When the actual world is not even possible

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Abstract

Approaches to quantum gravity often involve the disappearance of space and time at the fundamental level. The metaphysical consequences of this disappearance are profound, as is illustrated with David Lewis's analysis of modality. As Lewis's possible worlds are unified by the spatiotemporal relations among their parts, the non-fundamentality of spacetime—if borne out—suggests a serious problem for his analysis: his pluriverse, for all its ontological abundance, does not contain our world. Although the mere existence—as opposed to the fundamentality—of spacetime must be recovered from the fundamental structure in order to guarantee the empirical coherence of the non-spatiotemporal fundamental theory, it does not suffice to salvage Lewis's theory of modality from the charge of rendering our actual world impossible.

1 Introduction

Various approaches to formulating a quantum theory of gravity either presuppose or entail that fundamentally, there is neither space nor time. Instead, space and time emerge from the more fundamental, non-spatiotemporal structure of quantum gravity very much in the way that tables and chairs emerge from the more fundamental, non-chairtabular structure of quantum particle physics.

This may have cataclysmic consequences for metaphysics: some philosophical analyses of causation, laws of nature, persistence, personal and material identity, and even modality crucially seem to rely on the fundamental existence of space and time. For instance, David Lewis (1986) characterizes possible worlds as unified by the spatiotemporal relations among their parts but as spatiotemporally isolated from other possible worlds. If borne out, the disappearance of space and time would motivate a new—fatal—objection to Lewis's account of modality: his pluriverse, for all its ontological abundance, does not contain our world.

Apart from questioning the truth of theories denying the fundamental existence of spacetime, the Lewisian may respond in one of two ways to this shock: either the empirical coherence of any theory denying the fundamental existence of space and time can be questioned, or it can be argued that Lewisian modality only requires the existence, but not the fundamentality, of space and time. This second response may succeed as long as it is assumed that no sensible theory of fundamental physics questions the—perhaps non-fundamental—existence of space and time.

This essay contends that the first strategy fails, even though it unveils an important foundational task for the defender of a theory-sans-fundamental spacetime. In order to avoid the charge of empirical incoherence, and thus to salvage the possibility of its own epistemic justification, such a theory must be shown to admit emergent spacetime. Thus, in circumventing the first Lewisian

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response the assumption of the second response—that spacetime exists at some ontological level—is granted. However, this by no means entails that the second response succeeds. In fact, it is argued that the merely emergent existence of space and time comes at an unpalatably high cost to Lewis.

§2 explicates the need for a quantum theory of gravity and argues that such a theory will be our most fundamental theory of gravity, at least to date. In §3, I will show how spacetime may not be part of the fundamental furniture of the world by introducing the conceptually simple and clean case of causal set theory. §4 acts as a reminder of the fact that it is spatiotemporal relations that unify and isolate worlds in Lewis's pluriverse, and §5 parses out the trouble it encounters if those spatiotemporal relations are absent. §6 and §7 articulate and discuss the two Lewisian strategies in responding to the challenge presented, respectively. I offer some brief conclusions in §8

2 Quantum gravity and fundamentality

Today, there are two incumbent theories in physics with a serious claim to be not just true, but fundamental theories: the standard model of particle physics and general relativity. The former describes the structure of what are (so far) the smallest scales at which physics makes reliable predictions and concerns the constitution of matter; the latter encodes the large-scale structure of our universe and its history. Physicists believe that there exist four fundamental forces operating in the physical realm; the standard model captures three of them, and general relativity the fourth. Both theories make eminently accurate predictions and both stand unrefuted, at least if evidence is restricted to direct tests of these theories.

Yet not everything is well in fundamental physics. For starters, the standard model and general relativity stand in significant conceptual tension. The standard model radically reconceptualizes matter and energy from the way they figured in pre-quantum theories, but general relativity relies on these obsolete notions. General relativity proposes an equally radically novel understanding of space and time and their joint interaction with their energy and matter content, departing from the pre-relativistic physics of space and time presupposed by the standard model.[1] This conceptual disunity is philosophically unattractive, but the clincher for their joint untenability is that there exist phenomena for which we have compelling reason to believe that their successful description must involve both quantum and relativistic, i.e., gravitational, effects. In other words, a theory is needed which commands the resources to *combine* the quantum with relativity. This is the theory the field of quantum gravity seeks to formulate, and I shall call a *quantum theory of gravity* any such theory with the explanatory ambition to account for these phenomena.

One might complain that the term 'phenomena' is ill-chosen given that they concern, e.g., the physics of the very early universe and the hitherto unobserved evaporation of black holes. Of course, it remains true that no *observation*, which is unambiguously quantum-gravitational in the sense necessary to justify the need for quantum gravity, has knowingly been made to date. But while the case for the evaporation of black holes may be more tenuous, the reasons to believe that our universe started out in a very dense state and that this state can only be correctly captured by a theory attending to both quantum and gravitational effects are firmly anchored in our currently best physical theories, the standard model and general relativity. The argument which translates these reasons into a need for a quantum theory of gravity requires little more in terms of assumptions about the actual world we inhabit beyond these reigning theories. So for present purposes I shall assume that a quantum theory of gravity is needed.

A quantum theory of gravity will be a fundamental theory, at least more fundamental than any other theory in physics currently or previously held to be true. Presently, I use fundamentality to

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[1] Or at least from the not *fully* relativistic assumptions made in the standard model.

denote a relation between theories, partially ordering the true theories of physics, perhaps including merely approximately true theories, or perhaps even including theories which have, or had, some currency in science. The fact that a theory is more fundamental than another in no way entails that the first theory is fundamental *simpliciter*. The relation I am interested in here is thus more appropriately termed 'relative fundamentality', although I will often suppress the qualification in what follows. Let us state what the relevant sort of fundamentality is.

To *establish* whether a given pair of theories exemplifies the relation of relative fundamentality may be a highly non-trivial matter, particularly once one abandons the exclusive focus on physics. For present purposes, I shall put aside this question and the more general debate on reductionism it invites; both deserve greater care than I can devote to them here. For the sake of the present argument, then, I rely on the conjunctive assumption that all hitherto relevant theories form part of a partially ordered set and that this set has at least one minimal element in the sense that it contains at least one theory that is not less fundamental than any other theory in the set of theories considered. Let us call this set T. For T to contain at least one minimal element it suffices that every totally ordered subset of T has a lower bound in T.[2] To have a lower bound means that for any subset of theories which are totally ordered with respect to fundamentality, there exists a theory t in T—and not necessarily in the subset—which is the most fundamental.[3] To demand that T contain at least one minimal element does not rule out that there exist several distinct theories with a justified claim to being the most fundamental.

My insistence that there exists at least one minimal element of T is usually given expression in the stipulation that the partial ordering of T be 'well-founded'. A binary relation which induces a partial order on its domain is *well-founded* just in case every non-empty subset of the domain has a minimal element with respect to the relation.[4] [5] Well-foundedness results in the present case from the demand that "all priority claims terminate." (Schaffer 2009, 500) In other words, if from anywhere in the relevant set one starts asking the question whether there exists a theory which is relatively more fundamental than the one at hand, and then whether there exists a theory which is relatively more fundamental than *that* theory, etc, we must reach a negative answer within a finite number of steps. This means that all subsets which are totally ordered must be finite.

My characterization of fundamentality differs from others prevalent in the literature. At least in the philosophical literature, fundamentality is usually understood in ontological or ideological terms rather than as a relation between theories. More specifically, fundamentality typically gets explicated by relations of ontological dependence obtaining among objects or structures or by mereological relations of parthood or by relations of supervenience holding among properties or some combination of these, e.g. in that properties of objects which ontologically depend upon, or are mereological complexes of, more basal or simple objects supervene on the properties exemplified by the basal or simple objects. The relevant relations are then taken to induce a partial ordering on their domain. To take theories to be the primary target of considerations regarding fundamentality instead of the objects they describe and their properties reflects my conviction that in fundamental physics theories indeed precede their ontological commitments and that it is thus more fruitful to

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[2] This follows, mutatis mutandis, from Zorn's lemma (Zorn 1935). For a subset A of a partially ordered set $(B, \leq)$, an element x of B is a *lower bound* of A just in case for all $a \in A$, $x \leq a$.

[3] At the danger of appearing excessively finicky, since the relevant ordering relation is irreflexive, and the ordering hence a strict partial order, t is more fundamental, rather than just no less fundamental, than those in the subset considered.

[4] More precisely, a partial order is well-founded if and only if the corresponding *strict* order is induced by a well-founded relation. This precisification, however, adds nothing to the case at stake since 'relative fundamentality', as stated above, is an irreflexive relation.

[5] Note also that in a subset consisting of just two theories, neither of which is more fundamental than the other, both of its elements, rather than none, are minimal.

address the question at the level of theories. Of course, the fundamentality relations as they obtain among theories will entail relations of ontological dependence among the objects or structures they postulate and relations of supervenience among the properties they ascribe to these objects or structures. In fact, one would hope that judgments about the fundamentality of theories are precisely mirrored by judgments concerning other ways in which fundamentality is considered.

Under the current description, the attempted quantum theory of gravity will certainly be more fundamental than general relativity if its ambition will be actualized. After all, this is its stated goal: to offer a theory of gravity which cannot only deal with the relativistic aspects of gravity, and hence of spacetime, but which also incorporates pertinent quantum effects. In other words, it endeavours to deliver a fundamental theory of gravity, and as such will be more fundamental than our currently most fundamental theory of gravity. Depending on the particular features of a candidate theory of quantum gravity, it may or may not be more fundamental than the standard model of particle physics. As stated above, the standard model offers our currently most fundamental theory of the three non-gravitational forces. Thus, if the quantum theory of gravity to be does not only provide a fundamental theory of gravity, but instead a unified theory of all forces, then it will also be more fundamental than the standard model. Such is the ambition harboured by string theory, for example. If, however, it only amounts to a fundamental theory of gravity, then it will not be more fundamental than the standard model. This will be the case, e.g., for most approaches trying to quantize general relativity such as loop quantum gravity and for most approaches starting out from more revisionary assumptions such as causal set theory. It should be noted, however, that it is also not the case that the standard model is more fundamental than these approaches.

In either case, the quantum theory of gravity will be a minimal element of T and thus among the most fundamental theories, at least to date. Let us proceed on this premise.

3 The disappearance of space and time

According to most approaches to quantum gravity, spacetime is not part of the fundamental furniture of our world, but merely 'emergent'. What I mean by this is that whatever fundamental structure a theory of quantum gravity postulates, it is importantly dissimilar from the structure 'spacetime' refers to in general relativity or other, non-fundamental theories of gravity or of spacetime. For instance, in a vast class of approaches to quantum gravity, the fundamental structure is discrete.[6] In so-called canonical approaches to quantum gravity, there is a strong suggestion that at least time is not fundamental.[7] Dualities in string theory may be interpreted to mean that space(time) is not fundamental in string theory either. The so-called Weyl symmetry of the 'internal metric' already present in plain vanilla string theories is often interpreted, at least among physicists, as rendering unnecessary the background spacetime in which the string was at first assumed to propagate.[8] Taking the theory ontologically seriously, as string theorists tend to, thus means accepting the vanishing of the spacetime at the fundamental level. In non-commutative geometry, an approach related to string theory, the fundamental constituents replacing the familiar spatiotemporal quantities in different dimensions do not commute, i.e. where measuring quantities along different dimension depends on the *order* in which these measurement are made. Despite its spatiotemporal vestige, the non-commutativity of the approach renders the fundamental structure profoundly different from the ordinary conception of spacetime. And the examples could be multiplied.

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[6] Smolin (2009, 549).

[7] Huggett et al. (2013, §2).

[8] Witten (1996), but cf. also (Huggett et al. 2013, §3).

But rather than fully establishing the assertion that approaches to quantum gravity generically deny the fundamental existence of space and time, I shall content myself with offering a representative yet tractable example: causal set theory. Causal set theory is a still inchoate, but conceptually clean approach to quantum gravity, which may serve as a perfect illustration for how radically a discrete fundamental structure differs from our usual conception of spacetime. Causal set theory is based on the assumption that the fundamental structure is a discrete set of featureless basal events partially ordered by causality. It is motivated by theorems in general relativity by Stephen Hawking et al. (1976) and David Malament (1977) which establish that given the causal structure and some volume information, the metric of the spacetime manifold is determined, as is its dimension, topology, and differential structure. In other words, the causal structure determines the geometry, albeit not the 'size' of the spacetime. Based on these theorems, causal set theory asserts that the fundamental structure is a 'causal set' and thus that causality is prior to space and time. Furthermore, the presupposition of the discreteness of the fundamental structure is justified through its technical and conceptual utility.

Slightly more formally, causal set theory postulates that the fundamental structure is a *causal set* $\mathcal{C}$, i.e. an ordered pair $\langle C, \preceq \rangle$ consisting of a set C of elementary events and a relation, denoted by the infix $\preceq$, defined on C satisfying two conditions: first, $\preceq$ induces a partial order on C (i.e. $\preceq$ is reflexive, antisymmetric, and transitive); second, $\mathcal{C}$ is discrete in that the number of elements of C which are causally 'between' any two points in C is finite.[9] That the discreteness is *stipulated* is not in itself a problem, as long as it is ultimately vindicated by the scientific success of the theory. Thus, it is a *feature* of the theory.

Causal set theory is plagued by two major challenges. First, like other discrete relational approaches to quantum gravity, it suffers from what is known as the 'entropy crisis', viz. that the vast majority of basic structures satisfying the above postulate cannot be approximated by, or physically related to, a relativistic spacetime. In other words, for most causal sets in causal set theory, no spacetime even remotely resembling ours emerges from it. Second, causal set theory as it has been articulated so far is a classical theory—it does not take quantum interference effects into account. This is hardly satisfactory when the goal was to produce a *quantum* theory of gravity. The hitherto unfulfilled hope is to kill both birds with one stone: both problems would be solved if only one could formulate additional constraints on the causal sets which exactly pick those causal sets with general-relativistic pendants and simultaneously act as independently justified principles governing an appropriately quantum dynamics. Thus, the goal is to formulate the dynamics in some principled and physically motivated way such that the dynamics will 'drive' the causal sets to those which do in fact approximate a general-relativistic spacetime.

It must be emphasized, however, that philosophers should not be misled by the presence of these difficulties to dismiss causal set theory altogether and much less to shrug off its basic tenet that the fundamental is discrete and thus dissimilar from spacetime as we find it in all prior and less fundamental theories. As pronounced above, the discreteness of the fundamental structure is quite common. In fact, it is so strongly expected that entire research programmes—and not just causal set theory—are based on the presupposition that the fundamental structure in quantum gravity—whatever else it might exactly turn out to be—is discrete. So like all other approaches to quantum gravity, causal set theory is not without its share of foundational problems. Pointing these out does not, however, make for an interesting objection against the argument about to be offered as the disappearance of spacetime at the fundamental level is quite generic in quantum theories of gravity. In this sense, the following argument does not depend on the truth of causal set theory in particular. Causal set theory permits an easily tractable illustration of much more general features of quantum

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[9] More technically still, the second axiom demands that $\forall a, b \in C$, $\mathrm{card}\{x \in C | a \preceq x \preceq b\} < \infty$.

theories of gravity. The argument only relies on the recognition that the non-fundamentality of spacetime is generically either an assumption or a consequence of quantum theories of gravity, at least for extant ones.

Furthermore, it could be complained that all causal set theory would establish, if borne out, was that spacetime just looked a bit different from what we expected when we thought that general relativity was our best theory of it. Sure, the complaint admits, spacetime is not the continuum that general relativity taught us it was; instead, we learn that spacetime is a discrete structure. But why should such progress in learning about the properties of spacetime—if progress it is—have any consequences for metaphysics, the complaint rhetorically asks.

If the point is simply to insist that we call 'spacetime' whatever fundamental structure gives rise to space and time as experienced by ordinary humans, then we have no debate. But the complaint would be mistaken if it purported that the fundamental structure shares the essential properties of ordinary space and time and hence deserves the honorific title of 'spacetime'. First, almost all physical theories taught today demonstrably rely on the assumption that space and time are (infinitely) extended continua, as do a number of metaphysical theories of (diachronic) personal, or generally material, identity, of causation, of laws of nature, as well as characterizations of determinism, etc. Thus, discrete spacetime would conceptually diverge from what is assumed to be evidently the case in all these theories and could no longer serve as their basis. Second, and more seriously, it takes hard—and controversial—work by mathematical physicists to even *define* the concepts necessary to attribute to discrete structures properties that we so routinely see exemplified by continuous spacetimes. The affine and differentiable structure evaporates, usual topological concepts become inapplicable, metric properties must be entirely revamped and redefined and dissertations are necessary to work out reasonable notions of the discrete correlate of the dimension of a smooth manifold. All these usual concepts developed to articulate the properties of continuous spacetimes, including, most importantly, metric properties of duration and length, become inapplicable. Worse, the discrete structures of causal set theory arguably do not possess metric properties at all.

Just as it would be presumptuous to call phlogiston 'oxygen' and attribute the differences simply to the ignorance of phlogiston theorists about what oxygen really is, it would be mistaken to label these discrete structures 'spacetime', given these profound differences. Terms assume their meaning in the holistic context of the theories in which they operate; if these contexts shift radically, it makes no sense to insist on the same term. In fact, using homonyms for profoundly distinct concepts only invites misapprehensions.

Finally, there are two concerns regarding the fundamentality of a quantum theory of gravity. First, one might worry that other fundamental theories might frustrate the conclusion that spacetime does not exist, fundamentally. As elaborated in §2, the requisite fundamentality is not jealous—there could be many minimal theories in the partially ordered set T as the well-foundedness did not require uniqueness. So even if no other theory is more fundamental than our quantum theory of gravity, it may still be the case that this is also true of many other theories. In particular, as may be welcomed by some, this notion of fundamentality also permits the failure of reductionism in that the existence of entirely disparate subsets of theories, i.e., subsets of T across which no fundamentality relations hold. For example, physical theories may be ordered according to fundamentality among themselves and likewise for biological theories, with relative fundamentality not exemplified by any physico-biological pair of relata. Closer to the case at hand, if our quantum theory of gravity is such as to remain silent about the other three forces, it can only hope to be more fundamental than general relativity, but not than the standard model. It is certainly conceivable that if there is a plurality of minimal elements in T, only one of which is our quantum theory of gravity, then different minimal elements may return different verdicts regarding the status of spacetime. So even if our quantum theory of gravity declared that there is no spacetime, other theories might postulate its existence and by virtue of their fundamentality insist that it does so fundamentally. How to regulate the jurisdictions in such a case?

In response to this worry, the first point to note is that whatever set of minimal theories we have, it must be consistent. I take this not to imply that different elements of T cannot assert different, and perhaps even inconsistent, propositions. What it does imply at a minimum, however, is that should such a case arise, then not both theories can be strictly true. At best, one of them is true, and the other one only approximately so. This means that one theory takes precedence over the other, at least with respect to the claims at stake. It also means that we should ideally possess a procedure to adjudicate the dispute over conflicting jurisdictions. Unfortunately, I know of no principled way of doing so generally. But we do not need a generally valid procedure; it suffices to provide an argument that a quantum theory of gravity's pronouncements regarding the fate of spacetime trumps those of other fundamental theories. And such an argument is readily available: our hitherto most accurate theory of spacetime is general relativity, our quantum theory of gravity is more fundamental than general relativity, so the nature of that fundamental structure which gives rise to relativistic spacetimes is most authoritatively described by the quantum theory of gravity.

A second worry someone might voice trades on the fact that even though fundamentality is intended without regard of the contingencies of the current state of science, it may be that as yet unformulated theories will revert the verdict on spacetime as it is handed to us by extant quantum theories of gravity. As stated in §2, I would not presently want to be bound by the claim that the discovery of ever more fundamental theories peters out at some finite level. Perhaps it *is* turtles all the way down. Regardless of how many more fundamental levels will be uncovered, the possibility which instigates the present worry forces me to stipulate that at the most fundamental one or, in the case of infinitely many, all which are more fundamental than *some* level do not reverse the outcome of §3, viz. that fundamentally, there is no spacetime. As long as this condition is honoured, the contingencies of future science do not affect the argument below. I believe that current research in fundamental physics warrants the tentative imposition of this condition and thus underwrites that there is no spacetime at the fundamental level, but I also understand that this warrant is fallible.

Let us then, at least for the purposes of this essay, accept that spacetime does not exist, fundamentally.

4 What unifies a Lewisian world

Assuming familiarity at least with the basic ideas of Lewis's account of modality in terms of possible worlds as it is articulated in his 1986 et passim, let me remind you of what Lewis claims unifies possible worlds. In the section entitled 'Isolation' (§1.6), he addresses the question of by virtue of what two possibilia are 'worldmates', i.e. denizens of the same possible world, and thus of what unifies possible worlds. For him, possible worlds are unified by the spatiotemporal relations holding among possibilia, and are distinct by virtue of being spatiotemporally isolated from one another. In his own words,

> ... things are worldmates iff they are spatiotemporally related. A world is unified, then, by the spatiotemporal interrelation of its parts. There are no spatiotemporal relations across the boundary between one world and another; but no matter how we draw the boundary within a world, there will be spatiotemporal relations across it. (1986, 71)

Lewis himself identifies three problems with his account.[10] These problems, or perhaps rather *limitations* of the account, concern the necessity of the condition much more so than its sufficiency. First, it does not permit a world to "consist of two or more completely disconnected spacetimes." (71) This limits Lewis's account insofar as a world which can be represented as the union of at least two disjoint non-empty spacetimes is deemed impossible. The relevant sense in which these spacetimes are disjoint, of course, must be that no spatiotemporal relations are exemplified by relata located in topologically separate parts of this world. As Lewis admits, and regrets, the impossibility of such worlds, he offers substitutes, i.e. possible worlds which exhibit some features emulating what a proponent of the possibility of disconnected spacetimes might have cared about. For instance, worlds might possess additional, covert, spatiotemporal dimensions along which seemingly disconnected regions of spacetime connect; or otherwise disjoint regions might be connected by virtue of their being embedded in a single spacetime such that topological or metric oddities effectively cordon off the regions from one another.

This limitation might worry a naturalist who takes recently popular multiverse proposals metaphysically serious. However varied these proposals may be, they all essentially defend the theses that our universe is just one among many, and that these alternate universes are causally disconnected from ours. If this were true of our world—and if there indeed are no spatiotemporal relations obtaining across universes in a way that cannot be cured by substitutes—, then Lewis's account of modality would rule out our actual world as impossible. The trouble with this conditional, unlike the one defended in the main argument of this essay, is that I find little reason to accept either—let alone both—of the antecedents. While an analysis of the evidential claims made in favour of multiverse hypotheses would require an entirely separate paper, the more plausible versions of it do not sanction the second antecedent as they assume that all the universes, as causally separate as they may be, are bubbles embedded in a larger spacetime, connecting them in the presently relevant way. I conclude that this limitation does not pose a serious threat to Lewis's analysis from a naturalistic perspective.

The second issue with his analysis, on Lewis's own view, is that it is impossible that there is nothing. There needs to be at least a tiny bit of "homogeneous unoccupied spacetime, or maybe only a single point of it" (73) for there to be a world at all. Lewis accepts that on his view, it comes out as a necessary truth that there is something rather than absolutely nothing at all. However, he repudiates that this *explains* in any way why this is so, as for him, explanation is etiological, i.e. in the business of providing a causal account, as is evidently not the case here. So long as this consequence is not mistaken as an *explanation*, Lewis is happy to accept it as a feature of his analysis, and so am I.

Thirdly, and most disconcertingly for Lewis himself, it seems as if a world in which events are related by two distinct types of 'distance'—temporal and spatial—such as a Newtonian world ought to be possible, even though events in it are not related by ontologically fundamentally distinct *spatiotemporal* relations, as demanded by the condition. Of course, one might also want to include worlds in which, e.g., spatial distances are not naturally part of the story of how this world is interrelated, but are instead replaced by three distances: that along the left-right axis, along the up-down axis, and along the front-back axis. It might just turn out that these distances are essentially different in this world and in others like it, thus resulting in four, and not just two, distances by which objects in those worlds are related. Since there is no good reason to think that such worlds are impossible, the generalized version of this complaint continues, to demand that worlds are spatiotemporally bound is overly narrow. What we dub 'spatiotemporal relations'

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[10] In fact, Lewis briefly addresses a fourth worry concerning the possibility of spirits living outside of space altogether. I shall ignore it here.

is grounded in their behaviour in the actual world. The question which then arises is whether those other multiple distance relations obtaining in these other worlds are really nothing but our spatiotemporal relations which “double up” to deliver several distances, or whether the latter are different relations altogether, which “take the place” of our spatiotemporal relations. If the former, we need not worry, Lewis assures us, quite correctly in my view. If the latter, however, a can of worms in the metaphysics of relations is opened. He offers to deal with this objection by accepting that

> [w]hat I need to say is that each world is interrelated (and is maximal to such interrelation) by a system of relations which, if they are not the spatiotemporal relations rightly so called, are at any rate analogous to them. (75)

He then goes on to state some preliminary points of analogy these ‘analogically spatiotemporal’ relations must satisfy. The relations, at a minimum, must be ‘natural’ (i.e. not gerrymandered or disjunctive), ‘pervasive’ (i.e. if relata are connected by a chain of relations, they are also directly so related), ‘discriminating’ (i.e. in sufficiently large worlds, the relata are possibly identified by a unique place in the structure of relations), and ‘external’ (i.e. not supervenient on the intrinsic natures of the relata). Lewis also considers, but ultimately rejects, a simplification circumventing the difficulty of having to deal with this messy business by relaxing the condition that worlds must be unified by at least analogically spatiotemporal relations to the condition that they can be unified merely by virtue of *some* natural external relation. However the further details of the metaphysics of analogically spatiotemporal relations in Lewis’s pluriverse look like, let me emphasize the dilemma Lewis faces at this juncture. Either the candidate relations (such as Newtonian spatial and temporal distances) unifying what he takes to be a nomologically distant world are nothing but our spatiotemporal relations behaving somewhat differently; or else they really are non-spatiotemporal, in which case the world they are supposed to unify is only possible if they are *analogically* spatiotemporal.

5 Unification failure

But now another problem arises in the light of the findings in §3. If borne out, the disappearance of space and time would motivate a new and, I would claim, fatal objection to Lewis’s account of modality: despite its modal *embarras de richesses*, his pluriverse does not contain our world. In other words, if all there is is contained in the pluriverse of all possible worlds, *then the actual world would not exist*, as it would not be a possible world. This follows immediately from the fact that in no possible causal set, we have at least a tiny bit of “homogeneous unoccupied spacetime”. The basic constituents of a causal set stand in *causal*, but not *spatiotemporal*, relations to one another. This seems to leave open the possibility of a world consisting of just one basal event, which perhaps could “double up” as a single point of spacetime. By virtue of what would this single event be a single point of spacetime? Spacetime, according to causal set theory, is an emergent, not a fundamental concept. But even if a single basal event doubling up as a point of spacetime is granted for the sake of argument, it seems as if at most one basal and structureless event populates a possible world.

In other words, all the basic elements of a causal set must live in different possible worlds according to the Lewisian condition, or so it seems. If, per the usual assumption in causal set theory, one such element of the fundamental structure corresponds to a Planck-sized volume of spacetime, then the observable part of our actual universe corresponds to something like a gargantuan 10^{185} basic elements. If there is no doubling up or analogically spatiotemporal about such a causal set,

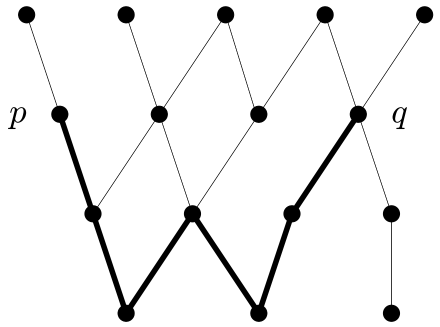

Figure 1: As shown in this Hasse diagram of (a section of) a causal set, events p and q are connected by a chain of causal relations (bold lines), yet do not stand in a direct causal relation. A Hasse diagram represents the antisymmetry of the partial ordering by vertical stacking.

then it appears that these elements must constitute numerically distinct worlds. In order to form something like the causal set representing the fundamental structure of our actual universe, an even more gargantuan number of causal relations must obtain across worlds. But Lewis rejects trans-world causation, and quite plausibly so. This injunction of causal isolation leaves us with a large number of possible worlds populated by just one basal event. But since these basal events obtain their identity only structurally,[11] we stand bereft of any power to ground their identity and hence their numerical plurality.

Whether all these 10^{185} elements give rise to numerically distinct possible worlds, or whether they are numerically one and the same shall not concern us here. What *is* pertinent is the fact that a possible world containing nothing but a single basal element of a causal set looks nothing at all like our actual world. Hence, the actual world, if fundamentally as causal set theory asserts, is not a possible world in the Lewisian pluriverse.

Or at least not if the causal relations exemplified in the fundamental structure either do not double up as spatiotemporal relations or are not analogically spatiotemporal. We can safely dismiss the first possibility: the causal relations featured in causal set theory are expressly *not* the spatiotemporal relations just behaving a little differently. They *give rise* to spatiotemporal relations, as we will see below, but differ markedly in both their extensional properties as well as their intensional role in the theory, as explained in §3. Thus, we are left with the option that the causal relations at work in the fundamental structure are analogically spatiotemporal. Unfortunately, this will not work, at least not of the four criteria of analogy listed above are intended as necessary for the analogy to succeed. The condition that they be discriminating will only be honoured thinly, in that for large causal sets, we generically expect them to fail to discriminate among all basal events (BLINDED). But the true culprit to fail the analogy is the demanded pervasiveness, which is routinely violated. Even though all events are integrated into the structure of a causal set and are typically causally linked to all other events in the causal set via long chains of causal relations, it is generally not the case that any given pair of them also stands in a direct causal relation, as is illustrated in Figure 1. This failure occurs for instance in 'space-like related' events and results from the conjoined effects of the asymmetry of the fundamental relation and the partiality of the order it induces.[12]

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[11] BLINDED.

[12] It should also be added that even though Lewis also considers causal relations, and particularly the possibility of their obtaining across worlds, he does not entertain them as solely sufficient to bind worlds. Lewis also declines to introduce a primitive worldmate relation vested with all the necessary unifying power.

Thus, the causal relations present in the fundamental structure of our world as conceived by causal set theory neither double up with spatiotemporal relations nor are they analogically spatiotemporal. Since it is necessary, and not merely sufficient, for worldmates to stand either in spatiotemporal relations or in external relations which double for spatiotemporal relations or in analogically spatiotemporal relations, non-trivial causal sets do not constitute possible worlds in Lewis's sense. Therefore, because what is not possible cannot exist—for both the actualist and the modal realist—we have a new problem: the non-existence of the actual world.[13] Instead of many non-actual possible worlds, we have a non-possible actual world!

6 The charge of empirical incoherence

There are two rather immediate ways to retort to the remonstrance against Lewis's account of modality as formulated in the previous section. First, the Lewisian may argue that the mere existence—as opposed to the fundamentality—of spacetime, and of spatiotemporal relations, suffices to hold together worlds; surely, the mere existence of spacetime should be granted. To deny the very existence of space and time could perhaps take the form of idealism about them; but an entirely different sort of charge would have to be sounded against Lewis if such a conclusion were the goal of the argument. This essay accepts a rather plain form of realism and has no interest in pursuing this line of attack. A second Lewisian rebuttal arges that spacetime is necessary in order to guarantee the empirical coherence of any theory in the empirical sciences. Let us address the responses of the sufficiency of the non-fundamental existence of spacetime and of the empirical incoherence in reverse order.

Let me explain, then, how the absence of spacetime may incur a worry about the empirical coherence of a theory-sans-spacetime. If one believes, with Tim Maudlin (2007), that any realist interpretation of quantum mechanics must include 'local beables' in its ontology, then one might be tempted to charge any theory denying the fundamental existence of spacetime with empirical incoherence. Coined by John Bell (1987), 'local beables' in quantum mechanics are basic existents "which are definitely associated with particular space-time regions." (234) They represent the physical content of the universe, they make up the 'stuff' we find in our world. Whatever else their properties, Maudlin (2007) makes it explicit that they require space and time when he asserts that "local beables do not merely exist: they exist somewhere." (3157) Now abstracting away from non-relativistic quantum mechanics, insisting that the basic ontology of a theory includes local beables thus entails a commitment to spacetime itself being a part of the basic furniture of the world (and hence not emergent at a higher level). Whatever else confirmation in an empirical theory involves, the observation of something located in some place at some time will play an ineliminable role in it.

Thus, it appears as if denying such fundamental existence to spacetime threatens a theory's 'empirical coherence' in the sense of Jeff Barrett (1999), who defines a theory to be *empirically incoherent* just in case the truth of the theory undermines our empirical justification for accepting it as true.[14] A theory denying the fundamental existence of spacetime is thus alleged to be empirically incoherent because the empirical justification of a theory derives only from our observation of the effects of local beables situated in spacetime; yet if such a theory were true, there could be no such local beables.

To be sure, Maudlin (2007, 3161) accepts that it would be sufficient in principle to *derive* the

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[13] The reader is reminded that we proceeded in §3 on the assumption that causal set theory or a relevantly similar theory is a true fundamental theory of our world.

[14] Cf. Barrett (1999, §4.5.2).

structure of the local beables and the spacetime that contains them from a fundamental ontology free of spacetime and hence of local beables. Such a derivation would permit an empirical grounding of the theory-sans-spacetime, *as long as it is "physically salient (rather than merely mathematically definable)."* (ibid.; emphasis added) Maudlin implies, however, that we do not even *know how* such a "physically salient" derivation could be undertaken, let alone having it at our disposal.

To summarize then, a necessary condition to circumvent the charge of empirical incoherence is to offer a "physically salient" derivation of spacetime from the basic, non-spatiotemporal structure. In other words, it must be shown how general-relativistic spacetime emerges as an approximation in quantum theories of gravity. The task of recovering the smooth relativistic spacetime from the discrete fundamental structure is also important in the 'context of justification', as its discharge provides an account of why the classical spacetime theory—general relativity—is as successful as it is. Understanding how spacetime emerges is thus doubly urgent: to establish the fundamental theory's empirical coherence by connecting it to evidence in the form of spatiotemporally located beables, and to provide an explanation of the successes—and failures—of the predecessor theory by relating the fundamental structures to relativistic spacetimes.

The necessity of recovering spacetime in a well-understood approximation arises for every quantum theory of gravity which denies its fundamental existence. As I have asserted above, this applies to most approaches in quantum gravity. How exactly spacetime can be recovered depends on the particular approach taken and remains an open question for many of them. I have explicated the general idea for completing this task—cum attendant difficulties—as it appears in causal set theory in BLINDED. For the present purpose, however, let us assume that the endeavour is both acknowledged and can be successfully completed.

7 Emergent salvation?

Given the outcome of §6, the existence of spacetime is assured, albeit not at the fundamental level. Spacetime, just like tables and chairs, is non-fundamental, but existent. The non-fundamentality clearly does not entail the non-existence of spacetime, just as it didn't for tables and chairs. In fact, by acknowledging the necessity of a "physically salient" emergence of spacetime, its existence was accepted, at least in an appropriate approximation. The needed spatiotemporal relations exemplified by pairs of worldmates, the thought goes, may well be ontologically dependent upon some more fundamental structure. Consequently, their relata need not, and in general will not, be fundamental objects either.

Of course, this existential acknowledgement would only save the Lewisian if the non-fundamental existence of spacetime were sufficient to unify (and isolate) worlds. Unfortunately, this is not the case. There are two principal reasons why this emergent salvation of Lewis's theory of modality fails. First, a metaphysician will hardly be satisfied by being offered what physicists call an 'effective theory', i.e. an at best approximately true theory which ignores the fundamental reality in favour of a description of emergent phenomena. One might have thought that the argument in §6 established that any fundamental theory must be compatible with a theory of emergent spacetime *which is true simpliciter* in order to guarantee its empirical coherence. After all, how could a false theory ground our empirical judgments? Unfortunately, this will simply not be borne out: the relevant theory of emergent spacetime is general relativity, and general relativity's failure in regimes with strong gravitational fields where quantum effects matter was the very reason a quantum theory of gravity was sought in the first place. Thus, general relativity will be false in its pronouncements in these regimes. And it will only be approximately true in more placid domains. A remarkably accurate approximation, but an approximation nevertheless. Just as most of our statements about tables and chairs are true simpliciter, many statements concerning the observable domain entailed by general relativity will be true simpliciter. But just as in our folk theories of tables and chairs, not all will, frustrating the theory's truth simpliciter. So that's the non-fundamentalist Lewisian's first problem: worlds are unified by relations furnished by theories which are strictly speaking false.

Secondly, a Lewisian theory of modality trading in emergent spatiotemporal relations must remain impotent in binding the basic constituents of actuality into one and the same possible world. The reason for this impotence is simple: there are no spatiotemporal relations exemplified in the fundamental structure of a causal set, and the basal elements of this structure cannot be the relata of the emergent spacetime relations.

This latter claim needs explicating. If spatiotemporal relations did obtain between elementary events, then propositions asserting corresponding facts such as 'This event and that event are at such-and-such spatiotemporal distance' would be meaningful. But metrical (and many topological etc.) concepts are inapplicable at the fundamental level of causal sets, as was argued in §3. It is notoriously difficult to define geometric notions in purely fundamental terms, i.e., in terms of causal set theory. There are various difficulties. First, even if successful, we should expect that our familiar geometric notions will only approximately track the fundamental magnitudes, even if the analogy is as strong as it can be. Secondly, even for those magnitudes for which the fit is reasonably close, we should expect that they will give out in exactly those regimes for which general relativity was found wanting. Thirdly, many of the fundamental notions defined in the literature rely in their ability to successfully emulate emergent geometric notions on additional substantive assumptions about either the fundamental causal set or the emergent spacetime, or both. For instance, the notion of spatial distance has been plagued by all these difficulties (Rideout and Wallden 2009). The only somewhat promising ansatz to define a notion which gives rise to something like a spatial distance in the literature (offered in Rideout and Wallden 2009), can only (approximately) reproduce distance in Minkowski spacetime, but not in curved spacetimes. Since general relativity insists that our actual world is best understood as having a curved spacetime structure, this proposal does not help in saving the actual world from the impending impossibility.

One might be tempted to impose, by a strong hand, spatiotemporal relations to obtain between the basal events of a fundamental causal set, as follows. Suppose we have a relativistic spacetime and the fundamental causal set it approximates. Showing that the spacetime emerges from the causal set essentially amounts to finding a map f, satisfying certain conditions of well-behavedness that need not concern us here, from the causal set to the spacetime. For any two basal events a and b in the causal set, $f(a)$ and $f(b)$, being elements of the spacetime, surely stand in some spatiotemporal relation to one another. Since f is injective, i.e., every element of its domain is unambiguously mapped onto an element of its range, it seems as if this move is available for any pair of basal events of any causal set which has any prayer of giving rise to an emergent spacetime. Since by definition no spatiotemporal relations are defined at the fundamental level of a causal set, couldn't one introduce surrogate spatiotemporal relations by stipulating that any two events a and b of a causal set stand in the spatiotemporal relation R just in case the two spacetime points $f(a)$ and $f(b)$ stand in R?

Thus, one would start out from a fundamental causal set capturing the fundamental structure of a world such as ours, find the relativistic spacetime which emerges from it, and then extend the spatiotemporal relations as they are defined of the spacetime points to obtain of pairs of basal events, as suggested in the previous paragraph. Unfortunately, as straightforward as this procedure appears, it is marred with insurmountable difficulties. First, it turns out that most causal sets admitted by the basic axioms of the theory cannot be embedded into a spacetime, as stated in §3. For all these causal sets it would still be the case that they do not constitute possible worlds on the Lewisian analysis of modality. On the one hand, this would not be much of a loss, since none of these causal sets can describe the fundamental structure of our actual world, on pain of empirical incoherence. On the other hand, this would have the odd consequence that the theory correctly describing the fundamental structure of our world asserts nomological possibilities which are not metaphysically possible. Usually, the metaphysically possible worlds are considered a (proper) superset of the physically possible worlds. This would no longer be the case on this proposal.

A second difficulty of the proposal to extend the spatiotemporal relations to causal sets from which a spacetime emerges arises from our assumption of the unique existence of the embedding spacetime. Uniqueness may be granted, but the problem is, once again, that relativistic spacetimes only approximate causal sets. In exactly those domains for which a quantum theory of gravity proved necessary, it will not be possible to map the causal set into a spacetime which is also a model of general relativity. This should be expected to happen exactly in those circumstances general relativity gives out. It is simply not nomologically possible, as far as general relativity is concerned, to have a spacetime which faithfully describes our world in these regimes. And note that these domains are believed to be part of our actual world. So we should not expect to find a relativistic spacetime which emerges globally from the causal set and truthfully describes the fundamental structure of our world.

Thus, if causal set theory is true of our world, then the basic elements of reality do not stand in spatiotemporal relations and thus cannot populate the same possible world in Lewis's analysis. And as we have seen in §5, they also do not exemplify merely analogically spatiotemporal relations.

Given our failed attempts to precisely identify *spatiotemporal* relations at the fundamental level, couldn't the causal relation, which *is* fundamentally exemplified, be reinterpreted as a relation of *temporal precedence*? After all, it partially orders the elementary events, which is exactly what time does in special relativity. But this escape falls short on two counts. First, the theorems in §3 strongly suggest an interpretation of the fundamental relation as being closely related to *causal*, not *temporal*, structure familiar from relativistic physics. Secondly, even if these connections are ignored, we only recover temporal relations, and not spatiotemporal ones as demanded by Lewis. This, in turn, would seriously exacerbate the third objection he discusses (1986, 73f; cf. §4 above), which he already took to be the by far most troubling, as we would no longer have at least spatial *and* temporal but only temporal relations. Therefore, the non-fundamental existence of spacetime does not save the Lewisian analysis of modality.

8 Summary and outlook

In conclusion, if a true fundamental theory such as causal set theory ruled out spacetime as being ontological part and parcel of the fundamental reality described by the theory, and if it was shown how spacetime emerges as an approximation from the fundamental structure thus connecting the theory to known physics and rendering it empirically coherent, then our actual world is not possible according to Lewis's analysis of modality. Or at least if this analysis is supposed to regiment more than just emergent modality. Of course, the Lewisian could content herself with a theory of modality restricted to the emergent level, thus covering almost all of human modal discourse. Nothing of what I said precludes this restriction, but suffice it to say that such a restricted analysis would neither be globally applicable as to include our theorizing about fundamental reality nor be based on a fundamental and true understanding of our world. For both these reasons, the move to emergent modality reeks of desperation. I see no principled reason why we couldn't do better than that.

The conclusion that on Lewis's analysis, the actual world is impossible only depended on the assumption that according to the most fundamental theory of gravity spacetime does not form part of the furniture of the world and on what I take to be a mild form of naturalism, viz. that the fundamental structure of our world is best described by our best physical theories. Obviously, my conclusion can be resisted by repudiating either of these premises. However, the first assumption is supported—but of course not guaranteed—by recent developments in fundamental physics, and in particular in quantum gravity. Given that there exists the serious possibility that spacetime is absent from fundamental reality, I hope you agree, esteemed reader, that it is worth considering its metaphysical consequences, at least if you share my mild naturalism. I hope to have shown that these consequences may be significant.

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