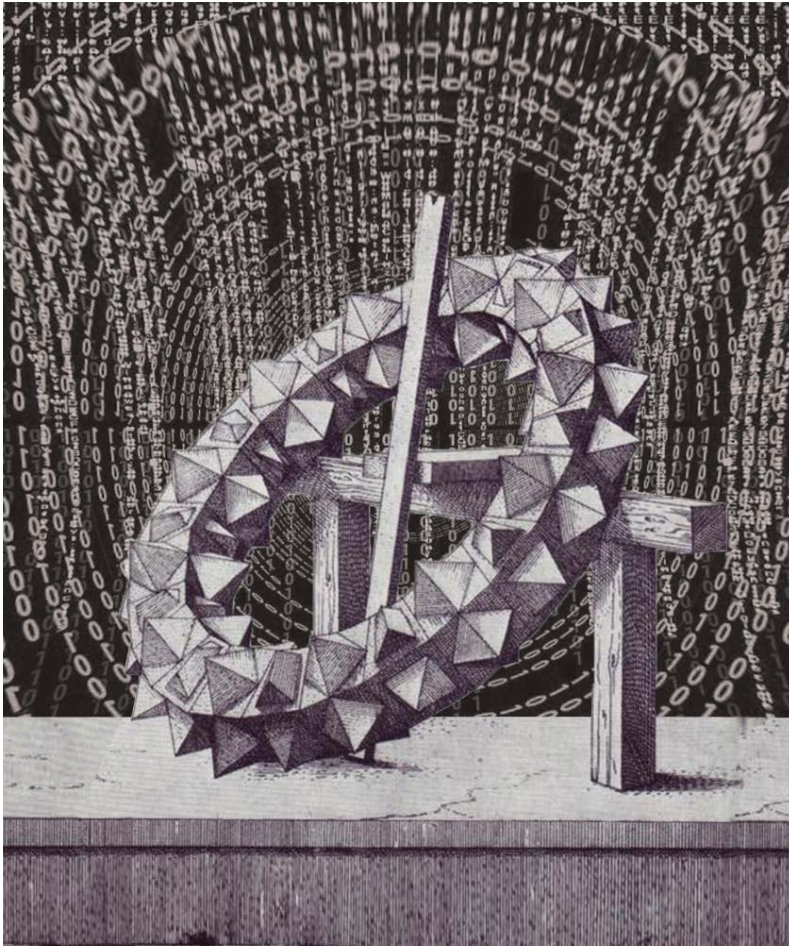


# IN DEFENSE OF PLATONIC ESSENTIALISM ABOUT NUMBERS



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## ABSTRACT

In defense of anti-essentialism, pragmatist Richard Rorty holds that we may think of all objects as if they were numbers. I find that Rorty's metaphysics hinges on two rather weak arguments against the essences of numbers. In contrast, Plato's metaphysics offers a plausible definition of essentiality by which numbers do have essential properties. Further, I argue that Rorty's argumentative mistake is mischaracterizing Plato's definition. I conclude that Plato's definition of "essential" is a robust one which implies that many properties, beyond those we might intuitively think of, can count as essential properties of objects.



## I. INTRODUCTION

The overarching project of Plato's *Phaedo* is to offer an account of the immortality of the soul, prompted by Socrates' imminent death. To construct this argument, Socrates is pressed by his interlocutors to defend the existence of abstract objects like the soul, the Good, and Beauty. To do so, Plato asserts the existence of entities called Forms. Though he is not explicit in *Phaedo* about the nature of the relationship, Plato suggests that Forms are related to properties of material objects despite not being the same as the objects. For example, despite having hotness as a property, fire is not the same entity as "the hot" itself.<sup>1</sup> Further, Plato describes how objects that contain a particular Form cannot take on the properties of the opposite Form without perishing—that is, a flame cannot admit the form of the Cold.<sup>2</sup> This suggests that the properties of objects to which Forms are related are properties that are essential to the object: without such properties or being related to the Form in the same way, the object would no longer be itself. This view of Forms makes Plato out to be a metaphysical essentialist; his theory of Forms implies that objects have natures which can be described by unchanging, abstract properties.

In stark contrast to Plato's essentialism, American pragmatists, such as Richard Rorty, broadly reject any appeals to metaphysical essential natures, preferring a metaphysics where all properties of objects are merely external relations—that is, the property is external to the object itself because properties are really describing something about the object which is useful for "human needs or consciousness or language."<sup>3</sup> For a pragmatist like Rorty, the sentence, "X is blue," communicates that it is useful to think of X as blue rather than describing anything intrinsic to the object X.<sup>4</sup> As Rorty tells it, one motivation behind such a pragmatist metaphysical project is to do away with distinctions like "subject versus object" and "mind versus body," which have stoked seemingly irresolvable disputes throughout the history of Western philosophy.<sup>5</sup> More broadly, Rorty's move towards pragmatism is responsive to the linguistic turn in the nineteenth century; pragmatism acquiesces to the worry that our knowledge of things, in themselves, is fundamentally altered by the language with which we capture that knowledge.<sup>6</sup> Implications of Rorty's pragmatism

1 Plato, "Phaedo," in *Plato: Complete Works*, trans. G.M.A. Grube (Indianapolis: Hackett, 1997), 103c8-d3.

2 Plato, "Phaedo," 104c1.

3 Richard Rorty, "A World without Substances or Essences," in *Philosophy and Social Hope* (Westminster: Penguin Books, 1999), 50.

4 Rorty, "Substances or Essences," 51.

5 Rorty, "Substances or Essences," 47.

6 Rorty, "Substances or Essences," 50.

include the obliteration of the correspondence theory of truth, because pragmatists instead posit that truth is a matter of social utility; further, pragmatism entails that ethics is reducible to mere norms.

To sketch out the broad, metaphysical picture that his pragmatist view entails, Rorty suggests that his readers "think of everything as if it were a number," because, as he argues, numbers are not the type of thing which have essential natures.<sup>7</sup> If Rorty is incorrect and numbers can have essential properties, then his entire anti-essentialist metaphysical theory becomes significantly less plausible—both because he is substantively incorrect that no object can have essential properties, and because the picture of the world he takes to be the most compelling reflects the opposite of what he believes.

Contra Rorty, in *Phaedo*, Plato asserts the existence of essential properties of numbers by defining a theory of Forms that can cause certain essential, non-relational properties in particular objects, such as numbers, which instantiate them. I outline Plato's account of numbers as having essential properties, and I argue that Rorty's objections—that all properties of numbers are really relationships between the number and other numbers and that numbers have no essential natures—ultimately fail, in large part because Rorty mischaracterizes Plato's definition of "essential." I finally argue that there might be even more types of essential properties than those Plato suggests, by arguing that some scalar relations between particulars can also be essential by the provisional definition of "essential" that Plato gives in *Phaedo*.

## II. NUMBERS IN PLATO'S PHAEDO

Plato begins his argument by describing the relationship between Forms and the objects which instantiate their properties. He writes that, "it is through Bigness that big things are big ... smaller things are made small by Smallness," thereby suggesting that Bigness is an essential property of big objects, because they were "made" to be that way by the form of the Big.<sup>8</sup> Then, Plato puts forward the contradiction that if a big object was said to be bigger than something else by an amount smaller than itself, such as a "man [who] is taller than another by a head," then that man would be made bigger by something small—the Small would be what causes Bigness.<sup>9</sup> This would contradict Plato's initial account of the causal powers of Forms because the Forms, in this case, cause the opposite of themselves—the Form of the Small causes the man to be bigger, despite the initial assertion that big things are big because of the Form of the Big. To avoid this contradiction,

7 Rorty, "Substances or Essences," 52.

8 Plato, "Phaedo," 100e5-6.

9 Plato, "Phaedo," 100e7-101a1.

Plato concludes that it is not the relative difference or the relationship between two material objects which gives them a property like bigness or smallness. Rather, it is Forms which cause properties like Bigness to appear in objects. In the context of numbers, Plato's argument implies that it is not a comparative relationship between numbers, like addition or division for example, that can cause a property like Twoness—besides the Form of Two, there is no “other cause of becoming two except by sharing in Twoness.”<sup>10</sup> For example, the number 2 is not the number 2 because it is one more than the number 1, or because it is half of 4—it is the number 2 because it possesses the property of Twoness, which is given by the Form of Two. In so arguing, Plato suggests that Forms (such as the magnitude of numbers) are what give numbers their properties and that those properties are intrinsic to the object, not dependent on comparison to other objects.

Plato then goes on to explain the apparent contradiction between the different relationships that objects can have to other objects. Plato reiterates that, “the opposite could never become opposite to itself,” because a property will “retreat before [its opposite] or be destroyed.”<sup>11</sup> Then, Plato characterizes particular instantiations of Forms as “something else that is not the Form but has its character whenever it exists.”<sup>12</sup> For example, the number 3 is not the Odd but has the property of being Odd: “it must always be called both by its own name [three] and by that of the Odd.”<sup>13</sup> Note here Plato's equivocation of the copula: saying that the number 3 is 3 is to state an identity relationship between 3 and itself, but also saying that 3 is Odd is to apply the propositional description of Oddness to the number 3. To illustrate how instantiations of Forms will “perish or give way” before admitting their opposite, Plato describes how “three will perish or undergo anything before, while remaining three, becoming even.”<sup>14</sup> In other words, making 3 even (perhaps by adding or subtracting 1) will cause 3 to perish and become 2 or 4; there is no way to make 3 even while retaining its three-ness. This suggests, importantly, a definition of essential properties of objects: a property of an object is essential if replacing that property with its opposite would cause the object to no longer be the same object.

Assuming the existence of forms, one argument against Plato's theory of Forms is that it is unclear what things are Forms and which are not, and therefore, it is unclear which of an object's properties can

be said to be essential. For example, Plato discusses the number 3 as if it is a particular that instantiates the Form of the Odd in the second section of his argument,<sup>15</sup> but also refers to things like “Twoness” as being an essence of the number 2.<sup>16</sup> That raises the question of whether numbers are particular objects or are themselves abstract entities like Forms. While *Phaedo* is not particularly explicit about this apparent ambiguity, there are ways to reconcile this tension. We might hold that individual numbers are particulars, because they are instantiations of Forms like the Odd that Plato is committed to and that they also instantiate other forms like Twoness. If we think that the form of Twoness causes the property of having two elements in its instantiations, then this holds for the number 2, because it has two elements, each with a size of one. This view still allows us to hold that numbers, like 2, are themselves objects that are distinct from their Form in the same way that a cup is distinct from the Form of the Cup, despite being more abstract than physical objects like cups. Fortunately, given that Plato believes in the existence of Forms themselves, he would likely also be willing to maintain that individual numbers exist distinctly as abstract particulars.

Another issue with Plato's argument is his conflation of binary and scalar properties of objects. When explaining why the properties of objects which Forms instantiate cannot be relational and must be essential, Plato uses the example of Simmias being “taller than Socrates but shorter than Phaedo” to conclude that Simmias has both Tallness and Shortness—which Socrates concludes is a contradiction.<sup>17</sup> However, it is only because Plato seems to hold that Forms are essential that there is a contradiction; if we hold that Tallness is a primarily relational or scalar type of property (i.e., to say something is tall implies that it is tall in relation to other things), then Simmias' relation of tallness does not produce a contradiction. This interpretation of the property of tallness is most consistent with how we describe height in ordinary language. For example, under Plato's interpretation of tallness as an essential, non-relational property, we could never say that a child is tall because they are taller than other children their age (despite being shorter than almost all adults). The Platonist might respond by bracketing out all properties which seem relational (such as Tallness, Bigness, or Warmness) from the set of things that are legitimately Forms. However, doing so would mean that Forms are essential, not relational, just because they have been so defined and not because of a metaphysical fact about their nature.

10 Plato, “Phaedo,” 101c4-5.

11 Plato, “Phaedo,” 103b2-3; 103d6.

12 Plato, “Phaedo,” 103e3-4.

13 Plato, “Phaedo,” 104a5-6.

14 Plato, “Phaedo,” 104c1; 104c2-3.

15 Plato, “Phaedo,” 103-104.

16 Plato, “Phaedo,” 101c5.

17 Plato, “Phaedo,” 102b3-4.



### III. AGAINST RORTY'S ANTI-ESSENTIALISM

It is the force of this final objection that motivates a pragmatist response to Plato's theory of numbers. As part of his broader project of sketching out an antiessentialist metaphysics, Rorty suggests we "think of everything as if it were a number," because he holds that numbers do not have essential properties.<sup>18</sup> Despite only providing two arguments against essentialism about numbers, Rorty's claim that numbers are both essence-less and like all other objects in existence has powerful implications. To take Rorty at his word would first mean committing to the view that no material objects have any essential properties and are, only truly describable in relation to other objects. Second, Rorty's view implies that even other entities that are as abstract as numbers—beliefs, moral maxims, or perhaps even colors, for example—do not have essences either; they, too, are mere conventions. Adopting Rorty's view would have powerful implications for the way we as humans think about our material, moral, and epistemic roles in the world, to say the least. Despite the strength of Rorty's conclusion, however, I argue that his two arguments in favor of it are in fact relatively weak.

Rorty's first argument for why numbers, like 17, do not have essential properties is that there are many ways to describe 17 in terms of operations that can be done on other numbers, but none of them seem to capture the essential properties of 17 better than any of the other descriptions. Rorty's second argument is that because each way we might describe 17 does specify "all its relations to *all* the other numbers," a mathematician attempting to describe an essential property of 17 would have to refer to arithmetic and set theoretic axioms that specify the relations between all numbers.<sup>19</sup> However, Rorty holds, these axioms do not uniquely describe 17; "they are equally the essence of 1, or 2, of 289, and of 1,678,922."<sup>20</sup>

Overall, I am skeptical of whether Rorty's arguments against essentialism constitute actual problems for the essentialist. His first argument is that no essential description of 17 exists because no description can capture "the intrinsic seventeeness of 17—the unique feature which makes it the very number that it is."<sup>21</sup> Rorty's claim is ultimately that essences of objects cannot exist, because no description of an object could describe a property that makes it unique from other objects. This is a clear conflation of essential properties and unique properties. Rorty's argument is analogous to the claim that an essential description of the cup-ness of a cup on my desk must capture the

essence of that particular cup rather than a shared essence of cups in general. Even if Rorty is correct that essential properties could only be true of one object, it is still possible for there to be properties of a number, like 17, that are unique to it and essential by Plato's definition of "essential." For example, every integer has a unique decomposition into prime factors, and if that decomposition were different, the number itself would have to be a different number.<sup>22</sup> But, besides Rorty's conflation, his argument does not pose a significant challenge for Plato's view. Returning to *Phaedo*, Socrates describes how "two and four and the whole other column of numbers; each of them, while not being the same as the Even, is always even."<sup>23</sup> Under Plato's own view, Evenness is a Form because it gives the essential property of being even to the set of numbers given by the series (2, 4, 6, ...), and no even number is more even than any other. Forms are general and abstract because they cause properties that many different particulars may share, and particulars therefore cannot be the same as the Forms whose properties they possess. Rorty's objection is, therefore, resolved by Plato's characterization of Forms as abstract entities which can give essential properties to multiple objects.

Ultimately, Rorty is conflating essential properties of numbers with unique properties of numbers. Rorty seems to want a description of the essence of 17 (i.e., Seventeen-ness) that is completely different than Twoness or the essence of any other number, but Plato's theory of Forms does give an account of the essential properties of numbers, even if those essential properties are not unique to any one particular number. Recall the provisional definition of "essential" given by Socrates: that the object could not admit the opposite of its Form without perishing.<sup>24</sup> By this account, Oddness and Prime-ness are essential properties of 17, because 17 would have to be an altogether different number to be even or not prime. Moreover, this means that even those relations between 17 and other numbers are essential relations. For example, 17 is related to 18 by being smaller by 1. If 17 were smaller than 18 by 2, it would no longer be 17, it would be 16, and it would perish. We can infer from this definition that relations between 17 and other numbers are internal relations and therefore, are essential to Seventeen-ness.

Rorty's second argument (against the mathematician who claims that set theoretic axioms offer an essential description of 17) is that set

18 Rorty, "Substances or Essences," 52.

19 Rorty, "Substances or Essences," 53.

20 Rorty, "Substances or Essences," 53.

21 Rorty, "Substances or Essences," 53.

22 Seventeen is decomposable into  $17 * 1$ . A number, like 20 for example, is decomposable into  $2^2 * 5$ . It could not be the case that 20 decomposes into  $2 * 5$ , because the original 20 would perish and be replaced by 10.

23 Plato, "Phaedo," 104b2-3.

24 Plato, "Phaedo," 104c2-3.



theoretical axioms apply to all other numbers. In other words, Rorty's charge against Plato's essentialism is that the theory of Forms needs to offer an account of essential properties where different objects cannot share any properties in common. This is not a wholly reasonable objection, because two different objects can have some properties in common and other properties that are different, whether those properties are essential or not. For example, even though 17 and 2 are clearly different numbers and therefore, have some things about them that are different, they may also share certain properties by virtue of both being numbers. For example, 17 is odd and 2 is even, so Seventeen-ness would not be able to admit the Even, while Twoness would, which offers a partial explanation of why 17 and 2 cannot be the same number. Despite this difference, we can still hold that both Seventeen-ness and Twoness both admit the Forms of the Integer and the Prime. All this is to demonstrate that distinct objects, and therefore, distinct forms, can share some properties while having some properties that are different. This disproves Rorty's objection because, under Plato's view, it can be true that an object has an essential property that other objects also have. There is no reason why an essential property has to be unique to a certain particular. It is possible for set theoretic axioms to be essential properties of the number 17, because if 17 were not equal to the set of all natural numbers less than it, then it would either be a different natural number or not a natural number at all—both of which would essentially change the nature of 17. Therefore, set theoretic axioms do constitute an essential feature of numbers.

Plato's essentialism about numbers is thus resistant to Rorty's objections. I return, then, to the problem posed earlier in Plato's proof by contradiction—that Plato's account of Forms must be that the properties given by Forms to objects are intrinsic to the object, not describing the relationship between it and something else. Perhaps, it is even not impossible for relational attributes to be essential *qua* Plato's definition of "essential." We can make sense of this by considering my previous argument that the relationships between 17 and other numbers are essential, because changing those relationships would require us to define the number 17 differently—that is, the number 17 would perish. With reference to the example of Simmias being taller than Socrates yet shorter than Phaedo, this understanding of relations as essential would hold that Simmias would not be the same Simmias if he were, for example, shorter than Socrates rather than taller. Of course, it is slightly harder to find this example to be intuitive, because defining personal identity is not necessarily clear. This view requires a fairly strong conception of personal identity such that changing one's height,

personality, ethnicity, etc. would constitute a change in a person's identity.

An antiessentialist might broadly reply to the claim that relations between objects could be essential by pointing out the existence of relations that, if changed, clearly do not make an object perish. Neither this argument for essential relations nor Plato's essentialism denies that there can be properties of objects that are not essential, so the existence of non-essential relations between objects does not constitute an objection to either theory of essentialism. For example, ownership of my sweater is a relationship between me and my sweater, but if my sweater were owned by someone else, it would still be the same sweater even though it would not be related to me in the same way. All that would show is that ownership is the type of relation between objects that is internal, and therefore, ownership does not constitute an essential property of an object; this supports neither the conclusion that no relations are internal nor that relationships cannot be internal.

#### IV. CONCLUSION

I have outlined and defended a Platonic account of the essential natures of numbers. In doing so, I have identified a definition of essential properties given in *Phaedo* and used that definition of essential to respond to Rorty's antiessentialist arguments about numbers. That analysis suggests that perhaps even relations between numbers are essential to them—though they seem external—and that if it is possible for relations between objects to be essential to those objects, then essentialism about numbers and properties might even be stronger than what Plato suggests in *Phaedo*. In particular, if it is the case that relational attributes can be essential properties, and that essential properties must be given by Forms, there are infinitely many possible ways to relate an object to other objects. This might imply the existence of infinitely many Forms, which would indeed result in a very bountiful, yet cluttered metaphysics.

Some final questions that merit further investigation are what sorts of standards might be used to determine whether a particular object perishes in the face of its opposite, or by how much an object must change to be said to be no longer the same object. Despite these minor ambiguities in Plato's definition, preserving the view that abstract entities like numbers can have essential properties gives us cause for optimism about the existence of other physical properties like color. The highly essentialist contours of Plato's philosophy, therefore, offer a promising and compelling alternate view to pragmatist metaphysics.





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