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Research Article

Adaptive Feedback Control for Synchronization of Chaotic Neural Systems with Parameter Mismatches

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This work pertains to the study of the synchronization problem of a class of coupled chaotic neural systems with parameter mismatches. By means of an invariance principle, a rigorous adaptive feedback method is explored for synchronization of a class of coupled chaotic delayed neural systems in the presence of parameter mismatches. Finally, the performance is illustrated with simulations in a two-order neural systems.

1. Introduction

Pecora and Carroll firstly addressed the chaotic synchronization in systems and used the drive-response conception in [1]. The idea is to control the response signal by using the output signal of drive system such that the two kinds of signals synchronize. The problems of synchronization in chaotic dynamical systems have received increasing attention in the control areas [2–6]. Different approaches including adaptive design control [2], intermittent control [3], adaptive-impulsive control [4], and sliding mode control [6] have been proposed. In particular, Liu et al. [3] obtained novel synchronization criteria for exponential synchronization of chaotic systems with time delays via periodically intermittent control. By an adaptive feedback control technique, the synchronization of a class of chaotic systems with unknown parameters is achieved via mimicking model reference adaptive control-like structure in [2]. Tam et al. [5] addressed adaptive synchronization of complicated chaotic systems with unknown parameters via a set of fuzzy modeling-based adaptive strategy. Chen et al. [6] designed a sliding mode control scheme for adaptive synchronization of multiple response systems under the effects of external disturbances.

Recently, there has sprung up hot research topics in the synchronization of chaotic neural systems (CNSs) due to possible chaotic behaviors in such systems [7–13]. For instance, synchronization of coupled delayed CNSs and applications to memristive CNSs in [12] have resulted in a theoretical condition under an irreducible assumption on coupling matrix. Cao and Lu [13] proposed a simple adaptive method for the synchronization of uncertain CNSs with or without variable delay via invariant principles. In particular, some efforts have been devoted to adaptive synchronization of CNSs [10–12, 14]. However, most existing works were applicable only for the CNSs with parameter matching. While in practical implementation of synchronized CNSs, it is well known that parameter mismatch in systems is generally inevitable [15–17], which will result in poor performance or loss of synchronization [17, 18]. For example, Zhang et al. [18] discussed asymptotic synchronization for delayed CNSs with fully unknown parameters based on the Lyapunov method and the inverse optimal method. Therefore, it is of importance to explore the effects of parameter mismatch in synchronization of CNSs.

In this paper, we present theoretical analysis and numerical simulations of the parameter mismatch effect on synchronization for a class of coupled CNSs. By using adaptive control approaches [13, 19, 20] instead of traditional linear coupling scheme, and combining the invariance principle, we show that adaptive synchronization of such CNSs with parameter mismatches under loose conditions can be rapidly achieved. In addition, by adjusting the update gain of coupling strength introduced in this work, one can control the synchronization speed.

The organization of this work is as follows. In Section 2, we present needed formulation of synchronization of CNSs. Section 3 presents an adaptive control scheme in CNSs and provides two criteria for synchronization for CNSs. In Section 4, numerical simulations on a two-order CNS are provided to show the effectiveness of the proposed results. Section 5 concludes the paper.

Notation 1. Throughout the paper, we denote A^{T} and A^{-1} the transpose and the inverse of any square matrix A. $A > 0\ (A < 0)$ denotes a positive- (negative-) definite matrix A; and I is used to denote the $n \times n$ identity matrix. $\|A\|$ denotes the spectral norm of matrix A. Let $\mathbf{R}$ denote the set of real numbers, $\mathbf{R}^n$ denotes the n-dimensional Euclidean space, and $\mathbf{R}^{n\times m}$ denotes the set of all $n \times m$ real matrices. $\lambda_{\max}(\cdot)$ or $\lambda_{\min}(\cdot)$ denotes the largest or smallest eigenvalue of a matrix, respectively.

2. Formulation of Synchronization in Neural Networks

Consider the following CNS in a general form:

$$\dot{x}(t) = -C_1 x(t) + A_1 f(x(t)) + B_1 f(x(t-\tau(t))) + J, \tag{1}$$

where $x(t) = (x_1(t), \ldots, x_n(t))^{\mathrm{T}} \in \mathbf{R}^n$ denotes the state vector; C_1 represents a diagonal matrix with $c_{1i} > 0, i = 1, 2, \ldots, n$; $A_1 = (a_{1ij})_{n\times n}$ denotes the weight matrix; $B_1 = (b_{1ij})_{n\times n}$ denotes the delayed weight matrix; $J = (J_1, \ldots, J_n)^{\mathrm{T}} \in \mathbf{R}^n$ is the input vector function; $\tau(t)$ represents the transmission variable delay; and $f(x(t)) = [f_1(x_1(t), \ldots, f_n(x_n(t)))]^{\mathrm{T}}$ represents the activation function.

Throughout the paper, we have the following two assumptions:

(**A1**) Each $f_j\colon \mathbf{R} \to \mathbf{R}$ satisfies the Lipschitz condition, that is, there exist positive scalars $k_j > 0$ such that

$$\left| f_j(x) - f_j(y) \right| \le k_j |x - y|, j = 1, 2, \ldots, n, \tag{2}$$

for any $x, y \in \mathbf{R}$.

(**A2**) $\tau(t) \ge 0$ is a function satisfying $\tau^* = \max_t(\tau(t))$ and $0 \le \dot{\tau}(t) \le \sigma < 1$, for all t.

$x_i(t) = \phi_i(t) \in \mathscr{C}([-\tau^*, 0], \mathbf{R})$ denotes initial conditions of (1), where $\mathscr{C}([-\tau^*, 0], \mathbf{R})$ represents the set of continuous functions from $[-\tau^*, 0]$ to $\mathbf{R}$.

To synchronize the drive (or master) system (1), the controlled response (or slave) system is given by

$$\dot{y}(t) = -C_2 y(t) + A_2 f(y(t)) + B_2 f(y(t-\tau(t))) + J + u(t), \tag{3}$$

where $u(t)$ is the driving signal, $y(t) = (y_1(t), \ldots, y_n(t))^{\mathrm{T}} \in \mathbf{R}^n$. C_2, A_2, and B_2 are generally different from C_1, A_1, and B_1, respectively. Namely, parameter mismatches exist between the drive system and the response system. The initial conditions of system (3) denote $y_i(t) = \psi_i(t) \in \mathscr{C}([-\tau^*, 0], \mathbf{R})$ $(i = 1, 2, \ldots, n)$. Denote the mismatch errors by $\Delta C = C_2 - C_1$, $\Delta A = A_2 - A_1$, and $\Delta B = B_2 - B_1$.

We aim to design an appropriate controller $u(t)$ in order to make the coupled CNSs remain synchronized in the presence of even large parameter mismatches. First, consider the feedback controller $u(t) = \epsilon \circ (y(t) - x(t))$, where $\epsilon = (\epsilon_1, \ldots, \epsilon_n)^{\mathrm{T}} \in \mathbf{R}^n$ is the coupling strength, and the symbol $\circ$ is defined as

$$\epsilon \circ (y(t) - x(t)) \triangleq [\epsilon_1(y_1(t) - x_1(t)), \ldots, \epsilon_n(y_n(t) - x_n(t))]^{\mathrm{T}}. \tag{4}$$

Define the synchronization error as $e(t) = y(t) - x(t)$, which leads to the following synchronization error system:

$$\begin{aligned} \dot{e}(t) = {} & -C_2 e(t) + A_2 g(e(t)) + B_2 g(e(t-\tau(t))) \\ & - (C_2 - C_1) x(t) + (A_2 - A_1) f(x(t)) \\ & + (B_2 - B_1) f(x(t-\tau(t))) + \epsilon \circ e(t), \end{aligned} \tag{5}$$

or

$$\begin{aligned} \dot{e}(t) = {} & -C_2 e(t) + A_2 g(e(t)) + B_2 g(e(t-\tau(t))) - \Delta C x(t) \\ & + \Delta A f(x(t)) + \Delta B f(x(t-\tau(t))) + \epsilon \circ e(t), \end{aligned} \tag{6}$$

where

$$\begin{aligned} e(t) &= (e_1(t), \ldots, e_n(t))^{\mathrm{T}}, \\ g(e(t)) &= (g(e_1(t)), \ldots, g(e_n(t)))^{\mathrm{T}}, \end{aligned} \tag{7}$$

with $g_i(e_i(t)) = f_i(e_i(t) + x_i(t)) - f_i(x_i(t)), i = 1, 2, \ldots, n$.

Obviously, using the assumption (**A1**), $g_i(\cdot)$ has the following properties:

$$|g_i(e_i)| \le k_i |e_i|. \tag{8}$$

3. Adaptive Control Scheme

In this section, based on Lyapunov function and an invariance principle by combining an adaptive control approach, we consider the adaptive synchronization for two CNSs with time-varying delay and parameter mismatches.

Theorem 1. *Suppose that $\chi = \{x \in \mathbf{R}^n \mid \|x\| \le \alpha_1\}$ and the parameter mismatches satisfy $[\|\Delta C\| + (\|\Delta A\| + 1/1-\sigma\|\Delta B\|) K^2] \le \alpha_2$, where $K = \max_{1\le i\le n}\{k_i\}$. Under the assumptions (**A1**) and (**A2**), let $\alpha = \alpha_1 \cdot \alpha_2$ and the controller $u(t) = \epsilon \circ (y(t) - x(t)) = \epsilon \circ e(t)$ with the following update law:*

$$\dot{\epsilon}_i = -\delta_i \left(e_i^2 + \frac{\alpha}{\epsilon_i + l} x_i^2(t) \right), \tag{9}$$

where $\delta_i > 0$ $(i = 1, 2, \dots, n)$ are arbitrary constants, and $l > 0$ is a constant to be determined. Then, the controlled uncertain response system (3) will globally synchronize with the drive system (1).

Proof 1. Consider the following Lyapunov function for the error dynamical system:

$$\begin{aligned} V(t) &= e^{\mathrm{T}}(t)e(t) + \sum_{i=1}^{n} \frac{1}{\delta_i}(\epsilon_i + l)^2 + \frac{1}{1-\sigma}\int_{t-\tau(t)}^{t} g^{\mathrm{T}}(e(s))g(e(s))ds \\ &\quad + \frac{1}{1-\sigma}\int_{t-\tau(t)}^{t} f^{\mathrm{T}}(x(s))\Delta B^{\mathrm{T}}\Delta B f(x(s))ds. \end{aligned} \tag{10}$$

Calculating the derivative of (10) along the trajectories of (6), we have

$$\begin{aligned} \dot{V}(t) &= 2e^{\mathrm{T}}(t)\dot{e}(t) - 2\sum_{i=1}^{n}(\epsilon_i + l)\left(e_i^2 + \frac{\alpha}{\epsilon_i + l}x_i^2(t) \right) \\ &\quad + \frac{1}{1-\sigma} g^{\mathrm{T}}(e(t))g(e(t)) + \frac{1}{1-\sigma} f^{\mathrm{T}}(x(t))\Delta B^{\mathrm{T}}\Delta B f(x(t)) \\ &\quad - \frac{1-\dot{\tau}(t)}{1-\sigma} g^{\mathrm{T}}(e(t-\tau(t)))g(e(t-\tau(t))) \\ &\quad - \frac{1-\dot{\tau}(t)}{1-\sigma} f^{\mathrm{T}}(x(t-\tau(t)))\Delta B^{\mathrm{T}}\Delta B f(x(t-\tau(t))) \\ &\le 2e^{\mathrm{T}}(t)(-C_2 e(t) + A_2 g(e(t)) + B_2 g(e(t-\tau(t)))) \\ &\quad - 2e^{\mathrm{T}}(t)\Delta C x(t) + 2e^{\mathrm{T}}(t)\Delta A f(x(t)) \\ &\quad + 2e^{\mathrm{T}}(t)\Delta B f(x(t-\tau(t))) + 2e^{\mathrm{T}}(t)\epsilon e(t) \\ &\quad - 2\sum_{i=1}^{n}(\epsilon_i + l)\left(e_i^2 + \frac{\alpha}{\epsilon_i + l}x_i^2(t) \right) + \frac{1}{1-\sigma} g^{\mathrm{T}}(e(t))g(e(t)) \\ &\quad + \frac{1}{1-\sigma} f^{\mathrm{T}}(x(t))\Delta B^{\mathrm{T}}\Delta B f(x(t)) \\ &\quad - g^{\mathrm{T}}(e(t-\tau(t)))g(e(t-\tau(t))) \\ &\quad - f^{\mathrm{T}}(x(t-\tau(t)))\Delta B^{\mathrm{T}}\Delta B f(x(t-\tau(t))) \\ &\le 2e^{\mathrm{T}}(t)(-C_2 e(t) + A_2 g(e(t)) + B_2 g(e(t-\tau(t)))) \\ &\quad - 2e^{\mathrm{T}}(t)\Delta C x(t) + 2e^{\mathrm{T}}(t)\Delta A f(x(t)) \\ &\quad + 2e^{\mathrm{T}}(t)\Delta B f(x(t-\tau(t))) - 2\alpha\sum_{i=1}^{n} x_i^2(t) - l e^{\mathrm{T}}(t)e(t) \\ &\quad + \frac{1}{1-\sigma} g^{\mathrm{T}}(e(t))g(e(t)) + \frac{1}{1-\sigma} f^{\mathrm{T}}(x(t))\Delta B^{\mathrm{T}}\Delta B f(x(t)) \\ &\quad - g^{\mathrm{T}}(e(t-\tau(t)))g(e(t-\tau(t))) \\ &\quad - f^{\mathrm{T}}(x(t-\tau(t)))\Delta B^{\mathrm{T}}\Delta B f(x(t-\tau(t))). \end{aligned} \tag{11}$$

Using (8) and the Lemma 2 in [21], one can obtain

$$2e^{\mathrm{T}}(t)A_2 g(e(t)) \le e^{\mathrm{T}}(t)A_2A_2^{\mathrm{T}}e(t) + g^{\mathrm{T}}(e(t))g(e(t)), \tag{12}$$

$$\begin{aligned} 2e^{\mathrm{T}}(t)B_2 g(e(t-\tau(t))) &\le e^{\mathrm{T}}(t)B_2B_2^{\mathrm{T}}e(t) \\ &\quad + g^{\mathrm{T}}(e(t-\tau(t)))g(e(t-\tau(t))), \end{aligned} \tag{13}$$

$$-2e^{\mathrm{T}}(t)\Delta C x(t) \le \|e(t)\| + \|\Delta C\|\|x(t)\|, \tag{14}$$

$$2e^{\mathrm{T}}(t)\Delta A f(x(t)) \le \|e(t)\| + \|\Delta A\|K^2\|x(t)\|, \tag{15}$$

$$\begin{aligned} 2e^{\mathrm{T}}(t)\Delta B f(x(t-\tau(t))) &\le \|e(t)\| + f^{\mathrm{T}}(x(t-\tau(t)))\Delta B^{\mathrm{T}}\Delta B f \\ &\quad \cdot (x(t-\tau(t))), \end{aligned} \tag{16}$$

$$\frac{1}{1-\sigma} f^{\mathrm{T}}(x(t))\Delta B^{\mathrm{T}}\Delta B f(x(t)) \le \frac{1}{1-\sigma}\|\Delta B\|K^2\|x(t)\|. \tag{17}$$

Substituting (12), (13), (14), (15), (16), and (17) into (11) and combining

$$\left[\|\Delta C\| + \left(\|\Delta A\| + \frac{1}{1-\sigma}\|\Delta B\|\right)K^2\right]\|x(t)\| \le 2\alpha \tag{18}$$

yield

$$\begin{aligned} \dot{V}(t) &\le e^{\mathrm{T}}(t)\left(-2C_2 + A_2A_2^{\mathrm{T}} + B_2B_2^{\mathrm{T}} + (3-l)I\right)e(t) \\ &\quad + g^{\mathrm{T}}(e(t))g(e(t)) + \frac{1}{1-\sigma} g^{\mathrm{T}}(e(t))g(e(t)) \\ &\le e^{\mathrm{T}}(t)\Bigg(2\lambda_{\max}(-C_2) + \lambda_{\max}\left(A_2A_2^{\mathrm{T}}\right) + \lambda_{\max}\left(B_2B_2^{\mathrm{T}}\right) \\ &\quad + K^2 + \frac{1}{1-\sigma}K^2 + 3 - l\Bigg)e(t). \end{aligned} \tag{19}$$

We properly choose the constant l as

$$\begin{aligned} l &= 2\lambda_{\max}(-C_2) + \lambda_{\max}\left(A_2A_2^{\mathrm{T}}\right) + \lambda_{\max}\left(B_2B_2^{\mathrm{T}}\right) \\ &\quad + K^2 + \frac{1}{1-\sigma}K^2 + 4, \end{aligned} \tag{20}$$

then we have $\dot{V} \le -e^{\mathrm{T}}(t)e(t)$.

It is obvious that $\dot{V} = 0$ if and only if $e_i = 0, i = 1, 2, \dots, n$. It implies that the set $E = \{[e(t), \epsilon]^{\mathrm{T}} \in \mathbf{R}^{2n} : e(t) = 0, \epsilon = \epsilon_0 \in \mathbf{R}^n\}$ is the largest invariant set included in $M = \{\dot{V} = 0\}$ for system (6). Then, using the well-known Lyapunov-LaSalle-type theorem, the error converges asymptotically to E, that is, $e(t) \to 0$ and $\epsilon \to \epsilon_0$ as $t \to \infty$. Therefore, the synchronization of the CNSs (1) and (3) is achieved under the coupling (9). The proof is completed.

For the coupled CNSs without time-varying delay (i.e., $B_1 = 0$ in (1) and B_2 in (3)), one can easily derive the

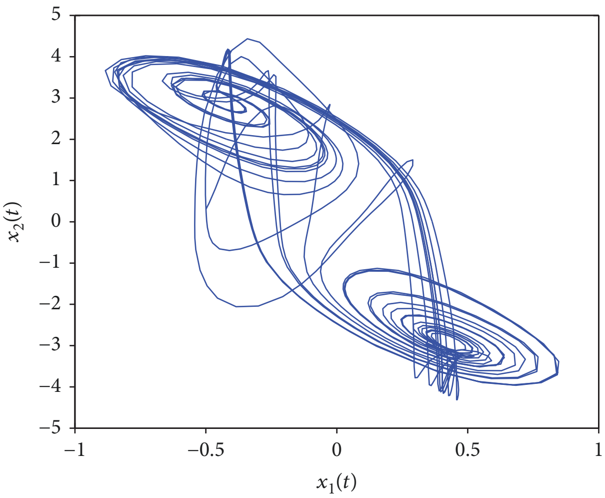


FIGURE 1: Attractor of neural network model [21].

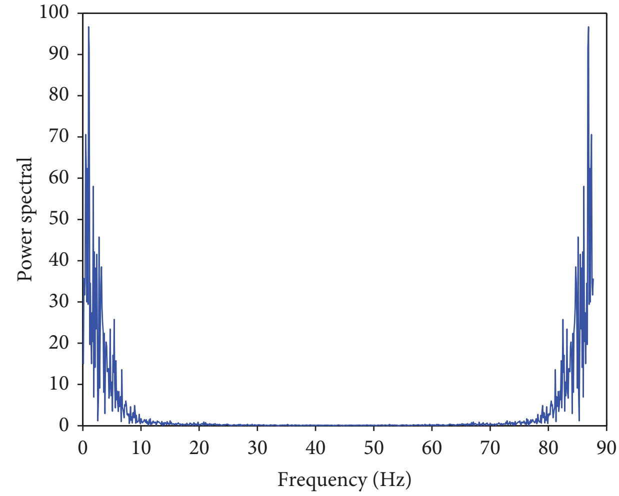


FIGURE 2: Power spectral plot of neural network model [21].

following corollary for two CNSs without delay (drive system and response system, resp.):

$$\dot{x}(t) = -C_1 x(t) + A_1 f(x(t)) + J, \tag{21}$$

and

$$\dot{y}(t) = -C_2 y(t) + A_2 f(y(t)) + J + \epsilon \circ (y(t) - x(t)). \tag{22}$$

Corollary 1. *Suppose that $\chi = \{x \in \mathbf{R}^n \mid \|x\| \le \alpha_1\}$ and the parameter mismatches satisfy $(\|\Delta C\| + \|\Delta A\| K^2) \le \alpha_2$, where $K = \max_{1\le i\le n}(k_i)$. Under the assumptions ($\mathbf{A1}$) and ($\mathbf{A2}$), set $\alpha = \alpha_1 \cdot \alpha_2$ and the controller $u(t) = \epsilon \circ (y(t) - x(t)) = \epsilon \circ e(t)$ with the following update law:*

$$\dot{\epsilon}_i = -\delta_i \left(e_i^2 + \frac{\alpha}{\epsilon_i + l} x_i^2(t) \right), \tag{23}$$

where $\delta_i > 0$ $(i = 1, 2, \ldots, n)$ are arbitrary constants; l is chosen as

$$l = 2\lambda_{\max}(-C_2) + \lambda_{\max}\left(A_2 A_2^{\mathrm{T}}\right) + K^2 + \frac{1}{1-\sigma} K^2 + 3. \tag{24}$$

Then, the controlled uncertain response system (3) will globally synchronize with the drive system (1).

Remark 1. It is noted from Theorem 1 that one can choose the constant δ_i properly to adjust the synchronization speed. Large adaptive gain δ_i will lead to fast synchronization, while small adaptive gain δ_i will result in slow synchronization. In addition, such a way is robust against the effect of noise. An extension effort that extends the results for systems with hybrid characteristics as in [7, 22–24] is possible, which remains an open problem.

Remark 2. In [9, 11, 13], the derived results are applicable for CNSs with parameter matches. While the results here are suitable for parameter mismatches. Therefore, our results have more expansive application foreground. In addition, using adaptive feedback method, the criterion obtained here improves and extends the results reported in [9, 11, 13].

4. Numerical Simulations

In this section, a numerical example is employed to illustrate our results. Simulation results show that the proposed adaptive synchronization scheme is valid.

Example 1. Consider the following two-order CNSs with time-varying delay:

$$\dot{x}(t) = -C_1 x(t) + A_1 f(x(t)) + B_1 f(x(t - \tau(t))) + J, \tag{25}$$

with

$$C_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, A_1 = \begin{bmatrix} 2.1 & -0.12 \\ -5.1 & 3.2 \end{bmatrix}, B_1 = \begin{bmatrix} -1.6 & -0.1 \\ -0.2 & -2.4 \end{bmatrix}, J = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \tag{26}$$

and

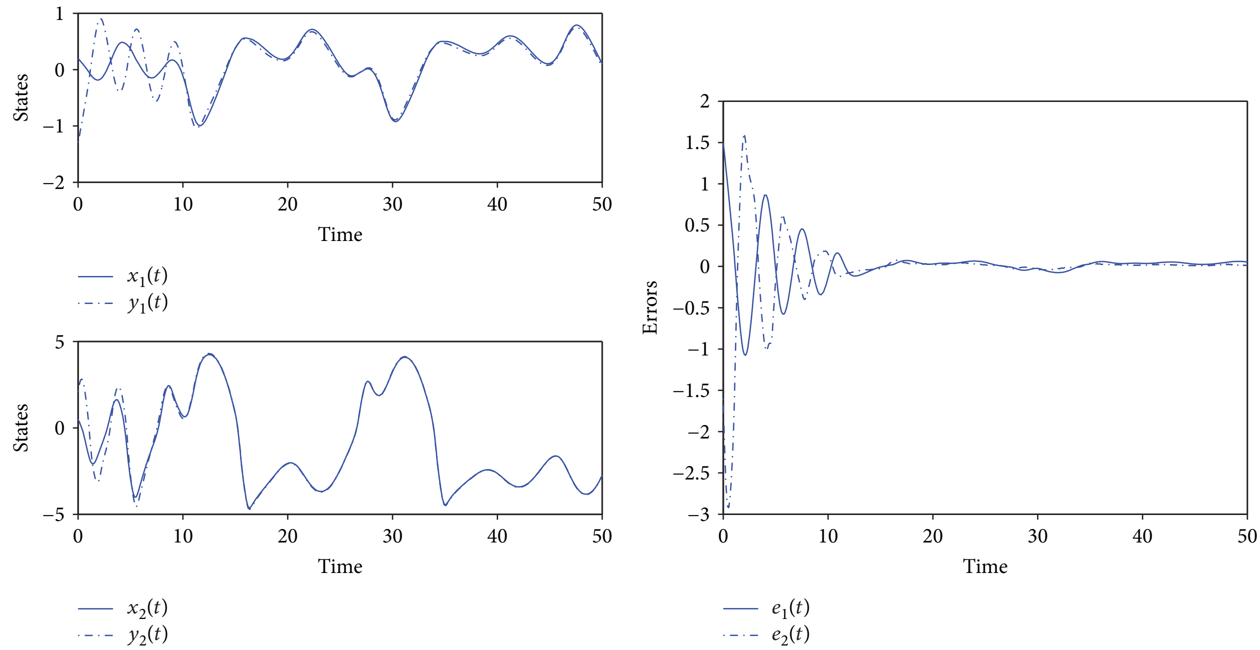
$\tau(t) = e^t/(1+e^t)$, where $x(t) = [x_1(t), x_2(t)]^{\mathrm{T}}$, $f(x(t)) = [\tanh(x_1(t)), \tanh(x_2(t))]^{\mathrm{T}}$.

It is seen that $k_1 = k_2 = 1$, and thus $K = 1$. Moreover,

$$\tau^* = 1, \dot{\tau}(t) = \frac{e^t}{(1+e^t)^2} \in [0, 0.5], \tag{27}$$

that is, $\sigma = 0.5$. Therefore, ($\mathbf{A1}$) and ($\mathbf{A2}$) hold.

Note that the neural system in this example is chaotic. Figure 1 shows the phase plot of the CNS, and Figure 2



(a) System states

(b) Synchronization errors

FIGURE 3: The temporal evolution of each variable and the plot of synchronization errors when the adaptive gains are $\delta_1 = \delta_2 = 0.5$.

illustrates the power spectral with initial values $\phi_1(s) = -0.5$ and $\phi_2(s) = 0.3, \forall s \in [-1, 0]$. It is found in Figure 1 that the double-scroll attractor is confined within the set.

$$\chi = \left\{ x = (x_1, x_2)^{\mathrm{T}} \mid -1 \le x_1 \le 1, -5 \le x_2 \le 5 \right\}. \tag{28}$$

In this case, it is verified that $\alpha_1 \simeq 5.0990$.

To verify the effectiveness of the proposed method, consider the output signals of drive system in CNS (25). Then, the controlled response system is given by

$$\dot{y}(t) = -\tilde{C}y(t) + \tilde{A}f(y(t)) + \tilde{B}f(y(t - \tilde{\tau}(t))) + J + u(t), \tag{29}$$

where $\tilde{\tau}(t) = 1 + 1/2 \sin(t)$, $y(t) = (y_1(t), y_2(t))^{\mathrm{T}}$ and

$$C_2 = \begin{bmatrix} 1.1 & 0 \\ 0 & 0.8 \end{bmatrix}, A_2 = \begin{bmatrix} 2.2 & -0.1 \\ -5.0 & 3.1 \end{bmatrix}, B_2 = \begin{bmatrix} -1.7 & -0.05 \\ -0.3 & -2.3 \end{bmatrix}, J = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \tag{30}$$

Therefore, we obtain

$$\|\Delta C\| + \left(\|\Delta A\| + \frac{1}{1-\sigma} \|\Delta B\| \right) K^2 \le \alpha_2 = 0.7128, \tag{31}$$

then $\alpha = \alpha_1 \cdot \alpha_2 = 3.6345$.

By Theorem 1, since

$$2\lambda_{\max}(-C_2) + \lambda_{\max}(A_2 A_2^{\mathrm{T}}) + \lambda_{\max}(B_2 B_2^{\mathrm{T}}) + K^2 + \frac{1}{1-\sigma} K^2 + 4 = 48.9486, \tag{32}$$

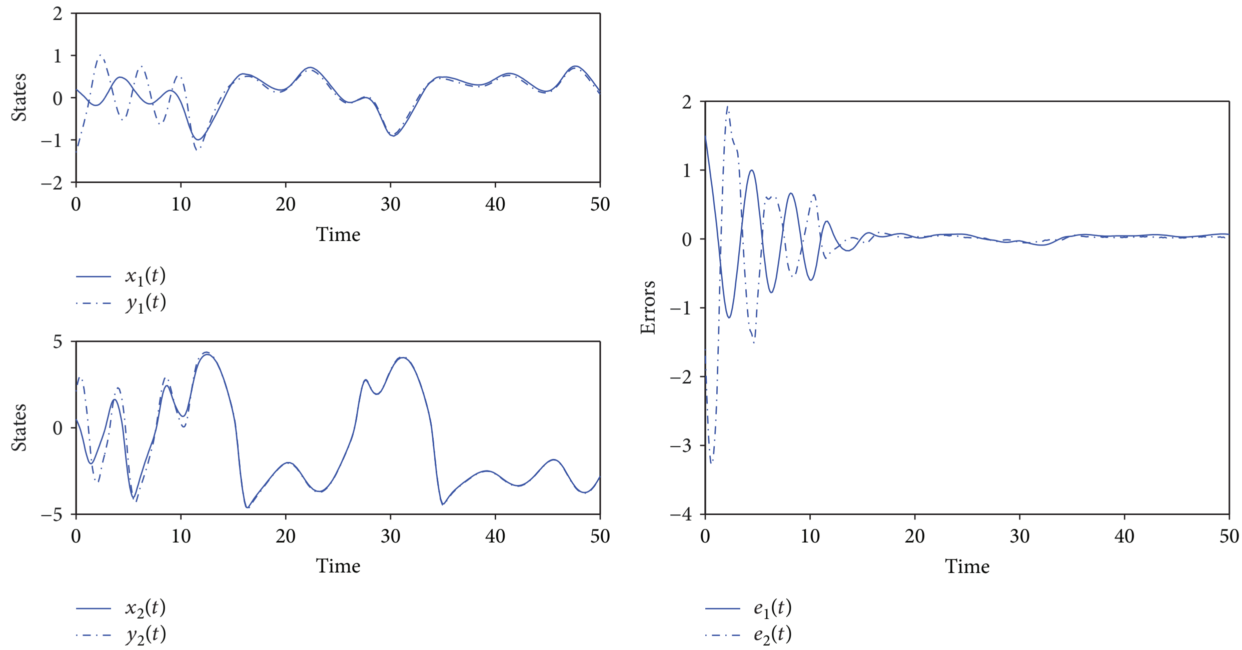
take $l = 50$. Then, we can design the controller $u(t) = \epsilon \circ (y(t) - x(t))$ with the adaptive update laws

$$\begin{aligned} \dot{\epsilon}_1 &= -0.5 \left((y_1(t) - x_1(t))^2 + \frac{\alpha}{\epsilon_1 + l} x_1^2(t) \right), \\ \dot{\epsilon}_2 &= -0.5 \left((y_2(t) - x_2(t))^2 + \frac{\alpha}{\epsilon_2 + l} x_2^2(t) \right). \end{aligned} \tag{33}$$

Here, the adaptive gains are taken as $\delta_1 = \delta_2 = 0.5$. Next, suppose the initial conditions are

$$\begin{aligned} (\phi_1(s), \phi_2(s))^{\mathrm{T}} &= (0.2, 0.5)^{\mathrm{T}}, = (\psi_1(s), \psi_2(s))^{\mathrm{T}} \\ &= (-1.3, 2.1)^{\mathrm{T}}, s \in [-1, 0], \end{aligned} \tag{34}$$

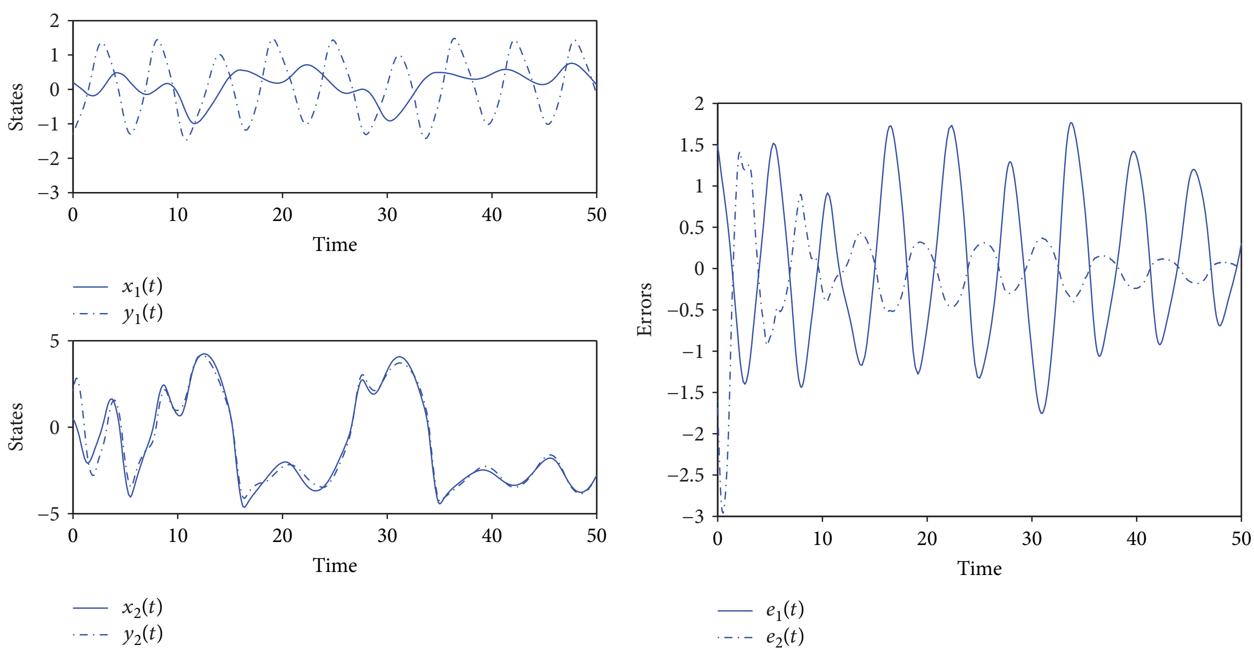
respectively, and $\epsilon_1(0) = \epsilon_2(0) = 0$. The simulation results are depicted in Figures 3–6. Figure 3 shows the temporal evolution of states and errors for $\delta_1 = \delta_2 = 0.5$. When $\delta_1 = \delta_2 = 0.3$, namely, decreasing the update gain of coupling strength, Figure 4 shows the corresponding simulation results and it is revealed that it takes longer to achieve synchronization. From Figures 3 and 4, we found that less time is needed to achieve synchronization when larger δ_1 and δ_2 are taken. When $\delta_1 = 0$ and $\delta_2 = 0.5$, Figure 5 shows the results for the case that only x_2 is chosen



(a) System states

(b) Synchronization errors

FIGURE 4: The temporal evolution of each variable and the plot of synchronization errors when the adaptive adaptive gains are $\delta_1 = \delta_2 = 0.3$.



(a) System states

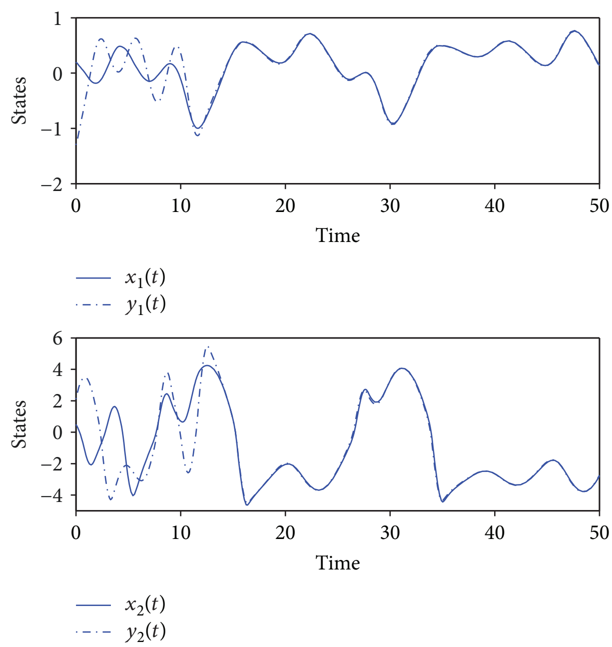
(b) Synchronization errors

FIGURE 5: The temporal evolution of each variable and the plot of synchronization errors driven only by the signal x_2, and the synchronization cannot be achieved in Example 1.

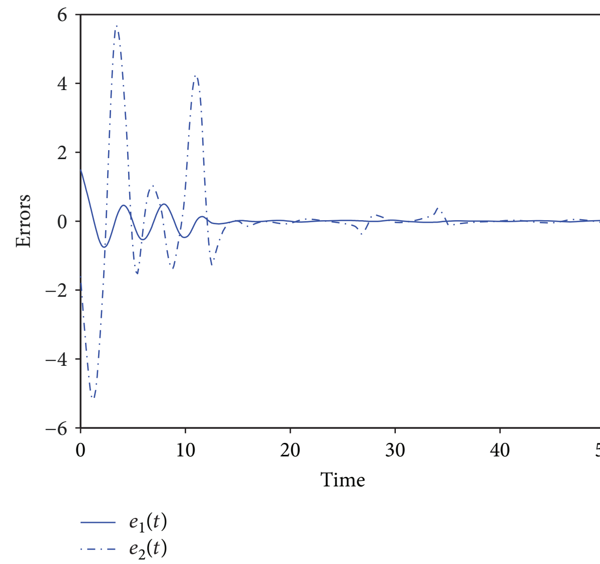
as the drive signal. When $\delta_1 = 0.5$ and $\delta_2 = 0$, Figure 6 shows the results for the case that only x_1 is chosen as the drive signal. It can be seen that the coupling between x_1 and y_1 would drive the two CNSs (25) and (29) synchronized, while the coupling between x_2 and y_2 is invalid.

5. Conclusions

This paper has analyzed the adaptive synchronization between two coupled CNSs with parameter mismatches by applying an invariance principle and a simple adaptive



(a) System states



(b) Synchronization errors

FIGURE 6: The temporal evolution of each variable and the plot of synchronization errors driven only by the signal x_1, and the synchronization is achieved in Example 1.

feedback approach. Practical and less restrictive conditions have been presented for adaptive synchronization of CNSs. Numerical simulations of two-order-coupled CNSs have also been provided to verify the usefulness and practicability of proposed theoretical results.

Data Availability

The main results of our work are proved in detail, which can be seen in the context. All data related to the simulation part of our results are given in Section 4. The readers can replicate the analysis clearly.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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