Hindawi
Complexity
Volume 2018, Article ID 8094292, 16 pages
https://doi.org/10.1155/2018/8094292

Research Article

H_∞ Synchronization of Semi-Markovian Jump Neural Networks with Randomly Occurring Time-Varying Delays

Mengping Xing,[1] Hao Shen ,[1] and Zhen Wang [2]

[1]*School of Electrical and Information Engineering, Anhui University of Technology, Ma'anshan 243002, China*
[2]*College of Mathematics and Systems Science, Shandong University of Science and Technology, Qingdao 266590, China*

Correspondence should be addressed to Zhen Wang; zhenwang_sd@126.com

Received 23 January 2018; Accepted 14 June 2018; Published 12 September 2018

Academic Editor: Bernhard C. Geiger

Based on the Lyapunov stability theory, this paper mainly investigates the H_∞ synchronization problem for semi-Markovian jump neural networks (semi-MJNNs) with randomly occurring time-varying delays (TVDs). The continuous-time semi-MJNNs, where the transition rates are dependent on sojourn time, are introduced to make the issue under our consideration more general. One of the main characteristics of our work is the handling of TVDs. In addition to using the improved Jensen inequality and the reciprocal convexity lemma to deal with the integral inequality, we also employ Schur complement and the projection lemma to achieve the decoupling between the square term of TVDs. Finally, we verify the validity and feasibility of our method by a couple of simulation examples.

1. Introduction

In recent years, with the unceasingly thorough research on large data and artificial intelligence, the theory and application of neural networks have been greatly developed. It has tremendous application prospect, especially in robotics [1], pattern recognition [2, 3], associative memory [4–6], identification [7, 8], and combinatorial optimization [9–12]. Neural networks can be simply divided into the deterministic neural networks and stochastic neural networks based on whether they are disturbed by outside noise [13]. When the system is undisturbed, the deterministic neural network can describe the actual system accurately [13]. Nevertheless, as far as we know, the actual system is generally uncertain and most of the physical system will be affected by random parameter variation and structure change [14–16]. These changes may be caused by some sudden phenomena, such as components or connection failure and the deviation of parameter. In this circumstance, the stochastic neural networks can be described by a hybrid model, where a discrete stochastic variable called mode or pattern is attached to continuous state variables to describe the random jump of system parameters as well as the appearance of discontinuous points. It allows policymakers to respond to discrete events, which significantly perturb or alter the normal working condition of the system, by combining the empirical knowledge of events and the statistical information of their rates, adequately [17, 18].

Markovian jump neural networks (MJNNs), as we all know, as a kind of typical hybrid dynamic systems are widely used in the field of aerospace, industrial production, and biological, medical, and social construction in the past few decades due to its strong modeling ability and therefore draw great attention from researchers. For instance, the stability analysis, state estimation, filter design, passivity analysis, and stochastic synchronization for MJNNs were discussed in [19, 20], respectively. But one obvious drawback of MJNNs is that its jump time obeys the exponential distribution, which is a memoryless distribution and makes the transition probability of jump system an invariant function matrix; that is, the transition probability of the system obeys a stochastic process which is not relevant with the mode of the past [21]. Because of this limitation, it brings great restriction to the application of MJNNs. Therefore, semi-MJNNs were put forward later, where a transition probability matrix and a fixed dwell time probability density function matrix are used to represent the stochastic neural networks [22]. It has a wider range of application background due to the relaxation to the constraint condition where the probability density distribution function obeys exponential distribution. Compared with the abundant research achievements on MJNNs, the research efforts devoted to semi-MJNNs are relatively scarce. The robust stochastic stability condition for semi-MJNNs was derived in [23] and the relevant controller was also designed there. The synchronization controller for the semi-MJNNs was designed in [24] where the semi-MJNNs were transformed into associated MJNNs. An exponential passive filter was designed, and a cone complementarity linearization method was applied to manage the nonconvex feasibility issue in [25]. As mentioned above, the semi-MJNNs have more extensive application, such as in complex medical procedures [26].

Due to the finite signal transition speed as well as the limited switching speed of hardware facilities, the time-delay phenomenon exists in various practical industrial control systems widely, such as chemical system, process control system, and network control system [27–36]. It is known that delay argument existing in the system is often unknown or time-varying and the occurrence of delay tends to be random, which makes the analysis and control of the system more difficult. Also, the existence of time-delay tends to result in the degradation of the performance index and can even make the system unstable [37]. Therefore, it has important theoretical significance and practical applying value to study the system with time-varying delays (TVDs). As the system that considers that time-delay is more in line with the actual situation, an increasing number of researches have been made on the time-delay systems in recent years, and considerable results have been presented. To mention a few, Gun and Niculescu discussed the problem of stability analysis for the systems with time-delay and gave a summary on literature about the stability analysis and controller design of systems with time-delay in [38]. Park and Wan Ko studied the stability and robust stability criteria for TVD systems in [39], and then the reciprocally convex approach and the second-order reciprocally convex approach were proposed for stability analysis of TVD systems in [40].

Synchronization refers to two or more dynamic systems whose properties are identical or close to each other. Through the interaction between the systems, the state of the dynamic system that evolves under different initial conditions is gradually close to each other and finally reaches the same. Synchronization analysis is particularly important in many dynamic behaviors of neural networks and therefore has been widely studied. Exponential synchronization, adaptive synchronization, finite-time H_∞ synchronization, mixed H_∞/passive synchronization, and new delay-dependent exponential H_∞ synchronization were considered in [41, 42], respectively.

This paper mainly studies the H_∞ synchronization of semi-MJNNs with randomly occurring TVDs. First of all, by Lyapunov stability theory, we can get that the key point to establish a Lyapunov functional is to contain more useful information about the delays, which is useful to obtain the results with less conservatism [43]. As a result of the existence of TVDs, some novel inequality techniques derived from the Park inequality and the improved Jensen inequality [44] are employed to handle the time-varying items. At the same time, considering the existence of the square term of TVDs in the formulas, we use the projection lemma to achieve the decoupling between time-varying items. By using convex optimization techniques, the synchronization control of semi-MJNNs is investigated in this paper. The corresponding main results are presented by three theorems: Theorem 1 provides sufficient conditions for the stochastic stability and H_∞ synchronization of the closed-loop dynamic error system; Theorem 2 conducts the decoupling arithmetic; and Theorem 3 is then presented to get strict LMI-based conditions, and a numerical method to calculate controller gains is presented, which is simple and easily conducted.

Compared with the existing literature, this article has the following characteristics: (1) Different from the previous literature, a more general system model is introduced in this paper, in which both the semi-MJNNs and the random TVDs are taken into account simultaneously; (2) with the introduction of some advanced inequalities, combining with Schur complement lemma and projection lemma, H_∞ synchronization conditions with less conservatism are derived; (3) we use the LMI control toolbox to carry out the relevant simulation, and the corresponding controller can be obtained which can verify the correctness and feasibility of the proposed method. Throughout this work, the notations used are standard.

2. Problem Formulation

Firstly, given the following semi-MJNNs with randomly occurring TVDs (Σ),

$$\begin{aligned}\dot{x}(t) = {} & -A(\xi(t))x(t) + B(\xi(t))f(x(t)) \\ & + \beta(t)B_\theta(\xi(t))f(x(t-\theta(t))) + I(t),\end{aligned} \tag{1}$$

$$z(t) = C(\xi(t))x(t), \tag{2}$$

where $x(t) = [x_1^T(t), x_2^T(t), \ldots, x_n^T(t)]^T \in \mathbb{R}^n$ is the system state vector which is associated with the n neurons; $f(x(t)) = [f_1^T(x_1(t)), f_2^T(x_2(t)), \ldots, f_n^T(x_n(t))]^T \in \mathbb{R}^n$ denotes the neuron activation functions of the system, which is assumed to be bounded and satisfies

$$l_q^- \leq \frac{f_q(a) - f_q(b)}{a-b} \leq l_q^+, \quad q = 1, 2, \ldots, n, \tag{3}$$

where $f_q(0) = 0$, $a, b \in \mathbb{R}$, $a \neq b. l_q^-$, and l_q^+ are real known scalars, and they could be zero, positive, or negative. For the purpose of simplifying the symbols, we set

$$\begin{aligned} L_1 &\triangleq \operatorname{diag}\left\{l_1^+ l_1^-, l_2^+ l_2^-, \ldots, l_n^+ l_n^-\right\}, \\ L_2 &\triangleq \operatorname{diag}\left\{\frac{l_1^+ + l_1^-}{2}, \frac{l_2^+ + l_2^-}{2}, \ldots, \frac{l_n^+ + l_n^-}{2}\right\}. \end{aligned} \tag{4}$$

$I(t)$ stands for external input; $\theta(t)$ denotes the TVDs satisfying $0 \leq \theta_1 \leq \theta(t) \leq \theta_2 < \infty$ and $\dot{\theta}(t) \leq \mu < \infty$, where the nonnegative scalars θ_1 and θ_2 refer to the minimum and maximum time-delay, respectively. $\{\xi(t), \sigma\}_{t\geq 0} = \{\xi_m, \sigma_m\}_{m\in \mathbb{M}_{\geq 1}}$ ($\mathbb{M}$ is a positive integer) represents a continuous-time and discrete-state homogeneous semi-Markovian process whose trajectories are right continuous. Assuming $\xi(t)$ takes value in a finite state space $\Upsilon = \{1, 2, \ldots, \mathcal{N}\}$, the transition rate matrix $\overline{\Lambda}(\sigma) \triangleq \{\rho_{ij}(\sigma)\}_{\mathcal{N}\times\mathcal{N}}$ can be given by

$$\begin{aligned}
&\Pr\{\xi_{m+1} = j, \sigma + \varepsilon \geq \sigma_{m+1} \mid \xi_m = i, \sigma < \sigma_{m+1}\} \\
&\quad = \rho_{ij}(\sigma)\varepsilon + o(\varepsilon), i \neq j, \\
&\Pr\{\xi_{m+1} = j, \sigma + \varepsilon < \sigma_{m+1} \mid \xi_m = i, \sigma < \sigma_{m+1}\} \\
&\quad = 1 + \rho_{ii}(\sigma)\varepsilon + o(\varepsilon), i = j,
\end{aligned} \tag{5}$$

where $\varepsilon > 0, \lim_{\varepsilon\to 0}(o(\varepsilon)/\varepsilon) = 0$ and $\rho_{ij}(\sigma) \geq 0(\xi(t) = i, \xi(t+\varepsilon) = j)$; $i \neq j$ stands for the transition rates from i to j, and $\rho_{ii}(\sigma) = -\sum_{j=1, j\neq i}^{\mathcal{N}} \rho_{ij}(\sigma)$. $\beta(k)$ is a Bernoulli-distributed white sequence that takes values of 0 and 1 and obeys the following probability distribution laws

$$\begin{aligned}
&\Pr\{\beta(k) = 1\} = \overline{\beta}, \mathscr{E}\{\beta(k)\} = \overline{\beta}, \\
&\Pr\{\beta(k) = 0\} = 1 - \overline{\beta},
\end{aligned} \tag{6}$$

where $\overline{\beta} \in [0, 1]$ is a known constant.

Remark 1. Different from the previous literature, a more general system model is introduced in this paper, in which both the semi-MJNNs and the random TVDs are taken into account. The time-delay phenomenon, which occurs randomly and tends to be time-varying, exists in various practical neural networks. Therefore, the stochastic variable $\beta(t)$ is introduced to express the randomly occurring TVDs in this paper to make the issue under consideration more practical and more reasonable.

In this paper, the slave system $(\widehat{\Sigma})$ could be represented as the following forms

$$\begin{aligned}
\dot{\widehat{x}}(t) = &-A(\xi(t))\widehat{x}(t) + B(\xi(t))f(\widehat{x}(t)) \\
&+ \beta(t)B_\theta(\xi(t))f(\widehat{x}(t-\theta(t))) \\
&+ I(t) + D(\xi(t))\omega(t) + u(t),
\end{aligned} \tag{7}$$

$$\widehat{z}(k) = C(\xi(t))\widehat{x}(k), \tag{8}$$

where $\widehat{x}(t)$ and $\widehat{z}(k)$ are the response state vector and the response output, respectively; $u(t)$ is the advisable control input.

For presenting a better explanation to the addressed problem, we introduce $\varrho(t) = x(t) - \widehat{x}(t)$ as the synchronization error vector, $\tilde{\varrho}(t) = z(t) - \widehat{z}(t) = C(\xi(t))\varrho(t)$ as the output error, and $f(\varrho(t)) = f(x(t)) - f(\widehat{x}(t))$ as the nonlinear error. Then, the following error system is obtained:

$$\begin{aligned}
\dot{\varrho}(t) = &-A(\xi(t))\varrho(t) + B(\xi(t))f(\varrho(t)) \\
&+ \beta(t)B_\theta(\xi(t))f(\varrho(t-\theta(t))) \\
&+ D(\xi(t))\omega(t) + u(t).
\end{aligned} \tag{9}$$

The controller input $u(t)$ for the error system is established as

$$u(t) = K(\xi(t))\varrho(t), \tag{10}$$

where the controller gain matrix $K(\xi(t)) \in \mathbb{R}^{n\times n}$ will be designed in the sequel. In order to simplify the notation, $A(\xi(t)), B(\xi(t)), B_\theta(\xi(t)), C(\xi(t)), D(\xi(t))$ and $K(\xi(t))$ are denoted by A_i, B_i, $B_{\theta i}$, C_i, D_i, and K_i, for each $i \in \Upsilon$, respectively. Then, the closed-loop dynamic error system $(\overline{\Sigma})$ can be obtained:

$$\begin{aligned}
\dot{\varrho}(t) &= -A_i\varrho(t) + B_if(\varrho(t)) + \beta(t)B_{\theta i}f(\varrho(t-\theta(t))) \\
&\quad + D_i\omega(t) + K_i\varrho(t) \\
&= -(A_i - K_i)\varrho(t) + B_if(\varrho(t)) \\
&\quad + \beta(t)B_{\theta i}f(\varrho(t-\theta(t))) + D_i\omega(t).
\end{aligned} \tag{11}$$

Before making further derivation, we need the definition and lemmas as shown in the following.

Lemma 1. *[24] The following inequalities hold for any diagonal matrices $V_{li} > 0, l = 1, 2$:*

$$\begin{aligned}
&\begin{bmatrix} \varrho(t) \\ f(\varrho(t)) \end{bmatrix}^T \begin{bmatrix} -L_1V_{1i} & L_2V_{1i} \\ * & -V_{1i} \end{bmatrix} \begin{bmatrix} \varrho(t) \\ f(\varrho(t)) \end{bmatrix} \geq 0, \\
&\begin{bmatrix} \varrho(t-\theta(t)) \\ f(\varrho(t-\theta(t))) \end{bmatrix}^T \begin{bmatrix} -L_2V_{2i} & L_2V_{2i} \\ * & -V_{2i} \end{bmatrix} \begin{bmatrix} \varrho(t-\theta(t)) \\ f(\varrho(t-\theta(t))) \end{bmatrix} \geq 0.
\end{aligned} \tag{12}$$

Lemma 2. *[45] (projection lemma) For a symmetric matrix F, two matrices U, V with the same column dimension of F, the problem*

$$F + U^TX^TV + V^TXU < 0, \tag{13}$$

can be solved with respect to matrix X if and only if

$$N_U^TFN_U < 0, \quad N_V^TFN_V < 0, \tag{14}$$

where N_U and N_V are any basis of the nullspace of U, V.

Lemma 3. *[30] (Schur complement lemma). Given constant matrices M_{11}, M_{12}, and M_{22}, where $M_{11} = M_{11}^T < 0$ and $M_{22} = M_{22}^T < 0$, then, $M_{11} - M_{12}M_{22}^{-1}M_{12} < 0$ if and only if*

$$\begin{aligned}
&\begin{bmatrix} M_{11} & M_{12} \\ * & M_{22} \end{bmatrix} < 0 \\
\text{or } &\begin{bmatrix} M_{22} & M_{12}^T \\ * & M_{11} \end{bmatrix} < 0.
\end{aligned} \tag{15}$$

Lemma 4. *[40] Given a matrix $\mathscr{R} > 0$, a differentiable function $\{x(u) \mid u \in [a_1, a_2]\}$, it is easy to obtain the following inequalities*

$$\int_{a_1}^{a_2} x^T(\varpi)\mathscr{R}x(\varpi)d\varpi \geq \frac{1}{a_2-a_1}\Omega_1^T\mathscr{R}\Omega_1 + \frac{3}{a_2-a_1}\Omega_2^T\mathscr{R}\Omega_2,$$
$$\int_{a_1}^{a_2} \dot{x}^T(\varpi)\mathscr{R}\dot{x}(\varpi)d\varpi \geq \frac{1}{a_2-a_1}\Omega_3^T\mathscr{R}\Omega_3 + \frac{3}{a_2-a_1}\Omega_4^T\mathscr{R}\Omega_4 + \frac{5}{a_2-a_1}\Omega_5^T\mathscr{R}\Omega_5,$$
$$\int_{a_1}^{a_2}\int_{a_1}^{\beta} \dot{x}^T(\varpi)\mathscr{R}\dot{x}(\varpi)d\varpi d\beta \geq 2\Omega_8^T\mathscr{R}\Omega_8 + 4\Omega_9^T\mathscr{R}\Omega_9, \tag{16}$$

where

$$\Omega_1 = \int_{a_1}^{a_2} x(\varpi)d\varpi,$$
$$\Omega_2 = \int_{a_1}^{a_2} x(\varpi)d\varpi - \frac{2}{a_2-a_1}\int_{a_1}^{a_2}\int_{\beta}^{a_2} x(\varpi)d\varpi d\beta,$$
$$\Omega_3 = x(a_2) - x(a_1),$$
$$\Omega_4 = x(a_2) + x(a_1) - \frac{2}{a_2-a_1}\int_{a_1}^{a_2} x(\varpi)d\varpi,$$
$$\Omega_5 = x(a_2) - x(a_1) + \frac{6}{a_2-a_1}\int_{a_1}^{a_2} x(\varpi)d\varpi - \frac{12}{(a_2-a_1)^2}\int_{a_1}^{a_2}\int_{\beta}^{a_2} x(\varpi)d\varpi d\beta,$$
$$\Omega_6 = x(a_2) - \frac{1}{a_2-a_1}\int_{a_1}^{a_2} x(\varpi)d\varpi,$$
$$\Omega_7 = x(a_2) + \frac{2}{a_2-a_1}\int_{a_1}^{a_2} x(\varpi)d\varpi - \frac{6}{(a_2-a_1)^2}\int_{a_1}^{a_2}\int_{\beta}^{a_2} x(\varpi)d\varpi d\beta,$$
$$\Omega_8 = x(a_1) - \frac{1}{a_2-a_1}\int_{a_1}^{a_2} x(\varpi)d\varpi,$$
$$\Omega_9 = x(a_1) - \frac{4}{a_2-a_1}\int_{a_1}^{a_2} x(\varpi)d\varpi + \frac{6}{(a_2-a_1)^2}\int_{a_1}^{a_2}\int_{\beta}^{a_2} x(\varpi)d\varpi d\beta. \tag{17}$$

Lemma 5. *[40] (reciprocal convexity lemma) For any vector $\xi \in \mathbb{R}^m$; matrices $R_1, R_2 \in \mathbb{S}_n^+$, $S \in \mathbb{R}^{n\times n}$, $W_1, W_2 \in \mathbb{R}^{n\times m}$; and nonnegative real scalars α and β meeting $\alpha + \beta = 1$, the following inequality holds*

$$\frac{1}{\alpha}\xi^T W_1^T R_1 W_1 \xi + \frac{1}{\beta}\xi^T W_2^T R_2 W_2 \xi \geq \xi^T \begin{bmatrix} W_1 \\ W_2 \end{bmatrix}^T \begin{bmatrix} R_1 & S \\ S^T & R_2 \end{bmatrix} \begin{bmatrix} W_1 \\ W_2 \end{bmatrix} \xi, \tag{18}$$

subject to $\begin{bmatrix} R_1 & S \\ S^T & R_2 \end{bmatrix} \geq 0.$

Definition 1. (H_∞ synchronization) The system $(\overline{\Sigma})$ is said to be H_∞ synchronization with the disturbance attenuation γ under the condition that the following requirements are met:

(1) The system $(\overline{\Sigma})$ is stochastically stable when the disturbance input $\omega(t)$ is always equal to 0.

(2) For a positive scalar γ, the following inequality is satisfied under zero initial conditions

$$\Theta = \int_0^\infty \left[\tilde{\varrho}^T(t)\tilde{\varrho}(t) - \gamma^2\omega^T(t)\omega(t)\right]dt \leq 0 \left(\text{i.e.,} \sup_{\omega\neq 0, \omega\in L_2[0,\infty)} \frac{\|\tilde{\varrho}(t)\|_2}{\|\omega(t)\|_2} \leq \gamma\right). \tag{19}$$

3. Main Results

Theorem 1. *Given scalars μ, $\gamma > 0$, $0 \leq \theta_1 \leq \theta_2 < \infty$, $\theta = \theta_2 - \theta_1$, $\delta_1 > 0$, $\delta_2 > 0$, and $\overline{\beta} \in [0, 1]$, the considered system $(\overline{\Sigma})$ is stochastically stable and H_∞ synchronized; if there exist symmetric matrices $P_i \in \mathbb{R}^{3n} > 0$, positive definite matrices Q_{1i}, Q_{2i}, Q_{3i}, R_1, R_2, G_{1i}, G_{2i}, Z_1, Z_2, positive definite diagonal matrices V_{1i}, V_{2i}, matrices Y_i, and $S_{ij} \in \mathbb{R}^n$, $i, j \in \Upsilon$, such that for each $i \in \Upsilon$, the following matrix inequalities hold*

$$\widehat{\Phi}_i = \overline{\Phi}_i + \psi_i(\theta(t)) + \Pi(\theta(t))\left(\sum_{k=1}^{\mathcal{N}} \overline{\rho}_{ik}P_k\right)\Pi^T(\theta(t)) < 0, \tag{20}$$

$$\Psi_i = \begin{bmatrix} G_{2i}+W_{2i} & 0 & 0 & S_{11} & S_{12} & S_{13} \\ 0 & 3(G_{2i}+W_{2i}) & 0 & S_{21} & S_{22} & S_{23} \\ 0 & 0 & 5(G_{2i}+W_{2i}) & S_{31} & S_{32} & S_{33} \\ S_{11}^T & S_{21}^T & S_{31}^T & G_{2i} & 0 & 0 \\ S_{12}^T & S_{22}^T & S_{32}^T & 0 & 3G_{2i} & 0 \\ S_{13}^T & S_{23}^T & S_{33}^T & 0 & 0 & 5G_{2i} \end{bmatrix} > 0, \tag{21}$$

$$M_{1i} = R_1 + R_2 - \sum_{k=1}^{\mathcal{N}} \overline{\rho}_{ik}Q_{1k} - \sum_{k=1}^{\mathcal{N}} \overline{\rho}_{ik}Q_{2k} - \sum_{k=1}^{\mathcal{N}} \overline{\rho}_{ik}Q_{3k} > 0, \tag{22}$$

$$M_{2i} = R_2 - \sum_{k=1}^{\mathcal{N}} \overline{\rho}_{ik} Q_{2k} - \sum_{k=1}^{\mathcal{N}} \overline{\rho}_{ik} Q_{3k} > 0, \tag{23}$$

$$M_{3i} = R_2 - \sum_{k=1}^{\mathcal{N}} \overline{\rho}_{ik} Q_{2k} > 0, \tag{24}$$

$$W_{1i} = Z_1 - \theta_1 \sum_{k=1}^{\mathcal{N}} \overline{\rho}_{ik} G_{1k} > 0, \tag{25}$$

$$W_{2i} = Z_2 - \theta \sum_{k=1}^{\mathcal{N}} \overline{\rho}_{ik} G_{2k} > 0, \tag{26}$$

where

$$\begin{aligned}
\overline{\Phi}_i = {} & Sym\{E_1P_{11i}E_2^T + \theta_1E_1P_{12i}E_8^T + E_2P_{12i}E_2^T - E_3P_{12i}E_2^T + \theta_1E_2P_{22i}E_8^T - \theta_1E_3P_{22i}E_8^T + E_3P_{13i}E_2^T - E_5P_{13i}E_2^T \\
& + \theta_1E_3P_{23i}E_8^T - \theta_1E_5P_{23i}E_8^T\} + E_2Q_iE_2^T - (1-\mu)E_4Q_{3i}E_4^T - E_5Q_{2i}E_5^T + E_1G_iE_1^T - (E_2-E_3)G_{1i}(E_2-E_3)^T \\
& - 3(E_2+E_3-2E_8)G_{1i}(E_2+E_3-2E_8)^T - 5(E_2-E_3+6E_8-6E_{11})G_{1i}(E_2-E_3+6E_8-6E_{11})^T \\
& - 2(E_2-E_8)W_{1i}(E_2-E_8)^T - 4(E_2+2E_8-3E_{11})W_{1i}(E_2+2E_8-3E_{11})^T - \frac{\theta}{\theta_1}(E_2-E_3)W_{2i}(E_2-E_3)^T \\
& - \frac{3\theta}{\theta_1}(E_2+E_3-2E_8)W_{2i}(E_2+E_3-2E_8)^T - \frac{5\theta}{\theta_1}(E_2-E_3+6E_8-6E_{11})W_{2i}(E_2-E_3+6E_8-6E_{11})^T \\
& - 2(E_3-E_9)W_{2i}(E_3-E_9)^T - 4(E_3+2E_9-3E_{12})W_{2i}(E_3+2E_9-3E_{12})^T - 2(E_4-E_{10})W_{2i}(E_4-E_{10})^T \\
& - 4(E_4+2E_{10}-3E_{13})W_{2i}(E_4+2E_{10}-3E_{13})^T - \Gamma_1\widehat{\Psi}_i\Gamma_1^T - Sym\Big\{\delta_1E_1Y_iE_1^T + \delta_1E_1Y_iA_iE_2^T - \delta_1E_1Y_iK_iE_2^T \\
& - \delta_1E_1Y_iB_iE_6^T - \delta_1\overline{\beta}E_1Y_iB_{\theta i} \times E_7^T - \delta_1E_1Y_iD_iE_{14}^T + \delta_2E_2Y_iE_1^T + \delta_2E_2Y_iA_iE_2^T - \delta_2E_1Y_iK_iE_2^T - \delta_2E_2Y_iB_iE_6^T \\
& - \delta_2\overline{\beta}E_2 \times Y_iB_{\theta i}E_7^T - \delta_2E_2Y_iD_iE_{14}^T\Big\} - E_2L_1V_{1i}E_2^T + E_4L_1V_{2i}E_4^T + Sym\{E_2L_2V_{1i}E_6^T + E_4L_2V_{2i}E_7^T\} \\
& - E_6V_{1i}E_6^T - E_7V_{2i}E_7^T + E_2C_i^TC_iE_2^T - \gamma^2E_{14}E_{14}^T, \\
\psi_i(\theta(t)) = {} & Sym\{(\theta(t)-\theta_1)E_1P_{13i}E_9^T + (\theta_2-\theta(t))E_1P_{13i}E_{10}^T + (\theta(t)-\theta_1)E_2P_{23i}E_9^T + (\theta_2-\theta(t)) \\
& \times E_2P_{23i}E_{10}^T + (\theta(t)-\theta_1)E_3(P_{33i}-P_{23i})E_9^T + (\theta_2-\theta(t))E_3(P_{33i}-P_{23i})E_{10}^T - (\theta(t)-\theta_1)E_5P_{33i}E_9^T \\
& - (\theta_2-\theta(t))E_5P_{33i}E_{10}^T\} - (\theta(t)-\theta_1)E_9M_{2i}E_9^T - 3(\theta(t)-\theta_1)(E_9-E_{12}) \times M_{2i}(E_9-E_{12})^T \\
& - (\theta_2-\theta(t))E_{10}M_{3i}E_{10}^T - 3(\theta_2-\theta(t))(E_{10}-E_{13})M_{3i}(E_{10}-E_{13})^T, \\
\Pi(\theta(t)) = {} & [E_2\theta_1E_8(\theta(t)-\theta_1)E_9 + (\theta_2-\theta(t))E_{10}],
\end{aligned} \tag{27}$$

with

$$E_i = \Big[\underbrace{0\dots0}_{(i-1)n}I_n\underbrace{0\dots0}_{(14-i)n}\Big]^T \in \mathbb{R}^{14n\times n},$$

$$Q_i = \theta_1R_1 + \theta_2R_2 + Q_{1i} + Q_{2i} + Q_{3i},$$

$$G_i = \theta_1^2G_{1i} + \theta^2G_{2i} + \frac{\theta_1^2}{2}Z_1 + \frac{\theta_2^2-\theta_1^2}{2}Z_2,$$

$$\Gamma_1 = [(E_3-E_4)(E_3+E_4-2E_9)(E_3-E_4+6E_9-6E_{12}) \cdot (E_4-E_5)(E_4+E_5-2E_{10})(E_4-E_5+6E_{10}-6E_{13})],$$

$$\widehat{\Psi}_i = \begin{bmatrix} G_{2i} & 0 & 0 & S_{11} & S_{12} & S_{13} \\ 0 & 3G_{2i} & 0 & S_{21} & S_{22} & S_{23} \\ 0 & 0 & 5G_{2i} & S_{31} & S_{32} & S_{33} \\ S_{11}^T & S_{21}^T & S_{31}^T & G_{2i} & 0 & 0 \\ S_{12}^T & S_{22}^T & S_{32}^T & 0 & 3G_{2i} & 0 \\ S_{13}^T & S_{23}^T & S_{33}^T & 0 & 0 & 5G_{2i} \end{bmatrix}. \tag{28}$$

Proof 1. We firstly consider a stochastic semi-Markovian Lyapunov-Krasovskii functional as follows:

$$V(\varrho_s(t),\xi(t)) = \sum_{m=1}^{5} V_m(\varrho_s(t),\xi(t)), \tag{29}$$

where

$$\begin{aligned}
V_1(\varrho_s(t),\xi(t)) &= \overline{\varrho}^T(t)P_i\overline{\varrho}(t),\\
V_2(\varrho_s(t),\xi(t)) &= \int_{t-\theta_1}^{t} \varrho(\varpi)Q_{1i}\varrho(\varpi)d\varpi\\
&\quad + \int_{t-\theta_2}^{t} \varrho^T(\varpi)Q_{2i}\varrho(\varpi)d\varpi\\
&\quad + \int_{t-\theta(t)}^{t} \varrho^T(\varpi)Q_{3i}\varrho(\varpi)d\varpi,\\
V_3(\varrho_s(t),\xi(t)) &= \int_{-\theta_1}^{0}\int_{t+\beta}^{t} \varrho^T(\varpi)R_1\varrho(\varpi)d\varpi d\beta\\
&\quad + \int_{-\theta_2}^{0}\int_{t+\beta}^{t} \varrho^T(\varpi)R_2\varrho(\varpi)d\varpi d\beta,\\
V_4(\varrho_s(t),\xi(t)) &= \theta_1\int_{-\theta_1}^{0}\int_{t+\beta}^{t} \dot{\varrho}^T(\varpi)G_{1i}\dot{\varrho}(\varpi)d\varpi d\beta\\
&\quad + \int_{-\theta_1}^{0}\int_{y}^{0}\int_{t+\beta}^{t} \dot{\varrho}^T(\varpi)Z_1\dot{\varrho}(\varpi)d\varpi d\beta dy,\\
V_5(\varrho_s(t),\xi(t)) &= \theta\int_{-\theta_2}^{-\theta_1}\int_{t+\beta}^{t} \dot{\varrho}^T(\varpi)G_{2i}\dot{\varrho}(\varpi)d\varpi d\beta\\
&\quad + \int_{-\theta_2}^{-\theta_1}\int_{y}^{0}\int_{t+\beta}^{t} \dot{\varrho}^T(\varpi)Z_2\dot{\varrho}(\varpi)d\varpi d\beta dy,
\end{aligned} \tag{30}$$

with

$$P_i = \begin{bmatrix} P_{11i} & P_{12i} & P_{13i}\\ * & P_{22i} & P_{23i}\\ * & * & P_{33i}\end{bmatrix},$$
$$\overline{\varrho}(t) = \left[\varrho^T(t)\int_{t-\theta_1}^{t}\varrho^T(\varpi)d\varpi\int_{t-\theta_2}^{t-\theta_1}\varrho^T(\varpi)d\varpi\right]^T. \tag{31}$$

First of all, we define

$$\varsigma_1(t) = \begin{bmatrix} \dot{\varrho}(t)\\ \varrho(t)\\ \varrho(t-\theta_1)\\ \varrho(t-\theta(t))\\ \varrho(t-\theta_2)\\ f(\varrho(t))\\ f(\varrho(t-\theta(t)))\end{bmatrix},$$

$$\varsigma_2(t) = \begin{bmatrix} \frac{1}{\theta_1}\int_{t-\theta_1}^{t}\varrho(\varpi)d\varpi\\ \frac{1}{\theta(t)-\theta_1}\int_{t-\theta(t)}^{t-\theta_1}\varrho(\varpi)d\varpi\\ \frac{1}{\theta_2-\theta(t)}\int_{t-\theta_2}^{t-\theta(t)}\varrho(\varpi)d\varpi\\ \frac{2}{\theta_1^2}\int_{-\theta_1}^{0}\int_{t+\beta}^{t}\varrho(\varpi)d\varpi d\beta\\ \frac{2}{(\theta(t)-\theta_1)^2}\int_{-\theta(t)}^{-\theta_1}\int_{t+\beta}^{t-\theta_1}\varrho(\varpi)d\varpi d\beta\\ \frac{2}{(\theta_2-\theta(t))^2}\int_{-\theta_2}^{-\theta(t)}\int_{t+\beta}^{t-\theta(t)}\varrho(\varpi)d\varpi d\beta\\ \omega(t)\end{bmatrix},$$

$$\varsigma(t) = \begin{bmatrix}\varsigma_1(t)\\ \varsigma_2(t)\end{bmatrix},$$

$$E_i = \Big[\underbrace{0\ldots0}_{(i-1)n}I_n\underbrace{0\ldots0}_{(14-i)n}\Big]^T \in \mathbb{R}^{14n\times n},$$

$$\overline{\rho}_{ij} = \mathscr{E}\{\rho_{ij}(\sigma)\} = \int_0^{\infty}\rho_{ij}(\sigma)g_i(\sigma)d\sigma, \tag{32}$$

where $g_i(\sigma)$ is the probability density function of sojourn time σ resting on mode i.

Before further analysis, we consider the weak infinitesimal operator $\mathscr{L}$ as the following forms

$$\mathscr{L}V(\varrho_s(t),\xi(t)) = \lim_{\varepsilon\to0^+}\frac{1}{\varepsilon}[\mathscr{E}\{V(\varrho_s(t+\varepsilon),\xi(t+\varepsilon)) \mid \varrho_s(t), \xi(t)=i\} - V(\varrho_s(t),\xi(t))], \tag{33}$$

and we have

$$\begin{aligned}
&\mathscr{L}V_1(\varrho_s(t),\xi(t))\\
&\quad = Sym\{\varsigma^T(t)[E_1E_2 - E_3E_3 - E_5]P_i^T\Pi(\theta(t))\varsigma(t)\}\\
&\qquad + \varsigma^T(t)\Pi(\theta(t))\left(\sum_{k=1}^{\mathscr{N}}\overline{\rho}_{ik}P_k\right)\Pi^T(\theta(t))\varsigma(t),
\end{aligned} \tag{34}$$

$$\begin{aligned}
&\mathscr{L}V_2(\varrho_s(t),\xi(t))\\
&\quad = \varrho^T(t)Q_{1i}\varrho(t) - \varrho^T(t-\theta_1)Q_{1i}\varrho(t-\theta_1)\\
&\qquad + \int_{t-\theta_1}^{t}\varrho^T(\varpi)\left(\sum_{k=1}^{\mathscr{N}}\overline{\rho}_{ik}Q_{1k}\right)\varrho(\varpi)d\varpi\\
&\qquad + \varrho^T(t)Q_{2i}\varrho(t) - \varrho^T(t-\theta_2)Q_{2i}\varrho(t-\theta_2)\\
&\qquad + \int_{t-\theta_2}^{t}\varrho^T(\varpi)\left(\sum_{k=1}^{\mathscr{N}}\overline{\rho}_{ik}Q_{2k}\right)\varrho(\varpi)d\varpi\\
&\qquad + \varrho^T(t)Q_{3i}\varrho(t) - (1-\mu)\varrho^T(t-\theta(t))Q_{3i}\varrho(t-\theta(t))\\
&\qquad + \int_{t-\theta(t)}^{t}\varrho^T(\varpi)\left(\sum_{k=1}^{\mathscr{N}}\overline{\rho}_{ik}Q_{3k}\right)\varrho(\varpi)d\varpi,
\end{aligned} \tag{35}$$

$$\mathscr{L}V_3(\varrho_s(t),\xi(t)) = \varrho^T(t)(\theta_1 R_1 + \theta_2 R_2)\varrho(t) - \int_{t-\theta_1}^{t} \varrho^T(\varpi)R_1\varrho(\varpi)d\varpi - \int_{t-\theta_2}^{t} \varrho^T(\varpi)R_2\varrho(\varpi)d\varpi. \quad (36)$$

Considering the integral items in (35) and (36), we can get that

$$\begin{aligned}
&\int_{t-\theta_1}^{t} \varrho^T(\varpi)\left(\sum_{k=1}^{\mathscr{N}}\overline{\rho}_{ik}Q_{1k}\right)\varrho(\varpi)d\varpi \\
&\quad + \int_{t-\theta_2}^{t} \varrho^T(\varpi)\left(\sum_{k=1}^{\mathscr{N}}\overline{\rho}_{ik}Q_{2k}\right)\varrho(\varpi)d\varpi \\
&\quad + \int_{t-\theta(t)}^{t} \varrho^T(\varpi)\left(\sum_{k=1}^{\mathscr{N}}\overline{\rho}_{ik}Q_{3k}\right)\varrho(\varpi)d\varpi \\
&\quad - \int_{t-\theta_1}^{t} \varrho^T(\varpi)R_1\varrho(\varpi)d\varpi - \int_{t-\theta_2}^{t} \varrho^T(\varpi)R_2\varrho(\varpi)d\varpi \\
&= -\int_{t-\theta_1}^{t} \varrho^T(\varpi)\left(R_1 + R_2 - \sum_{k=1}^{\mathscr{N}}\overline{\rho}_{ik}Q_{1k} - \sum_{k=1}^{\mathscr{N}}\overline{\rho}_{ik}Q_{2k} - \sum_{k=1}^{\mathscr{N}}\overline{\rho}_{ik}Q_{3k}\right)\varrho(\varpi)d\varpi \\
&\quad - \int_{t-\theta(t)}^{t-\theta_1} \varrho^T(\varpi)\left(R_2 - \sum_{k=1}^{\mathscr{N}}\overline{\rho}_{ik}Q_{2k} - \sum_{k=1}^{\mathscr{N}}\overline{\rho}_{ik}Q_{3k}\right)\varrho(\varpi)d\varpi \\
&\quad - \int_{t-\theta_2}^{t-\theta(t)} \varrho^T(\varpi)\left(R_2 - \sum_{k=1}^{\mathscr{N}}\overline{\rho}_{ik}Q_{2k}\right)\varrho(\varpi)d\varpi \\
&= -\int_{t-\theta_1}^{t} \varrho^T(\varpi)M_{1i}\varrho(\varpi)d\varpi - \int_{t-\theta(t)}^{t-\theta_1} \varrho^T(\varpi)M_{2i}\varrho(\varpi)d\varpi \\
&\quad - \int_{t-\theta_2}^{t-\theta(t)} \varrho^T(\varpi)M_{3i}\varrho(\varpi)d\varpi.
\end{aligned} \quad (37)$$

For the integral items $-\int_{t-\theta_1}^{t} \varrho^T(\varpi)M_{1i}\varrho(\varpi)d\varpi$, $-\int_{t-\theta(t)}^{t-\theta_1} \varrho^T(\varpi)M_{2i}\varrho(\varpi)d\varpi$, and $-\int_{t-\theta_2}^{t-\theta(t)} \varrho^T(\varpi)M_{3i}\varrho(\varpi)d\varpi$ in (37), by using Lemma 4, we can obtain that

$$\begin{aligned}
&-\int_{t-\theta_1}^{t} \varrho^T(\varpi)M_{1i}\varrho(\varpi)d\varpi \\
&\quad \le -\frac{1}{\theta_1}\left(\int_{t-\theta_1}^{t}\varrho(\varpi)d\varpi\right)^T M_{1i}\left(\int_{t-\theta_1}^{t}\varrho(\varpi)d\varpi\right) \\
&\quad - \frac{3}{\theta_1}\left(\int_{t-\theta_1}^{t}\varrho(\varpi)d\varpi - \frac{2}{\theta_1}\int_{t-\theta_1}^{t}\int_{\beta}^{t}\varrho(\varpi)d\varpi d\beta\right)^T \\
&\quad \times M_{1i}\left(\int_{t-\theta_1}^{t}\varrho(\varpi)d\varpi - \frac{2}{\theta_1}\int_{t-\theta_1}^{t}\int_{\beta}^{t}\varrho(\varpi)d\varpi d\beta\right) \\
&\quad = \varsigma^T(t)\left[-4\theta_1 E_8 M_{1i}E_8^T + Sym\{3\theta_1 E_8 M_{1i}E_{11}^T\} - 3\theta_1 E_{11}M_{1i}E_{11}^T\right]\varsigma(t).
\end{aligned} \quad (38)$$

Similarly, it is easy for us to get the following conditions:

$$\begin{aligned}
&-\int_{t-\theta(t)}^{t-\theta_1} \varrho^T(\varpi)M_{2i}\varrho(\varpi)d\varpi \le -(\theta(t)-\theta_1)\varsigma^T(t) \\
&\quad \cdot \left[E_9 M_{2i}E_9^T + 3(E_9 - E_{12})M_{2i}(E_9 - E_{12})^T\right]\varsigma(t), \\
&-\int_{t-\theta_2}^{t-\theta(t)} \varrho^T(\varpi)M_{3i}\varrho(\varpi)d\varpi \le -(\theta_2-\theta(t))\varsigma^T(t) \\
&\quad \cdot \left[E_{10} M_{3i}E_{10}^T + 3(E_{10} - E_{13})M_{3i}(E_{10} - E_{13})^T\right]\varsigma(t).
\end{aligned} \quad (39)$$

Synthesizing the above results, we can get that

$$\begin{aligned}
&\mathscr{L}V_2(\varrho_s(t),\xi(t)) + \mathscr{L}V_3(\varrho_s(t),\xi(t)) \\
&\quad \le \varsigma^T(t)\Big\{E_2 Q_i E_2^T - E_3 Q_{1i}E_3^T - E_5 Q_{2i}E_5^T \\
&\qquad - (1-\mu)E_4 Q_{3i}E_4^T - 4\theta_1 E_8 M_{1i}E_8^T \\
&\qquad + Sym\{3\theta_1 E_8 M_{1i}E_{11}^T\} - (\theta(t)-\theta_1) \\
&\qquad \cdot \left(E_9 M_{2i}E_9^T + 3(E_9 - E_{12})M_{2i}(E_9 - E_{12})^T\right) \\
&\qquad - 3\theta_1 E_{11}M_{1i}E_{11}^T - (\theta_2 - \theta(t)) \\
&\qquad \cdot \left(E_{10} M_{3i}E_{10}^T + 3(E_{10} - E_{13})M_{3i}(E_{10} - E_{13})^T\right)\Big\}\varsigma(t).
\end{aligned} \quad (40)$$

$$\begin{aligned}
\mathscr{L}V_4(\varrho_s(t),\xi(t)) &= \theta_1^2\dot{\varrho}^T(t)G_{1i}\dot{\varrho}(t) + \frac{\theta_1^2}{2}\dot{\varrho}^T(t)Z_1\dot{\varrho}(t) \\
&\quad - \theta_1\int_{t-\theta_1}^{t}\dot{\varrho}^T(\varpi)G_{1i}\dot{\varrho}(\varpi)d\varpi \\
&\quad + \theta_1\int_{-\theta_1}^{0}\int_{t+\beta}^{t}\dot{\varrho}^T(\varpi)\left(\sum_{k=1}^{\mathscr{N}}\overline{\rho}_{ik}G_{1k}\right)\dot{\varrho}(\varpi)d\varpi d\beta \\
&\quad - \int_{-\theta_1}^{0}\int_{t+\beta}^{t}\dot{\varrho}^T(\varpi)Z_1\dot{\varrho}(\varpi)d\varpi d\beta,
\end{aligned} \quad (41)$$

$$\begin{aligned}
\mathscr{L}V_5(\varrho_s(t),\xi(t)) &= \theta^2\dot{\varrho}^T(t)G_{2i}\dot{\varrho}(t) + \frac{\theta_2^2-\theta_1^2}{2}\dot{\varrho}^T(t)Z_2\dot{\varrho}(t) \\
&\quad - \theta\int_{t-\theta_2}^{t-\theta_1}\dot{\varrho}^T(\varpi)G_{2i}\dot{\varrho}(\varpi)d\varpi \\
&\quad + \theta\int_{-\theta_2}^{-\theta_1}\int_{t+\beta}^{t}\dot{\varrho}^T(\varpi)\left(\sum_{k=1}^{\mathscr{N}}\overline{\rho}_{ik}G_{2k}\right)\dot{\varrho}(\varpi)d\varpi d\beta \\
&\quad - \int_{-\theta_2}^{-\theta_1}\int_{t+\beta}^{t}\dot{\varrho}^T(\varpi)Z_2\dot{\varrho}(\varpi)d\varpi d\beta.
\end{aligned} \quad (42)$$

Considering the single integral items in (41) and (42), we can get that

$$\begin{aligned}
&-\theta_1\int_{t-\theta_1}^{t}\dot{\varrho}^T(\varpi)G_{1i}\dot{\varrho}(\varpi)d\varpi - \theta\int_{t-\theta_2}^{t-\theta_1}\dot{\varrho}^T(\varpi)G_{2i}\dot{\varrho}(\varpi)d\varpi \\
&\quad = -\theta_1\int_{t-\theta_1}^{t}\dot{\varrho}^T(\varpi)G_{1i}\dot{\varrho}(\varpi)d\varpi - \theta\int_{t-\theta_2}^{t-\theta(t)}\dot{\varrho}^T(\varpi)G_{2i}\dot{\varrho}(\varpi)d\varpi \\
&\qquad - \theta\int_{t-\theta(t)}^{t-\theta_1}\dot{\varrho}^T(\varpi)G_{2i}\dot{\varrho}(\varpi)d\varpi.
\end{aligned} \quad (43)$$

For the integral items $-\theta_1\int_{t-\theta_1}^{t}\dot{\varrho}^T(\varpi)G_{1i}\dot{\varrho}(\varpi)d\varpi$, $-\theta\int_{t-\theta_2}^{t-\theta(t)}\dot{\varrho}^T(\varpi)G_{2i}\dot{\varrho}(\varpi)d\varpi$, and $-\theta\int_{t-\theta(t)}^{t-\theta_1}\dot{\varrho}^T(\varpi)G_{2i}\dot{\varrho}(\varpi)d\varpi$ in (43), by using Lemma 4, we can obtain that

$$\begin{aligned}
&-\theta_1\int_{t-\theta_1}^{t}\dot{\varrho}^T(\varpi)G_{1i}\dot{\varrho}(\varpi)d\varpi \\
&\leq -(\varrho(t)-\varrho(t-\theta_1))^T G_{1i}(\varrho(t)-\varrho(t-\theta_1)) \\
&\quad -3\left(\varrho(t)+\varrho(t-\theta_1)-\frac{2}{\theta_1}\int_{t-\theta_1}^{t}\varrho(\varpi)d\varpi\right)^T G_{1i} \\
&\quad \cdot\left(\varrho(t)+\varrho(t-\theta_1)-\frac{2}{\theta_1}\int_{t-\theta_1}^{t}\varrho(\varpi)d\varpi\right) \\
&\quad -5\left(\varrho(t)-\varrho(t-\theta_1)+\frac{6}{\theta_1}\int_{t-\theta_1}^{t}\varrho(\varpi)d\varpi\right. \\
&\qquad \left.-\frac{12}{\theta_1^2}\int_{t-\theta_1}^{t}\int_{\beta}^{t}\varrho(\varpi)d\varpi d\beta\right)^T G_{1i} \\
&\quad \times\left(\varrho(t)-\varrho(t-\theta_1)+\frac{6}{\theta_1}\int_{t-\theta_1}^{t}\varrho(\varpi)d\varpi\right. \\
&\qquad \left.-\frac{12}{\theta_1^2}\int_{t-\theta_1}^{t}\int_{\beta}^{t}\varrho(\varpi)d\varpi d\beta\right) \\
&= -\varsigma^T(t)\Big[(E_2-E_3)G_{1i}(E_2-E_3)^T \\
&\qquad +3(E_2+E_3-2E_8)G_{1i}(E_2+E_3-2E_8)^T \\
&\qquad +5(E_2-E_3+6E_8-6E_{11})G_{1i} \\
&\qquad \cdot(E_2-E_3+6E_8-6E_{11})^T\Big]\varsigma(t).
\end{aligned} \tag{44}$$

By using a similar method, it is not difficult to get that

$$\begin{aligned}
&-\theta\int_{t-\theta_2}^{t-\theta(t)}\dot{\varrho}^T(\varpi)G_{2i}\dot{\varrho}(\varpi)d\varpi \\
&\leq -\frac{\theta}{\theta(t)-\theta_1}\varsigma^T(t) \\
&\quad \cdot\Big[(E_3-E_4)G_{2i}(E_3-E_4)^T \\
&\qquad +3(E_3+E_4-2E_9)G_{2i}(E_3+E_4-2E_9)^T \\
&\qquad +5(E_3-E_4+6E_9-6E_{12})G_{2i} \\
&\quad \cdot(E_3-E_4+6E_9-6E_{12})^T\Big]\varsigma(t),
\end{aligned} \tag{45}$$

$$\begin{aligned}
&-\theta\int_{t-\theta(t)}^{t-\theta_1}\dot{\varrho}^T(\varpi)G_{2i}\dot{\varrho}(\varpi)d\varpi \\
&\leq -\frac{\theta}{\theta_2-\theta(t)}\varsigma^T(t) \\
&\quad \cdot\Big[(E_4-E_5)G_{2i}(E_4-E_5)^T \\
&\qquad +3(E_4+E_5-2E_{10})G_{2i}(E_4+E_5-2E_{10})^T \\
&\qquad +5(E_4-E_5+6E_{10}-6E_{13})G_{2i} \\
&\quad \cdot(E_4-E_5+6E_{10}-6E_{13})^T\Big]\varsigma(t).
\end{aligned} \tag{46}$$

Considering the double integral items in (41) and (42), we can get that

$$\begin{aligned}
&\theta_1\int_{-\theta_1}^{0}\int_{t+\beta}^{t}\dot{\varrho}^T(\varpi)\left(\sum_{k=1}^{\mathcal{N}}\overline{\rho}_{ik}G_{1k}\right)\dot{\varrho}(\varpi)d\varpi d\beta \\
&\quad -\int_{-\theta_1}^{0}\int_{t+\beta}^{t}\dot{\varrho}^T(\varpi)Z_1\dot{\varrho}(\varpi)d\varpi d\beta \\
&\quad +\theta\int_{-\theta_2}^{-\theta_1}\int_{t+\beta}^{t}\dot{\varrho}^T(\varpi)\left(\sum_{k=1}^{\mathcal{N}}\overline{\rho}_{ik}G_{2k}\right)\dot{\varrho}(\varpi)d\varpi d\beta \\
&\quad -\int_{-\theta_2}^{-\theta_1}\int_{t+\beta}^{t}\dot{\varrho}^T(\varpi)Z_2\dot{\varrho}(\varpi)d\varpi d\beta \\
&= -\int_{-\theta_1}^{0}\int_{t+\beta}^{t}\dot{\varrho}^T(\varpi)\left(Z_1-\theta_1\sum_{k=1}^{\mathcal{N}}\overline{\rho}_{ik}G_{1k}\right)\dot{\varrho}(\varpi)d\varpi d\beta \\
&\quad -\int_{-\theta_2}^{-\theta_1}\int_{t+\beta}^{t}\dot{\varrho}^T(\varpi)\left(Z_2-\theta\sum_{k=1}^{\mathcal{N}}\overline{\rho}_{ik}G_{2k}\right)\dot{\varrho}(\varpi)d\varpi d\beta \\
&= -\int_{-\theta_1}^{0}\int_{t+\beta}^{t}\dot{\varrho}^T(\varpi)W_{1i}\dot{\varrho}(\varpi)d\varpi d\beta \\
&\quad -\int_{-\theta_2}^{-\theta_1}\int_{t+\beta}^{t}\dot{\varrho}^T(\varpi)W_{2i}\dot{\varrho}(\varpi)d\varpi d\beta.
\end{aligned} \tag{47}$$

For $-\int_{-\theta_1}^{0}\int_{t+\beta}^{t}\dot{\varrho}^T(\varpi)W_{1i}\dot{\varrho}(\varpi)d\varpi d\beta$ in (47), by using Lemma 4, we can obtain that

$$\begin{aligned}
&-\int_{-\theta_1}^{0}\int_{t+\beta}^{t}\dot{\varrho}^T(\varpi)W_{1i}\dot{\varrho}(\varpi)d\varpi d\beta \\
&\leq -2\left[\varrho(t)-\frac{1}{\theta_1}\int_{t-\theta_1}^{t}\varrho(\varpi)d\varpi\right]^T W_{1i} \\
&\quad \cdot\left[\varrho(t)-\frac{1}{\theta_1}\int_{t-\theta_1}^{t}\varrho(\varpi)d\varpi\right] \\
&\quad -4\left[\varrho(t)+\frac{2}{\theta_1}\int_{t-\theta_1}^{t}\varrho(\varpi)d\varpi-\frac{6}{\theta_1^2}\int_{t-\theta_1}^{t}\int_{\beta}^{t}\varrho(\varpi)d\varpi d\beta\right]^T W_{1i} \\
&\quad \cdot\left[\varrho(t)+\frac{2}{\theta_1}\int_{t-\theta_1}^{t}\varrho(\varpi)d\varpi-\frac{6}{\theta_1^2}\int_{t-\theta_1}^{t}\int_{\beta}^{t}\varrho(\varpi)d\varpi d\beta\right] \\
&= \varsigma^T(t)\Big[-2(E_2-E_8)W_{1i}(E_2-E_8)^T \\
&\qquad -4(E_2+2E_8-3E_{11})W_{1i}(E_2+2E_8-3E_{11})^T\Big]\varsigma(t).
\end{aligned} \tag{48}$$

For $-\int_{-\theta_2}^{-\theta_1}\int_{t+\beta}^{t}\dot{\varrho}^T(\varpi)W_{2i}\dot{\varrho}(\varpi)d\varpi d\beta$ in (47), we firstly divide it into two parts

$$\begin{aligned}
&-\int_{-\theta_2}^{-\theta_1}\int_{t+\beta}^{t}\dot{\varrho}^T(\varpi)W_{2i}\dot{\varrho}(\varpi)d\varpi d\beta \\
&\quad = -\int_{-\theta_2}^{-\theta_1}\int_{t-\theta_1}^{t}\dot{\varrho}^T(\varpi)W_{2i}\dot{\varrho}(\varpi)d\varpi d\beta \\
&\qquad -\int_{-\theta_2}^{-\theta_1}\int_{t+\beta}^{t-\theta_1}\dot{\varrho}^T(\varpi)W_{2i}\dot{\varrho}(\varpi)d\varpi d\beta.
\end{aligned} \tag{49}$$

It is easy to find that the first integral term in (49) can be reduced to a single integral. So we can use Lemma 4 to handle it:

$$\begin{aligned}
&-\int_{-\theta_2}^{-\theta_1}\int_{t-\theta_1}^{t}\dot{\varrho}^T(\varpi)W_{2i}\dot{\varrho}(\varpi)d\varpi d\beta \\
&\quad = -\theta\int_{t-\theta_1}^{t}\dot{\varrho}^T(\varpi)W_{2i}\dot{\varrho}(\varpi)d\varpi \\
&\quad \leq -\frac{\theta}{\theta_1}\varsigma^T(t)\Big[(E_2-E_3)W_{2i}(E_2-E_3)^T \\
&\qquad + 3(E_2+E_3-2E_8)W_{2i}(E_2+E_3-2E_8)^T \\
&\qquad + 5(E_2-E_3+6E_8-6E_{11})W_{2i} \\
&\qquad \cdot (E_2-E_3+6E_8-6E_{11})^T\Big]\varsigma(t).
\end{aligned} \tag{50}$$

For the second integral term in (49), we firstly divide it into three parts:

$$\begin{aligned}
&-\int_{-\theta_2}^{-\theta_1}\int_{t+\beta}^{t-\theta_1}\dot{\varrho}^T(\varpi)W_{2i}\dot{\varrho}(\varpi)d\varpi d\beta \\
&\quad = -\int_{-\theta(t)}^{-\theta_1}\int_{t+\beta}^{t-\theta_1}\dot{\varrho}^T(\varpi)W_{2i}\dot{\varrho}(\varpi)d\varpi d\beta \\
&\qquad - \int_{-\theta_2}^{-\theta(t)}\int_{t+\beta}^{t-\theta(t)}\dot{\varrho}^T(\varpi)W_{2i}\dot{\varrho}(\varpi)d\varpi d\beta \\
&\qquad - (\theta_2-\theta(t))\int_{t-\theta(t)}^{t-\theta_1}\dot{\varrho}^T(\varpi)W_{2i}\dot{\varrho}(\varpi)d\varpi.
\end{aligned} \tag{51}$$

Then, we consider using Lemma 4 to deal with (51) and the following inequalities can be obtained:

$$\begin{aligned}
&-\int_{-\theta(t)}^{-\theta_1}\int_{t+\beta}^{t-\theta_1}\dot{\varrho}^T(\varpi)W_{2i}\dot{\varrho}(\varpi)d\varpi d\beta \\
&- \int_{-\theta_2}^{-\theta(t)}\int_{t+\beta}^{t-\theta(t)}\dot{\varrho}^T(\varpi)W_{2i}\dot{\varrho}(\varpi)d\varpi d\beta \\
&\leq \varsigma^T(t)\Big[-2(E_3-E_9)W_{2i}(E_3-E_9)^T \\
&\quad - 4(E_3+2E_9-3E_{12})W_{2i}(E_3+2E_9-3E_{12})^T \\
&\quad - 2(E_4-E_{10})W_{2i}(E_4-E_{10})^T \\
&\quad - 4(E_4+2E_{10}-3E_{13})W_{2i}(E_4+2E_{10}-3E_{13})^T\Big]\varsigma(t),
\end{aligned} \tag{52}$$

$$\begin{aligned}
&-(\theta_2-\theta(t))\int_{t-\theta(t)}^{t-\theta_1}\dot{\varrho}^T(\varpi)W_{2i}\dot{\varrho}(\varpi)d\varpi \\
&\leq -\frac{\theta}{\theta(t)-\theta_1}\varsigma^T(t)\Big[(E_3-E_4)W_{2i}(E_3-E_4)^T \\
&\quad + 3(E_3+E_4-2E_9)W_{2i}(E_3+E_4-2E_9)^T \\
&\quad + 5(E_3-E_4+6E_9-6E_{12})W_{2i}(E_3-E_4+6E_9-6E_{12})^T\Big]\varsigma(t) \\
&\quad + \varsigma^T(t)\Big[(E_3-E_4)W_{2i}(E_3-E_4)^T + 3(E_3+E_4-2E_9)W_{2i} \\
&\qquad \cdot (E_3+E_4-2E_9)^T + 5(E_3-E_4+6E_9-6E_{12})W_{2i} \\
&\qquad \cdot (E_3-E_4+6E_9-6E_{12})^T\Big]\varsigma(t).
\end{aligned} \tag{53}$$

Let us define $\alpha=(\theta(t)-\theta_1)/\theta$ and $\beta=(\theta_2-\theta(t))/\theta$. Using Lemma 5, we can obtain the following relation from inequalities (45), (46), and (53):

$$\begin{aligned}
&-\frac{1}{\alpha}\varsigma^T(t)\Big[(E_3-E_4)(G_{2i}+W_{2i})(E_3-E_4)^T \\
&\qquad + 3(E_3+E_4-2E_9)(G_{2i}+W_{2i})(E_3+E_4-2E_9)^T \\
&\qquad + 5(E_3-E_4+6E_9-6E_{12})(G_{2i}+W_{2i}) \\
&\qquad \cdot (E_3-E_4+6E_9-6E_{12})^T\Big]\varsigma(t) - \frac{1}{\beta}\varsigma^T(t) \\
&\quad \cdot \Big[(E_4-E_5)G_{2i}(E_4-E_5)^T \\
&\quad + 3(E_4+E_5-2E_{10})G_{2i}(E_4+E_5-2E_{10})^T \\
&\quad + 5(E_4-E_5+6E_{10}-6E_{13})G_{2i} \\
&\quad \cdot (E_4-E_5+6E_{10}-6E_{13})^T\Big]\varsigma(t) + \varsigma^T(t) \\
&\quad \cdot \Big[(E_3-E_4)W_{2i}(E_3-E_4)^T \\
&\quad + 3(E_3+E_4-2E_9)W_{2i}(E_3+E_4-2E_9)^T \\
&\quad + 5(E_3-E_4+6E_9-6E_{12})W_{2i} \\
&\quad \cdot (E_3-E_4+6E_9-6E_{12})^T\Big]\varsigma(t) \leq -\Gamma_1\widehat{\Psi}_i\Gamma_1^T.
\end{aligned} \tag{54}$$

Synthesizing inequalities (41), (42), (44), (48), (50), (52), and (54), we can get that

$$\begin{aligned}
&\mathscr{L}V_4(\varrho_s(t),\xi(t)) + \mathscr{L}V_5(\varrho_s(t),\xi(t)) \\
&\leq \varsigma^T(t)\Big\{-(E_2-E_3)G_{1i}(E_2-E_3)^T - 3(E_2+E_3-2E_8)G_{1i} \\
&\quad \cdot (E_2+E_3-2E_8)^T - 5(E_2-E_3+6E_8-6E_{11})G_{1i} \\
&\quad \cdot (E_2-E_3+6E_8-6E_{11})^T - 2(E_2-E_8)W_{1i} \\
&\quad \cdot (E_2-E_8)^T - 4(E_2+2E_8-3E_{11})W_{1i} \\
&\quad \cdot (E_2+2E_8-3E_{11})^T - \frac{\theta}{\theta_1}\Big[(E_2-E_3)W_{2i}(E_2-E_3)^T \\
&\quad + 3(E_2+E_3-2E_8)W_{2i}(E_2+E_3-2E_8)^T \\
&\quad + 5(E_2-E_3+6E_8-6E_{11})W_{2i} \\
&\quad \cdot (E_2-E_3+6E_8-6E_{11})^T\Big] - 2(E_3-E_9)W_{2i} \\
&\quad \cdot (E_3-E_9)^T - 4(E_3+2E_9-3E_{12})W_{2i} \\
&\quad \cdot (E_3+2E_9-3E_{12})^T - 2(E_4-E_{10})W_{2i}(E_4-E_{10})^T \\
&\quad - 4(E_4+2E_{10}-3E_{13})W_{2i}(E_4+2E_{10}-3E_{13})^T \\
&\quad - \Gamma_1\widehat{\Psi}_i\Gamma_1^T + E_1G_iE_1^T\Big\}\varsigma(t).
\end{aligned} \tag{55}$$

After that, some free-weight matrices are utilized. For any scalars δ_1 and δ_2 and the matrix Y_i of appropriate dimensions, we have

$$\begin{aligned}
&2\left(\delta_1\dot{\varrho}^T(t)Y_i+\delta_2\varrho^T(t)Y_i\right)\\
&\cdot\Big[-\dot{\varrho}(t)-(A_i-K_i)\varrho(t)+B_if(\varrho(t))\\
&\quad+\overline{\beta}B_{\theta i}f(\varrho(t-\theta(t)))+D_i\omega(t)\Big]\\
&=\varsigma^T(t)\Big[Sym\Big\{\delta_1E_1Y_iE_1^T+\delta_1E_1Y_iA_iE_2^T-\delta_1E_1Y_iK_iE_2^T\\
&\quad-\delta_1E_1Y_iB_iE_6^T-\delta_1\overline{\beta}E_1Y_iB_{\theta i}E_7^T-\delta_1E_1Y_iD_iE_{14}^T\\
&\quad+\delta_2E_2Y_iE_1^T+\delta_2E_2Y_iA_iE_2^T-\delta_2E_1Y_iK_iE_2^T\\
&\quad-\delta_2E_2Y_iB_iE_6^T-\delta_2\overline{\beta}E_2Y_iB_{\theta i}E_7^T\\
&\quad-\delta_2E_2Y_iD_iE_{14}^T\Big\}\Big]\varsigma(t).
\end{aligned}\tag{56}$$

On the other hand, considering the nonlinear part and according to Lemma 1, one has that for any matrices $V_{ki}>0$, $k=1,2$:

$$\begin{aligned}
&\begin{bmatrix}\varrho(t)\\ f(\varrho(t))\end{bmatrix}^T\begin{bmatrix}-L_1V_{1i} & L_2V_{1i}\\ * & -V_{1i}\end{bmatrix}\begin{bmatrix}\varrho(t)\\ f(\varrho(t))\end{bmatrix}\\
&=\varsigma^T(t)\left[-E_2L_1V_{1i}E_2^T+Sym\left\{E_2L_2V_{1i}E_6^T\right\}\right.\\
&\quad\left.-E_6V_{1i}E_6^T\right]\varsigma(t)\ge 0,\\
&\begin{bmatrix}\varrho(t-\theta(t))\\ f(\varrho(t-\theta(t)))\end{bmatrix}^T\begin{bmatrix}-L_2V_{2i} & L_2V_{2i}\\ * & -V_{2i}\end{bmatrix}\begin{bmatrix}\varrho(t-\theta(t))\\ f(\varrho(t-\theta(t)))\end{bmatrix}\\
&=\varsigma^T(t)\left[-E_4(L_1V_{2i})E_4^T+Sym\left\{E_4L_2V_{2i}E_7^T\right\}\right.\\
&\quad\left.-E_7V_{2i}E_7^T\right]\varsigma(t)\ge 0.
\end{aligned}\tag{57}$$

Furthermore, considering the H_∞ synchronization, in view of Definition 1, we can get

$$\tilde{\varrho}^T(t)\tilde{\varrho}(t)-\gamma^2\omega^T(t)\omega(t)=\varsigma^T(t)\left[E_2C_i^TC_iE_2^T-\gamma^2E_{14}E_{14}^T\right]\varsigma(t).\tag{58}$$

Above all, we can get that

$$\begin{aligned}
&\mathscr{E}\left\{\mathscr{L}V(\varrho_s(t),\xi(t))+\tilde{\varrho}^T(t)\tilde{\varrho}(t)-\gamma^2\omega^T(t)\omega(t)\right\}\\
&\quad\le\varsigma^T(t)\left[\overline{\Phi}_i+\prod(\theta(t))\left(\sum_{k=1}^{\mathscr{N}}\overline{\rho}_{ik}P_k\right)\prod^T(\theta(t))\right]\varsigma(t).
\end{aligned}\tag{59}$$

According to (20), one can obtain that

$$\mathscr{E}\left\{\mathscr{L}V(\varrho_s(t),\xi(t))+\tilde{\varrho}^T(t)\tilde{\varrho}(t)-\gamma^2\omega^T(t)\omega(t)\right\}<0.\tag{60}$$

With $\omega(t)=0$, we can get from (59) that $\mathscr{L}V(\varrho_s(t),\xi(t))<0$, which implies that the system $(\overline{\Sigma})$ is stochastically stable.

Furthermore, considering the system $(\overline{\Sigma})$, from (60) one could obtain the following inequality under zero initial conditions for any $\gamma>0$:

$$\begin{aligned}
&\mathscr{E}\left\{\int_0^\infty\left[\tilde{\varrho}^T(t)\tilde{\varrho}(t)-\gamma^2\omega^T(t)\omega(t)\right]dt\right\}\\
&\le\mathscr{E}\left\{\int_0^\infty\left[\mathscr{L}V(\varrho_s(t),\xi(t))+\tilde{\varrho}^T(t)\tilde{\varrho}(t)-\gamma^2\omega^T(t)\omega(t)\right]dt\right\}\\
&\le 0,
\end{aligned}\tag{61}$$

which means that

$$\Theta=\int_0^\infty\left[\tilde{\varrho}^T(t)\tilde{\varrho}(t)-\gamma^2\omega^T(t)\omega(t)\right]dt\le 0.\tag{62}$$

Therefore, the condition in Definition 1 holds.

In view of the above, we can conclude that the considered system is stochastically stable and satisfies H_∞ synchronization with the disturbance attenuation γ.

From the stochastic stability and H_∞ synchronization analysis criterion for the system in Theorem 1, it is not difficult to find that the inequalities may not be easily verified as the existence of TVD term $\theta(t)$ and nonlinear delay term $\theta^2(t)$, especially the quadratic term $\prod(\theta(t))(\sum_{k=1}^{\mathscr{N}}\overline{\rho}_{ik}P_k)\prod^T(\theta(t))$. So, we utilize projection lemma and Schur complement to achieve matrix decoupling.

Remark 2. For the quadratic term $\prod(\theta(t))(\sum_{k=1}^{\mathscr{N}}\overline{\rho}_{ik}P_k)\prod^T(\theta(t))$, it is not difficult to find that $\overline{\rho}_{ii}, i\in\Upsilon$ are negative terms while $\overline{\rho}_{ij}, i,j\in\Upsilon, i\ne j$ are positive terms. Therefore, we use different methods to deal with this problem. When $i\ne j, \overline{\rho}_{ij}>0$, we can use Schur complement to handle it. When $i=j, \overline{\rho}_{ii}<0$, it is quite clear that Schur complement cannot be applied to it directly. So we use projection lemma to cope with it. By using this way, we can achieve the decoupling between nonlinear terms properly.

Remark 3. From stochastic Lyapunov functional (29) as well as inequality (16), sufficient conditions which guarantee the stochastically stable and H_∞ synchronization of system $(\overline{\Sigma})$ are obtained. The term $\overline{\varrho}(t)=[\varrho^T(t)\int_{t-\theta_1}^{t}\varrho^T(\varpi)d\varpi\int_{t-\theta_2}^{t-\theta_1}\varrho^T(\varpi)d\varpi]^T$ in $V_1(\varrho_s(t),\xi(t))$ is used to include more information about state delays. The semi-Markovian parameters are introduced to $V_2(\varrho_s(t),\xi(t))$, $V_4(\varrho_s(t),\xi(t))$, and $V_5(\varrho_s(t),\xi(t))$ for the consideration of mode-dependent Lyapunov functional. Inequality (16) is used to deal with the integral terms. By using this way, we can obtain sufficient conditions with less conservatism.

Theorem 2. *Given scalars μ, $\gamma>0$, $0\le\theta_1\le\theta_2<\infty$, $\theta=\theta_2-\theta_1$, $\delta_1>0$, $\delta_2>0$, and $\overline{\beta}\in[0,1]$, the considered system $(\overline{\Sigma})$ is stochastically stable and H_∞ synchronized; if there exist symmetric matrices $P_i\in\mathbb{R}^{3n}>0$, positive definite matrices Q_{1i}, Q_{2i}, Q_{3i}, R_1, R_2, G_{1i}, G_{2i}, Z_1, Z_2, positive definite diagonal matrices V_{1i}, V_{2i}, matrices Y_i and $S_{ij}\in\mathbb{R}^n$, $i,j\in\Upsilon$ such that for each $i\in\Upsilon, k=1,2$, conditions (21), (22), (23), (24), (25), and (26) and the following inequality hold*

$$\tilde{\Phi}_i^{(k)} = \begin{bmatrix} \overline{\Phi}_i + \psi_i^{(k)} + Sym\left\{\Pi^{(k)}X\right\} & -X^T & \Pi^{(k)}N_i\mathscr{P}_i \\ * & \overline{\rho}_{ii}P_i & 0 \\ * & * & -\mathscr{P}_i \end{bmatrix} < 0, \tag{63}$$

where

$$\begin{aligned}
\psi_i^{(1)} &= sym\left\{\theta E_1 P_{13i} E_{10}^T + \theta E_2 P_{23i} E_{10}^T + \theta E_3 (P_{33i} - P_{23i}) E_{10}^T - \theta E_5 P_{33i} E_{10}^T\right\} \\
&\quad - \theta E_{10} M_{3i} E_{10}^T - 3\theta (E_{10} - E_{13}) M_{3i} (E_{10} - E_{13})^T, \\
\psi_i^{(2)} &= Sym\left\{\theta E_1 P_{13i} E_9^T + \theta E_2 P_{23i} E_9^T + \theta E_3 (P_{33i} - P_{23i}) E_9^T - \theta E_5 P_{33i} E_9^T\right\} \\
&\quad - \theta E_9 M_{2i} E_9^T - 3\theta (E_9 - E_{12}) M_{2i} (E_9 - E_{12}) T, \\
\Pi^{(1)} &= [E_2 \theta_1 E_8 \theta E_{10}], \\
\Pi^{(2)} &= [E_2 \theta_1 E_8 \theta E_9], \\
N_i &= \left[\sqrt{\overline{\rho}_{i1}}I, \sqrt{\overline{\rho}_{i2}}I, \dots, \sqrt{\overline{\rho}_{i,i-1}}I, \sqrt{\overline{\rho}_{i,i+1}}I, \dots, \sqrt{\overline{\rho}_{i\mathscr{N}}}I\right], \\
\mathscr{P}_i &= \text{diag}\left\{P_1, P_2, \dots, P_{i-1}, P_{i+1}, \dots, P_{\mathscr{N}}\right\},
\end{aligned} \tag{64}$$

with the other notations are the same as in Theorem 1.

Proof 2. From Theorem 1, we can know that when conditions (20), (21), (22), (23), (24), (25), and (26) are satisfied, the considered system $(\overline{\Sigma})$ is stochastically stable. In order to achieve decoupling between nonlinear time-delay terms in (20), further processing will be made for inequality (20).

First of all, the inequality (20) can be rewritten as

$$N_P^T \Xi_i N_P < 0, \tag{65}$$

where

$$N_P = \left[I_{14n} \quad \Pi(\theta(t))\right]^T,$$
$$\Xi_i = \begin{bmatrix} \overline{\Phi}_i + \psi_i(\theta(t)) + \Pi(\theta(t))\left(\sum_{k=1,k\neq i}^{\mathscr{N}} \overline{\rho}_{ik}P_k\right)\Pi^T(\theta(t)) & 0 \\ 0 & \overline{\rho}_{ii}P_i \end{bmatrix}, \tag{66}$$

From inequality (65) we can also get from quadratic form knowledge that $\Xi_i < 0$, then we can obtain the following inequality

$$N_Q^T \Xi_i N_Q < 0, \tag{67}$$

where $N_Q = \left[0 \quad I_{3n}\right]^T$.

We define $P = \left[\Pi^T(\theta(t)) \quad -I_{14n}\right]$, $Q = \left[I_{14n} \quad 0\right]$. It is not difficult to know that $PN_P = 0$, $QN_Q = 0$. Therefore, N_P and N_Q are the elements of the nullspace of P and Q.

Then, by using projection lemma to inequalities (65) and (67), we can know that

$$\Xi_i + sym\left\{P^T X Q\right\} < 0, \tag{68}$$

which means

$$\begin{bmatrix} \overline{\Phi}_i + \psi_i(\theta(t)) + \Pi(\theta(t))\left(\sum_{k=1,k\neq i}^{\mathscr{N}} \overline{\rho}_{ik}P_k\right)\Pi^T(\theta(t)) + sym\{\Pi(\theta(t))X\} & -X^T \\ * & \overline{\rho}_{ii}P_i \end{bmatrix} < 0. \tag{69}$$

Then, we use Schur complement to deal with the coupling term $\Pi(\theta(t))(\sum_{k=1,k\neq i}^{\mathscr{N}} \overline{\rho}_{ik}P_k)\Pi^T(\theta(t))$ in (69) and we can obtain that

$$\begin{bmatrix} \overline{\Phi}_i + \psi_i(\theta(t)) + sym\{\Pi(\theta(t))X\} & -X^T & \Pi(\theta(t))N_i\mathscr{P}_i \\ * & \overline{\rho}_{ii}P_i & 0 \\ * & * & -\mathscr{P}_i \end{bmatrix} < 0, \tag{70}$$

where

$$\begin{aligned}
N_i &= \left[\sqrt{\overline{\rho}_{i1}}I, \sqrt{\overline{\rho}_{i2}}I, \dots, \sqrt{\overline{\rho}_{i,i-1}}I, \sqrt{\overline{\rho}_{i,i+1}}I, \dots, \sqrt{\overline{\rho}_{i\mathscr{N}}}I\right], \\
\mathscr{P}_i &= \text{diag}\left\{P_1, P_2, \dots, P_{i-1}, P_{i+1}, \dots, P_{\mathscr{N}}\right\}.
\end{aligned} \tag{71}$$

By using the above decoupling method, we achieve that there is only linear TVD term $\theta(t)$ in inequality (70). As $0 \le \theta_1 \le \theta(t) \le \theta_2 < \infty$, therefore inequality (70) is satisfied under the condition that $\theta(t) = \theta_1$, $\theta(t) = \theta_2$,, respectively. It can guarantee that once inequality (70) is satisfied under the condition that $\theta_1 \le \theta(t) \le \theta_2$, then the correctness of inequality (70) can be readily deduced as (63) holds. So, (20) holds if inequality (63) is satisfied. This completes the proof.

Since a stochastic stability and H_∞ synchronization analysis criterion for the system in Theorem 2 is developed, the procedure of H_∞ synchronization can be developed in Theorem 3 in the further analysis. Therefore, in this section we derive H_∞ synchronization conditions for the drive-response dynamic systems. The main result about the design of a desired controller will be presented, and the controller gain will be presented.

Theorem 3. *Fixed scalars* μ, $\gamma > 0$, $0 \le \theta_1 \le \theta_2 < \infty$, $\theta = \theta_2 - \theta_1$, $\delta_1 > 0$, $\delta_2 > 0$, *and* $\overline{\beta} \in [0,1]$, *the considered system* $(\overline{\Sigma})$ *is stochastically stable and* H_∞ *synchronized, if there exist symmetric matrices* $P_i \in \mathbb{R}^{3n} > 0$, *positive definite matrices* $Q_{1i}, Q_{2i}, Q_{3i}, R_1, R_2, G_{1i}, G_{2i}, Z_1, Z_2$, *positive definite diagonal matrices* V_{1i}, V_{2i}, *matrices* Y_i *and* $\tilde{K}_i$, *and* $S_{ij} \in \mathbb{R}^n$, $i, j \in \Upsilon$, *such that for each* $i \in \Upsilon$, *and* $k = 1, 2,$, *the conditions (21), (22), (23), (24), (25), and (26) and the following inequality hold:*

$$\begin{bmatrix} \check{\Phi}_i + \psi_i^{(k)} + sym\left\{\Pi^{(k)}X\right\} & -X^T & \Pi^{(k)}N_i\mathscr{P}_i \\ * & \overline{\rho}_{ii}P_i & 0 \\ * & * & -\mathscr{P}_i \end{bmatrix} < 0, \quad (72)$$

where

$$\begin{aligned}\check{\Phi}_i = {} & Sym\{E_1P_{11i}E_2^T + \theta_1E_1P_{12i}E_8^T + E_2P_{12i}E_2^T - E_3P_{12i}E_2^T \\ & + \theta_1E_2P_{22i}E_8^T - \theta_1E_3P_{22i}E_8^T + E_3P_{13i}E_2^T \\ & - E_5P_{13i}E_2^T + \theta_1E_3P_{23i}E_8^T - \theta_1E_5P_{23i}E_8^T\} \\ & + E_2Q_iE_2^T - (1-\mu)E_4Q_{3i}E_4^T - E_5Q_{2i}E_5^T \\ & + E_1G_iE_1^T - (E_2 - E_3)G_{1i}(E_2 - E_3)^T \\ & - 3(E_2 + E_3 - 2E_8)G_{1i}(E_2 + E_3 - 2E_8)^T \\ & - 5(E_2 - E_3 + 6E_8 - 6E_{11})G_{1i}(E_2 - E_3 + 6E_8 - 6E_{11})^T \\ & - 2(E_2 - E_8)W_{1i}(E_2 - E_8)^T - 4(E_2 + 2E_8 - 3E_{11})W_{1i} \\ & \cdot (E_2 + 2E_8 - 3E_{11})^T - \frac{\theta}{\theta_1}(E_2 - E_3)W_{2i}(E_2 - E_3)^T \\ & - \frac{3\theta}{\theta_1}(E_2 + E_3 - 2E_8)W_{2i}(E_2 + E_3 - 2E_8)^T \\ & - \frac{5\theta}{\theta_1}(E_2 - E_3 + 6E_8 - 6E_{11})W_{2i}(E_2 - E_3 + 6E_8 - 6E_{11})^T \\ & - 2(E_3 - E_9)W_{2i}(E_3 - E_9)^T - 4(E_3 + 2E_9 - 3E_{12})W_{2i} \\ & \cdot (E_3 + 2E_9 - 3E_{12})^T - 2(E_4 - E_{10})W_{2i}(E_4 - E_{10})^T \\ & - 4(E_4 + 2E_{10} - 3E_{13})W_{2i}(E_4 + 2E_{10} - 3E_{13})^T - \Gamma_1\widehat{\Psi}_i\Gamma_1^T \\ & - Sym\Big\{\delta_1E_1Y_iE_1^T + \delta_1E_1Y_iA_iE_2^T - \delta_1E_1\tilde{K}_iE_2^T \\ & \quad - \delta_1E_1Y_iB_iE_6^T - \delta_1\overline{\beta}E_1Y_iB_{\theta i} \times E_7^T \\ & \quad - \delta_1E_1Y_iD_iE_{14}^T + \delta_2E_2Y_iE_1^T + \delta_2E_2Y_iA_iE_2^T \\ & \quad - \delta_2E_1\tilde{K}_iE_2^T - \delta_2E_2Y_iB_iE_6^T - \delta_2\overline{\beta}E_2Y_i \\ & \quad \times B_{\theta i}E_7^T - \delta_2E_2Y_iD_iE_{14}^T\Big\} - E_2L_1V_{1i}E_2^T \\ & + E_4L_1V_{2i}E_4^T + Sym\{E_2L_2V_{1i}E_6^T + E_4L_2V_{2i}E_7^T\} \\ & - E_6V_{1i}E_6^T - E_7V_{2i}E_7^T - E_2C_i^TC_iE_2^T - \gamma^2E_{14}E_{14}^T,\end{aligned} \quad (73)$$

then the system $(\overline{\Sigma})$ *is stochastically stable. In this regard, the available gain of the controller can be calculated by*

$$K_i = Y_i^{-1}\tilde{K}_i, \quad i \in \Upsilon. \quad (74)$$

Proof 3. We define

$$\tilde{K}_i = Y_iK_i. \quad (75)$$

By using this way, the freedom variable Y_i and the controller gain K_i can be merged, which makes sure inequality (72) can be solved by dealing with a convex optimization problem. This completes the proof.

4. Numerical Examples

In this section, two numerical examples are given for demonstrating the feasibility and validity of the proposed method. In the first example, the existence of the desired H_∞ synchronization controller is verified. The second example gives a set of figures to reveal the correctness of our method.

Example 1. For the proposed system (Σ) and $(\widehat{\Sigma})$ with three modes $(i = 1, 2, 3)$, the parameters are chosen as follows:

$$A_1 = \begin{bmatrix} 2.20 & 0 \\ 0 & 1.80 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0.20 & 0 \\ 0 & 3.40 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 1.00 & 0 \\ 0 & 0.80 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.80 & 0.40 \\ -0.20 & 0.10 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 0.70 & 1.10 \\ 0.20 & -0.05 \end{bmatrix},$$

$$B_3 = \begin{bmatrix} 1.00 & 1.00 \\ -1.00 & -1.00 \end{bmatrix},$$

$$B_{\theta 1} = \begin{bmatrix} 1.20 & 1.00 \\ -0.20 & 0.30 \end{bmatrix},$$

$$B_{\theta 2} = \begin{bmatrix} -2.40 & -4.80 \\ -0.32 & 2.00 \end{bmatrix},$$

$$B_{\theta 3} = \begin{bmatrix} 0.88 & 1.00 \\ 1.00 & 1.00 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 0.10 & 0 \\ 0 & 0.10 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 0.20 & -0.30 \\ -0.30 & 0.20 \end{bmatrix},$$

$$C_3 = \begin{bmatrix} -0.80 & 1.00 \\ 1.00 & 0.60 \end{bmatrix},$$

$$D_1 = \begin{bmatrix} 0.20 & -0.10 \\ -0.10 & 0.20 \end{bmatrix},$$

$$D_2 = \begin{bmatrix} 0.20 & -0.32 \\ -0.32 & 0.20 \end{bmatrix},$$
$$D_3 = \begin{bmatrix} 0.10 & 0.10 \\ 0.10 & 0.10 \end{bmatrix}. \quad (76)$$

Considering the activation functions $f_i(s)$, $i = 1, 2, 3$ with the parameters $l_q^- = 0$ and $l_q^+ = 0.4$, then it is easy to get that

$$L_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$
$$L_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}. \quad (77)$$

The expectation of $\beta(t)$ is taken as $\overline{\beta} = 0.6$. Assuming that $I(t) = 0$, and the TVDs $\theta(t) = 1.1 + \sin(0.5t)$, then we can obtain that $0.1 \leq \theta(t) \leq 2.1$ and $\dot{\theta}(t) \leq 0.5$, which means $\theta_1 = 0.1$, $\theta_2 = 2.1$, and $\mu = 0.5$. Besides, the transition rates of $\xi(t)$ are selected as the same as in [24], where the probability distribution function is also chosen as Weibull distribution. Therefore, the corresponding mathematical expectation of $\overline{\Lambda}(\sigma)$ is

$$\mathscr{E}\{\overline{\Lambda}(\sigma)\} = \begin{bmatrix} -1.7724 & 0.8862 & 0.8862 \\ 1.7725 & -3.5450 & 1.7725 \\ 2.6587 & 2.6587 & -5.3174 \end{bmatrix}. \quad (78)$$

We set $\delta_1 = 1$, $\delta_2 = 2$, and $\gamma^2 = 0.4$. Then, solving LMIs (21), (22), (23), (24), (25), (26) and (72) by using the LMI control toolbox, the H_∞ synchronization controller gain matrixes can be obtained as follows:

$$K_1 = \begin{bmatrix} -8.6398 & 3.1022 \\ 3.1631 & -7.3662 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} -28.0246 & -13.5891 \\ -13.6663 & -17.7993 \end{bmatrix}, \quad (79)$$
$$K_3 = \begin{bmatrix} -10.4938 & -2.2998 \\ -2.7691 & -9.9076 \end{bmatrix}.$$

Remark 4. When we set $\overline{\beta} = 1$ and $D_1 = D_2 = D_3 = 0$ and do not consider the H_∞ performance index and the other parameters remain the same, then our system is the same as Example 13 in [24]. In this condition, we can get the controller gain matrixes as follows:

$$K_1 = \begin{bmatrix} -1.5988 & -0.0753 \\ 0.0621 & -0.3183 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} -6.9360 & 1.1315 \\ 1.4177 & 0.8950 \end{bmatrix},$$
$$K_3 = \begin{bmatrix} -1.6030 & -0.3402 \\ 0.0094 & -1.3144 \end{bmatrix}. \quad (80)$$

Then, we can find that when $\mu = 0.5$, $\theta_1 = 0.1$, the maximum allowable upper delay bound θ_2 in [24] is 0.984, while we set $\theta_2 = 2.1$. The desired controller gains can also be obtained in this paper. This indicates that our method is superior to [24] and can lead to less conservatism for time-delay systems.

Example 2. The parameters of the systems (Σ) and $(\widehat{\Sigma})$ are given with $\mathscr{N} = 3$ as follows:

$$A_1 = \begin{bmatrix} 1.00 & 0 \\ 0 & 1.00 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 1.10 & 0 \\ 0 & 0.90 \end{bmatrix},$$
$$A_3 = \begin{bmatrix} 1.20 & 0 \\ 0 & 0.80 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 1.40 & -0.30 \\ 1.05 & 1.50 \end{bmatrix},$$
$$B_2 = \begin{bmatrix} 1.75 & -0.40 \\ 0.95 & 1.65 \end{bmatrix},$$
$$B_3 = \begin{bmatrix} 1.25 & -0.25 \\ 1.10 & 1.30 \end{bmatrix},$$
$$B_{\theta 1} = \begin{bmatrix} -2.20 & 1.00 \\ -0.80 & -2.10 \end{bmatrix},$$
$$B_{\theta 2} = \begin{bmatrix} -2.70 & 1.10 \\ -0.70 & -2.30 \end{bmatrix},$$
$$B_{\theta 3} = \begin{bmatrix} -2.80 & 1.20 \\ -0.50 & -2.10 \end{bmatrix},$$
$$C_1 = \begin{bmatrix} 0.10 & 0 \\ 0 & 0.10 \end{bmatrix},$$
$$C_2 = \begin{bmatrix} 0.20 & -0.30 \\ -0.30 & 0.20 \end{bmatrix},$$
$$C_3 = \begin{bmatrix} -0.80 & 1.00 \\ 1.00 & 0.60 \end{bmatrix},$$
$$D_1 = \begin{bmatrix} 0.20 & -0.10 \\ -0.10 & 0.20 \end{bmatrix},$$
$$D_2 = \begin{bmatrix} -0.20 & -0.32 \\ -0.32 & -0.20 \end{bmatrix},$$

$$D_3 = \begin{bmatrix} 0.10 & 0.10 \\ 0.10 & 0.10 \end{bmatrix}. \quad (81)$$

The activation functions are assumed as $f_i(s) = \tanh(s)$, $i \in \Upsilon$ with the parameters $l_q^- = 0$ and $l_q^+ = 1$; then, it is easy to get that

$$L_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}. \quad (82)$$

The expectation of $\beta(t)$ is chosen as $\overline{\beta} = 1$. Assuming that $I(t) = 0$, $\omega(t) = [1/1+t^2 \quad 1/1+t^2]^T$ and the TVDs $\theta(t) = 1 + 0.3 \sin(2t)$, then we can obtain that $\theta_1 = 0.7$, $\theta_2 = 1.3$, and $\mu = 0.6$. The mathematical expectation of the transition rate matrix is taken as the same as Example 1. Then, we set $\delta_1 = 1$, $\delta_2 = 5$, $\gamma^2 = 0.4$, and we can get the desired controller gain matrixes as follows:

$$K_1 = \begin{bmatrix} -14.8258 & -0.4952 \\ -0.4985 & -14.6980 \end{bmatrix}, \quad K_2 = \begin{bmatrix} -22.0351 & -1.4217 \\ -0.7621 & -20.2739 \end{bmatrix}, \quad K_3 = \begin{bmatrix} -24.5083 & -0.6844 \\ 0.2891 & -21.1654 \end{bmatrix}. \quad (83)$$

Under the obtained gain matrixes and $x(0) = [0.2 \quad 0.25]^T$ and $\hat{x}(0) = [0.05 \quad -0.15]^T$, we simulate the system and obtain a set of figures. Figure 1 shows the mode transitions $\xi(t)$ of the system. Figures 2 and 3 show the state vector's behaviors of systems (Σ) and $(\widehat{\Sigma})$, respectively. Figure 4 shows the state responses of the system $(\overline{\Sigma})$. In view of Figure 4, we can observe that although under different initial conditions, the error between systems (Σ) and $(\widehat{\Sigma})$ gradually tends to zero which indicates that the gain matrixes derived from this paper can guarantee the synchronization between the systems (Σ) and $(\widehat{\Sigma})$ effectively. Therefore, it confirms the validity of the developed method.

5. Conclusion

In this paper, the H_∞ synchronization problem for semi-MJNNs, where the randomly occurring TVDs have been considered to make the neural networks under consideration more practical, has been investigated. By constructing an appropriate semi-Markovian Lyapunov-Krasovskii functional, combining with the derivation of an infinitesimal generator for the Lyapunov functional and the sufficient H_∞ synchronization condition for semi-MJNNs has been

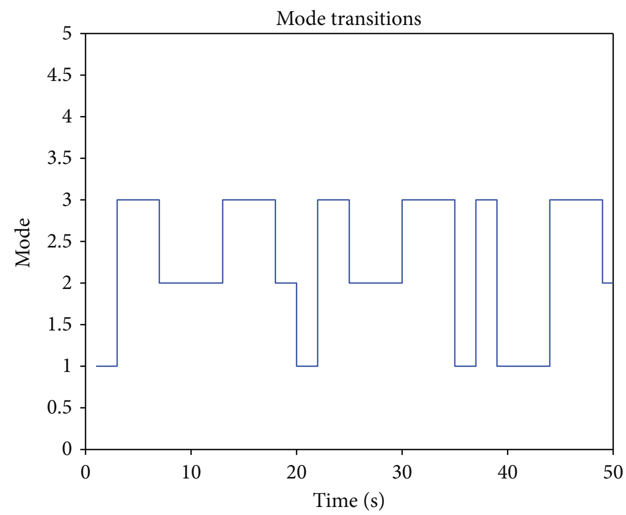

Figure 1: Modes of transitions of semi-MJNNs.

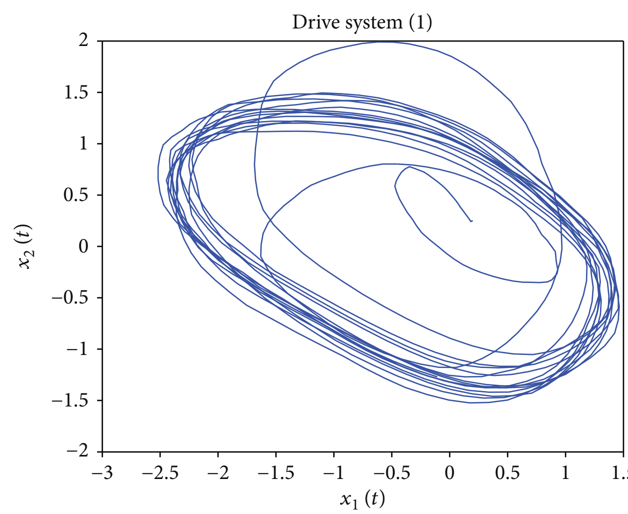

Figure 2: The state vector's behaviors of the drive system (1).

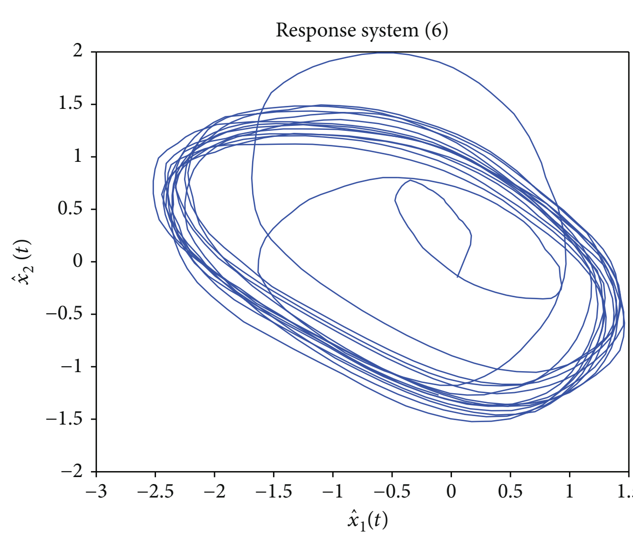

Figure 3: The state vector's behaviors of the response system (7).

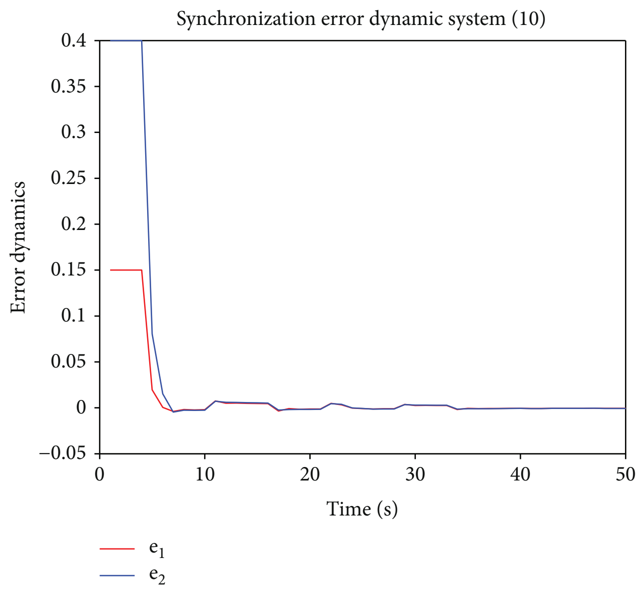

FIGURE 4: The synchronization error between the drive system and the response system.

established. To deal with the TVD items, some improved inequalities, together with Schur complement lemma and projection lemma, have been introduced. By using a linearization technique, the desired controller has been designed and the existence of the desired controller can be verified by the feasibility of a set of LMIs. Finally, two meaningful examples have been given to validate the feasibility and validity of the developed approach. Considering that the conditions established in this paper are sufficient, how to reduce the conservatism of current results will therefore be our future work. In addition, how to extend our results to cope with the controller design problem for more complex networks, such as genetic regulatory networks, will also be one of our future research directions.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

Acknowledgments

This work was supported by the National Natural Science Foundation of China under Grants 61304066, 61703004, 61573008, and 61473178; National Natural Science Foundation of Anhui Province under Grants 1708085M-F165 and 1808085QA18; China Postdoctoral Science Foundation under Grant 2018M632206; and SDUST Research Fund under Grant 2014TDJH102.

References

[1] B. Horne, M. Jamshidi, and N. Vadiee, “Neural networks in robotics: a survey,” *Journal of Intelligent and Robotic Systems*, vol. 3, no. 1, pp. 51–66, 1990.

[2] H. B. Azad, S. Mekhilef, and V. G. Ganapathy, “Long-term wind speed forecasting and general pattern recognition using neural networks,” *IEEE Transactions on Sustainable Energy*, vol. 5, no. 2, pp. 546–553, 2014.

[3] Q. Ma, G. Feng, and S. Xu, “Delay-dependent stability criteria for reaction-diffusion neural networks with time-varying delays,” *IEEE Transactions on Cybernetics*, vol. 43, no. 6, pp. 1913–1920, 2013.

[4] Y. V. Pershin and M. Di Ventra, “Experimental demonstration of associative memory with memristive neural networks,” *Neural Networks*, vol. 23, no. 7, pp. 881–886, 2010.

[5] L. Li, Z. Wang, Y. Li, H. Shen, and J. Lu, “Hopf bifurcation analysis of a complex-valued neural network model with discrete and distributed delays,” *Applied Mathematics and Computation*, vol. 330, pp. 152–169, 2018.

[6] J. Xia, G. Chen, and W. Sun, “Extended dissipative analysis of generalized Markovian switching neural networks with two delay components,” *Neurocomputing*, vol. 260, pp. 275–283, 2017.

[7] K. S. Narendra and K. Parthasarathy, “Identification and control of dynamical systems using neural networks,” *IEEE Transactions on Neural Networks*, vol. 1, no. 1, pp. 4–27, 1990.

[8] Y. Guo, “Global asymptotic stability analysis for integro-differential systems modeling neural networks with delays,” *Zeitschrift für Angewandte Mathematik und Physik*, vol. 61, no. 6, pp. 971–978, 2010.

[9] K. Smith, M. Palaniswami, and M. Krishnamoorthy, “Neural techniques for combinatorial optimization with applications,” *IEEE Transactions on Neural Networks*, vol. 9, no. 6, pp. 1301–1318, 1998.

[10] X. Huang, Y. Fan, J. Jia, Z. Wang, and Y. Li, “Quasi-synchronisation of fractional-order memristor-based neural networks with parameter mismatches,” *IET Control Theory and Applications*, vol. 11, no. 14, pp. 2317–2327, 2017.

[11] C. Ma, T. Li, and J. Zhang, “Consensus control for leader-following multi-agent systems with measurement noises,” *Journal of Systems Science and Complexity*, vol. 23, no. 1, pp. 35–49, 2010.

[12] C. Ma and J. Zhang, “On formability of linear continuous-time multi-agent systems,” *Journal of Systems Science and Complexity*, vol. 25, no. 1, pp. 13–29, 2012.

[13] R. Sakthivel, R. Sakthivel, B. Kaviarasan, C. Wang, and Y. K. Ma, “Finite-time nonfragile synchronization of stochastic complex dynamical networks with semi-Markov switching outer coupling,” *Complexity*, vol. 2018, Article ID 8546304, 13 pages, 2018.

[14] J. Wang, K. Liang, X. Huang, Z. Wang, and H. Shen, “Dissipative fault-tolerant control for nonlinear singular perturbed systems with Markov jumping parameters based on slow state feedback,” *Applied Mathematics and Computation*, vol. 328, pp. 247–262, 2018.

[15] W. Qi, Y. Kao, X. Gao, and Y. Wei, “Controller design for time-delay system with stochastic disturbance and actuator saturation via a new criterion,” *Applied Mathematics and Computation*, vol. 320, pp. 535–546, 2018.

[16] J. Xia, H. Gao, M. Liu, G. Zhuang, and B. Zhang, “Non-fragile finite-time extended dissipative control for a class of uncertain discrete time switched linear systems,” *Journal of the Franklin Institute*, vol. 355, no. 6, pp. 3031–3049, 2018.

[17] Y. Guo, “Mean square exponential stability of stochastic delay cellular neural networks,” *Electronic Journal of Qualitative Theory of Differential Equations*, no. 34, pp. 1–10, 2013.

[18] R. Yang, Z. Zhang, and P. Shi, “Exponential stability on stochastic neural networks with discrete interval and distributed delays,” *IEEE Transactions on Neural Networks*, vol. 21, no. 1, pp. 169–175, 2010.

[19] H. Shen, M. Xing, S. Huo, Z. G. Wu, and J. H. Park, “Finite-time H_∞ asynchronous state estimation for discrete-time fuzzy Markov jump neural networks with uncertain measurements,” *Fuzzy Sets and Systems*, 2018.

[20] S. Jiao, H. Shen, Y. Wei, X. Huang, and Z. Wang, “Further results on dissipativity and stability analysis of Markov jump generalized neural networks with time-varying interval delays,” *Applied Mathematics and Computation*, vol. 336, pp. 338–350, 2018.

[21] X. Song and J. H. Park, “Linear minimum mean-square estimation for discrete-time measurement-delay systems with multiplicative noise and markov Jump,” *IET Control Theory and Applications*, vol. 10, no. 10, pp. 1161–1169, 2016.

[22] Y. Wei, J. H. Park, J. Qiu, L. Wu, and H. Y. Jung, “Sliding mode control for semi-Markovian jump systems via output feedback,” *Automatica*, vol. 81, pp. 133–141, 2017.

[23] J. Huang and Y. Shi, “Stochastic stability and robust stabilization of semi-Markov jump linear systems,” *International Journal of Robust and Nonlinear Control*, vol. 23, no. 18, pp. 2028–2043, 2013.

[24] Y. Wei, J. H. Park, H. R. Karimi et al., “Improved stability and stabilization results for stochastic synchronization of continuous-time semi-Markovian jump neural networks with time-varying delay,” *IEEE Transactions on Neural Networks and Learning Systems*, vol. 29, no. 6, pp. 2488–2501, 2018.

[25] P. Shi, F. Li, L. Wu, and C.-C. Lim, “Neural network-based passive filtering for delayed neutral-type semi-Markovian jump systems,” *IEEE Transactions on Neural Networks and Learning Systems*, vol. 28, no. 9, pp. 2101–2114, 2017.

[26] Y. Foucher, E. Mathieu, P. Saint-Pierre, J.-F. Durand, and J.-P. Daurès, “A semi-Markov model based on generalized Weibull distribution with an illustration for HIV disease,” *Biometrical Journal*, vol. 47, no. 6, pp. 825–833, 2005.

[27] H. Gao, J. Xia, G. Zhuang, Z. Wang, and Q. Sun, “Non-fragile finite-time extended dissipative control for a class of uncertain switched neutral systems,” *Complexity*, vol. 2017, Article ID 6581308, 22 pages, 2017.

[28] G. Liu, S. Xu, Y. Wei, Z. Qi, and Z. Zhang, “New insight into reachable set estimation for uncertain singular time-delay systems,” *Applied Mathematics and Computation*, vol. 320, pp. 769–780, 2018.

[29] Y. Wei, J. Qiu, P. Shi, and M. Chadli, “Fixed-order piecewise-affine output feedback controller for fuzzy-affine-model-based nonlinear systems with time-varying delay,” *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 64, no. 4, pp. 945–958, 2017.

[30] S. Xu and J. Lam, “On equivalence and efficiency of certain stability criteria for time-delay systems,” *IEEE Transactions on Automatic Control*, vol. 52, no. 1, pp. 95–101, 2007.

[31] Y. Guo and S. T. Liu, “Global exponential stability analysis for a class of neural networks with time delays,” *International Journal of Robust and Nonlinear Control*, vol. 22, no. 13, pp. 1484–1494, 2012.

[32] S. Xu, J. Lam, and X. Mao, “Delay-dependent H_∞ control and filtering for uncertain Markovian jump systems with time-varying delays,” *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 54, no. 9, pp. 2070–2077, 2007.

[33] Y. Guo, C. Xu, and J. Wu, “Stability analysis of neutral stochastic delay differential equations by a generalisation of Banach’s contraction principle,” *International Journal of Control*, vol. 90, no. 8, pp. 1555–1560, 2016.

[34] Z. Wang, “A numerical method for delayed fractional-order differential equations,” *Journal of Applied Mathematics*, vol. 2013, Article ID 256071, 7 pages, 2013.

[35] Z. Wang, X. Huang, and J. Zhou, “A numerical method for delayed fractional-order differential equations: based on G-L definition,” *Applied Mathematics & Information Sciences*, vol. 7, no. 2L, pp. 525–529, 2013.

[36] Z. Wang, X. Huang, and G. Shi, “Analysis of nonlinear dynamics and chaos in a fractional order financial system with time delay,” *Computers & Mathematics with Applications*, vol. 62, no. 3, pp. 1531–1539, 2011.

[37] T. H. Lee, J. H. Park, and S. Xu, “Relaxed conditions for stability of time-varying delay systems,” *Automatica*, vol. 75, pp. 11–15, 2017.

[38] K. Gu and S.-I. Niculescu, “Survey on recent results in the stability and control of time-delay systems,” *Journal of Dynamic Systems, Measurement, and Control*, vol. 125, no. 2, pp. 158–165, 2003.

[39] P. G. Park and J. Wan Ko, “Stability and robust stability for systems with a time-varying delay,” *Automatica*, vol. 43, no. 10, pp. 1855–1858, 2007.

[40] P. Park, W. I. Lee, and S. Y. Lee, “Auxiliary function-based integral inequalities for quadratic functions and their applications to time-delay systems,” *Journal of the Franklin Institute*, vol. 352, no. 4, pp. 1378–1396, 2015.

[41] L. Chao, “Hybrid delayed synchronizations of complex chaotic systems in modulus-phase spaces and its application,” *Journal of Computational and Nonlinear Dynamics*, vol. 11, no. 4, article 041010, 2016.

[42] H. R. Karimi and Huijun Gao, “New delay-dependent exponential H_∞ synchronization for uncertain neural networks with mixed time delays,” *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, vol. 40, no. 1, pp. 173–185, 2010.

[43] T. H. Lee and J. H. Park, “A novel Lyapunov functional for stability of time-varying delay systems via matrix-refined-function,” *Automatica*, vol. 80, pp. 239–242, 2017.

[44] J.-H. Kim, “Further improvement of Jensen inequality and application to stability of time-delayed systems,” *Automatica*, vol. 64, pp. 121–125, 2016.

[45] H. D. Tuan, P. Apkarian, and T. Q. Nguyen, “Robust and reduced-order filtering: new LMI-based characterizations and methods,” *IEEE Transactions on Signal Processing*, vol. 49, no. 12, pp. 2975–2984, 2001.