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ON THE TYPE-TOKEN RELATIONSHIPS

The complete text of this paper will appear in *Zeszyty Naukowe Wyższej Szkoły Pedagogicznej w Opolu*. Some extended ideas of it will also be presented in a paper submitted in *Studia Filozoficzne*.

The two-fold ontological character of linguistic objects revealed due to the distinction between "type" and "token" introduced by Peirce can be a base of the two-fold, both theoretical and axiomatic, approach to the language. In [1] referring to some ideas included in A. A. Markov's work [2] and in some earlier papers of the author ([4], [5] and [7]), the problem of formalization of the concrete and abstract words theories raised by J. Słupecki was solved. The construction of the theories presented in the above mentioned papers has two levels. The axiomatic theory of label-tokens: material, physical linguistic objects, constitutes the first one. Label-types, according to the literature of the subject, are defined on the other level as equivalence classes of equiform lebel-tokens. Assuming the opposite point of view, one can accept that theory of lebel-types: abstract labels, in which the theory it is possible to define the notion of label-token as well as the derivative notions should become the basis of formalization of the theory of linguistic labels and the theory of language in general. The axioms and definitions of both theories of labels: Tk and Tp representing the other approach to the ontology of language are included in the sequel of the abstract. The foundations of the theory of labels Tk in which the primary assumption as to the label-types existence is superfluous have been referred on the basis of the monography [7]. The basis of the theory of labels Tp which takes into account the other position has to be presented here for the first time.

The theories Tk and Tp are added to the theory of functional calculus with identity and to the theory of sets. The primitive notions of the former are:

$$Lb, \approx, c, V,$$

i.e. respectively: the set of all label-tokens, binary equiformity relation and ternary concatenation relation defined in the set Lb, the vocabulary of word-tokens.

The primitive notions of the theory Tp are:

$$\overline{Lb}, \cdot, \overline{V},$$

i.e. respectively: the set of all label-types, a binary function of concatenation of label-types and vocabulary of word-types.

Writing the axioms of the theory Tk down (resp. Tp) we assume that the variables

$$p,\ q,\ r,\ s,\ t,\ u,\ v \text{ (resp. } \overline{p},\ \overline{q},\ \overline{r},\ \overline{s},\ \overline{t},\ \overline{u},\ \overline{v}),$$

with subscripts or without them, run over the set Lb (resp. $\overline{Lb}$), while the letter X (resp. $\overline{X}$), with a subscript or without – the family 2^{Lb} (resp. $2^{\overline{Lb}}$).

We read the expression "$p \approx q$" as: "label-tokens p and q are equiform", or shortly: "labels p and q are equiform".

We read the expression "$c(p, q, r)$" as: "label-token r is a concatenation of label-tokens p and q" and the expression "$\overline{r} = \overline{p} \cdot \overline{q}$" as: "label-type $\overline{r}$ is a concatenation of label-types $\overline{p}$ and $\overline{q}$".

Let us note that the concatenation relation c need not be a function because it is possible to obtain many equiform labels as concatenation of two labels.

In the notation of some axioms of the theories Tk and Tp we shall use terms: "W" and "$\overline{W}$" which denote the set of all word-tokens and the set of all word-types, respectively. They are defined as follows:

in the theory Tk
$D1$. $W = \bigcap\{X \mid V \subseteq X \wedge \forall r \forall p, q \in X(c(p, q, r) \Rightarrow r \in X)\}$,

in the theory Tp
$\overline{D}1$. $\overline{W} = \{\overline{X} \mid \overline{V} \subseteq \overline{X} \wedge \forall \overline{p}, \overline{q} \in \overline{X}(\overline{p} \cdot \overline{q} \in \overline{X})\}$.

The sets W and $\overline{W}$ are the smallest sets of appropriate labels closed with respect to a suitable concatenation.

The following expressions are the axioms of the theory Tk:

$A1.$ a) $p \approx p$,
b) $p \approx q \Rightarrow q \approx p$,
c) $p \approx q \wedge q \approx r \Rightarrow p \approx r$,
$A2.$ $\exists r c(p,q,r)$,
$A3.$ $c(p,q,r) \Rightarrow \sim (r \approx p) \wedge \sim (r \approx q)$,
$A4.$ $c(p,q,t) \wedge c(r,s,u) \wedge p \approx r \wedge q \approx s \Rightarrow t \approx u$,
$A5.$ $c(p,q,s) \wedge c(s,r,t) \wedge c(q,r,v) \wedge c(p,v,u) \Rightarrow t \approx u$,
$A6.$ $c(p,q,t) \wedge c(r,s,t) \Rightarrow (p \approx r \Leftrightarrow q \approx s)$,
$A7.$ $c(p,q,r) \wedge s \approx r \Rightarrow c(p,q,s)$,
$A8.$ $c(p,q,r) \wedge c(r,s,u) \wedge t \approx u \Rightarrow [p \approx r \vee \exists v(c(r,v,p) \vee c(p,v,r))]$,
$A9.$ $\emptyset \neq V \subseteq Lb$,
$A10.$ $p \in V \wedge q \approx p \Rightarrow q \in V$,
$A11.$ $c(p,q,r) \Rightarrow r \notin V$,
$A12.$ $r \in W \backslash V \Rightarrow \exists p,q \in W c(p,q,r)$,
$A13.$ $r \in W \wedge c(p,q,r) \Rightarrow p,q \in W$,

The following expressions are the axioms of the theory Tp:

$\overline{A}1.$ $\cdot : \overline{Lb} \times \overline{Lb} \rightarrow \overline{Lb}$ – the concatenation $\cdot$ is a binary function in the set $\overline{Lb}$,
$\overline{A}2.$ $\overline{p} \cdot \overline{q} \neq \overline{p} \wedge \overline{p} \cdot \overline{q} \neq \overline{q}$,
$\overline{A}3.$ $(\overline{p} \cdot \overline{q}) \cdot \overline{r} = \overline{p} \cdot (\overline{q} \cdot \overline{r})$,
$\overline{A}4.$ $\overline{p} \cdot \overline{q} = \overline{r} \cdot \overline{s} \Rightarrow (\overline{p} = \overline{r} \Leftrightarrow \overline{q} = \overline{s})$,
$\overline{A}5.$ $\overline{p} \cdot \overline{q} = \overline{r} \cdot \overline{s} \Rightarrow [\overline{p} = \overline{r} \vee \exists \overline{u}(\overline{p} = \overline{r} \cdot \overline{u} \vee \overline{r} = \overline{p} \cdot \overline{u})]$,
$\overline{A}6.$ $\emptyset \neq \overline{V} \subseteq \overline{Lb}$,
$\overline{A}7.$ $\overline{p} \cdot \overline{q} \notin \overline{V}$,
$\overline{A}8.$ $\overline{r} \in \overline{W} \backslash \overline{V} \Rightarrow \exists \overline{p}, \overline{q} \in \overline{W}(\overline{r} = \overline{p} \cdot \overline{q})$,
$\overline{A}9.$ $\overline{p} \cdot \overline{q} \in \overline{W} \Rightarrow \overline{p}, \overline{q} \in \overline{W}$.

The relation $\approx$ is an equivalence relation in the set Lb of label-tokens ($A1$ a-c). By $[p]$ we denote the equivalence class of the relation $\approx$ determined by p.

It is possible to define the notions of the theory Tp in the theory Tk – the sets of label-types $\overline{Lb}, \overline{V}, \overline{W}$, and also the function of concatenation $\cdot$. We add the following definitions to the axioms of the theory Tk:

$\overline{D}2.\ \overline{p} \in \overline{Lb} \Leftrightarrow \exists p(\overline{p} = [p])$,
$D3.\ \overline{r} = \overline{p} \cdot \overline{q} \Leftrightarrow \exists p, q, r(\overline{p} = [p] \wedge \overline{q} = [q] \wedge \overline{r} = [r] \wedge c(p, q, r))$,
$D4.\ \overline{p} \in \overline{V} \Leftrightarrow \exists p \in V(\overline{p} = [p])$,
$D5.\ \overline{p} \in \overline{W} \Leftrightarrow \exists p \in W(\overline{p} = [p])$.

We define the notions of the theory Tk in the theory Tp – the set of label-tokens Lb, V, W, and the relations of equiformity $\approx$ and concatenation c. Hence we add two axioms and definitions of the above mentioned notions to the axioms of the theory Tp:

$\overline{A}10.\ \overline{p} \neq \emptyset$,
$\overline{A}11.\ \overline{p} \in \overline{q}_1 \wedge p \in \overline{q}_2 \Rightarrow \overline{q}_1 = \overline{q}_2$.

$\overline{D}2.\ p \in Lb \Leftrightarrow \exists \overline{p}(p \in \overline{p})$,
$\overline{D}3.\ p \approx q \Leftrightarrow \exists \overline{p}(p, q \in \overline{p})$,
$\overline{D}4.\ c(p, q, r) \Leftrightarrow \exists \overline{p}, \overline{q}, \overline{r}(p \in \overline{p} \wedge q \in \overline{q} \wedge r \in \overline{r} \wedge \overline{r} = \overline{p} \cdot \overline{q})$,
$\overline{D}5.\ p \in V \Leftrightarrow \exists \overline{p} \in \overline{V}(p \in \overline{p})$,
$\overline{D}6.\ p \in W \Leftrightarrow \exists \overline{p} \in \overline{W}(p \in \overline{p})$.

It can be shown that the theories: Tk and Tp are equivalent. The axioms $A1 - A13$ and definitions $D1 - D5$ are the theorems of the theory Tp while the axioms $\overline{A}1 - \overline{A}11$ and definitions $\overline{D}1 - \overline{D}6$ are the theorems of the theory Tk. This testifies that

1^o notions of the label-tokens and label-types, and also of the concatenation relations c and $\cdot$ are mutually definable. Let us note additionally that the theories Tk and Tp are consistent (cf. [4], [5] and [7]). It is possible, after making a suitable assumption as to the form of the set V, to reconstruct all the axioms of Tarski's metascience [3] formulated for word-types in them.

Theory Tk is the core of the theory of the languages treated in [6] and [7]. These theories give full axiomatic, syntactic characteristic of the languages, first on the level of tokens and then, on the level of types. Enriching theory Tp in a visible way we can give full axiomatic characteristic of the languages assuming that the basic language foundation consists of expression-types, the derivative one – of expression-tokens. Then, both approaches to the theory of languages are equivalent. It could result from above that

2^o in purely syntactic theoretical researches of language philosophical aspects referring to the nature of linguistic objects may be omitted,

however, the possibility of constructing a theory of languages as the theory which does not require initial assumptions as to abstract linguistic objects (theory Tk) shows, at the same time that

3^o the assumption that there exist languages of expression-types is superfluous.

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