Reflexive, Symmetric and Transitive Scientific Representations

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Abstract

Theories of scientific representation, following Chakravartty's categorization, are divided into two groups. Whereas *cognitive-functional* views emphasize agents' intentions, *informational* theories stress the objective relation between represented and representing. In the first part, a modified structuralist theory is introduced that takes into account agents' intentions. The second part is devoted to dismissing a criticism against the structural account of representation on which similarity as the backbone of representation raises serious problems, since it has definite logical features, i.e. reflexivity, symmetry and transitivity, which representation lacks. Drawing on the representational relation between quantum and statistical field theories, I argue that scientific representation displays these logical features, although depending on the context they may be used or not.

1. Scientific Representation and Its Theories

It seems that all philosophers of science, whether realist or anti-realist, agree that successful scientific theories and models represent the world.[1] By looking at all kinds of representations, three elements can be recognized easily: 1.What represents, called the source of representation or briefly the source, 2.what is represented or the target and, 3.the relation of representation established between the source and the target. For instance, Watson-Crick model of DNA, as a source, represents the molecular structure in cells as

[1]It is obvious that anti-realists restrict representation to the observable aspects of the world.

the target or billiard model of ideal gases represents the behaviour of atoms in a gas. These models are known as *iconic* models. Besides these models, there are more abstract ones that constitute or characterize scientific theories.[2] Take, for example, the theory of classical particle mechanics. This theory *is* (*characterized* by) set-theoretic structures or models satisfying a given set-theoretic predicate (McKinsey, Sugar, and Suppes 1953). Let's call this group "theoretical" models.

But *in virtue of* what, these models are representational? To answer this question, theories of scientific representation attempt to find the necessary and sufficient conditions for representation, and complete the sentence:

"Source S represents target T iff . . . "

Chakravartty (2010) has divided these theories into two groups: cognitive-functional and informational. Cognitive-functional view holds scientific representation as a cognitive tool helping an agent to infer, interpret, understand, etc. some properties of a target. For instance, an architect may use the properties of a bridge made of pasta *to infer* a real bridge's features. In this view, there is no place for objective connection between a source and its target; the relation in which only agents' intentions get involved. One may express the cognitive-functional view as follows:

COGNITIVE-FUNCTIONAL DEFINITION OF SCIENTIFIC REPRESENTATION: S represents T iff a cognitive agent C, with regard to some properties of S, intends to infer, interpret, understand etc. some properties of T, and achieves them.

But on the other side, informational theorists envisage scientific representation in a completely different way. They accept that agents use representation towards their cognitive aims, but those aims and intentions they argue, have no constitutional role in representation. Besides, scientific representation is a kind of similarity which is realized by structural relations e.g. isomorphism (Van Fraassen 1980, Ch.3), homomorphism or partial isomorphism.[3] Consequently, a structuralist may fill the above bi-conditional as follows:

[2]In this article, we follow the non-linguistic, semantic or model-theoretic approach to scientific theories. Scientific theories, according to this view, are constituted (Van Fraassen 1989, p.222) or characterized (da Costa and French 2003, p.34) by models. This view is in contrast with the syntactic approach which adopts them as the collection of propositions.

[3]Although several philosophers have considered isomorphism too strong to be a relation inducing representation, and instead dealt with weaker structural relations such as homomorphism (Bartels 2006) and partial isomorphism (da Costa and French 2003, Ch.3), for simplicity and the sake of argument we take it as a paradigmatic structural relation on which representation takes place.

STRUCTURALIST DEFINITION OF SCIENTIFIC REPRESENTATION: S represents T iff $\mathfrak{S}$ and $\mathfrak{T}$ are isomorphic.

We have $S = \mathfrak{S} = \langle \mathcal{S}, K \rangle$ and $T = \mathfrak{T} = \langle \mathcal{T}, L \rangle$ provided that S and T are structures themselves, otherwise $\mathfrak{S}$ and $\mathfrak{T}$ are structures exemplified or instantiated by S and T respectively. K and L are n-tuple relations defined on sets $\mathcal{S}$ and $\mathcal{T}$ respectively. Now, structures $\mathfrak{S}$ and $\mathfrak{T}$ are isomorphic if there is a bijective map $f : \mathcal{S} \longrightarrow \mathcal{T}$ such that for $(s_1, \ldots, s_n) \in \mathcal{S}^n$ and $(f(s_1), \ldots, f(s_n)) \in \mathcal{T}^n$, $K(s_1, \ldots, s_n) \longleftrightarrow L(f(s_1), \ldots, f(s_n))$.
As we see in the structuralist definition, intentions apparently do not have any job to do, that is in contrast to the cognitive-functional definition. There are several arguments attempting to show that, even in the structuralist definition, intentions get involved (Frigg 2006; Hendry and Psillos 2007; Muller 2011). The implicit role of language in the structuralist representations and its relationship with intentionality are two assumptions which these arguments are based on. In the following, a modified structuralist theory is introduced which takes into account intentions although it does not deal with the role of language.
The point is that structuralists are likely to accept that representation, even objectively established, is a relation which scientists use to achieve cognitive goals. If so, the clause "there *just* is a bijective map" is not enough to achieve these goals. We should discover this map to use it. On the other hand, this map may be not unique. For instance, consider two maps f and g through which T is represented by S. Suppose further that with discovering f definite properties of T can be inferred, interpreted etc. and with discovering g other ones.[4] It is intuitive to consider these representations as different ones. To explain why distinct maps lead to different representations, an analogy with the Tarskian form of linguistic representation may be helpful. Consider a world with two brothers named Joe and Jim as objects and brotherhood as a relation. In this world, *Joe is Jim's brother* and *Jim is Joe's brother*. Does

[4]It should be argued that this is possible. In the case of mathematical structures, there are some cases in which different properties may be inferred via distinct isomorphisms between two structures. For instance, consider the isomorphism between vector structure V and its dual V^*. One possible isomorphism is $f : V \longrightarrow V^*$ such that $\forall v_i \in A$ and $\forall v_j^* \in B, f(v_i) = v_j^*$ where $(v_i.v_j^*) = \delta_{ij}$. A and B are bases of V and V^* respectively. $(\, . \,)$ indicates inner product and δ_{ij} Kronecker delta function. If we consider V^* as a representation of V, it can be inferred that *the basis of* V *is* A. However, there are other options for defining isomorphism. For example, suppose map $g : V \longrightarrow V^*$ such that $\forall u_i \in A$ and $\forall u_j^* \in B, g(u_i) = u_j^*$ such that $(u_i.u_j^*) = \delta_{ij}$. This time, C and D are bases of V and V^* respectively and it can be inferred that *the basis of* V *is* C. Mathematicians in these situations say isomorphism is not natural (Mac Lane and Brikhoff 1999, p.209).

sentence "A is B's brother" represent this world linguistically? Yes, but in two different ways. If A and B interpreted as Joe and Jim respectively, the sentence represents *Joe is Jim's brother*, but if A and B interpreted as Jim and Joe respectively, it represents this time *Jim is Joe's brother*. Therefore, choosing different interpretations leads to distinct linguistic representations. In the same manner, selecting out different maps leads to distinct scientific representations. So there is a striking similarity between the way of looking at scientific representation and the Tarskian form of linguistic representation. As in the Tarskian truth different interpretations change the identity of truth, in the structuralist form of scientific representation different maps alter the identity of representation. If so, not only the source and the target but the bijective map constitutes representation. Having equipped with the Tarskian truth as a triplet relation:

TARSKIAN TRUTH: Open sentence $F(x_1, \ldots, x_n)$ is true in model $\mathfrak{M}$ regarding *interpretation* $I : x_1 \to a_1, \ldots, x_n \to a_n$, $T(F, I, \mathfrak{M})$, iff there is a *satisfaction* of $F(a_1, \ldots, a_n)$ in $\mathfrak{M}$, where $a_1, \ldots, a_n$ are objects of $\mathfrak{M}$.

we will have the following modified structuralist account of representation:

MODIFIED STRUCTURALIST THEORY OF SCIENTIFIC REPRESENTATION: S represents T regarding bijection map of *fixing* $f : \mathcal{S} \longrightarrow \mathcal{T}$, iff there is *fitness* $K(s_1, \ldots, s_n) \leftrightarrow L(f(s_1), \ldots, f(s_n))$, where $\mathcal{S}, \mathcal{T}, (s_1, \ldots, s_n), K$ and L are defined as before.

Fixing and fitness in scientific representation resemble interpretation and satisfaction in linguistic representation respectively. Moreover, as truth in Tarskian sense is (or may be defined as) a 3-tuple relation between sentence, model and interpretation, our formulation of scientific representation is a 3-tuple relation between source, target and fixing map. Therefore, insofar as the fixing is indefinite it is meaningless to ask whether S represents T or not. It is easy to see how intentions get involved in the fixing process. Depending on the target's properties we intend to be inferred, interpreted or understood, the appropriate fixing function is discovered. Remember the linguistic representation discussed above. In a similar vein, depending on our intentions for inferring either *Joe is Jim's brother* or *Jim is Joe's brother*, different interpretations are chosen.

Although Bartels (2006), in the structuralist camp, has acknowledged the role of intentions, the place of intentions in my account differs from what he has commented on. According to Bartels:

> For example, one can use a road map to correctly represent ones way home, if one intentionally takes a certain red curve on the map to stand for the highway which one has to pass etc. Since the road map is endowed with the relevant structure, it entails a *potential* representation of his or her way home that can be exploited by means of an intentional representational mechanism. Thus, we shall claim that A being homomorphic to B is sufficient for A to be potentially represented by B, i.e. that the extensions of the relations "to be homomorphic to" and "to represent potentially" coincide. In order for B to be also a correct *actual* representation of A, A has to be selected as the target of the representation from the set of objects potentially represented by B (i.e., from the content of B) by some representational mechanism connecting A with B.[5] (Bartels 2006, pp.11-12)

He adds that intentionality is one of the representational mechanisms through which the transformation from potential to actual representations occurs. But in the modified definition, intentions get involved even in the potential representation, since without considering the map associated with homomorphism, which is tantamount to ignoring intentions (like the case of isomorphism), homomorphism cannot take place. In other words, the map associated with homomorphism seems to be an essential part of scientific representation in the account of Bartels as well.
Before exploring the next section, a further point is worthy of note. With respect to the logical properties, i.e. reflexivity, symmetry and transitivity, the modified definition is on a par with the first structuralist definition. Since S represents S with the identity fixing map (reflexivity), if S represents T with fixing map f, then T represents S with f^{-1} (symmetry), and finally if S represents T with f, and T represents W with g, then S represents W with fixing map $g \circ f$ (transitivity). These are the logical "odd" features which are involved in the structuralist theories except homomorphism, and structuralism has been criticized severely in virtue of having them. In the next section, we will trace back these allegedly odd features to Goodman's work on the philosophy of art.

2. Science in the Shadow of Art

Although the notion of representation in the literature of philosophy of science is rather new, other areas of philosophy such as philosophy of mind,

[5]The first emphasis in the quote is mine.

language and art have been acquainted with thinking about it long ago. For instance, Goodman criticizes the notion of artistic representation as resemblance in his seminal work on the philosophy of art named *Languages of Art*. One of the main reasons he outlines to reject artistic representation in this sense is three already mentioned logical features which resemblance possesses and consequently representation should have, but artworks lack. Concerning reflexivity and symmetry, he writes:

> Some of the faults [of representation as resemblance] are obvious enough. An object resembles itself to the maximum degree but rarely represents itself; resemblance, unlike representation, is reflexive. Again, unlike representation, resemblance is symmetric: B is as much like A as A is like B, but while a painting may represent the Duke of Wellington, the Duke doesn't represent the painting. Furthermore, in many cases neither one of a pair of very like objects represents the other: none of the automobiles off an assembly line is a picture of any of the rest; and a man is not normally a representation of another man, even his twin brother. Plainly, resemblance in any degree is no sufficient condition for representation (Goodman 1976, p.6)

For rejecting transitivity as a feature of artistic representation, he uses an example. Goodman asks us to imagine two paintings. The first is a realistic picture painted in ordinary perspective and having normal color. The second picture is just like the first except that the perspective is reversed and each color replaced by its complementary. The second picture is in structural similarity with the first and the first with reality. Therefore, regarding the transitivity of similarity, the second should be similar with reality and represent it pictorially. But our intuition suggests that the second does not represent reality. As a result, Goodman argues that representation cannot be reduced to resemblance or similarity.
Following Goodman's argument against artistic representation as resemblance, philosophers of science have put forward their arguments against scientific representation as structural similarity. For instance, Frigg argues:

> The first and simple reason why representation cannot be explained in terms of isomorphism is that the latter has the wrong formal properties: isomorphism is symmetric and reflexive while representation is not.(Frigg 2006, p.54)

Or Suarez claims that:

> Representation in general is an essentially non-symmetric phenomenon: a source is not represented by a target merely in virtue of the fact that the source represents the target ... Merely because an equation represents a phenomenon, the phenomenon cannot be said to stand for the equation. Representation is also non-transitive and non-reflexive.(Suarez, 2003)

Suarez then argues that scientific representation is not identified with isomorphism. For it possesses logical properties, i.e. reflexivity, symmetry and transitivity, which scientific representation lacks. His reason for considering representation without these features may be stated as follows. First of all, he concludes that, following Goodman's arguments, a true theory of representation in general does not imply the logical properties for representation. Secondly, he considers scientific representation as a particular type of representation *in general*. Therefore, a true theory of scientific representation does not imply these logical properties, and consequently, scientific representation is not identified with isomorphism. However, there are some points about this argument.

Firstly, why does Suarez presuppose a general type of representation which scientific and artistic representations belong to? The possibility of such general type needs a further argument which he does not try to put forward.

Secondly, even if we suppose, for the sake of argument, the possibility of such general type of representation, we are not justified in projecting the properties of a particular type, either scientific or artistic, to a general type. That particular type of representation may have its own properties which representation in general does not possess. However, Suarez, following Goodman, gives some examples from pictorial representation, and after investigating their properties (in particular this property that a true theory of representation does not imply reflexivity, symmetry and transitivity), projects them to representation in general. However, it seems that he is not justified in doing that.

Thirdly, it seems that he misinterprets the material implication in the definition of the logical properties. It is true that according to the structuralist definitions, if S represents T then T represents S, but "then" has no any sense tied in with *in virtue of* and *because of*.

This is what Suarez puts forward against the structuralist definitions regarding the logical properties. However, I would like to take a further step and strengthen what he wants to show. Besides the argument criticized above, he apparently has some intuitions supporting this idea that a true theory of scientific representation should not imply the logical features. On the one hand, it seems that intuitions suggested by artistic representations which are non-reflexive, non-symmetric and non-transitive strengthen his idea. However, this kind of intuitions has nothing to do with scientific representation as long as the relation between scientific and artistic representations is uncovered.

On the other hand, some examples from scientific representation also may support his idea. For instance, Suarez holds representing a phenomenon by an equation should not imply representing the equation by the phenomenon. He probably keeps in his mind that scientists usually use scientific representation directionally; from scientific models, equations, diagrams, theories etc. to the world. For example, scientists use equations to discover some facts about phenomena, not vice versa.

But the problem here is that considering such scientific examples supports, at most, the claim that "there are some genuine examples of scientific representation wherein S represents T, but T does not represent S". But what Suarez needs is the idea that "according to a true theory of scientific representation it is not true that if S represents T, then T represents S". However, our scientific examples do not support the latter.

Suarez himself accepts that there are some contexts in which the logical properties, e.g. symmetry, obtain. If we take the former claim seriously, we should accept that these examples and related contexts are exceptional and to some extent bogus. However, in the next sections, I will try to show that the representational relation between quantum field theory (QFT)[6] and statistical field theory (SFT), which can be regarded importantly from a scientific point of view, realizes the logical features.

Before discussing the relation between SFT and QFT, scrutinizing the role of context is worthy of note. Until now, we have found that the logical properties are realized in some contexts of scientific representation, and they are not in others. The question is that how context influences on the realization of these properties. It seems to me that the role of context is tied in with the function of representation not representation *simpliciter*. In other words, whereas the function of representation possesses the logical properties in some contexts, it does not in others.

For example, consider a scientist having an exact equation with the full order of approximation. She uses this equation, as a source of representation, to infer some facts about the behaviour of an object in the world. In this case, the function of representation, i.e. inference from the equation to the world,

[6] In this article, QFT means the so-called conventional quantum field theory which physicists use in practice. Therefore, algebraic or axiomatic approaches to this theory are excluded. There is a real battle between philosophers of science over which approaches is appropriate for philosophical scrutinizing. To be familiar with the arguments of both sides see, for example, Wallace (2006),(2011) and Fraser (2009),(2011).

is non-symmetric. Now consider again a scientist having an equation complete only to the third-order approximation. She uses this equation, up to the third-order, to infer something about the behaviour of an object. However, she also may turn to the world to find the behaviour of the equation in higher-orders. So the function in this case is symmetric.

The difference between the first and the second contexts lies in the difference between the functions of representation. Whilst in the first case inferring some properties of the world is important, in the second case inferring the behaviour of the equation is as important as inferring some properties of the world. According to a structuralist, the representational relations in both cases are symmetric, though its function is non-symmetric in the first and symmetric in the second.

Scientists usually make use of representation towards their cognitive goals, and representation, because of that, goes hand in hand mainly with its functions. Associating representation with its functions may lead us to mistaking the properties of function for representation, and projecting wrongly the properties of the former to the latter. For instance, although representation *simpliciter* may be a symmetric and a non-directional relation, it is conceivable that its function is non-symmetric and directional. If so, we may subsequently think of representation as a non-symmetric and directional relation that is not true.

But, if functions usually accompany a representation, how can we discriminate the properties of representation *simpliciter*? It seems that one way is considering different scenarios or contexts in which functions' properties depart from the properties of representation. We suspect there are contexts in which we can easily do it. For instance, outlining the representational relation between QFT and STF shows that non-reflexivity, non-symmetry and non-transitivity are not essential properties of scientific representation and possibly just pertain to its functions.

It seems to me that outlining such scenarios weaken the idea that scientific representation does not have the logical properties. Nevertheless, as a possible response, a proponent of cognitive-functional view might say that it is possible to define concepts, including scientific representation, in terms of their functions. If so, functions of representation constitute representation and outlining such scenarios is deemed to fail, since defining functionally leads to the identity of representation's with function's properties. Therefore, the departures appealed by a structuralist would not happen at all. The point is that, however, taking functions of representation as its constituent is what she should show and to do that, the identity cannot be presupposed. In other words, she is not justified in objecting the structuralist definitions by invalidating such scenarios. She needs a further argument in which func-

tional definition of scientific representation is not presupposed.
A theory of representation should be such that those logical features can be realized. In the next sections, we will see that QFT and SFT represent each other in accordance with the structuralist definitions and represent each other *structurally.* Moreover, there is a representational relation between them in scientific sense, i.e. by having one theory we can interpret, infer, understand etc. *some* properties of the other one. We will show that this notion of representation between QFT and SFT, which we call representation in *scientific* sense, is reflexive, symmetric and transitive. If this evidence is conclusive and the logical features are not so odd, the structuralist definitions cannot be criticized due to having them, since there is a genuine example of representation in scientific sense in which these logical features are realized.

3. QFT and SFT Represent each other Structurally

We now discuss the relation between QFT and SFT. When philosophers of science talk about scientific representation, they often bear in mind that the world and its properties can be represented by scientific theories and models. In other words, this relation is essentially established between scientific theories and models on the one side and the world on the other side. Scientific representation may be established, however, between theories themselves. There are several cases in the history of science wherein scientists employ a theory or model to understand, interpret, infer etc. some properties of another theory or model. This has led to a more precise representation and modeling of the world.
Let us focus on the relation between QFT and SFT in which source and target are theories themselves. Physicists use one of them to represent another one. Should we expand the domain of targets to scientific theories and models, some new features of representation would be revealed, e.g. the departure of representation's from functions' properties.
Before examining the relation between QFT and SFT, some words are in order to give a brief account of each theory. SFT is a phenomenological theory which regardless of microscopic properties of systems, either quantum mechanical or classical, aims to describe their statistical and thermodynamic behaviours. As the title of statistical field theory suggests, two elements play the key roles in this theory: 1.Field-theoretic physical magnitudes, i.e. the physical magnitudes with infinite degrees of freedom which their values do

not depend on the microscopic structure of the system, 2.statistics which may be classical or quantum mechanical. This theory deals with several questions, e.g. what phase matter is in, in what temperatures phase transitions occur, how correlation length changes in different temperatures and other questions concerning the thermodynamic and statistical features of physical systems.
QFT deals with various topics from elementary particles decay to describing superconductivity. This theory has at least two ingredients: 1.Field-theoretic physical magnitudes[7] and 2.field quantizations.[8] In this article, we will only be concerned with the main aspects of these theories and their details will not be scrutinized.
There are two ways to see the representational relation between QFT and SFT. The first one is showing that they are structurally similar and then represent each other structurally. However, one may object that structural representation is not concerned with the scientific or pre-philosophical sense of representation which is connected to interpretation, understanding, inferring, etc. Elucidating these latter relations between QFT and SFT without discussing about the structuralist definitions is the second way, which will of course not satisfy a structuralist. To overcome these two problems, we will show that QFT and SFT represent each other in two ways, i.e. they represent each other both *structurally* and *scientifically*.
To show the structural similarity between QFT and SFT, the first step is to express statistical properties of a physical system field-theoretically. Articulating QFT in path integral formulation is the second one.[9] Consider a lattice system possessing physical magnitude s with value s_i in the i^{th} site of lattice. s may either be scalar or vector. For simplicity we take it to be

[7]This is in contrast with the particle theories which attribute physical magnitudes with finite degrees of freedom to particles. However, it is challenging that whether QFT should be considered as a *foundationally* particle or field theory. Our discussions and results here are, however, independent of how this question answered.

[8]This question that what is quantum field theory is challenging itself. For example Fraser (2009) has argued that quantum field theory is *by definition* a theory that best unifies quantum theory and the special theory of relativity. But this definition excludes the quantum field theory used in condensed matter physics, since being field-theoretic is more important than obeying special relativity in this physics.

[9]For details of SFT see (Mussardo 2010; Yeomans 1992), regarding path integral formulation of QFT see (Peskin and Schroeder 1995, Chap.9-13; Greiner and Reinhardt 1996, Chap.11-12). Regarding the relation between SFT and QFT see (Bellac 1991; Zinn-Justin 2002). Second quantization is another but rather old formulation of QFT which is not pursued in this article. Path integral formulation have some advantages over second quantization, e.g. the former is independent of employing perturbative or non-perturbetive methods, but the latter is dependent on perturbetive ones, or the renormalization group methods are applicable in QFT provided that the theory is best formulated via path integrals.

scalar. Changing s in each site leads to different system configurations with definite probabilities given by the Gibbs distribution. For configuration $\{s_i\}$ we have the probability:

$$P_{\{s_i\}} = \frac{1}{Z} e^{-\beta E_{\{s_i\}}} \tag{1}$$

where $E_{\{s_i\}}$ is the energy of configuration $\{s_i\}$. β is defined as $\frac{1}{K_B T}$ which K_B and T are Boltzmann's constant and temperature respectively. The normalizing factor or *partition function* Z is:

$$Z = \sum_{i=1} e^{-\beta E_{\{s_i\}}} \tag{2}$$

If we consider s as a field-theoretic physical magnitude, i.e. a variable that is defined on continuum x rather discrete parameter i, we deal with field ϕ instead of s and write the Gibbs distribution as:

$$P_{\{\phi\}} = \frac{1}{Z} e^{-\beta S_{\{\phi\}}} \tag{3}$$

S is the action of the theory, given by an integral on a Lagrangian density. This time field-theoretic partition function is:

$$Z = \int \mathcal{D}\phi \, e^{-\beta S_{\{\phi\}}} \tag{4}$$

which $\mathcal{D}\phi$ can be defined:

$$\mathcal{D}\phi = \prod_{0 \leq |k| \leq \frac{1}{a}} d\phi(k) \tag{5}$$

a is the lattice spacing and k field wavevector. Requirement $0 \leq |k| \leq \frac{1}{a}$ is imposed to ensure integral (4) to be well defined. Lattice spacing a acts as an ultraviolet cut-off that permits us to control ultraviolet divergences. In other words, it tells us that field ϕ is well-defined only around lengths greater than a. There is no guarantee when $a \to 0$ the field remains to be well-defined. Physicists use cut-off to control ultraviolet divergences not only in SFT but in QFT as well. This interpretation of cut-off quantity in SFT, that every field is well-defined provided that the momentum or wavenumber is smaller than a given cut-off, can be applied in QFT and makes sense the appearance of divergences in this theory. By considering cut-off in this way, we interpret QFT via SFT or, philosophically, QFT *is represented* by SFT. We will return back to this issue in the next section.[10]

[10] Of course, cutoff is a method for removing unphysical divergences in QFT, *it is* "regularization" method. There are other regularization methods which essentially do the same job. Perhaps, cutoff is the one with more physical interpretation.

By changing temperature, field value in each point of the continuum changes and accordingly thermal averaging or *correlation function* as observables of SFT will be:

$$< \phi(x_1)\,\phi(x_2)\ldots\phi(x_n) >= \frac{1}{Z}\int \mathcal{D}\phi\,\phi(x_1)\,\phi(x_2)\ldots\phi(x_n)e^{-\beta S_{\{\phi\}}} \tag{6}$$

It can be shown that if we couple ϕ with external field J, the correlation function is:

$$< \phi(x_1)\,\phi(x_2)\ldots\phi(x_n) >= \frac{1}{Z[0]}\,\frac{\delta^n Z[J]}{\delta J(x_1)J(x_2)\ldots J(x_n)}\,|_{J=0} \tag{7}$$

Where $Z[J]$ is the partition function with external field J and $Z[0]$ with $J = 0$. This is the first step in SFT. We should now present QFT such that its structural similarity with SFT is made apparent.

Although quantum and statistical fields are completely different physical objects[11], similar relations with (3)-(6) might be explored about quantum fields. Consider quantum field $\psi(x)$. The probability amplitude for transition from $\psi(x)$ to $\psi(x')$ configuration is given by *generating functional*:

$$W = \int \mathcal{D}\psi\, e^{iS_{\{\psi\}}} \tag{8}$$

S, like the action of statistical field, is given by the Lagrangian density of quantum field. Not only from definitional point of view Z and W are similar, but also they play similar roles in constructing observables of their own theories. For instance, the correlation function in QFT as an observable is given by:

$$< \psi(x_1)\,\psi(x_2)\ldots\psi(x_n) >= \frac{1}{W}\int \mathcal{D}\psi\,\psi(x_1)\,\psi(x_2)\ldots\psi(x_n)e^{iS_{\{\psi\}}} \tag{9}$$

If the field coupled with external field J, it can be shown that:

$$< \psi(x_1)\,\psi(x_2)\ldots\psi(x_n) >= \frac{1}{W[0]}\,(-i)^n\frac{\delta^n W[J]}{\delta J(x_1)J(x_2)\ldots J(x_n)}\,|_{J=0} \tag{10}$$

To show that relations (11) with (5) and (12) with (6) are *structurally* similar, the final step should be taken that is establishing a fixing map relating them to each other. In fact, the Wick rotation is the fixing map we are seeking for. If the statistical field is defined on a 4-dimensional Euclidean space

[11]For instance, quantum, fields fluctuate due to quantum fluctuations and statistical fields as a result of thermal ones.

$E = \{(x_{E_1}, x_{E_2}, x_{E_3}, x_{E_4})\}$ and quantum field on a 4-dimensional Minkowski space $M = \{(x_0, x_1, x_2, x_3)\}$, the Wick rotation as a fixing map facilitates the transition from SFT to QFT:

$$\begin{gathered} f : E \longrightarrow M \\ f(x_E) = f(x_{E_1}, x_{E_2}, x_{E_3}, x_{E_4}) = (-ix_{E_4}, x_{E_1}, x_{E_2}, x_{E_2}) = (x_0, x_1, x_2, x_3) \end{gathered} \tag{11}$$

and from QFT to SFF transition:

$$\begin{gathered} f^{-1} : M \longrightarrow E \\ f^{-1}(x) = f^{-1}(x_0, x_1, x_2, x_3) = (x_1, x_2, x_3, ix_0) = (x_{E_1}, x_{E_2}, x_{E_3}, x_{E_4}) \end{gathered} \tag{12}$$

These fixing maps provide the translations from one theory to another or structural similarity between them.[12] Relations (4), (6) and (7) in SFT play the same roles which (10), (11) and (12) do in QFT respectively.

Concerning the structural similarity between QFT and SFT, there are some important remarks to be made. First of all, this similarity is *partial*, i.e. it does not hold for all domains of the theories. For instance, although thermal fluctuations in SFT can be switched off in principle, quantum fluctuations in QFT, even in principle, cannot. In other words, isomorphism between QFT and SFT is just partial.[13] This article does not deal with this partiality and concerns only with the validity of structural representation, regardless of its totality or partiality.

Secondly, the Wick rotation or the fixing map is not established, at least *prima facie*, for showing the structural similarity between SFT and QFT. Physicists use the Wick rotation to be sure about the convergence of correlations functions and subsequently to compute them.[14] Along with, the structural similarity is discovered. Therefore, as the case in which target is the world and fixing map has a heuristic nature that should be discovered, in our case wherein the target is a theory, it should be discovered as well.

Thirdly, establishing and uncovering the structural similarity provided by the Wick rotation is not just a mathematical game and has a physical significance:

[12] In fact, there is a further structural similarity, maybe more importantly, between these two theories based on the space or manifold constructed by the coupling constants rather Euclidean or Minkowski spaces. The dynamics of this new space is represented by the renormalization group flow which is indeed a dynamics through energy scales. The author is preparing an independent paper on the structural similarity between SFT and QFT due to the renormalization group methods.

[13] For the definition of partial isomorphism and a structuralist theory of scientific representation in terms of partial isomorphism see (Bueno and French 2011).

[14] For its details see (Greiner and Reinhardt 1996, pp.371-375).

> In essence, it [the structural similarity between QFT and SFT] adds to our reserves of knowledge a completely new source of intuition about how field theory expectation values should behave. This intuition will be useful in imaging the general properties of loop diagrams and it will give important insights will help us correctly understand the role of ultraviolet divergences in [quantum] field theory calculations. (Peskin and Schroeder 1995, p.296)

As we see, there is a representational relation between QFT and SFT according to the structuralist definitions. What we should do in the following is showing that they represent each other according to what a proponent of cognitive-functional view keeps in mind. The next section is devoted to this issue.

4. QFT and SFT Represent each other Scientifically

In this section, we will employ superconductivity to show that QFT may be used for interpreting a statistical field theory, i.e. the Ginzburg-Landau field theory of superconductivity. Moreover, we will mention using SFT to interpret renormalization method in QFT, although its details will not be spelled out here.[15] If this evidence is conclusive, it can be concluded that the representational relation between SFT and QFT is not restricted to the structural sense and covers the meaning which scientists bear in mind.
Meissner-Ochsenfeld effect is a phenomenon in which second-order phase transition from normal conductivity to superconductivity occurs.[16] There are several theories, from classical London theory (1935) to quantum field-theoretic account of Bardeen, Cooper and Schrieffer (BCS) (1957), aim to explain this phenomenon. Among these theories, the Ginzburg-Landau model (1950) has a middle situation, since it is a phenomenological theory which can be interpreted microscopically.[17] To see how this model works, consider

[15] David Wallace (2011) has recently discussed about it.

[16] To become familiar with this effect briefly, consider a sample of material which is held at temperature $T > T_C$ and placed in a small magnetic field. In this case, the magnetic field will easily penetrate into the sample. But if we cool the sample to below T_C, the magnetic field is expelled. Different materials have their own critical temperature T_C at which the phase transition occurs. For details see (Annet 2004).

[17] The application of the Ginzburg-Landau theory is beyond describing superconductivity and used in particle physics such as Higgs mechanism.

its free energy:

$$F_{LG}[\Psi(r)] = \int dr(\frac{\hbar^2}{2m^*} \mid (\frac{\hbar}{i}\nabla + q\overrightarrow{A})\Psi(r) \mid^2 + a \mid \Psi(r) \mid^2 + \frac{b}{2} \mid \Psi(r) \mid^4) \quad (13)$$

where $\overrightarrow{A}$ is the electromagnetic vector potential and, a and b are parameters to be determined. Field Ψ is the order parameter which is continuous at the critical point T_C and determines the nature of the second-order phase transition. The order parameter is zero at $T > T_C$ and non-zero at $T < T_C$. Like other models of SFT, the probability of configuration $\{\Psi\}$ is:

$$P_{\{\Psi\}} = \frac{1}{Z_{LG}} e^{-\beta F_{LG}\{\Psi\}} \quad (14)$$

and its partition function:

$$Z_{LG} = \int \mathcal{D}\Psi \, \mathcal{D}\Psi^* \, e^{-\beta F_{LG}\{\Psi\}} \quad (15)$$

Given the partition function, other physical properties such as internal energy and specific heat of a superconductor near critical temperature are known. For instance, it will be known that the specific heat is divergent at $T = T_C$. What is important in the Ginzburg-Landau model and makes it a phenomenological model in nature is that it is unknown what microscopic properties Ψ, m^* and q refer to. Bardeen, Cooper and Schrieffer developed the first microscopic theory of superconductivity in 1957 and two years later in 1959, Gor'kov (1959) could derive the Ginzburg-Landau model from BCS field theory and more importantly, he could interpret those terms in the Ginzburg-Landau field theory which their interpretations were unknown until that time:

> It is shown that the phenomenological Ginzburg-Landau equations follow from the theory of superconductivity [BCS] in the London temperature region in the neighborhood of Tc. In these equations there occurs, however, twice the electronic charge; this is related to the physical meaning of $\Psi(x)$ as the wave function for Cooper pairs. (Gor'kov 1959, p.1364)

Thanks to the Gor'kov's paper, it is known that $\Psi(x)$ is the wave function of Cooper pairs, q its charge and m^* its effective mass. Concerning this achievement, one proponent of cognitive-functional view is likely to accept that the BCS theory is used to interpret and consequently represent the

Ginzburg-Landau model. Strictly speaking, a quantum field theory represents a statistical field theory.[18]
The contribution of SFT in understanding QFT may be more than what QFT gives us to understand SFT. It is now a established fact that renormalization of fields and parameters provides a consistent set-up to deal with the divergences which appear once one uses standard field theory computational tools; renormalization of parameters is a well tested prediction of QFT and perhaps one of its biggset triumphs.[19]
Renormalization group methods in QFT, which initially developed from the statistical field-theoretic methods in condensed matter physics by Wilson and Kogut in the early 1970s (Wilson and Kogut 1974; Wilson 1975), produced an explanatory framework which has been very fruitful for QFT. Although discussing this topic in detail is beyond the goal of this paper and needs to be tackled in an independent paper, we will mention it briefly.[20]
According to the renormalization group based interpretation of QFT, it is a large-scale theory which breaks down at some short lengthscales. This latter in turn gives rise to the ultraviolet divergences. Before employing the renormalization group methods, physicists got rid of infinities by a tricky mathematical method which had no physical interpretation. Roughly speaking, they imposed cut-offs in the integrals associated with scattering amplitudes to immune physical magnitudes from divergences. As Wallace (2011) has argued, imposing cut-off becomes meaningful due to the renormalization group. According to this method, cut-off expresses the bound of applicability of QFT or the range of spacetime in which QFT is defined. Therefore, as employing SFT in condensed matter physics in the ranges smaller than the lattice spacing gives rise to divergences, using QFT in arbitarily short lengthscales results in the ultraviolet divergences. In the case of SFT, we have theories such as non-relativistic quantum mechanics which works well in the ranges smaller than the lattice spacing. For QFT, however, we have not yet had a well-established theory for some short lenghtscales. In sum, for interpreting QFT, or philosophically for representing QFT, SFT is used as the source of representation.

[18]Although BCS developed their theory in terms of second quantization method of QFT, it might be presented according to the path-integral method as well (Fletcher 1990).

[19]There are still some misconceptions about this point. For instance, Fraser (2006; 2011) has argued against the conventional quantum field theory because the existence of infinities which this theory is confronted with. Her alternative is the algebraic or axiomatic quantum field theory.

[20]Wallace (2011) has argued that employing the renormalization group methods in QFT provides a physical explanation for removal of infinities, though Huggett (2002) introduced these methods earlier.

In sections 3 and 4, we tried to show that SFT and QFT represent each other not only structurally, but also according to the scientific sense which some proponents of cognitive-functional view may favour. Consequently, the representational relation between QFT and SFT is symmetric from both philosophical and scientific points of views.
The structuralist account of scientific representation sets the scene such that the symmetry of scientific representation is realized. The symmetry of representation in scientific area, possibly in contrast to the artistic representation, is not a strange or odd feature. It has, on the contrary, a central importance among the properties of scientific representation, in particular regarding the relation between QFT and SFT.

5. Scientific Representation with Other "Peculiar" Features

Until now, it has been shown that the symmetry may play a key role in scientific representation. In this section, I concern with two other apparently "odd" features of scientific representation, i.e. reflexivity and transitivity.
First, reflexivity. As mentioned earlier, it is usually claimed that a scientific theory or model cannot represent itself. In other words, the source and the target of representation cannot be identical. According to the structuralist definitions, however, there are four elements involved in representation which their identity should be explored: $S, T, \mathfrak{S}$ and $\mathfrak{T}$. Therefore, if we consider the identity between these four elements, the identities may be realized in three ways. In the first case, in which S with T and $\mathfrak{S}$ with $\mathfrak{T}$ are identical, we get *full representation*. Although there is a genuine representation in this way, we don't use it. Since if we have the source and its instantiated structure we know everything about its target, i.e. itself and the associated structure.
In the second case, S and T are identical but $\mathfrak{S}$ and $\mathfrak{T}$ are not. For instance, in the case of representation about superconductors discussed above, the same sample of material instantiates two different structures, i.e. one structure described by SFT and another one by QFT. Therefore, it is conceivable that one takes a sample with the structure associated with the BCS theory, and represents it with the structure described by the Ginzburg-Landau model. Thus, in this case of identity also, there is no problem with the reflexivity.
In the final case, the source and the target are different, but their related structures are identical. There are genuine examples of scientific representation which belong to this category. To take a concrete example, consider

ferromagnetic and anti-ferromagnet materials which both can be modelled by the Ising model.[21] In this model, the lattice may be assumed to be cubic, and spin number associated with each site of the lattice is either 1 or -1. The system's Hamiltonian would be:

$$H = \frac{1}{2} \sum_{ij} \mathcal{J}_{ij} s_i s_j - B \sum_i s_i \tag{16}$$

where B is an externally imposed field. $\mathcal{J}_{ij}$ is constant $\mathcal{J}$ if i and j are neighbouring sites and zero otherwise. The partition function can be written:

$$Z_{Ising} = \sum_{\{s_i\}} e^{[\beta(B \sum_i s_i - \frac{1}{2} \sum_{ij} \mathcal{J}_{ij} s_i s_j)]} \tag{17}$$

where $\{s_i\}$ indicates that the sum is on all configurations obtained by assigning 1 or -1 to the sites. With regard to describing both ferromagnetic and anti-ferromagnetic materials by this model, one can utilize thermodynamic properties of a ferromagnetic to infer the properties of an anti-ferromagnetic, and vice versa. For instance, it can be shown that their internal energies are equal. Consequently, reflexive representation in the third category, as well, is not irrelevant and includes some genuine examples of scientific representation.

The only apparently "odd" logical feature left to discuss is transitivity. Consider the last case of representation between ferromagnetic and anti-ferromagnetic materials. These materials are modelled not only by a common structure described above, but also by their own structures as well. These models are determined by assigning the value of $\mathcal{J}$. The structures associated with a ferromagnetic and an anti-ferromagnetic are the ones with $\mathcal{J} = c < 0$ and $\mathcal{J} = c' > 0$ respectively. We denote them by $\mathfrak{S}$ and $\mathfrak{T}$.

On the one hand, they are isomorphic and then represent each other. On the other hand, structure $\mathfrak{T}$ represents the properties of anti-ferromagnetic materials denoted by W. Up to now, we know $\mathfrak{S}$ represents $\mathfrak{T}$ and $\mathfrak{T}$ represents W. But as stated earlier, we can use the structure associated with the ferromagnetic materials, $\mathfrak{S}$, to represent the behaviour of anti-ferromagnetic materials, i.e. W. Therefore, the conditional if $\mathfrak{S}$ represents $\mathfrak{T}$ and $\mathfrak{T}$ represents W, then $\mathfrak{S}$ represents W is satisfied. So the transitivity also obtains.

[21] For details see (Binney *et al.* 1992).

Conclusion

After reviewing theories of scientific representation briefly, a modified structuralist definition of scientific representation has been introduced that takes into account agents' intentions. In the second part, some examples from SFT and QFT highlighted that reflexivity, symmetry and transitivity have central importance in scientific representation, and because of that, should be taken seriously. My arguments support this claim that scientific representation displays these logical features, although depending on the scientific context we may use them or not. The structuralist theories of scientific representation has this advantage that sets the scene such that these logical properties are possible to be realized.

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References

Annett, J.F. (2004), *Superconductivity, Superfluids and Condensates*, (Oxford University Press).

Bardeen, J., Cooper, L.N. , and Schrieffer, J.R. (1957), Theory of superconductivity, *Physical Review*, 108, 1175 1204.

Bartels, Andreas (2006), Defending the Structural Concept of Representation, *Theoria*, 21, 7-19.

Bellac, M.L. (1991), *Quantum and statistical field theory* (Oxford University Press).

Binney, J., et al. (1992), *The Theory of Critical Phenomena: An*

Introduction to the Renormalization Group (Clarendon Press).

Bueno, Otavio and French, Steven (2011), How Thories Represent, *British Journal of the Philosophy of Science*, 1-38.

Chakravartty, Anjan (2010), Informational versus functional theories of scientific representation, *Synthese*, 172, 197 213.

da Costa, N. C. A. and French, Steven (2003), *Science and Partial Truth: A Unitary Approach to Models and Scientific Reasoning* (Oxford: Oxford University Press).

Fletcher, G. (1990), Functional integrals and the BCS theory of superconductivity, *American Journal of Physics*, 58, 50-53.

Fraser, Doreen (2009), Quantum Field Theory: Underdetermination, Inconsistency, and Idealization, *Philosophy of Science*, 76, 536-67.

Fraser, Doreen (2011), How to take particle physics seriously: A further defence of axiomatic quantum field theory, *Studies In History and Philosophy of Science Part B: Studies In History and Philosophy of Modern Physics*, 42, 126-35.

Frigg, Roman (2006), Scientific representation and the semantic view of theories, *Theoria*, 50, 49 65.

Ginzburg, V.L and Landau, L (1950), On the theory of superconductivity, *Zhurnal Eksperimental'noi i Teoreticheskoi Fiziki*, 20, 1064-82.

Goodman, N. (1976), *Languages of art: An approach to a theory of symbols* (2 edn.; Indianapolis: Hackett).

Gor'kov, L.P. (1959), Microscopic derivation of the Ginzburg-Landau equations in the theory of superconductivity, *Zhurnal Eksperimental'noi i Teoreticheskoi Fiziki*, 36, 1918-23. English translation Soviet Phys. *Journal of Experimental and Theoretical Physics*, 9, 1364-1367 (1959).

Greiner, W. and Reinhardt, J. (1996), *Field Quantization* (Springer).

Hendry, R. F and Psillos, Stathis (2007), How to Do Things with Theories: An Interactive View of Languages and Models in Science, in J

Brzezinski, A Klawiter, and T. A. F Kuipers (eds.), *The Courage of Doing Philosophy: Essays Dedicated to Leszek Nowak* (New York: Rodopi,), 123-57.

Huggett, Nick (2002), Renormalization and the Disunity of Science, in Meinard Kuhlmann, Holger Lyre, and Andrew Wayne (eds.), *Ontological Aspects of Quantum Field Theory* (World Scientific), 255 277.

London, F. and London, H. (1935), The Electromagnetic Equations of the Supraconductor, *Proceedings of the Royal Society A*, 149, 71 88.

Mac Lane, S.M. and Birkhoff, G. (1999), *Algebra* (AMS Chelsea Publishing).

McKinsey, J. C. C., Sugar, A. C., and Suppes, P. (1953), Axiomatic Foundations of Classical Particle Mechanics, *Journal of Rational Mechanics* 2: 253 272.

Muller, F. (2011), Reflections on the revolution at Stanford, *Synthese*, 183 (1), 87-114.

Mussardo, G. (2009), *Statistical field theory: an introduction to exactly solved models in statistical physics* (Oxford University Press).

Peskin, M.E. and Schroeder, D.V. (1995), *An Introduction To Quantum Field Theory* (Addison-Wesley Publishing Company).

Surez, M. (2003), Scientific representation: against similarity and isomorphism, *International Studies in the Philosophy of Science*, 172, 225-44.

Van Fraassen , Bas C. (1989), *Laws and symmetry* (Oxford: Clarendon).

Wallace, David (2006), In Defence of Naivete: The Conceptual Status of Lagrangian Quantum Field Theory, *Synthese*, 151, 33 80.

Wallace, David (2011), Taking particle physics seriously: A critique of the algebraic approach to quantum field theory, *Studies In History and Philosophy of Science Part B: Studies In History and Philosophy of Modern Physics*, 42, 116-25.

Wilson, Kenneth G. (1975), 'The renormalization group: Critical phenomena and the Kondo problem', *Reviews of Modern Physics*, 47 (4), 773-840.

Wilson, Kenneth G. and Kogut, J. (1974), 'The renormalization group and the expansion', *Physics Reports*, 12 (2), 75-199.

Yeomans, J.M. (1992), *Statistical Mechanics of Phase Transitions* (Clarendon Press).

Zinn-Justin, J. (2002), *Quantum Field Theory and Critical Phenomena* (Clarendon Press).