Seeming Incomparability and Rational Choice

Abstract

We sometimes have to choose between options that are seemingly incomparable insofar as they seem to be neither better than, worse than, nor equal to each other. This often happens when the available options are quite different from one another. For instance, consider a choice between prioritizing either criminal justice reform or healthcare reform as a public policy goal. Even after the relevant details of the goals and possible reforms are filled in, it is plausible that neither goal is better than, worse than, nor equal to the other. Such seemingly incomparable options present a problem for rational choice since it is unclear how an agent might rationally choose between them. What we need are some principles to help govern rational choice when faced with seemingly incomparable options. I here present three such principles. While each principle is individually compelling, I show that they are jointly incompatible. I then argue that the correct response to this inconsistent triad is to reject the principle that rationally censures performing a sequence of choices one knows will result in a suboptimal outcome. The upshot is that when seeming incomparability is involved, an agent can money pump themselves without being less rational for it.

Key Words

1. Introduction

It is not difficult to think of cases where we feel an intuitive resistance towards claims of betterness, worseness, or equality between two options. Indeed, many choices in life involve such options. For instance, our choice of public policies to support, professional careers to pursue, romantic partners, vacation destinations, and even casual lunch plans often involve options that seem to be neither better than, worse than, nor equal to each other. In this way, we often have to choose between options that are seemingly incomparable to each other. Given the ubiquity of such cases, it would be helpful to have some principles to help govern rational choice between seemingly incomparable options.

In this paper, I present three such principles. The first principle focuses on the comparative rationality of sequences of choices (i.e., sets of multiple choices over time) rather than the rationality of a single choice. This principle, which I call the Comparative Money Pump Principle, roughly holds that it is less rational to knowingly perform a sequence of choices that will result in a suboptimal outcome than it is to perform a sequence of choices that will not result in a suboptimal outcome. The second and third principles focus on the supervenience base of the rationality of sequences of choices and individual choices, respectively. While each of these principles is individually compelling, I show that they are actually jointly incompatible. I then argue that the correct response to this inconsistent triad is to maintain the two supervenience principles, but reject the Comparative Money Pump Principle. So when seeming incomparability is involved, one can money pump themselves without being less rational for it.

This paper proceeds by first examining the phenomenon of seeming incomparability and two of its key features. After that, I present the three principles and demonstrate their joint

incompatibility. Finally, I consider the consequences of rejecting each of the principles before concluding that the one to reject is the Comparative Money Pump Principle.

2. The Phenomenon of Seeming Incomparability

In order to understand the phenomenon of seeming incomparability, it is helpful to start with a paradigmatic example provided by Ruth Chang (2002, p. 669):

> Suppose you must determine which of a cup of coffee and a cup of tea tastes better to you. The coffee has a full-bodied, sharp, pungent taste, and the tea has a warm, soothing, fragrant taste. It is surely possible that you rationally judge that the cup of Sumatra Gold tastes neither better nor worse than the cup of Pearl Jasmine and that although a slightly more fragrant Jasmine would taste better than the original, the more fragrant Jasmine would not taste better than the cup of coffee. In this case, it is plausible to suppose that you know everything that is relevant to comparing the drinks and that in this case you have first-person authority over which tastes better to you.

Here, there is an attempt to compare two particular items, the cup of Sumatra Gold coffee and the cup of Pearl Jasmine tea, with respect to a particular value, tastiness, that is applicable to both items. Now despite the value of tastiness clearly applying to both the Sumatra Gold and Pearl Jasmine, there can nonetheless be intuitive resistance to claims that either is better than, worse than, or equal to the other with respect to tastiness. That is, you can be reluctant to assent to all of the following claims:[1]

 [1] Pearl Jasmine is better than Sumatra Gold with respect to tastiness.
 [2] Pearl Jasmine is worse than Sumatra Gold with respect to tastiness.
 [3] Pearl Jasmine and Sumatra Gold are equal with respect to tastiness.[2]

This is not yet to say that you would hold these claims to all be false. That is a stronger commitment and one that is not, I take it, as intuitively clear. Rather, the phenomenology of such cases seems to be an intuitive reluctance to assent to claims of betterness, worseness, and equality rather than an outright rejection of those claims. So I take the first key feature of

seeming incomparability to be this intuitive resistance to claims of betterness, worseness, and equality.

The second key feature of seeming incomparability is that this intuitive resistance persists even if one of the items is slightly improved or worsened. So in addition to being reluctant to assent to claims [1] – [3] above, one can also be reluctant to assent to any of the following claims where the Pearl Jasmine has been replaced with a slightly better tea, call it Pearl Jasmine⁺:

[1*] Pearl Jasmine⁺ is better than Sumatra Gold with respect to tastiness.
[2*] Pearl Jasmine⁺ is worse than Sumatra Gold with respect to tastiness.
[3*] Pearl Jasmine⁺ and Sumatra Gold are equal with respect to tastiness.

Keeping these two features of seeming incomparability in mind, we turn now to the question of what to do in choice situations involving seemingly incomparable options.[3]


3. Three Principles

Given the possibility of choice situations involving seemingly incomparable options, it would be nice to have some principles to help govern rational choice in such cases.[4] I will not here defend a complete set of such rational principles, but rather will present three independently compelling principles that we would intuitively want to include in any such complete set.

3.1 Comparative Money Pump Principle

The first principle compares the rationality of performing different sequences of choices (i.e., sets of multiple choices over time). Before presenting the principle itself though, it's helpful to first look at two choice situations involving seemingly incomparable options that help motivate it:

Figure 1. Motivating Choice Situation 1
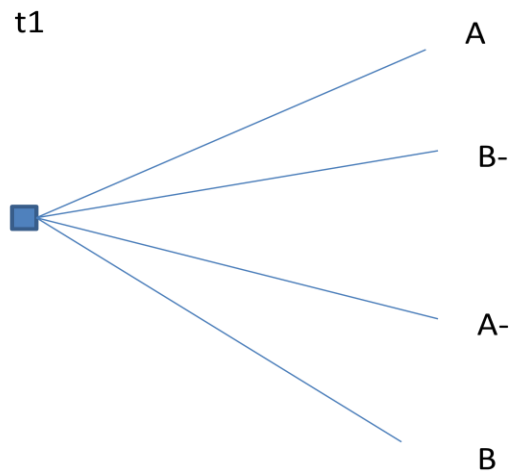
In this first choice situation, an agent is faced with a single choice at t1 between four options: A, B-, A-, and B. Here the A options are seemingly incomparable with the B options (e.g., you might imagine them to be Chang's cups of coffee and tea). So, each individual option is better than or worse than one of the other options, but seemingly incomparable with the other two. For example, A is better than A-, but seemingly incomparable with both B and B-.

Now, whatever one thinks about the rationality of choosing between seemingly incomparable options, it seems clearly less rational to choose A- or B- rather than A or B. After all, in choosing A- or B-, one is choosing an option that they know to be worse than some other available option and in this way would be suffering a sure loss. While A and B are not better than or equal to every other option, they are at least not worse than any other option and so are not suboptimal, unlike A- and B-. More generally, it seems less rational to choose options that are suboptimal than it is to choose options that are not suboptimal, even when seeming incomparability is involved.[5] This is of course not surprising. Take now a more complicated choice situation involving a sequence of two choices:

Figure 2. Motivating Choice Situation 2
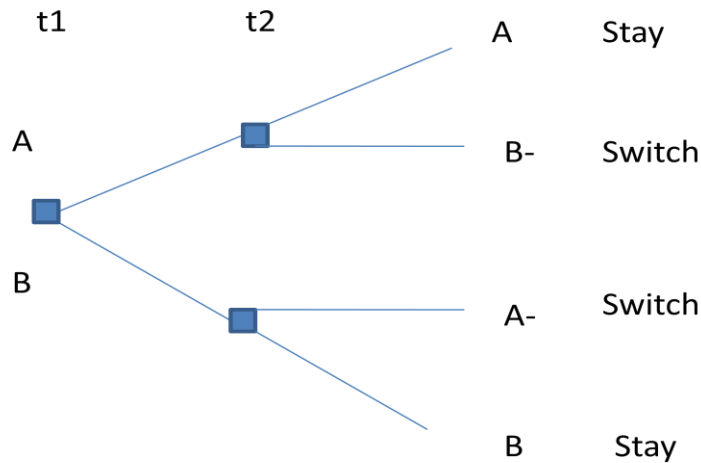
Here, an agent is faced with a choice between seemingly incomparable options A and B at t1. They then face another choice at t2 between keeping their previously chosen option (A or B), or, trading it for an option that is slightly worse than the option they did not choose at t1 (B- or A-). Importantly, the choice at t2 is also between seemingly incomparable options given the second key feature of seeming incomparability previously discussed. (You might image the agent facing an initial choice between Chang's coffee and tea, then immediately thereafter being offered to trade their chosen option for a slightly worse version of their unchosen option.) There are then four different sequences that the agent can perform in this choice situation: A/A, A/B-, B/A-, and B/B. Two of these sequences involve staying with the option chosen at t1 (A/A, B/B), while the other two involve switching from that option to one that is slightly worse than the option they rejected at t1 (A/B-, B/A-).[6]

Now whatever one thinks about the rationality of choosing between seemingly incomparable options, it seems clearly less rational to perform one of the Switch sequences rather than one of the Stay sequences. After all, in performing one of the Switch sequences, one would end up with an outcome that they know is definitely worse than another they could have had if only they had performed one of the Stay sequences. In performing one of the Stay

sequences on the other hand, one would not end up with an outcome that they know to be definitely worse than another they could have had by performing a different sequence. So agents who switch are making a series of choices they know will result in a suboptimal outcome, while agents who stay are not. In this way, agents who switch are in effect money pumping themselves (i.e., making a series of choices that results in a sure loss), while agents who stay are not.[7] It seems then that just as it is less rational to make a single choice whose outcome is suboptimal as compared to a single choice whose outcome is not suboptimal, so too when it comes to sequences of choices.[8] This seeming difference in the rational statuses of sequences of choices is plausibly explained by the following general principle:

Comparative Money Pump Principle
> In choice situations where risk is not involved, it is less rational to perform a sequence of choices whose outcome one knows in advance will be worse compared to that of some other available sequence than it is to perform a sequence of choices whose outcome one does not know will be worse compared to that of any other available sequence.

This principle captures the intuition that it is less rational to perform a Switch sequence than a Stay sequence in Figure 2. However, it is important to note that this principle only applies to choice situations where the outcomes are not subject to risk. That is, it only applies when the agent knows which outcomes will result from her choices with certainty and where those outcomes are not lotteries.[9] It is also important to note that this principle is entirely comparative in nature, making no claim whatsoever about the rationality or irrationality of performing any particular sequence of choices as such. It only claims that certain sequences of choices are *less* rational to perform than others. It does not follow from this that any sequence is fully rational or irrational to perform.

3.2 Sequential Rationality Supervenes on Constituent Rationality

The second principle claims that when evaluating the rationality of some sequence of choices, we need only look at the rationality of the individual choices within that sequence. So if we want to evaluate the rationality of a sequence of choices C consisting of two individual choices [c1, c2], we need only look at the rationality of those individual choices. That is, the rationality of c1 and c2 together determine the rationality of sequence C.

The general idea here is that a sequence of choices is nothing more than a series of particular, individually assessable choices, and that it is the rationality of these constituent choices that determine the rationality of the sequence of choices. So, if two sequences of choices differ in rational status, it must be because of some difference in the rationality of their constituent choices. More specifically, this idea can be understood as the following supervenience principle:

Sequential Rationality Supervenes on Constituent Rationality (Sequence-Constituent Principle)
> If one sequence of choices is more rational than another, then they must differ in the rational status of (some of) their constituent choices in a way that favors the more rational sequence.

This supervenience principle seems quite plausible. To deny it would be to endorse the possibility of two sequences of choices differing in rationality despite being composed of constituent choices that were identical in rational status. But that would be strange. After all, what else would the rationality of a sequence of choices depend on besides the rationality of their constituent choices?[10] As we will see in section 5 though, one might nonetheless hold that the rationality of a sequence of choices can depend on things besides the rationality of their constituent choices. However, I will argue there that this possibility is seriously problematic because it would result in either a picture of rationality that is fundamentally fractured, or, a

rational assessment of sequences of choices that is devoid of normative force, or, a commitment to the rejection of the next supervenience principle.

3.3 Rationality Supervenes on Available Actions Only

The third principle makes a claim about the rationality of choosing particular options in individual choice situations where only value considerations matter. These are choice situations where there are no non-value considerations (e.g., categorical imperatives, deontic side constraints, etc...) at play.[11] That is, the only thing that matters is how the options fare with respect to certain values that are stipulated to be relevant to the choice at hand. In such choice situations, this principle claims that the comparative rationality of choosing a particular option depends only on the comparative relations that obtain between the currently available options with respect to the relevant values. So in a choice situation involving options [o1, o2] and where value V is the only relevant consideration, the comparative rationality of choosing either option depends only on how o1 and o2 compare with respect to V.

The general idea here is that if there is a difference in the rationality of choosing between two available options in a choice situation, there has to be a reason that justifies that difference. In cases where only value considerations are relevant, this reason must appeal to some difference between the available options with respect to the relevant values that favors one option over the other. Options that are perfectly symmetrical with respect to the relevant values will not allow for any reason that could justify a rational difference in choosing one rather than the other. In this way, there must be some comparative asymmetry between the available options with respect to the relevant values that can then allow one option to be rationally favored over the others.[12] More specifically, this idea can be understood as the following supervenience principle:

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Rationality Supervenes on Available Options Only (Rationality-Availability Principle)
> If one option is more rational than another in a choice situation where only value considerations matter, then there must be some comparative asymmetry between the available options with respect to the relevant values that favors the more rational option.

So if there is a difference between the rationality of choosing o1 and o2, there must be some comparative asymmetry between the two with respect to V, favoring the more rational option. This supervenience principle also seems quite plausible. After all, in choice situations where only V matters, how could there be a rational difference between choosing o1 and o2 unless there was some sort of comparative asymmetry between them with respect to V? As we will see in section 5 though, one might object that this supervenience principle is overly restrictive insofar as it only considers currently available options and does not also include previously available options. However, I will argue that this restriction is entirely appropriate as broadening the scope to include previously available options amounts to committing the sunk cost fallacy.

In this section, I have identified three independently plausible principles that might be used to help govern rational choice between seemingly incomparable items. For ease of reference, these principles are restated together below:

Comparative Money Pump Principle
> In choice situations where risk is not involved, it is less rational to perform a sequence of choices whose outcome one knows in advance will be worse compared to that of some other available sequence than it is to perform a sequence of choices whose outcome one does not know will be worse compared to that of any other available sequence.

Sequential Rationality Supervenes on Constituent Rationality (Sequence-Constituent Principle)
> If one sequence of choices is more rational than another, then they must differ in the rational status of (some of) their constituent choices in a way that favors the more rational sequence.

Rationality Supervenes on Available Options Only (Rationality-Availability Principle)
> If one option is more rational than another in a choice situation where only value considerations matter, then there must be some comparative asymmetry between the available options with respect to the relevant values that favors the more rational option.

4. Incompatibility of the Three Principles

I take each of the three principles discussed in the previous section to be intuitively compelling. However, I will show that they are jointly incompatible. To do this, I will use two choice situations involving seemingly incomparable options and show that the three principles entail a contradiction. These choice situations are simply variants of Figure 2 above. They are structurally identical insofar as they both consist of a first choice between two options at t1, followed by a second choice at t2 between keeping their previously chosen option, or, switching it for one that is slightly worse than the option they did not previously choose. As with Figure 2, each of the choices at t1 and t2 are between seemingly incomparable options. That is, I am assuming that each of A+, A, and A- are seemingly incomparable with each of B+, B, and B- with respect to some value that is stipulated to be the only one that is relevant in this choice situation.[13] The choice nodes (n1–n6) and the possible choice sequences (Stay I-IV and Switch I-IV) are labeled for ease of reference.

Figure 3. Two Choice Situations
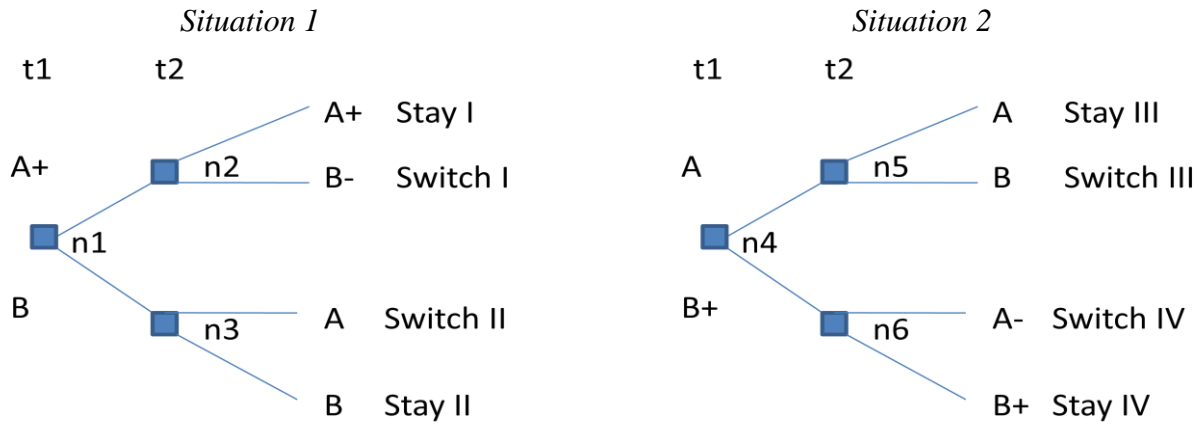
Incompatibility Argument

P1 – In choice situations where risk is not involved, it is less rational to perform a sequence of choices whose outcome one knows in advance will be worse compared to that of some other available sequence than it is to perform a sequence of choices whose outcome one does not know will be worse compared to that of any other available sequence. [Comparative Money Pump Principle]

C1 – It is less rational to perform Switch II than Stay II. [from P1 and Situation 1]

C2 – It is less rational to perform Switch III than Stay III. [from P1 and Situation 2]

P2 – If one sequence of choices is more rational than another, then they must differ in the rational status of (some of) their constituent choices in a way that favors the more rational sequence. [Sequence-Constituent Principle]

C3 – There must be some difference in the rationality of choosing between A and B at node n3, favoring B. [from C1 and P2]

C4 – There must be some difference in the rationality of choosing between A and B at node n5, favoring A. [from C2 and P2]

P3 – If one option is more rational than another in a choice situation where only value considerations matter, then there must be some comparative asymmetry between the available options with respect to the relevant values that favors the more rational option. [Rationality-Availability Principle]

C5 – There must be some comparative asymmetry between A and B, favoring B. [from C3 and P3]

C6 – There must be some comparative asymmetry between A and B, favoring A. [from C4 and P3]

∴ Contradiction

This argument begins by first invoking the Comparative Money Pump Principle (P1). This

principle is then applied to the two example choice situations to conclude that it is less rational to

perform certain sequences of choices rather than others in both cases (C1 and C2). Combining these conclusions with the Sequence-Constituent Principle (P2), it follows that there must be some difference in the rationality of choosing between particular choices within those sequences that explains their differing rational statuses. In particular, the rational difference must be between the choices made at t2 since the choices made at t1 are identical in the differing sequences. Therefore, there must be a difference in the rationality between choosing A and choosing B at node n3 favoring B, and between choosing A and choosing B at node n5, favoring A (C3 and C4). This is not yet a contradiction though. Rather, the contradiction arises only once we also invoke the Rationality-Availability Principle (P3) which requires that any difference in the rationality between choosing two options must be due to some comparative asymmetry between the available items favoring the more rational option. It follows that there must be both a comparative asymmetry between A and B that favors B (C5) and a comparative asymmetry between A and B that favors A (C6). But this is impossible, so the three principles are incompatible.

One point to note here is how this argument relates to the different accounts of seeming incomparability. These accounts purport to explain what is going on in cases of seeming incomparability. For instance one account holds that seemingly incomparability items are in fact either better than, worse than, or equal to each other, but appear to be incomparable because of our ignorance as to which of these relations holds between them.[14] Another account claims that it is determinately false that seemingly incomparable items are better than, worse than, or equal to each other, but that there is a fourth comparative relation of parity that holds between the items.[15] While interesting in its own right, a full discussion of these accounts is best saved for another time.[16] The important point for our purposes is that the argument presented here is independent

of any particular account of seeming incomparability since the premises invoked neither presuppose nor imply any particular account of seeming incomparability. In this way, the incompatibility of these three principles cannot be avoided by simply appealing to one account rather than another.[17] These three principles are incompatible regardless of which account of seeming incomparability is ultimately correct.[18]

Finally, it is worth reiterating that the three principles here only make claims about the *comparative* rationality of choosing between options or sets of options and what must be the case when they *differ* in rational status. The principles however make no claims about the particular rational statuses of those options and whether any are ever fully rational to choose. These principles then are incompatible regardless of whether choices between seemingly incomparable options can be fully rational to choose or instead always fall short of full rationality.[19]


5. Which Principle to Reject?

In this section, I consider rejecting each of the principles before concluding that the one we should reject is the Comparative Money Pump Principle.

5.1 Rejecting the Sequence-Constituent Principle?

The Sequence-Constituent Principle claims that the rationality of a sequence of choices supervenes on the rationality of the choices made within that sequence. So the rational statuses of two sequences of choices cannot differ if the rational statuses of their constituent choices are identical. While this principle seems quite plausible, one might try to deny it by claiming that the rationality of a sequence of choices is a rational assessment distinct from and independent of the

rational assessment of its constituent choices. How might this work? Take again Motivating Choice Situation 2, reproduced here with choice nodes and sequence names added:

Figure 4. Motivating Choice Situation 2 Redux
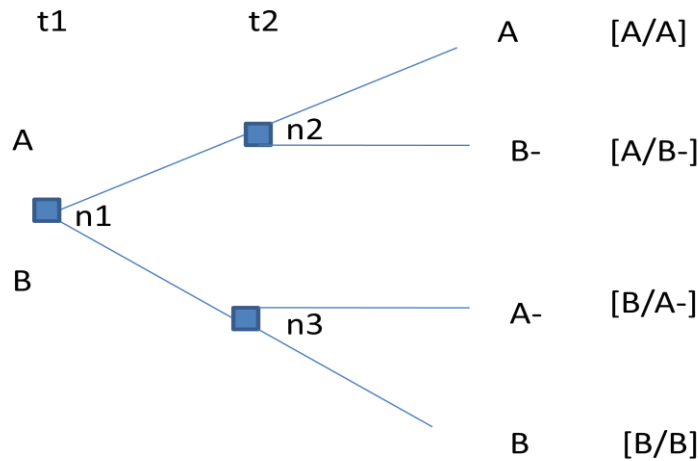
The idea here is that one might claim that the rational assessment of the agent's choices at t1 and t2 individually are distinct from and independent of the rational assessment of the agent's choices at t1 and t2 collectively. That is, there are three different rational assessments of the agent's choices in Figure 4. There is one rational assessment of the agent's choice at t1, another rational assessment of the agent's choice at t2, and a third rational assessment of the agent's choices at both t1 and t2. Importantly this third rational assessment does not supervene on the first two rational assessments. In this way, the rational assessments of all the possible choices the agent might make in Figure 4 could be as follows:

Figure 5: Three Rational Assessments of Motivating Choice Situation 2

| | Rational Assessment of the Agent's Choice(s) at: | | |
	t1	t2	t1 & t2
[A/A]	Rational	Rational	Rational
[A/B-]	Rational	Rational	Irrational
[B/A-]	Rational	Rational	Irrational
[B/B]	Rational	Rational	Rational

I am assuming here, purely for the sake of simplicity, that when faced with a choice between seemingly incomparable options, it is rational to choose either option.[20] Given this, all of the agent's possible choices assessed at t1 and t2 individually are rational. However, the agent's possible sequences of choices assessed at t1 and t2 jointly are not all rational. Rather, the performance of sequences [A/B-] and [B/A-] are irrational since each results in an outcome that is worse than that of some other sequence. These sequences are judged to be irrational even though each of the constituent choices within the sequences is individually rational. Now if this model is correct, then the Sequence-Constituent Principle is false.

There is some intuitive plausibility to this model, but it is actually incomplete as stated. There is a further question that needs to be answered, namely what the agent should rationally do in cases of conflict between these rational assessments. For instance, suppose the agent chose A at t1 and is now at node n2, facing a choice between A and B-. What rationally should she do now? On this model, there are two conflicting rational assessments of her current options. The assessment of her options at t2 alone holds that it is rational to choose either A or B-. The assessment of her options at t1 & t2 jointly holds that in choosing A, she would be completing a rational sequence of choices, whereas in choosing B-, she would be completing an irrational sequence of choices. Given these conflicting rational assessments, what should she rationally do *all-things-considered*? Is there even an all-things-considered rational assessment here?

There are two ways to go and both, I argue, are problematic. Either there is an all-things-considered rational assessment or there isn't. Take first the possibility that there is no all-things-considered rational assessment. If this is right, then in cases of conflict between different rational assessments, the agent will be simply left with the conflicting assessments and no further rational guidance as to which takes precedence or how they might be balanced. Rationality then offers no

univocal assessment, consisting instead of a fractured series of different assessments vulnerable to irresolvable conflict. Indeed, on this view when we ask what it is rational to do, we are actually asking an ambiguous question. We could be asking about the rationality of options assessed only at the time they are available or assessed as the completion of any sequence of choices of arbitrary length. After all, every choice we make results in the completion of a great many different sequences of choices, ranging from relatively short up to the sequence composed of every choice we have ever made. Rationality then consists of an awful lot of different rational assessments with no all-things-considered assessment of what the agent should rationally do. This splintered view of rationality is neither attractive nor acceptable.

Take now the possibility that there is an all-things-considered rational assessment. If this is right, then there are two possibilities of how it would rule in cases of conflict between different rational assessments. Either this all-things-considered rational assessment at a given time always agrees with the rational assessment of the individual choice at that time, or it does not. So in the case above where the agent is at node n2 and asking what it is all-things-considered rational for her to do, either that assessment at t2 agrees with the individual rational assessment of the agent's choices at t2, or it does not. If it always agrees with the rational assessment of the individual choice at a given time, then any conflicting rational assessment of some sequence of choices ending at that time is simply irrelevant for determining the all-things-considered rational assessment. In this way, the rational assessment of any sequence of choices carries no real rational weight since such assessments are effectively ignored with respect to all-things-considered rationality. The rational assessment of a sequence of choices then turns out to be rationally toothless.

However, the all-things-considered rational assessment at a given time might not always agree with the rational assessment of the individual choice at that time. Rather, the all-things-considered rational assessment at a given time might instead side with the rational assessment of some sequence of choices ending at that time. For instance, it may be that when faced with the conflicting rational assessments at node n2 above, the all-things-considered rational assessment sides with the rational assessment of the sequence of choices made at t1 and t2. This possibility is intuitively appealing since it would allow for an all-things-considered rational judgment that forbids the agent from switching and money pumping themselves. The problem with this possibility though is that such an all-things-considered rational assessment would violate the Rationality-Availability Principle. This is because it would allow for something besides the comparative values of the available options at the time to influence the rationality of choosing between them. After all, this is precisely what allows the rational assessment of some sequence of choices ending at a given time to differ from the rational assessment of the individual choice at that time. So this possibility is also committed to the rejection of the Rationality-Availability Principle. As I will argue in the next section, there are serious problems with rejecting the Rationality-Availability Principle, which this possibility then inherits.

The upshot here is that if one wants to reject the Sequence-Constituent Principle, they will have to either endorse a fundamentally splintered view of rationality, admit that the rational assessments of sequences of choices have no all-things-considered rational bite, or also be committed to rejecting the Rationality-Availability Principle.

5.2 Rejecting the Rationality-Availability Principle?

The Rationality-Availability Principle claims that in choice situations where only value considerations matter, the rationality of choosing a particular option supervenes on the comparative relations that obtain between the available options with respect to the relevant values. In particular, if there is some difference in the rationality of choosing among the available options, there must be some comparative asymmetry between the available options with respect to the relevant values that justifies this rational difference. While this principle seems quite plausible, one might try to reject it by claiming that there are other factors that can justify a difference in rational status among the available options. What might these other factors be? The most natural answer here is to appeal to the comparison of the currently available options to those options that were previously available to the agent. Indeed, what seems to be driving the intuition that it is less rational to perform the Switch sequences than the Stay sequences is that in performing the Switch sequences the agent ends up with an option that is worse than one they previously rejected. We might then conclude that the switching agent's rational mistake is at t2 because even though their chosen option is not worse than any of those currently available, it is worse than one that was previously available to the agent. That is, the Rationality-Availability Principle might be incorrect because the rationality of choosing a particular option depends not only on the comparative relations that obtain between the currently available options, but also on the comparative relations that obtain between both currently and previously available options as well.

The problem with this response though is that it is not clear why the comparative relations between currently available options and previously available options are at all relevant for determining the rationality of choosing between only currently available options. In deciding

which option it is rational for us to choose *now*, why should we consider how they compare to options that are no longer available? Given that previously available options are not currently available, they seem to be simply irrelevant to what we should rationally do now. Indeed, taking previously available options into consideration in this manner seems to be committing the sunk cost fallacy.

The sunk cost fallacy is a mistake where one irrationally takes into account previously sunk costs in determining what to do now. Take the following example provided by David Ramsey Steele (1996, p. 609):

> [I]f Hillary has paid to commence the building of a canal, which is now half completed, this is sometimes believed to provide a reason for Hillary to complete the canal, even if, in an alternative scenario, the half-completed canal existed as a natural geographical feature, and, knowing what Hillary now knows, she would not think it worthwhile to "complete" the canal. The economist says that these two scenarios are alike in all relevant respects: past expenditures do not justify future expenditures.[21]

The idea here is that the fact that Hillary has incurred a certain cost in the past is not relevant to whether or not she should complete the canal. Rather, the rationality of her decision to complete the canal remains the same regardless of how high or low her previous costs were. All that matters for her decision now are the expected future costs/benefits of each possible action.

Turning back to the sequential choice situations considered above, notice that by taking the options that the agent did not previously choose at t1 into account for the rationality of their choice at t2, one is effectively holding sunk costs to be rationally relevant. This is because at the t2 choice nodes, the option that the agent did not choose at t1 is effectively a sunk cost. To be sure, the sunk cost here takes the form of an opportunity cost wherein what is lost is an opportunity to receive an item rather than an item one already had. That is, in choosing A (or B),

they are forgoing the opportunity to receive B (or A). But since this opportunity cost is already sunk at t2, it should have no bearing on the rationality of their current possible choices.

Take a more concrete case, again using Chang's example of the seemingly incomparable coffee and tea. Here, the question can be formulated as whether in a choice at t2 between Sumatra Gold and Pearl Jasmine, an agent has more reason to choose Sumatra Gold in the scenario where she had previously chosen Sumatra Gold over Pearl Jasmine$^+$ at t1 as compared to the scenario where she was simply given the Sumatra Gold at t1. Now the agent in the first scenario has certainly incurred a cost at t1 (i.e., the opportunity of getting Pearl Jasmine$^+$) that the agent in the second scenario has not. But is this sunk cost relevant to the agent's choice between Sumatra Gold and Pearl Jasmine at t2? If so, why is it relevant here but not in the case of Hillary's canal? The upshot is that if one wants to reject the Rationality-Availability Principle by appealing to the comparative relations between currently available options and previously available options, one would have to either show how this is not an instance of the sunk cost fallacy, or, hold that the sunk cost fallacy is not a fallacy.

One possible response here is that there is a significant difference between appealing to previously available options when seeming incomparability is involved and when it is not. In traditional cases of the sunk cost fallacy not involving seeming incomparability, an agent chooses a suboptimal option (suboptimal at least when considering only currently available options) because they take previously available options to be rationally relevant. In cases involving seeming incomparability though, an agent who takes previously available options to be rationally relevant does not end up choosing a suboptimal option (again when considering only currently available options) since neither available option is worse than the other. Such an agent then is in effect using previously available options as a "tie-breaker" of sorts to decide between

two options, neither of which is worse than the other.[22] But appealing to previously available options in this way seems less problematic than it its use in traditional sunk cost cases. Given these differences, perhaps we should reject the Rationality-Availability principle in cases where seeming incomparability is involved, or so the objection might go.[23]

While there is certainly a distinction between the way in which previously available options are taken into account in the standard sunk cost fallacy cases and in cases involving seeming incomparability, this distinction does not actually justify a rejection of the Rationality-Availability Principle when seeming incomparability is involved. A few points of clarification may help explain why this is the case. The first is that the Rationality-Availability Principle does not forbid appealing to previously available options as *one* way to choose between seemingly incomparable options. Such a decision-making procedure may well be rationally permissible. What the principle does forbid though is *requiring* that agents decide in this way. It is this stronger requirement that is actually needed to prevent agents from money-pumping themselves in the sequential choice situations we have considered. However, the Rationality-Availability Principle holds that there cannot be a difference in the rationality between choosing two options unless there is some comparative asymmetry between those options. So while it may not be problematic to appeal to previously available options to decide between seemingly incomparable options, it is problematic to require appealing to previous options since it would in effect hold that there is a difference between the rationality of the current options despite there being no comparative asymmetry between them.

The difference here can be illustrated with an example not involving seeming incomparability. Suppose that given the half-built canal, Hillary's option of completing the canal and her option of not completing the canal are both equally good. Here it may well be rationally

permissible for her to appeal to the fact that she had previously completed the first half of the canal to help her now decide to complete the canal given that both currently available options are equally good.[24] However, it is surely too strong to require her to take her previous sunk costs into consideration in deciding whether to finish the canal now. Given that the currently available options are equally good, she would be rationally permitted to ignore her previous sunk costs and decide to not complete the canal. *Permitting* an agent to appeal to sunk costs (as an arbitrary choice mechanism) in certain situations is different from *requiring* that agent to appeal to sunk costs. While the former may be plausible, the latter is not, regardless of whether seeming incomparability is involved.

5.3 Rejecting the Comparative Money Pump Principle?

The Comparative Money Pump Principle claims that when comparing possible sequences of choices where risk is not a factor, if one sequence is known to result in a suboptimal outcome and another sequence is not, then the former is less rational to perform than the latter. While this principle seems compelling, I will argue that it is false. My view here is similar to Brian Hedden's (2015) insofar as we both hold that different time-slices of the same agent should be rationally assessed in the same manner as two entirely different agents, ceteris paribus. However, my discussion differs from his by focusing on the rationality of an agent's *actions* rather than the rationality of their *attitudes*. It is one thing to say that a set of attitudes is rational/irrational, it is another thing to say what is more/less rational for such agents to do given those attitudes. Moreover, my discussion here focuses on the rationality of actions stemming from seeming incomparability in particular rather than the more general category of what Hedden calls Tragic Attitudes, many of which have significantly different characteristics.[25] Finally, even if we adopt Hedden's Options-as-Decisions view, which I am sympathetic with, we still actually need the

Rationality-Availability Principle to demonstrate that the agent who money-pumps themselves is no less rational than the one who does not.

Turning back to the Comparative Money Pump Principle, we can first observe that in the realm of game theory, there are well known examples where two players can predictably and rationally end up with a collectively suboptimal outcome. The most famous is the prisoner's dilemma:

Figure 6. Prisoner's Dilemma

Figure 6 represents the choices (Cooperate or Defect) available to two players (1 and 2) and outcomes for each player in utility numbers (1 – 4, higher is better). For each outcome (box), the left number is Player 1's utility and the top number is Player 2's utility. Which outcome eventuates depends on the players' choices (i.e., when both Cooperate, both receive 3, when Player 1 Cooperates and Player 2 Defects, Player 1 receives 1 and Player 2 receives 4, etc…).

When first presented with the prisoner's dilemma, many have the intuition that it is irrational for the players to Defect since they can achieve an outcome that is better for both of them if they instead Cooperate. More technically, the [Defect/Defect] outcome is Pareto inferior

to the [Cooperate/Cooperate] outcome. However, the only rational strategy for the agents in such a one-shot prisoner's dilemma is for both to Defect. This is because [Defect/Defect] is the only Nash equilibrium strategy pair in the game (indicated by the asterisk). That is, it is the only set of strategies such that neither player can improve her outcome by unilaterally deviating. Moreover, Defect dominates Cooperate for both players in this case. That is, it is always in a player's interest to Defect regardless of what the other player does. To see this, take the perspective of each player in turn. In deciding what to do, Player 1 can reason that Player 2 will either Defect or Cooperate, but in each case it is better for Player 1 to Defect rather than Cooperate. If Player 2 Defects, then it is better for Player 1 to also Defect since a utility of 2 is better than a utility of 1. If Player 2 Cooperates, it is still better for Player 1 to Defect since a utility of 4 is better than a utility of 3. (This is represented by the one-way lines in Figure 6.) So it is better for Player 1 to Defect than Cooperate. Player 2 can reason similarly to conclude that it is better for her to Defect than Cooperate as well. Because of this, and contrary to our initial intuition, it is not irrational for the agents to both Defect even though the outcome of [Defect/Defect] is Pareto inferior to that of [Cooperate/Cooperate].[26] Now consider the following game:

Figure 7. Equilibria Everywhere

Player 2

|	Stay	Switch
A	A A *	B- B- *
B	B B *	A- A- *

This game is set up such that every strategy pair is a Nash equilibrium, but that the outcome of two pairs (i.e., [A, Switch] and [B, Switch]) are each individually Pareto inferior to the outcomes of two other pairs (i.e., [B, Stay] and [A, Stay] respectively). Here, both players take the A's to be seemingly incomparable to the B's and so do not prefer an A item over a B item or vice versa. Given this, neither player can knowingly improve her outcome by unilaterally deviating from any strategy pair. Rather, each player knows that whatever the other player does, she will not prefer the outcome of one of her choices over another. That is, Player 1 can reason that Player 2 will choose either Stay or Switch, but in each case it is neither better nor worse to choose A rather than B. If Player 2 chooses Stay, then Player 1's choice will determine whether she gets A or B. But A and B are seemingly incomparable to her, so she does not prefer one over the other. If Player 2 chooses Switch, then Player 1's choice will determine whether she gets B- or A-. But B- and A- are seemingly incomparable to her, so she does not prefer one over the other. So it is neither better nor worse for Player 1 to choose A rather than B. Player 2 can reason similarly to

conclude that it is neither better nor worse to choose Stay rather than Switch. (This is represented by the two-way lines in Figure 7.) In this way, both players have no incentive to choose one of their available choices rather than the other. Because of this, it is not irrational for the agents to choose [A/Switch] even though that outcome is Pareto inferior to that of [B/Stay]. Likewise, it is not irrational for the agents to choose [B/Switch] even though that outcome is Pareto inferior to that of [A/Stay]. So we should reject the idea that the players are less rational if they choose [A/Switch] or [B/Switch] rather than [A/Stay] or [B/Stay]. The intuition common to both Prisoner's Dilemma and Equilibria Everywhere is that it seems irrational for the players to end up with a Pareto inferior outcome. However, once we recognize that these outcomes result from Nash equilibria strategy pairs and that neither player has incentive to play their part of a Pareto optimal strategy pair, we must then reject that intuition.

If this is right, then we should also reject the idea that the single agent in Motivating Choice Situation 2 would be less rational in performing one of the Switch sequences rather than Stay sequences. This is because Motivating Choice Situation 2 is simply a sequential version of Equilibria Everywhere where the players are two time-slices of the same agent rather than different individuals (i.e., Player 1 is the agent at t1 and Player 2 is the same agent but at t2). This modification though does not change which choices it is rational for the players to make. Take first the change from two players making one choice each to one player making two choices. In decision theory, what is important is not the number or identity of the agents involved in a choice situation or game. Rather, it is the outcomes and preferences of the agents involved that matter. In Equilibria Everywhere, even though there are two players, the outcomes and preferences of those players are exactly the same. Their preferences are identical by stipulation and the payouts to each player are the same for each possible outcome. So even though each

player is only concerned with satisfying their own preferences, they are in effect acting as if they are just as concerned with satisfying the preferences of the other player as well. In this way, they could never be at cross purposes as they could in other games (e.g., Prisoner's Dilemma). Rather, the players in Equilibria Everywhere are already acting as if they were a single person interested in one preference ordering, but faced with two different decisions. We could make this even clearer by modifying the game to the following:

Figure 8. Third Party
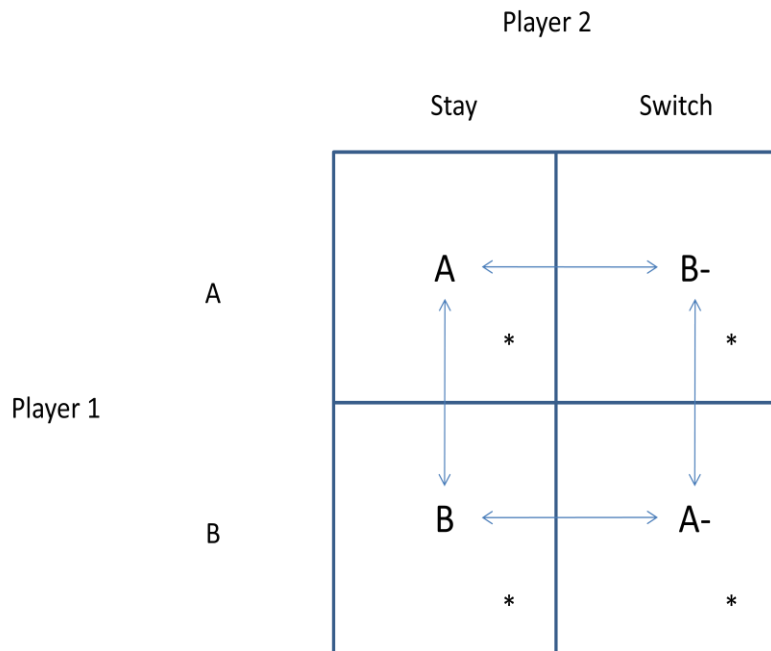
The only difference between this game and Equilibria Everywhere is that here the payout goes not to the two players, but to some third party. Moreover, each of the players have the same preference rankings as they did before, but now they have them with respect to what they want for this third party. For instance, imagine that Players 1 and 2 are parents who are choosing on behalf of their child and have exactly the same preferences for her. So even though there are two different agents making decisions here, the fact that the outcomes and preferences are the same

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for both agents makes the rationality of their choices the same as it would be if it were only a single agent making both choices.

Take now the change from the simultaneous nature of Equilibria Everywhere to the sequential nature of Motivating Choice Situation 2. The important difference here is that there is an epistemic asymmetry between the agent at t1 and t2 in Motivating Choice Situation 2 that does not exist between the players in Equilibria Everywhere. That is, the agent at t2 knows the decision she made at t1, whereas Player 2 does not know the decision made by Player 1. However, introducing this asymmetry into Equilibria Everywhere does not change the rationality of the player's choices. For instance, suppose that Player 2 gets to find out what Player 1 has chosen before making her choice. In this case, Player 2 will face a choice between A and B-, or, between B and A-, and she will know exactly which of these choices she faces. But this additional knowledge does not change the rationality of her possible strategies because it is still the case that both form equilibrium pairs with what Player 1 has chosen and that neither Stay nor Switch will result in an outcome that Player 2 prefers over the other. If she knows that Player 1 has chosen A, she knows that she cannot unilaterally improve her outcome by choosing Stay rather than Switch and vice versa. Likewise if she knows that Player 1 has chosen B.[27] Since the rational statuses of the player's strategies do not differ in the sequential and simultaneous versions of Equilibria Everywhere, this difference between Motivating Choice Situation 2 and Equilibria Everywhere is not relevant for determining the rationality of the agent's sequences of choices.

The upshot is that because we do not hold the players in Equilibria Everywhere to be less rational for playing strategy pairs that result in Pareto suboptimal outcomes, we should not hold the single agent in Motivating Choice Situation 2 to be less rational for performing a choice

sequence that they know will result in a suboptimal outcome. The agent in Motivating Choice Situation 2 is simply in a sequential version of Equilibria Everywhere where the players are different time-slices of the same agent. So if the two players are not less rational for playing a suboptimal strategy pair, then neither is the single agent in Motivation Choice Situation 2 when she performs a suboptimal sequence. The Comparative Money Pump principle then is false.

The appeal here to game theoretic principles also nicely explains why the Comparative Money Pump Principle seemed so compelling in the first place and why it is ultimately mistaken. The intuition that it is less rational to switch than stay in Motivating Choice Situation 2 is compelling for the same reason that it seemed clearly less rational to defect than cooperate in the prisoner's dilemma. In both cases, we see that one set of choices will result in a worse outcome by the lights of everyone involved than some other set of choices and so infer that the former must be less rational to perform than the latter.

However, this is a mistake since we are ignoring the rational status of each individual choice within those sets. In the prisoner's dilemma, we ignore the fact that both choices in the set [Defect, Defect] are rationally required and neither choice in the set [Cooperate, Cooperate] is rationally permitted. In Motivating Choice Situation 2, we ignore the fact that the rational statuses of each choice in all possible sets are the same since each possible individual choice never improves or worsens the agent's outcome. Ignoring the rational status of individual choices in a set has been realized to be a mistake when the choices are made by different agents, as in the prisoner's dilemma, but it should now also be realized to be a mistake when the choices are made by a single agent over time.

6. Conclusion

I have here discussed the phenomenon of seeming incomparability, identified two of its key features, and presented three intuitively compelling principles for governing rational choice between seemingly incomparable options: the Comparative Money Pump Principle, the Sequence-Constituent Principle, and the Rationality-Availability Principle. While each is individually plausible, I have demonstrated that they are jointly incompatible and argued that the correct response is to reject the Comparative Money Pump Principle. Despite its initial appeal, we should ultimately reject the idea that it need be less rational for an agent to perform a sequence of choices that she knows in advance will result in a suboptimal outcome.

The problem with the Comparative Money Pump Principle is that it takes an idea that is applicable in synchronic choice situations and illegitimately extends it to diachronic choice situations. While it is less rational to knowingly choose an option that will result in a suboptimal outcome in an individual choice situation, it need not be less rational to make a series of choices that will result in the same suboptimal outcome.[28] In particular, it is not less rational when none of the individual choices in the series involves choosing an option with a suboptimal outcome, which as we have seen is possible when seeming incomparability is involved. So even though the ultimate result of a sequence may be suboptimal, the performance of that sequence is not less rational if none of the individual choices that constitute the sequence are less rational. Just as individual choices made by different agents can result in a suboptimal outcome without rational defect, individual choices made by the same agent over time can also result in a suboptimal outcome without rational defect. Such suboptimal outcomes may be unfortunate and even avoidable, but neither of these things entails that the sequences leading to such outcomes are less rational to perform.

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Acknowledgements

Notes

1. The same goes for claims where the place of the items is switched (e.g., Sumatra Gold is better than Pearl Jasmine with respect to tastiness).
2. The implausibility of claim [3] is often also argued for using the Small Improvement Argument. See Chang (2002) pp. 667-8 for a presentation of this argument.
3. There is a third feature regarding the persistence of seeming incomparability that I take to be central to this phenomenon, but it is not included here since that feature does not bear on our present discussion. For a general discussion of the ways in which seeming incomparability may or may not persist though, see Herlitz (2020).
4. I should note here a similarity between seeming incomparability and imprecise credences in epistemology. Miriam Schoenfield (2014) has pointed out that both can be understood to be cases of orderings that cannot be represented using a single function. With seeming incomparability, there is no single function that assigns to each item a real number representing its value. With imprecise credences, there is no single function that assigns to each proposition a real number representing one's degree of belief in that proposition. Given this similarity, Schoenfield holds that investigations into how decision theory might be extended to accommodate each case will run parallel to each other. I agree with her here and think that much of what I say later on can be applied to the project of extending decision theory to cover imprecise credences. I mention some applications in note 18, but a full discussion of the parallels between these projects is beyond the scope of this paper.
5. Note that the intuition appealed to here is entirely comparative in nature. That is, it is only about the *relative* rationality of the available options. In this way, no stance is taken on whether choosing A or B in Figure 1 is fully rational, only that choosing A or B is *more* rational than choosing A- or B-.
6. It is important to note here that it is part of the stipulations of this choice situation that the A and B items remain seemingly incomparable throughout, regardless of what the agent chooses at t1 or t2. Here it may be helpful to imagine the agent as having a set of incomplete preferences that remains stable over the course of the choice situation. In this way, I am ruling out the possibility that the agent in this case can simply will herself to have a certain set of complete preferences or make commitments that change her preferences for future choices. I am then assuming that resolute choice is not possible in this particular choice situation. See Rabinowicz (1995) and McClennen (1990) for discussions of this resolute choice approach to dynamic decision making.

7. Chang (1997, 2021), Bader (manuscript), and Broome (2000, 2001) also discuss the intuitive irrationality of being money pumped in this way, though they all have different accounts of how problematic it is.

8. Another way to approach this is to ask yourself what advice you would give to someone about to face this choice situation. I take it that most of us would advise them to choose either A or B at t1 and stick with that choice at t2.

9. This is to rule out the applicability of this principle in cases where it seems more rational for an agent to accept a slightly suboptimal outcome for sure than accept a lottery that could result in the optimal outcome or one that is much worse (e.g., a choice between $1000 for sure, $999 for sure, or a lottery between $1000 or $0 with even chances).

10. It is worth noting that this principle in no way rules out the possibility of past choices affecting the rationality of future choices. Indeed, this principle says nothing at all about how the rationality of individual choices within sequences is to be determined.

11. I take it though that many, if not most or even all, choice situations are actually like this.

12. More technically, two items are comparatively asymmetrical with respect to some value just in case the truth value of *some* comparative claim between those items under that value (e.g., claims of betterness, worseness, equality, or perhaps parity) changes when switching the positions of those items in that claim. In contrast, two items are comparatively symmetrical with respect to some value just in case the truth value of *any* comparative claim between those items under that value does not change when switching the positions of those items in any of those claims.

13. Strictly speaking, I only need to assume the seeming incomparability of A and B to get the contradiction. It follows from that assumption that the choices at nodes n1, n3, n4, and n5 will all be between seemingly incomparable options and the sequences needed to derive the contradiction go through only these nodes.

14. See Regan (1997) for defense of such a view.

15. See Chang (1997) for defense of such a view.

16. See Chang (1997) for an introduction to the various accounts of seeming incomparability.

17. It is interesting though to note how different accounts of seeming incomparability might favor different ways of resolving the tension between these three principles. For instance, suppose that seeming incomparability is due to vagueness. That is, it is indeterminate whether A is better than B, A is worse than B, or A is equal to B, but it is determinately true that one of these relations holds (i.e., the disjunction of these claims is determinately true). Those who endorse this account of seeming incomparability might be tempted to resolve the incompatibility of the three principles by rejecting the Sequence-Constituent Principle. They might hold that while the rationality of each individual choice in the Switch and Stay sequences is indeterminate (because no determinate relation obtains between the items at each node), it can still be the case that the rationality of the sequence of choices is determinately irrational (since the combination of choices involve inconsistent sharpenings). Note though that this move must still answer all the challenges associated with rejecting the Sequence-Constituent Principle that I've laid out in Section 5.1. Moreover, the most likely response to those challenges would probably involve rejecting the Rationality-Availability Principle as well and so would also have to deal with the challenges associated with rejecting that principle laid out in section 5.2. An alternative account holds seeming incomparability to be due to ignorance. This epistemicist account of seeming incomparability holds that one of the three possible comparative relations (better than, worse than, or equal to) determinately holds between A and B, we just don't know which one. Furthermore, this could be combined with an externalist account of rational choice such that what it is rational to do depends on which of these relations in fact holds, despite our ignorance of the matter. Those who endorse this account of seeming incomparability would resolve the incompatibility of the three principles by rejecting the Comparative Money Pump Principle since they would have to hold that some Switch sequence will be more rational than some Stay sequence. For example, in Figure 3, the Comparative Money Pump Principle will hold that it is

less rational to perform Switch II than Stay II, and, that it is less rational to perform Switch III than Stay III. However, on the epistemicist/externalist view, it must be that A is either determinately better than, determinately worse than, or determinately equal to B. If A is determinately better than B, then switching to A at n3 is actually more rational than staying with B (assuming externalism), and so Switch II is actually more rational than Stay II (contra the Comparative Money Pump Principle). If A is determinately worse than B, then switching to B at n5 is actually more rational than staying with A, so Switch III is more rational than Stay III. If A is determinately equal to B, then both Stay and Switch Sequences are equally rational. So the epistemicist/externalist view must reject the Comparative Money Pump Principle. Thanks to an anonymous referee for pushing me on these points.

18. While my discussion here is focused on the incompatibility of these three principles given choice situations involving seeming incomparability, a similar incompatibility arises given choice situations involving imprecise credences as well. Elga (2010) comes close to identifying this incompatibility, but his focus there is importantly different. Elga is concerned with whether having imprecise credences is compatible with perfect rationality, arguing that they are incompatible because there is no adequate account of how such imprecise credences can constrain rational choice. My concern is not whether seeming incomparability is compatible with perfect rationality, but rather how we should rationally respond to choice situations involving seemingly incomparable options. The parallel question then would be how we should rationally respond to choice situations given imprecise credences, regardless of whether it is perfectly rational to have such credences. That said, many of the considerations Elga invokes against the compatibility of imprecise credences with perfect rationality mirror the three principles I discuss. Moreover, I think that various responses to Elga in the literature can be interpreted as rejecting one of these principles. For instance, I think we can interpret Rinard (2015) as rejecting the Sequence-Constituent Principle and Williams (2014) as rejecting the Rationality-Availability Principle.

19. My principles then are significantly weaker than those presented by Peterson (2007) in his argument against the possibility of rational choice between items that are on a par. There he explicitly assumes that it is rationally permissible to choose either of two items that are on a par. I take the generality of my weaker principles to be an asset as they better identify the source of the tension that arises when trying to determine what it is rational to do given seemingly incomparable options.

20. This assumption could be replaced with the claim that such choices are all irrational or indeterminately rational without affecting the ensuing discussion.

21. Steele actually takes there to be two forms of the sunk cost fallacy, with this example being an instance of what he calls the "Concorde form". We need not be concerned with the other form for our purposes here.

22. "Tie-breaker" is in quotes here since seemingly incomparable items are not strictly speaking tied insofar as they are not exactly equally good.

23. Thanks to an anonymous referee for pointing out this objection.

24. That is, appealing to sunk costs in such situations may be just as arbitrary as flipping a coin to decide.

25. For instance, seeming incomparability does not actually force an agent to perform a Tragic Sequence, unlike his main example of the Russian Nobleman's Preference Shifts. In this way, we might be tempted to think that an agent who money pumps themselves when faced with seemingly incomparable choices is less rational than another agent with the same preferences who does not, given that the money pump is unforced.

26. There are however those who reject this standard analysis of the Prisoner's Dilemma. For instance, David Gauthier (2013) holds that it is irrational to Defect in the Prisoner's Dilemma despite it dominating Cooperate because the result of both players Defecting is Pareto inferior to the result of both players Cooperating. Applied to sequential choice situations involving seeming incomparability, someone who endorsed such a non-traditional analysis of the Prisoner's

Dilemma would likely reject the Rationality-Availability Principle rather than the Comparative Money Pump Principle. Thanks to an anonymous referee for pointing this out.

27. More generally, changing a simultaneous game into a sequential game will not change the Nash equilibria in cases where each strategy pair in the simultaneous game is in equilibrium.

28. Compare the situations depicted in Figure 1 and Figure 2 on pages 5 and 6.

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