract are given by

$$\begin{aligned}p^{\mathrm{S}^*} &= \frac{\alpha bc}{(1-\phi)(b-1)}, \\ U_{\mathrm{A}}^{\mathrm{S}^*} &= \gamma\left(\phi p^{\mathrm{S}^*} - (1-\alpha)c\right)\left(p^{\mathrm{S}^*}\right)^{-b} + \psi.\end{aligned} \tag{46}$$

Based on the definition of supply chain coordination, it can be concluded that the supply chain is coordinated by $p_{\mathrm{A}}^{\mathrm{S}^*} = p_{\mathrm{A}}^{\mathrm{C}*}$ and $U_{\mathrm{A}}^{\mathrm{S}^*} = U_{\mathrm{A}}^{\mathrm{C}^*}$. Then, $\phi = 1-\alpha$, $\psi = \alpha\gamma(b-1)^{b-1}/(c^{b-1}b^b)$, which completes the proof of this theorem.

When the supply chain is coordinated, the profits of both members can be given as follows:

$$\begin{aligned}J_{\mathrm{M}}^{\mathrm{S}^*} &= \frac{\alpha a(b-1)^{b-1}}{\rho c^{b-1}b^b} + \frac{\alpha\gamma(b-1)^{b-1}G_0}{(\rho+\delta)c^{b-1}b^b} + \frac{\alpha\gamma^2(b-1)^{2(b-1)}}{\rho(\rho+\delta)^2 c^{2(b-1)}b^{2b}} \\ &\quad - \frac{\alpha\gamma(b-1)^{2(b-1)}G_0}{(\rho+\delta)c^{2(b-1)}b^{2b}} - \frac{\alpha\gamma^2(b-1)^{2(b-1)}}{\rho(\rho+\delta)^2 c^{3(b-1)}b^{3b}} - \frac{k}{\rho} \\ &= \alpha J_{\mathrm{C}}^* + \Delta - \frac{k}{\rho}, \\ J_{\mathrm{R}}^{\mathrm{S}^*} &= (1-\alpha)J_{\mathrm{C}}^* - \Delta + \frac{k}{\rho},\end{aligned} \tag{47}$$

where

$$\begin{aligned}\Delta &= \frac{\alpha\gamma^2(b-1)^{2(b-1)}}{\rho(\rho+\delta)^2 c^{2(b-1)}b^{2b}} - \frac{\alpha\gamma(b-1)^{2(b-1)}G_0}{(\rho+\delta)c^{2(b-1)}b^{2b}} \\ &\quad - \frac{\alpha\gamma^2(b-1)^{2(b-1)}}{\rho(\rho+\delta)^2 c^{3(b-1)}b^{3b}}.\end{aligned} \tag{48}$$

In order to ensure that both party members are involved in this supply chain coordination contract, the following conditions should be satisfied simultaneously: $J_{\mathrm{R}}^{\mathrm{S}^*} > J_{\mathrm{R}}^{\mathrm{D}*}$ and $J_{\mathrm{M}}^{\mathrm{S}^*} > J_{\mathrm{M}}^{\mathrm{D}*}$, then we can have the following conclusion.

Proposition 3. *When $k_1 < k < k_2$, both the members of supply chain are willing to involve in the implementation of this contract, where $k_1 = \rho(J_{\mathrm{R}}^{\mathrm{D}*} + \Delta - (1-\alpha)J_{\mathrm{C}}^*)$, $k_2 = \rho(\alpha J_{\mathrm{C}}^* + \Delta - J_{\mathrm{M}}^{\mathrm{D}*})$, and the interval (k_1, k_2) is called the feasible region of contract (ϕ, k, ψ) with $\phi = 1-\alpha$ and $\psi = \alpha\gamma(b-1)^{b-1}/(c^{b-1}b^b)$.*

Theorem 3 and Proposition 3 show that the retailer needs to set the constant revenue share for per unit sold based on the share of the channel cost when the supply chain is coordinated by the contract (ϕ, k, ψ), and each member of supply chain will participate in the implementation of contract when k locates between k_1 and k_2. Furthermore, the precise value of k can be determined by the Nash bargaining model by using further information, such as the risk preferences of the manufacturer and the retailer. Specifically, let λ_{m} and λ_{r} represent the risk preference coefficients of the manufacturer and the retailer, respectively; the utility functions for the manufacturer and the retailer are

$$\begin{aligned}u_{\mathrm{m}}(\Delta J_{\mathrm{M}}) &= (\Delta J_{\mathrm{M}})^{\lambda_{\mathrm{m}}}, \\ u_{\mathrm{r}}(\Delta J_{\mathrm{R}}) &= (\Delta J_{\mathrm{R}})^{\lambda_{\mathrm{r}}},\end{aligned} \tag{49}$$

where $\Delta J_{\mathrm{M}} = J_{\mathrm{M}}^{\mathrm{S}^*} - J_{\mathrm{M}}^{\mathrm{D}*} = (k_2 - k)/\rho$ and $\Delta J_{\mathrm{R}} = J_{\mathrm{R}}^{\mathrm{S}^*} - J_{\mathrm{R}}^{\mathrm{D}*} = (k - k_1)/\rho$ are the extraprofits for the manufacturer and the retailer with the state-dependent contract, and $\Delta J_{\mathrm{M+R}} = \Delta J_{\mathrm{M}} + \Delta J_{\mathrm{R}}$. Then we can establish Nash bargaining model as follows:

$$\begin{aligned}\max_{\Delta J_{\mathrm{M}}>0, \Delta J_{\mathrm{R}}>0} \quad & u_{\mathrm{m}}(\Delta J_{\mathrm{M}})u_{\mathrm{r}}(\Delta J_{\mathrm{R}}) = (\Delta J_{\mathrm{M}})^{\lambda_{\mathrm{m}}}(\Delta J_{\mathrm{R}})^{\lambda_{\mathrm{r}}} \\ \text{s.t.} \quad & \Delta J_{\mathrm{M}} + \Delta J_{\mathrm{R}} = \Delta J_{\mathrm{M+R}}.\end{aligned} \tag{50}$$

The Nash bargaining solutions are

$$\begin{aligned}\Delta J_{\mathrm{M}} &= \frac{\lambda_{\mathrm{m}}}{\lambda_{\mathrm{m}} + \lambda_{\mathrm{r}}}\Delta J_{\mathrm{M+R}}, \\ \Delta J_{\mathrm{R}} &= \frac{\lambda_{\mathrm{r}}}{\lambda_{\mathrm{m}} + \lambda_{\mathrm{r}}}\Delta J_{\mathrm{M+R}},\end{aligned} \tag{51}$$

which imply that the manufacturer and the retailer distribute the extraprofit proportionally related to their risk preference. Moreover, according to $\Delta J_{\mathrm{M}}/\Delta J_{\mathrm{R}} = \lambda_{\mathrm{M}}/\lambda_{\mathrm{R}}$, we have

$$k = \frac{\lambda_{\mathrm{m}}}{\lambda_{\mathrm{m}} + \lambda_{\mathrm{r}}}k_1 + \frac{\lambda_{\mathrm{r}}}{\lambda_{\mathrm{m}} + \lambda_{\mathrm{r}}}k_2. \tag{52}$$

Also, the change of parameters γ and b will affect the values of k_1 and k_2; hence, we will illustrate these effects in the next section.

Table 1: The impact γ onto the decisions and profits.

γ	p^{D*}	p^{C*}	φ^*	U_A^{D*}	U_A^{C*}	ΔU	J_M^{D*}	J_R^{D*}	J_C^*	ΔJ
1.0	31.43	19.46	0.504	0.033	0.061	0.028	12.340	24.912	46.477	9.226
1.5	31.43	19.46	0.504	0.049	0.092	0.043	12.428	25.082	46.806	9.297
2.0	31.43	19.46	0.504	0.066	0.122	0.056	12.518	25.255	47.144	9.371
2.5	31.43	19.46	0.504	0.082	0.153	0.071	12.611	25.431	47.492	9.450
3.0	31.43	19.46	0.504	0.098	0.183	0.085	12.707	25.610	47.849	9.532

Table 2: The impact b onto the decisions and profits.

b	p^{D*}	p^{C*}	φ^*	U_A^{D*}	U_A^{C*}	ΔU	J_M^{D*}	J_R^{D*}	J_C^*	ΔJ
1.50	85.80	33.00	0.692	1.199	1.934	0.735	118.84	379.05	618.30	120.41
1.75	53.04	25.67	0.612	0.485	0.835	0.350	59.05	151.38	262.40	51.96
2.00	39.60	22.00	0.555	0.210	0.379	0.069	29.19	64.45	118.14	23.50
2.25	32.47	19.80	0.512	0.096	0.177	0.081	14.50	29.67	55.14	10.97
2.50	28.11	18.33	0.478	0.045	0.085	0.040	7.24	13.88	26.36	5.24

6. Numerical Analysis

In this section, a simulation is given to illustrate the managerial insights by considering the following factors: (1) the effects from the effectiveness of the retailer's platform users γ and the effects from the price-elasticity index of demand b onto the equilibrium strategies and profits of the related supply chain and (2) the impacts from two parameters γ and b on coordination. Define the gap of profits between the centralized and decentralized scenarios $\Delta J = J_C^* - J_R^{D*} - J_M^{D*}$ and the gap of advertising efforts $\Delta U = U_A^{C*} - U_A^{D*}$. To address these questions, the following parameter values are chosen in the example: $\alpha = 0.8$, $a = 500$, $c = 11$, $\delta = 0.05$, $b = 1.5$, $\rho = 0.1$, $G_0 = 10$, and $\gamma = 2.3$.

Firstly, we focus on the effect from the effectiveness of the retailer's platform users γ on the corresponding decisions and profits under centralized and decentralized scenarios. We keep the values of α, a, c, δ, b, ρ, and G_0 unchanged and adjust the value of γ such that γ varies from 1 to 3 with step 0.5. According to Theorems 1 and 2, we can obtain Table 1. From Table 1, we can see that the parameter γ positively affects the advertising effort and the profits of supply chain but does not affect the retail price and the revenue share. The result indicates that, regardless of the centralization or decentralization, the higher effect from the number of the retailer's platform users onto the market demand induces a higher advertising effort and constant prices, which leads to a higher supply chain profits. Compared with the decentralized equilibria, the advertising effort and profits in the centralized scenario are relatively higher. Moreover, the gap of the profits among two scenarios and the gap of the advertising effort enlarge when γ increases. It means that the advertising effort and profits in both centralized and decentralized scenarios increase with γ, and the increase is more faster in the centralized scenario.

Next, we focus on the effect from the price-elasticity index of demand b onto the corresponding decisions and profits under the centralized and decentralized scenarios. When we adjust the value of b such that b varies from 1.5 to 2.5 with step 0.25, the conditions are satisfied in Proposition 1 based on the basic parameter values. As shown in Table 2, the increasing price-elasticity index of demand b leads to a lower retail price, the advertising effort and the profits in both centralized and decentralized scenarios. This result indicates that, regardless of the centralization and decentralization, the retailer is motivated by high price-elasticity index of demand and will invest less in the advertising effort and set a lower revenue share. In the same time, the manufacturer will set lower retail price, which results in a higher profit. Compared with the decentralized equilibria, the centralized ones are relatively higher, which is consistent with the results in Proposition 2. Moreover, the gap of the advertising effort and the gap of profits among two scenarios shrink with increasing price-elasticity index of demand, which means that the advertising effort and profits in both centralized and decentralized scenarios decrease with b, and the decrease is more faster in centralized scenario.

Finally, we aim to examine how the parameters γ and b affect the coordination contract proposed in Section 4. According to Theorems 1–3 and Proposition 1, we have Figures 1 and 2, which show that the supply chain is coordinated by the state-dependent contract (ϕ, k, ψ) when the contract parameter k is located between k_1 and k_2. Next, let $k^* = k_2 - k_1$ represents the span of the feasible region for contract. Figure 3 shows that the feasible region between k_1 and k_2 becomes larger as γ increases, which means that a larger γ will provide the retailer a greater degree of flexibility to coordinate the supply chain. Figure 2 depicts that the feasible region becomes smaller when the price-elasticity index of demand b increases. This implies that a larger b will provide the retailer a smaller degree of flexibility to coordinate the supply chain.

Remark 3. The problems of the coordinating pricing and advertising decisions for supply chain under consignment contract in the dynamic setting described in this paper are

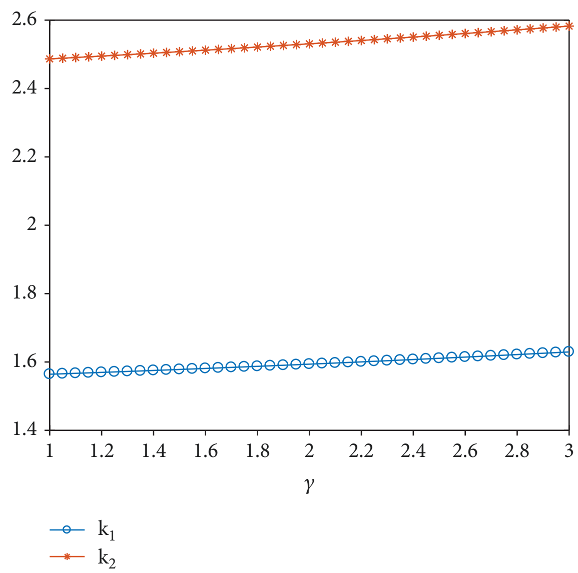

Figure 1: The impact of γ onto the span of the feasible region for contract.

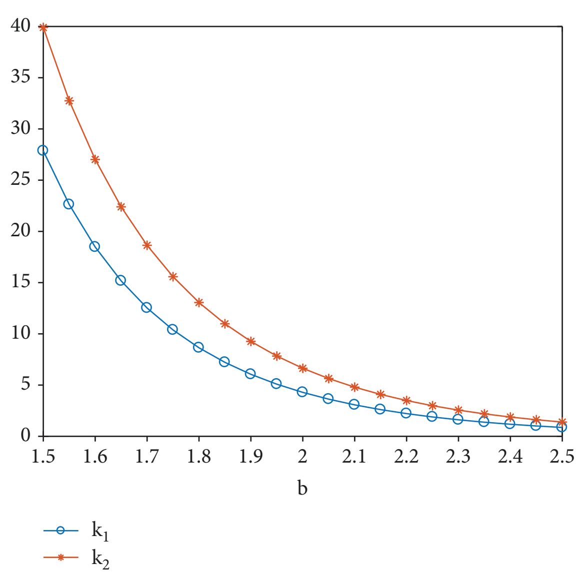

Figure 2: The impact of b onto the span of the feasible region for contract.

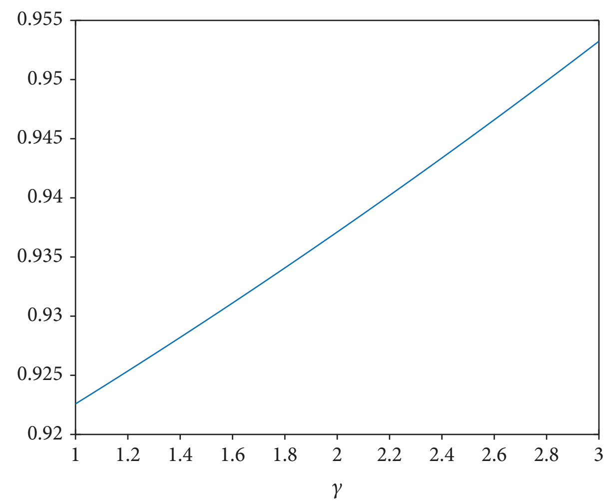

Figure 3: The impact of γ onto k^*.

new and more comprehensive than the established ones. A literature search indicates that there have been no existing pricing and advertising decision methods applied to the same problem addressed in this paper. In particular, it should be mentioned that this paper makes the first attempt to develop a new contract in the dynamic setting for the supply chain under the consignment background, which constitutes the major advantage compared to existing methods.

7. Conclusions

In this paper, we have discussed the decision problems of the supply chain under the consignment contract in a dynamic setting. By applying the optimal control theory, the strategies of optimal pricing and advertising have been presented in the decentralized and centralized circumstances. The main novelties lie in that (i) the effect from the number of the retailer's platform users onto the market demand has been considered and the number of the retailer's platform users has been set as a state variable and (ii) by constructing the differential game model, both the pricing and advertising strategies have been provided within the centralized and decentralized setting, and a state-dependent contract has been designed to coordinate the decentralized supply chain in a dynamic background. Finally, a numerical analysis has been conducted to illustrate the effectiveness of the number of the retailer's platform users and the effects from the price-elasticity index onto the equilibria and coordination. In particular, we have obtained the following results: (1) the optimal retail price is lower and the optimal advertising effort is higher in the centralized setting and (2) the state-dependent contract can effectively improve the performance of the decentralized supply chain under the consignment contract. Future research topics include the extension of the proposed strategies to the problems of the performance analysis and coordination scheme design for a supply chain with a monopolist retailer and price-competing duopolist manufacturers and to the coordination problem for a supply chain with lagged effect on the number of the retailer's platform users.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there is no conflict of interests regarding the publication of this paper.

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