Is There a Plural Object?

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> A class is the indefinite multitude of individuals denoted by a general name. — J. S. Mill (2002, I.ii.3)

> Sets are multiplicities which are also unities. — Gödel (Wang 1997, 254)

> . . . if we admit classes as objects, we must suppose that the same object can be both one and many, which seems impossible. — Whitehead and Russell (1962, 72)

A *plurality* or *plural object* is a single object, some one thing, that is also many.[1] Despite the contradictory character of this description, a wide range of authors hold that there is such an object. This view is implicit in a prominent approach to the semantics of plural constructions of natural languages, the one that takes a plural term (e.g., 'Venus and Serena') to refer to a composite object: a set or class, a fusion or mereological sum, a group or aggregate, or the like. I

[1] I use 'many' interchangeably with 'more than one' and 'two or more'.

think the approach is fundamentally mistaken because the notion of plurality is inconsistent. Nothing (no one object) whatsoever can be many, no matter how many are its members, elements, parts, etc. This is a logical truth, which we can see by attending to the logic of plural constructions (in short, plurals).

Call the thesis that there is a plurality or plural object *pluralitism*. In §1, I present usual arguments for the thesis and formulate its close relatives (e.g., the composition as identity thesis). Then I state some basic logical truths germane to plurals in §2, and argue that pluralitism and its relatives violate the logic of plurals in §§3-4. In §5, I conclude by relating the discussion of pluralitism and its relatives to treatments of plurals and the nature of number.

1. Routes to Pluralitism

Plato propounds pluralitism in the *Parmenides*. In the dialogue, he has Socrates challenge Parmenides and Zeno with an argument for the thesis:

> . . . if someone should demonstrate that I am one thing and many, what is astonishing about that? He will say, when he wants to show that I'm many, that my right side is different from my left . . . . But when he wants to show that I'm one, he will say I'm one person among the seven of us . . . . Thus he shows that both are true. (*Parm.* 129c-d)

Argued here is that a person, Socrates, is both one and many because (a) he is one person, and (b) his left and right sides differ from each other. The argument falls short. Because his left and right sides are different, *they* are indeed many (viz., two), but this does not mean that *Socrates himself* is so. Why would Plato take the argument to deliver this conclusion as well?

One can reach the conclusion if one assumes that his two sides (taken together) are *identical* with Socrates. And Plato holds this thesis elsewhere. In the *Theaetetus*, he holds that "when a thing has parts, the whole is necessarily all the parts" (*Theaet*. 204a). If so, Socrates, who has (proper) parts, would be identical with his parts (taken together).

David Lewis holds a closely related thesis, which he calls "the Thesis of *Composition as Identity*" (1991, 82; original italics).[2] While saying that the parts of something "compose" the thing, which he says is the "fusion" of the parts, he holds that the fusion of some cats, for example, "is nothing over and above the cats that compose it. It just *is* them. They just *are* it" (*ibid*., 81; original italics). And he continues:

> . . . composition — . . . the many-one relation of many parts to their fusion — is like identity. The 'are' of composition is, so to speak, the plural form of the 'is' of identity. (*Ibid*., 82)

The thesis formulated here is considerably weaker than Plato's. It just states that composition is "like" identity, and the qualification "so to speak" suggests that the 'are' of composition might not literally be the plural form of the identity predicate. By contrast, Plato's thesis states that the parts of a whole (taken together) are not just akin to but are literally identical with the whole.

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[2]Lewis (*ibid*., 84), erroneously I think, attributes the thesis to Baxter (1998a; 1998b). See my (1999b, 158, note 13).

Using this thesis, one can conclude that *the whole* is many from the observation that *its parts* are so. One cannot do so with Lewis's thesis. One who merely holds that 'The copper *is* the statue' is true on the grounds that 'is' might be used for constitution because constitution is akin to identity cannot use the claim to derive 'The copper was made yesterday' from 'The statue was made yesterday', or 'Alice likes the copper' from 'Alice likes the statue.' Similarly, one cannot derive 'Socrates is many' from 'Socrates's parts are many' by invoking the thesis that the parts compose Socrates while holding that one can express this thesis by 'The parts *are* Socrates' on the grounds that the plural form 'are' of the identity predicate might be used for composition because composition is *akin* to identity.

It is necessary, then, to distinguish two versions of the thesis that composition "is" identity: the *weak* and *strong composition theses*.[3] While the weak composition thesis merely holds that composition is akin to identity, the strong composition thesis holds that it *is* a kind of identity. On the latter thesis, the whole must have any property the parts (taken together) have. Not so on the former: the whole might lack a property the parts (taken together) have. Balking at the stronger thesis, Lewis settles for its emasculated cousin while calling it the *composition as identity thesis*.[4] This name better suits the stronger thesis, and I shall henceforth use the name for it. This is the thesis Plato assumes to argue for pluralitism.

Pluralitism, we have seen, results from the composition as identity thesis. It also results from a prominent approach to the logic and semantics of plurals, what I call the *plurality*

[3]See my (1999b, 145f), which distinguishes the two theses.

[4]Lewis (1991) uses it to argue that mereology is "ontologically innocent". I think the argument fails. See my (1999b).

approach. This approach takes a plural term to refer to some one thing, a composite object: a set or class, a fusion or mereological sum, an aggregate or agglomeration, a group or totality, etc. For example, 'Venus and Serena', on the approach, might be taken to refer to a set, fusion, group, etc. that somehow comprehends both Venus and Serena. And 'Venus and Serena are sisters', on the approach, attributes to the composite object a property signified by the predicate '*to* be sisters', and the sentence is true if the object has the property.

Suppose that proponents of the plurality approach take the plural term 'Venus and Serena' to refer to some one thing, a single object, be it a set, a fusion, or whatnot. Then they must take the one object (call it *Venuserena*) to be identical with the two tennis players (taken together); they must accept:

(1) *Venus and Serena* (taken together) are identical with *Venuserena*.

To see this, note that one cannot deny:

(2) 'Venus and Serena' refers to *Venus and Serena* (taken together).

And they hold:

(3) 'Venus and Serena' refers to *Venuserena*.

(2) and (3) imply (1), just as 'The term "Tully" refers to *Tully*' and 'The term "Tully" refers to

Cicero' imply '*Tully* is identical with *Cicero*.' The semantic analysis the approach gives to (1) yields the same conclusion. The statement is true if 'Venus and Serena' refers to the same thing as 'Venuserena'; and the two terms refer to the same thing on the approach. So the approach yields (1). And this yields pluralitism; Venuserena is one, and Venus and Serena are two.

Accordingly, many proponents of the approach hold that plural terms refer to "plural objects" or "pluralities".[5] This terminology accords by and large with mine. Some plural terms are atypical in that they do not refer to *two or more* objects (taken together). For example, 'Cicero and Tully' or 'the males identical with Cicero' refers to one object: Cicero (i.e., Tully); and 'the females identical with Cicero' does not refer at all. Call such plural terms *degenerate*. Then typical plural terms (e.g., 'Venus and Serena') are not degenerate. And the approach to plurals in question takes such a plural term to refer to a plurality (in my sense), an object that is both one and many.

I think the plurality approach is fundamentally mistaken because pluralitism is logically false. Dummett comes close to this view. He holds, correctly I think,[6] that pluralitism stems from a logical error. He says, "There is no such thing as a 'plurality', which is the misbegotten invention of a faulty logic" (1991, 93). But his objection to pluralitism is not that the thesis itself contravenes logic. He argues against it on the grounds that composite objects one might identify as pluralities (e.g., an army) might not exist while their components (e.g., its solders) do. But pluralitists might identify pluralities with sets (e.g., {Venus, Serena}), which arguably exist as long as their members do. Or they might decline to identify pluralities with sets, fusions, groups,

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[5]See, e.g., Link (1998, 1).

etc., but simply hold that ‘Venus and Serena’, for example, refers to a plurality, and that this is an object that exists as long as both Venus and Serena do. These versions of pluralitism are immune to Dummett’s objection.

I think pluralitism has a more serious problem. It is not that there happens to be “no such thing as a ‘plurality’”, but that there *cannot* be such an object. Logic rules out the existence of a plurality. (1), for example, is logically false (unless Venus is Serena) no matter what Venuserena is. To see this, it is necessary to attend to the logic of plurals. In the next section, I formulate some basic logical truths germane to plurals to prepare for subsequent discussions of pluralitism.

2. Plurals and Their Logic

The two routes to pluralitism presented above converge to a crucial intermediate thesis. Plato’s route proceeds via the thesis that Socrates’s parts, for example, are identical with Socrates, and the plurality approach route via the thesis that Venus and Serena, for example, are identical with some one thing (e.g., Venuserena). Both theses are instances of what I call the *many-one identity thesis*:

> *Many-one identity thesis*: Some things that are many (taken together) are identical with some one thing.

This thesis is an identical twin of pluralitism. They are equivalent, as it is straightforward to see.

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[6]But my diagnosis is different from his. See §5.

And both are logically false. To see this, it is useful to regiment the natural language plurals used to state the theses and other related statements. By doing so, we can give precise and concise formulations of basic logical truths germane to them.

2.1. Plural Languages

By regimenting basic plural constructions of natural languages, we can obtain regimented languages that result from adding refinements of those plurals to elementary languages, languages I call (*regimented*) *plural languages*. I have presented such languages in other works to account for the logic and meaning of natural language plurals.[7] Here let me give a sketch.

Elementary languages result from regimenting basic singular constructions (in short, *singulars*) of natural languages. They contain no counterparts of natural language plurals, and their constants, predicates, variables, and quantifiers are refinements of natural language singulars. The constants (e.g., 'a', 'b') amount to proper names (e.g., 'Ali', 'Baba'); the variables (e.g., 'x', 'y') to singular pronouns (e.g., 'he', 'she', 'it') as used anaphorically, as in 'A boy loves a girl, and *she* is happy'; the predicates (e.g., 'B^1', '$=$', 'L^2', 'G^3')[8] to verbs or verb phrases in the singular (e.g., 'is a boy', 'is identical with', 'loves', 'gives ... to'); and the quantifiers '$\exists$' and '$\forall$' to 'something' and 'everything' (or 'anything'). So they are all singular expressions.

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[7] See my (2002, Chapter 2; 2005; 2006). See also my (1998, §3; 1999a, §3; forthcoming).

[8] Superscripts on predicates indicate their arities. They are omitted when the arities are clear from the contexts.

We can obtain languages suitable for regimenting basic plurals (as well as singulars) by adding to elementary languages *plural cousins* of singular variables, quantifiers, and predicates:

(a) *plural variables*: '*xs*', '*ys*', '*zs*', etc.[9]

(b) *plural predicates*: '$\boldsymbol{C}^1$' ('*to* cooperate'), 'H^2' ('is one of'), '$\boldsymbol{D}^2$' ('*to* discover'), '$\boldsymbol{L}^2$' ('*to* lift'), '$\boldsymbol{W}^2$' ('*to* write'), etc.

(c) *plural quantifiers*: the existential '$\boldsymbol{\Sigma}$' ('some things') and the universal '$\boldsymbol{\Pi}$' ('any things')

Plural variables are refinements of the plural pronoun 'they' as used anaphorically (as in 'Some scientists worked in Britain, and *they* discovered the structure of DNA'). Plural quantifiers, which bind plural variables, are refinements of 'some things' and 'any things'. And plural predicates are refinements of natural language predicates (e.g., '*to* lift'), not their singular or plural forms. So they can combine with plural terms (e.g., 'they'); they have argument places that admit plural terms: the only argument place of '$\boldsymbol{C}^1$', the first of '$\boldsymbol{D}^2$', the second of 'H^2',[10] etc. (Call such argument places *plural argument places*, and predicates with plural argument places *plural predicates*.) By contrast, elementary language predicates are refinements of *singular forms* of natural language predicates (e.g., 'is funny', 'loves') and have only *singular* argument places, those that admit only singular terms. (Call such predicates *singular*

[9]I add '*s*' to lower-case letters of the Roman alphabet to write plural variables, but it is not a semantically significant component of the variables (so '*xs*' has no more tie to '*x*' than to '*y*').

predicates.)

Let me make two points about plural language predicates and quantifiers. Plural argument places might be *exclusively plural* (admitting only plural terms) or *neutral* (admitting singular terms as well). I think most natural language predicates have neutral argument places. The same predicate '*to* lift', for example, figures in 'He *lifts* Baba' and 'They *lift* Baba' (while taking its singular and plural forms), and has a passive form that figures in both 'Baba *is lifted by* him' and 'Baba *is lifted by* them.' Similarly, 'Cicero and Tully are one' and 'Cicero is one' have the same predicate (viz., '*to* be one'); likewise with 'Venus and Serena are many' and 'Cicero is not many.' Accordingly, the plural language predicate '$\boldsymbol{L}^2$', for example, has *neutral* argument places and figures in both of the plural language counterparts of 'He *lifts* Baba' and 'They *lift* Baba': '$\boldsymbol{L}(x, b)$' and '$\boldsymbol{L}(xs, b)$.'[11]

The plural quantifiers '$\boldsymbol{\Sigma}$' and '$\boldsymbol{\Pi}$' amount to 'some one or more things' and 'any one or more things', not to 'some two or more things' and 'any two or more things'. I think it is the same with the English 'some things' and 'any things' and their restricted cousins: 'some Romans', 'any Romans', etc.[12] For example, 'Some Romans are famous' follows from 'Cicero and Tully are famous Romans', which follows from 'Cicero is a famous Roman and Tully is a famous Roman'; and 'Any Romans can speak Latin' implies 'Any Roman can speak Latin.'[13]

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[10]The first argument place of '$\mathbf{H}^2$' is singular.

[11]Unlike English predicates, plural language predicates do not take singular or plural forms.

[12]So I disagree with, e.g., Link (1998, 17 & 21).

[13]By contrast, 'Any two or more Romans can speak Latin' does not; it is vacuously true if

Thus 'Cicero is a famous Roman' implies 'Some Romans are famous', and 'Any Romans can speak Latin' implies 'Cicero can speak Latin if he is a Roman.' It is the same with their plural language paraphrases.

Now, plural languages have a plural predicate of special significance, the plural language counterpart of 'is one of': '$\mathbf{H}^2$' (in short, '$\mathbf{H}$').[14] Like the singular identity predicate '=', it is a logical predicate. And we can use it to define the plural cousin of the singular identity predicate, '≈':

Def. 1 (*Plural Identity*):

$xs{\approx}ys \equiv: \forall x[\mathbf{H}(x, xs) \leftrightarrow \mathbf{H}(x, ys)]$. (These *are identical with* those ≡: something is one of these if and only if it is one of those.)[15]

This, like its singular cousin, supports substitution *salva veritate*:

Substitutivity of Plural Identity:[16]

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there is only one Roman.

[14]We can use it to define the *neutral expansion* π^N of a predicate π:

$\pi^N(xs) \equiv: \forall y[\mathbf{H}(y, xs) \rightarrow \pi(y)]$, where π is a predicate.

For example, the neutral expansion 'R^N' of the singular predicate 'R' (or 'is a Roman') amounts to the plural predicate '*to* be Roman(s)'.

[15]In English glosses on definitions, I use 'this', 'that', 'these', and 'those' (as well as 'it' and 'they') as counterparts of variables.

[16]I use Greek letters as metavariables.

$[\tau \approx \mu \wedge \phi] \rightarrow \phi'$, where ϕ' results from ϕ by replacing an occurrence of τ by μ and μ is substitutable for that occurrence of τ in ϕ.

And we can use '$\mathbf{H}^2$' to define predicates for being one and being many:[17]

Def. 2 (***One*** and ***Many***):

(a) **One**(xs) $\equiv$: $\exists y \forall z[\mathbf{H}(z, xs) \leftrightarrow z{=}y]$. (They *are one* $\equiv$: something is such that something is one of them if and only if the latter thing is the former thing.)

(b) **Many**(xs) $\equiv$: $\exists y \exists z[z{\neq}y \wedge \mathbf{H}(y, xs) \wedge \mathbf{H}(z, xs)]$. (They *are many* $\equiv$: there is something that is one of them and something else that is one of them.)

Meager plural languages result from adding to elementary languages the expressions discussed above: plural predicates, variables, and quantifiers. It is useful to augment those languages with operators for forming complex plural terms:[18]

(a) the term connective '**@**';

(b) the operator '< ... : —>' that yields plural definite descriptions of the form $<\nu: \phi>$, where ν is a singular variable;

(c) the operator '**I**' that yields plural definite descriptions of the form $(\mathbf{I}\omega)\phi$, where ω

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[17]They are one-place neutral predicates.

[18]The operators can be defined in meager plural languages. See my (2006, 244f).

is a plural variable.

The term connective ‘**@**’ amounts to the ‘and’ in the plural term ‘Venus and Serena’. Plural terms of the form $<\nu: \phi>$ are counterparts of one kind of plural definite descriptions: ‘the Romans identical with Cicero’, ‘the Residents of London’, etc. And those of the form $(\mathbf{I}\omega)\phi$ are counterparts of another kind of plural definite descriptions: ‘those who wrote *Principia Mathematica* (in short, *PM*)’, ‘the scientists who discovered the structure of DNA’, etc. Using the operators, we can paraphrase ‘Venus and Serena’, ‘the Romans identical with Cicero’, and ‘those who wrote *PM*’ by ‘$[v\boldsymbol{@}s]$’, ‘$<x: R(x) \wedge x=c>$’, and ‘$(\mathbf{I}xs)\boldsymbol{W}(xs, p)$’ (where ‘$p$’ is for *PM*),[19] respectively.

It is straightforward to paraphrase the following, for example, into plural languages:

(i) Venus and Serena are many.

(ii) Cicero is one.

(iii) Something is one of the Romans identical with Cicero if and only if it is a Roman identical with Cicero.

(vi) There are some things that are the Romans identical with Cicero if and only if there is a Roman identical with Cicero.

(v) The Romans identical with Cicero (taken together) *are* Cicero.

Here are their plural language paraphrases:

(i′) $\mathbf{Many}([v@s])$.

(ii′) $\mathbf{One}(c)$.

(iii′) $\forall x(\mathbf{H}(x, <x{:}\ R(x) \wedge x{=}c>) \leftrightarrow [R(x) \wedge x{=}c])$.

(vi′) $\mathbf{\Sigma} xs[xs \approx <x{:}\ R(x) \wedge x{=}c>] \leftrightarrow \exists x[R(x) \wedge x{=}c]$.

(v′) $<x{:}\ R(x) \wedge x{=}c> \approx c$.

And we can use the logic of plurals to see that the statements are either logical truths or logical consequences of ‘Venus is not Serena’ or ‘Cicero is a Roman.’ The logic is discussed next.

2.2. Plural Logic

Let me formulate some logical truths of plural languages for subsequent discussions:[20]

Th. 1. $\exists y\mathbf{H}(y, xs)$. (Given any things whatsoever, there is something that is one of them.)[21]

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[19]This paraphrase ignores the past tense in ‘wrote’.

[20]In stating them, I omit frontal universal quantifiers binding free variables. So the full formulation of, e.g., *Th*. 1 is ‘$\mathbf{\Pi} xs\exists y\mathbf{H}(y, xs)$.’ Note that substitutivity of plural identity, mentioned above, is a schema of logical truths.

[21]This is one of the basic logical truths that distinguishes the logic of plurals from higher-order logics. Its second-order analogue, ‘$\forall P\exists yP(y)$’, is logically false; its instances include ‘$\exists y\ y{\neq}y$.’

Th. 2. $\mathbf{H}(x, [ys@zs]) \leftrightarrow [\mathbf{H}(x, ys) \vee \mathbf{H}(x, zs)]$. (Something is one of these and those if and only if it is either one of these or one of those.)

Th. 3. $\mathbf{H}(x, [y@z]) \leftrightarrow [x=y \vee x=z]$. (Something is one of this and that if and only if it is either this or that.)

Th. 4. $\mathbf{H}(x, [ys@z]) \leftrightarrow [\mathbf{H}(x, ys) \vee x=z]$. (Something is one of these and that if and only if it is one of these or it is that.)

Th. 5. $\mathbf{H}(x, y) \leftrightarrow x=y$. (Something is one of something if and only if the former is the latter.)

Th. 6. $\mathbf{H}(x, \langle y\text{: } \phi(y)\rangle) \leftrightarrow \phi(x)$, if '$x$' is substitutable for '$y$' in $\phi(y)$. (Something is one of the so-and-so's if and only if it is a so-and-so.)

Th. 7. $\mathbf{\Sigma} xs\ xs \approx \langle x\text{: } \phi\rangle \leftrightarrow \exists x\phi$. (There are some things that are identical with the so-and-so's if and only if there is a so-and-so.)

Th. 8. $\forall x\mathbf{One}(x)$. (Anything whatsoever is one.)

We can use these to show that (i)–(v), discussed in §2.1, are true. 'Venus is not Serena' implies (i) by *Th*. 3. (ii) is an instance of *Th*. 8 (which follows from *Th*. 5). (iii) and (vi) are instances of *Th*. 6 and *Th*. 7, respectively. And (v) follows from 'Cicero is a Roman' by *Ths*. 5 & 7.[22]

The logical truths listed above are theorems of a system for the logic of plurals I have

[22]'$R(c)$' implies '$\forall x[(R(x) \wedge x=c) \leftrightarrow \mathbf{H}(x, c)]$' (*Th*. 5), and this is equivalent to (v′) by *Th*. 7. (To show this, it is necessary to use (iii′) as well as *Def*. 1.)

presented elsewhere.[23] For the present purpose, however, it is not necessary to present the system or the semantic characterization of the logic underlying the system. (I call the characterization *plural logic*.) I think those logical truths are intuitively clear; they merely codify logical truths we invoke, if implicitly, to reach commonly accepted statements, such as (i)–(v). Among them, *Th*. 5 might be the least familiar. One would rarely state the theorem or its instances.[24] But it is implicitly invoked in accepting its consequences, such as *Th*. 8 or its instances (e.g., 'Socrates is one'). And it follows from more familiar logical truths: *Ths*. 2 & 3.[25]

Some might object that *Th*. 5 is ill-formed because the English counterpart of '**H**', 'is one of', is an *exclusively plural* predicate, whose second argument place admits no singular terms. A problem with this objection is that the English counterparts of '**One**' and '**Many**' (i.e., '*to* be one' and '*to* be many') are clearly neutral. For example, 'Cicero *is one*, but Venus and Serena *are* not *one*' and 'Venus and Serena *are many*, but *Cicero is* not *many*' are well-formed. I think this gives a good, if indirect, reason to consider 'is one of' neutral, for *Def*. 2 yields good analyses of those predicates. Some might disagree. They might insist that one cannot give correct analyses of the predicates in terms of 'is one of' because they are neutral while 'is one of' is exclusively plural.

We can meet this objection by reformulating *Th*. 5 without assuming that '**H**' (or 'is one of') is neutral. To do so, it is useful to define its cousin, '$\mathbf{H}'$' (read as 'is-one-of'):

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[23]See my (1999a, §3; 2002, Chapter 2; 2006).

[24]But see Frege (1980, 40), which is discussed in §5.

[25]They have instances that imply *Th*. 5: '$\mathbf{H}(x, [y@y]) \leftrightarrow [x{=}y \lor x{=}y]$' and '$\mathbf{H}(x, [y@y]) \leftrightarrow [\mathbf{H}(x, y) \lor \mathbf{H}(x, y)]$.'

Def. 3. $\mathbf{H}'(x, ys)$ ≡: $\forall z\mathbf{H}(x, [ys@z])$. (It *is-one-of* them ≡: it is one of [them and anything].)

We can use this to reformulate the theorem:

Th. 5′. $\mathbf{H}'(x, y) \leftrightarrow x{=}y$. (Something is-one-of something if and only if the former is the latter.)

And we can show that this is a logical truth. It is incontrovertibly well-formed (for '$\mathbf{H}'$' is clearly neutral), and follows from *Th*. 3 (for '$\forall z[x{=}y \vee x{=}z]$' is equivalent to '$x{=}y$').

And we can reformulate the analyses of '*to* be one' and '*to* be many' underlying *Def*. 2 by giving alternative definitions of their plural language counterparts. We can use '$\mathbf{H}'$' to define cousins of '**One**' and '**Many**', '**One′**' and '**Many′**':

Def. 2′ (***One′*** and ***Many′***):

(a) $\mathbf{One}'(xs)$ ≡: $\exists y\forall z[\mathbf{H}'(z, xs) \leftrightarrow z{=}y]$. (They *are one* ≡: something is such that something is-one-of them if and only if the latter is the former.)

(b) $\mathbf{Many}'(xs)$ ≡: $\exists y\exists z[z{\neq}y \wedge \mathbf{H}'(y, xs) \wedge \mathbf{H}'(z, xs)]$. (They *are many* ≡: there is something that is-one-of them, and something else that is-one-of them.)

Like ‘$\mathbf{H}'$’, these are clearly neutral. And all three predicates agree with their senior cousins:

Th. 9 (*Agreement*):

(a) $\mathbf{H}'(x, ys) \leftrightarrow \mathbf{H}(x, ys)$.

(b) $\mathbf{One}'(xs) \leftrightarrow \mathbf{One}(xs)$.

(c) $\mathbf{Many}'(xs) \leftrightarrow \mathbf{Many}(xs)$.

Th. 9(a) follows from *Th*. 4 and implies *Ths*. 9(b) & 9(c).

Now, we can regain all the theorems listed above by reformulating them with ‘$\mathbf{H}'$’ and ‘$\mathbf{One}'$’. All of them except *Ths*. 5 & 8 imply their junior cousins by *Th*. 9. And *Th*. 5′ is a logical truth, as we have seen, and it implies the junior cousin of *Th*. 8:

Th. 8′. $\forall x\mathbf{One}'(x)$. (Anything whatsoever is one.)

3. Pluralitism and the many-one identity thesis

We can now turn to pluralitism and the many-one identity thesis. The theses can be formulated in plural languages:

[P] *Pluralitism*:

$\exists x[\mathbf{One}(x) \wedge \mathbf{Many}(x)]$. (There is something that is both one and many.)

[M] *Many-one Identity Thesis*:

$\Sigma xs \exists y[\mathbf{Many}(xs) \wedge \mathbf{One}(y) \wedge xs{\approx}y]$. (Some things that are many are identical with some one thing.)

These are logically false, for the following are logical truths:

Th. 10. (*Anti-pluralitism*):

(a) $\sim[\mathbf{One}(xs) \wedge \mathbf{Many}(xs)]$. (There are no things that are both one and many.)

(b) $\sim[\mathbf{One}(x) \wedge \mathbf{Many}(x)]$. (There is nothing that is both one and many.)

(c) $\sim\mathbf{Many}(x)$. (Nothing is many.)

(d) $\mathbf{Many}(xs) \leftrightarrow \sim\mathbf{One}(xs)$. (Some things are many if and only if they are not one.)

We can see that these are logical truths by applying the definitions of '**One**' and '**Many**'. Applying the definitions to '$\mathbf{One}(xs)$' and '$\mathbf{Many}(xs)$' yields logically incompatible sentences:

(i) $\exists y \forall z[\mathbf{H}(z, xs) \leftrightarrow z{=}y]$.

(ii) $\exists y \exists z[z{\neq}y \wedge \mathbf{H}(y, xs) \wedge \mathbf{H}(z, xs)]$.

So *Th*. 10(a) is logically true. Similarly, *Th*. 10(b) is logically true because the definitions of

‘**One**(x)’ and ‘**Many**(x)’ are incompatible.[26] *Th*. 10(c) follows from *Th*s. 8 & 10(b). And *Th*. 10(d) is logically true because (i) and (ii) are incompatible but one of them must hold.[27]

Using *Th*. 10, it is straightforward to see that [P] and [M] are logically false. [P] contradicts *Th*. 10(b). So does [M], which implies [P].[28]

One might give alternative formulations of pluralism and the many-one identity thesis:

[P′] $\exists x$**Many**(x). (There is something that is many.)

[P″] $\boldsymbol{\Sigma}xs$[**One**(xs) $\wedge$ **Many**(xs)]. (There are some things that are both one and many.)

[M′] $\boldsymbol{\Sigma}xs\exists y$[**Many**($xs$) $\wedge$ $xs{\approx}y$]. (Some things that are many are identical with something.)

[M″] $\boldsymbol{\Sigma}xs\boldsymbol{\Sigma}ys$[**Many**($xs$) $\wedge$ **One**(ys) $\wedge$ $xs{\approx}ys$]. (Some things that are many are identical with some things that are one.)

[P′] and [P″], which are alternatives to [M], are also logically false. [P′] and [P″] contradict *Th*. 10(c) and *Th*. 10(a), respectively. [M′] and [M″], which are alternatives to [M], are also logically false; they imply [P′] or [P″] by substitutivity.

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[26] And it follows from 10(a).

[27] For *Th*. 1 is logically true.

[28] And [M] directly contradicts both 10(a) and 10(b); ‘[**Many**(xs) $\wedge$ **One**(y) $\wedge$ $xs{\approx}y$]’ implies both ‘[**Many**(xs) $\wedge$ **One**(xs)]’ and ‘[**Many**(y) $\wedge$ **One**(y)]’ (substitutivity).

Some might object to the above formulations of pluralitism. They might argue that the thesis cannot be paraphrased by [P] or its ilk because the English predicates '*to* be one' and '*to* be many' cannot be analyzed as in *Def*. 2.[29] To do so, they might hold, for example, that a set can be said to be many in the sense that it has many members while said to be one in the sense that it is one set, or that a fusion can be said to be many in the sense that it has many parts while said to be one in the sense that it is one fusion.

I think it is wrong to say that a set or fusion *is* many simply because it *has* many members or parts. If a set (or fusion) has many members (or parts), *its members* (or *parts*) are many. But this does not mean that the set (or fusion) itself is many any more than my having two parents means that I myself am two. To conclude that the set (or fusion) itself is many, one might assume that it is identical with its members (or parts). But this assumption implies the many-one identity thesis (e.g., [M′]), and the thesis is logically false, as we have seen.

Accordingly, those who hold that there is something that is also many (in some sense) commit themselves to pluralitism (as I formulate it). To see this, note that one of the above formulations of the many-one identity thesis, [M′], does not involve the predicate '**One**', and that one can formulate the thesis without using the predicate '**Many**', either:

[M‴] $\boldsymbol{\Sigma} xs \exists y[\exists z \exists w(z \neq w \wedge \mathbf{H}(z, xs) \wedge \mathbf{H}(w, xs)) \wedge xs \approx y]$.

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[29]Cotnoir objects that the definition of '**One**' rules out some versions of pluralitism, which he formulates as the thesis that there are some things that are many that "are collectively identical to a single thing". But rejecting the definition does not help, because this thesis implies

This implies [P′]. And we can directly see that it is logically false: '$\exists z\exists w[z \neq w \wedge \mathbf{H}(z, xs) \wedge \mathbf{H}(w, xs)] \wedge xs \approx y$' implies '$\exists z\exists w[z \neq w \wedge \mathbf{H}(z, y) \wedge \mathbf{H}(w, y)]$', which is logically false (*Th*. 8).[30] Now, those who argue that there is something that is also many are committed to pluralitism because they must accept [M‴]. Proponents of the plurality approach, we have seen, must accept the following:

(4) $\exists x\, [v@s] \approx x$. (There is something that Venus and Serena are identical with.)

(5) $v \neq s$. (Venus is not Serena.)

And these imply [M‴] (*Th*. 3).[31] Similarly, Plato must accept [M‴], for he holds that Socrates's left and right sides, which differ from each other, are identical with Socrates.

Can pluralitists reject some of the theorems used in the argument? They might object that *Th*. 5 is ill-formed because the predicate '**H**' (or 'is one of') is exclusively plural. This objection does not directly challenge the argument. The argument does not rest on the theorem or other theorems that depend on it, for the proof of *Th*. 10(b), which contradicts [P], uses none

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[M′], where the predicate does not figure at all. See also the discussion below and note 38.

[30]We can show that without assuming that '**H**' is neutral: '$\exists z\exists w[z \neq w \wedge \mathbf{H}(z, xs) \wedge \mathbf{H}(w, xs)] \wedge xs \approx y$' implies '$\exists z\exists w[z \neq w \wedge \mathbf{H}'(z, xs) \wedge \mathbf{H}'(w, xs)] \wedge xs \approx y$' (*agreement*), and this implies '$\exists z\exists w(z \neq w \wedge \mathbf{H}'(z, y) \wedge \mathbf{H}'(w, y))$', which contradicts *Th*. 8′.

[31]We can directly see that (4) and (5) are contradictory: '$[v@s] \approx x$' implies '$\forall y[\mathbf{H}(y, [v@s]) \leftrightarrow \mathbf{H}(y, x)]$' (which results from '$\forall y[\mathbf{H}(y, [v@s]) \leftrightarrow \mathbf{H}(y, [v@s])]$' by substitutivity), and this implies '$\forall y[(y=v \vee y=s) \leftrightarrow y=x]$' (*Ths*. 3 & 5), which implies '$v=s$.' And we can prove it without assuming that '**H**' is neutral: '$[v@s] \approx x$' implies '$\forall y[\mathbf{H}'(y, [v@s]) \leftrightarrow \mathbf{H}'(y, x)]$', which implies '$\forall y[(y=v \vee y=s) \leftrightarrow y=x]$' (*Th*. 5′ and the junior cousin of *Th*. 3). For further discussion,

of the preceding theorems. But those who raise the objection might argue that [P] is not a correct formulation of pluralitism because it is ill-formed: '**One**' and '**Many**' take the singular 'x' in [P] while they must be exclusively plural if '**H**' is so.

We can meet this objection by reformulating pluralitism with cousins of '**One**' and '**Many**':

[P*] $\exists x[\mathbf{One}'(x) \wedge \mathbf{Many}'(x)]$. (There is something that is both one and many.)

This formulation of pluralitism is immune to the objection that [P] is ill-formed. And we can show that [P*] is logically false without assuming that '**H**' is neutral. Without invoking the assumption, we can prove the junior cousin of *Th*. 10:[32]

Th. 10′. (*Anti-pluralitism*):

(a) $\sim[\mathbf{One}'(xs) \wedge \mathbf{Many}'(xs)]$.

(b) $\sim[\mathbf{One}'(x) \wedge \mathbf{Many}'(x)]$.

(c) $\sim\mathbf{Many}'(x)$.

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see my (1999b, §2).

[32]The proof is the same as the proof of *Th*. 10. Applying *Def*. 2′ to the two conjuncts in 10′(b) yields incontrovertibly well-formed yet logically incompatible sentences:

(i′) $\exists y \forall z[\mathbf{H}'(z, x) \leftrightarrow z=y]$.
(ii′) $\exists y \exists z[z \neq y \wedge \mathbf{H}'(y, x) \wedge \mathbf{H}'(z, x)]$.

So 10′(b) is logically true. Likewise with 10′(a). 10′(c) follows from 10′(b) and *Th*. 8′. And

(d) $\mathbf{Many}'(xs) \leftrightarrow {\sim}\mathbf{One}'(xs)$.

And [P*] conflicts with *Th*. 10′(b).

Similarly, we can reformulate the many-one identity thesis:

[M*] $\Sigma xs \exists y[\mathbf{Many}'(xs) \wedge \mathbf{One}'(y) \wedge xs{\approx}y]$.

This implies [P*] and is logically false. Some might object that [M*] is also ill-formed because '$xs{\approx}y$' is so (its definition is '$\forall x[\mathbf{H}(x, xs) \leftrightarrow \mathbf{H}(x, y)]$'). But the argument against [M*] does not depend on defining '$\approx$' as in *Def*. 1. The predicate is a plural language counterpart of the English predicate '*to* be identical with', which proponents of the thesis use as a kind of identity predicate. So substitutivity must hold for the predicate '$\approx$' whether or not it is defined as in *Def*. 1. Moreover, we can give an alternative definition of the predicate:

Def. 1′: $xs{\approx}ys \equiv: \forall x[\mathbf{H}'(x, xs) \leftrightarrow \mathbf{H}'(x, ys)]$. (These *are identical with* those ≡: something is-one-of these if and only if it is-one-of those.)

On this definition, '$xs{\approx}y$' is incontrovertibly well-formed.

10′(d) is equivalent to 10(d) (*Th*. 9).

4. Composition and Identity

Pluralitism and the many-one identity thesis, we have seen, are logically false. So it is wrong to take the plurality approach to plurals. Its proponents must accept both of sentences (4) and (5): 'Venus and Serena (taken together) are identical with something' and 'Venus is not Serena.' These are incompatible, and directly imply the many-one identity thesis and pluralitism. How about the composition as identity thesis, which some might use to argue for pluralitism?

Unlike pluralitism, the composition as identity thesis falls short of being logically false. We can see that it follows from a thesis that trivializes the part-whole relation:

Part-Whole Triviality Thesis: Nothing has a *proper* part, a part not identical with itself.

This thesis is not quite a logical falsity. Moreover, it is compatible with unrestricted composition:

Unrestricted Composition Thesis: Any things whatsoever compose something.

Both follow from Eleatic monism: everything is identical with everything. This thesis also falls short of being logically false. But to say that a thesis is not logically false is not to say that one might well accept it. As proponents of composition as identity assume, monism and part-whole triviality are clearly false. But the composition as identity thesis implies part-whole triviality.

We can formulate the composition as identity thesis in plural languages. Use '$<$', 'O^2',

‘$\mathbf{Comp}^2$’ as counterparts of ‘is a part of’, ‘overlaps’, and ‘*to* compose’, respectively.[33] Then we can formulate definitions of the overlap and composition predicates as follows:

Def. 4 (*Overlap* and *Composition*):

(a) $O(x, y) \equiv: \exists z(z{<}x \wedge z{<}y)$. (This *overlaps* that ≡: something is a part of both this and that.)

(b) $\mathbf{Comp}(xs, y) \equiv: \forall z[\mathbf{H}(z, xs) \rightarrow z{<}y] \wedge \forall z(z{<}y \rightarrow \exists w[\mathbf{H}(w, xs) \wedge O(w, z)])$. (They *compose* it ≡: any one of them is a part of it and any part of it overlaps one of them.)

And we can formulate the thesis:

[C] *Composition as Identity Thesis*:

$\mathbf{Comp}(xs, y) \leftrightarrow xs{\approx}y$. (Some things compose something if and only if the former are identical with the latter.)

We can now see that the thesis trivializes the composition and part-whole relations. It implies the following:

[33]The first two are singular, the third plural (its first argument place is neutral).

(6) $\mathbf{Comp}(xs, y) \rightarrow \mathbf{One}(xs)$. (Any things that compose something are one.)[34]

(7) $x<y \rightarrow x=y$. (Any part of something is identical with it.)[35]

We can see that [C] implies (6) using a logical truth that follows from *Th.* 5 (or 5′):

Th. 11. $xs \approx y \rightarrow \mathbf{One}(xs)$. (Any things that are identical with something are one.)

And we can show that (6) implies (7) using basic truths about part-whole and composition:

(8) $x<x$. (Anything is a part of itself.)

(9) $\mathbf{Comp}(<x: x<y>, y)$. (The things that are parts of something compose it.)

(9) follows from (8), and (8) is an analytic truth any theory of part and whole must include as a theorem. And (8) and (9) together with (6) imply (7).[36]

On the composition as identity thesis, then, the part-whole and composition relations are totally trivial. Cicero (who is one) composes something (viz., himself); so do Cicero and Tully (taken together), for they are identical with Cicero; and likewise with the Romans identical with him. On the thesis, however, Venus and Serena (taken together) *cannot* compose anything because they are not one, and it is the same with any two or more things: Cicero and Venus,

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[34]The converse also holds. It follows from (8), below.

[35]This is equivalent to the part-whole triviality thesis.

Socrates's parts, etc. For nothing, on the thesis, can have Venus or anything else as a proper part.

Although these consequences of [C] fall short of being logically false, they are clearly incorrect. So their falsity is assumed by proponents of [C]. Most contemporary proponents of the thesis hold the unrestricted composition thesis:

Unrestricted Composition:

$\exists y\mathbf{Comp}(xs, y)$. (Any things whatsoever compose something.)

Combined with this, [C] implies monism. But they hold unrestricted composition not because they accept monism but because they think that Venus and Serena, for example, compose something of which both are proper parts.[37] Similarly, Plato assumes that Socrates has proper parts in arguing for pluralitism by holding that composition is identity. Likewise with others who hold this thesis. So proponents of the thesis have logically incompatible views.

Can they object? Some of them might object to (6) by rejecting *Th*. 5. And some might argue that [C] is not a correct formulation of their thesis because the plural identity predicate used to state it cannot be defined as in *Def*. 1 (or 1′). But the derivation of (6) from [C] does not depend on *Th*. 5 or the definition. We can prove *Th*. 11 without invoking the theorem or the definition:

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[36](6) and (9) imply '$\mathbf{One}(<x: x<y>)$', and this together with '$y<y$' implies (7).

[37]Although a Parmenides might combine composition as identity with unrestricted composition to argue for monism, the argument would give no good reason for holding monism.

Proof of Th. 11: ‘$\mathbf{One}'(y)$’ is a logical truth (*Th.* 8′). So ‘$xs{\approx}y$’ implies ‘$\mathbf{One}'(xs)$’ (substitutivity), which implies ‘$\mathbf{One}(xs)$’ (agreement).[38]

Notice that this proof invokes no definition of ‘≈’. It uses the substitutivity principle for the predicate, but the principle must hold as long as it is a predicate for a kind of identity, a plural version of ‘=’. So the cousin of *Th.* 11 matching any plural identity predicate must also be a logical truth. And the composition as identity thesis formulated with any such predicate must imply (6). Suppose it is formulated with a different plural identity predicate, ‘≡’:

[C*] $\mathbf{Comp}(xs, y) \leftrightarrow xs{\equiv}y.$

Then (6) follows from this as well, for the matching cousin of *Th.* 11 (i.e., ‘$xs{\equiv}y \rightarrow \mathbf{One}(xs)$’) is a logical truth. Moreover, the predicate ‘≡’ must agree with ‘≈’:

Agreement: $xs{\equiv}y \leftrightarrow xs{\approx}y.$

This is a logical truth, which we can prove using the substitutivity principle for the two

[38]Cotnoir objects to the derivation of (6) from [C] by disputing the definition of ‘**One**’. This is beside the point, for the use of the predicate merely facilitates formulations of relevant statements and proofs. We can formulate (6) without using the predicate: ‘$\mathbf{Comp}(xs, y) \rightarrow \exists y \forall z[\mathbf{H}(z, xs) \leftrightarrow z{=}y]$’ (likewise with *Th.* 11). This thesis, which trivializes the composition relation, follows from [C] and implies part-whole triviality, (7).

predicates. So [C] and [C*] must be logically equivalent. This means that one cannot defend the composition as identity thesis by giving an alternative definition of the plural identity predicate.[39]

Some might deny that substitutivity must hold for an identity predicate. Baxter (1988a) distinguishes two kinds of identity: identity in "a loose and popular sense", and identity in the strict sense. Although both are meant to be singular identities, identity relations singular predicates (e.g., '=') can signify, one might draw the same distinction for plural identity. One might then object that the above argument against the composition as identity thesis fails to address a weak version of the thesis (a version of the weak composition thesis): the composition relation is a kind of *loose* plural identity relation.[40] This is correct, for substitutivity would fail for the predicate for a loose plural identity. But the defense trivializes the thesis defended. To prove the weak thesis, one would simply need to propose to say that the composition relation is an identity relation in a loose sense.[41] Similarly, one could insulate Eleatic monism from any possible objection by holding that anything whatsoever is identical, albeit in a loose sense, with

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[39]Cotnoir (forthcoming), who holds composition as identity, defines a "generalized identity predicate", for which he uses '$\approx$' (without boldface), and points out that the predicate diverges from the "standard" plural identity predicate '$\boldsymbol{\approx}$' (boldface): in a situation where black and white tiles compose a black square and a white square, he holds, "the many tiles are *identical* to the many squares" (where 'identical' is used for his "general identity") is true while "for any *x*, *x* is one of the tiles if and only if *x* is one of the squares" is false (*ibid*., §2; original italics). This shows that the "general identity" predicate is not a kind of identity predicate. Otherwise, the first sentence must imply the second, which results from the logical truth 'for any *x*, *x* is one of the tiles if and only if *x* is one of the tiles' by replacing the underlined term with 'the squares'.

[40]Baxter and Cotnoir raise this objection while calling the strict identity "numerical identity".

[41]But this does not help to defend pluralitism. The weak version of composition as identity does not yield pluralitism for the same reason that Lewis's weak composition thesis does not.

anything. The issue is not whether one can formulate a weak version of monism with a terminological decision, but whether the ordinary version of monism is defensible. It is not, as is widely recognized, partly because there is a relation of singular identity in the strict sense. Similarly, what is important to see is that the non-trivializing composition as identity thesis is a virtual logical falsity because plural languages have a generalization of the singular identity predicate '=', '≈', that signifies strict plural identity.

5. Concluding Remarks

In plural languages, we have seen, we can state analyses of the predicates '*to* be one' and '*to* be many'. It is the same with other numerical predicates: '*to* be two', '*to* be three', etc. We can define, for example, the plural language counterpart of '*to* be two', '**Two**':

Def. 5 (***Two***):

> **Two**(xs) $\equiv$: $\exists x \exists y[x \neq y \wedge \forall z(\mathbf{H}(z, xs) \leftrightarrow (z=x \vee z=y))]$. (They *are two* $\equiv$: there is something and something else such that something is one of them if and only if it is the first or the second.)

I think the predicates corresponding to natural numbers signify properties of a special kind, *plural properties*: being one, being two, etc.[42] For example, being one is a property instantiated

[42]Similarly, '*to* be many' signifies the property of being many. Roughly, plural properties are properties signified by one-place plural predicates. See my (1999a; 2006, §5).

by any one thing, not by any two or more things; being two a property instantiated by any two things, no one of which can have the property. And I think natural numbers are the properties signified by those numerical predicates.[43] Pluralitism implies that those properties do not exclude each other, but we have seen that the thesis is logically false. Using the logic of plurals, we can prove that they exclude each other.

Frege (1980), who holds that numbers are not properties but objects, argues that there are no properties signified by numerical predicates. To do so, he argues that those predicates are not genuine predicates, and that properties signified by them would fail to be mutually exclusive. He argues that 'one' (unlike 'wise') "cannot be a predicate" because "We cannot say 'Solon and Thales were one'" (*ibid*., 40); that "every single thing should possess" the property signified by 'one' *if* such a property exists (*ibid*., 40); and that "while I am not in a position, simply by thinking of it differently, to alter the color or hardness of a thing in the slightest, I am able to think of the Iliad as one poem, or as 24 Books, or as some large number of verses" (*ibid*., 28).

It is straightforward to see that the arguments fail. First, we *can* say 'Solon and Thales were one' as we can say 'Solon and Thales were not one', although the former is rarely said because it is clearly false (Solon is not Thales). And 'Solon is one' and 'Thales is one' do not imply 'Solon and Thales are one', which we can see using the analysis of '*to* be one' (*Def*. 2). Second, Frege is right to observe that "every single thing" has the property of being one (*Th*. 8), but this does not mean that the content of the predicate '*to* be one' "must vanish altogether" or "its extension becomes all-embracing" (*ibid*., 40). This is a plural predicate, which is germane to any things whether they are one or many, and it does *not* denote any two or more things although

[43]See my (1998; 1999a; 2002, Chapter 4).

it denotes any one of them. So ‘Solon and Thales are one’ is well-formed but false, and Solon and Thales (taken together) fail to have the property of being one. Finally, one might certainly *think* of the same thing (or things) as one poem or as 24 books, but this does not mean that both thoughts would be correct. The *Iliad* is a poem *composed* of 24 books, which does not mean that it *is* 24 books or that the 24 books (taken together) *are identical with* the one poem. (To assume this is to hold the composition as identity thesis.) So the poem is not twenty four but one, and the books not one but twenty four.

In denying that numerical predicates are genuine predicates, Frege assumes an analysis of plurals falling under the plurality approach: a plural term refers to an aggregate. In his letter to Russell, he holds that ‘Bunsen and Kirchoff’ in ‘Bunsen and Kirchoff laid the foundation of spectral geometry’ refers to “a whole or system . . . consisting of parts” (1902, 140). On this analysis, the *Iliad*’s 24 books *are* identical with the aggregate thereof, which one might identify with the poem. We have seen, however, that the analysis leads to contradictions. To solve this problem, Frege denies that numerical predicates are genuine predicates. But his objections to this view are beside the point, we have seen, once the view is detached from the plurality approach. And banning numerical predicates does not help to solve the problem, for it is not necessary, as we have seen, to use the predicates to show that the approach leads to contradictions (it yields [M‴]).

Instead of banning numerical predicates altogether, it is usual to take them to be incomplete predicates. Frege gives an influential argument that seems to suggest this response:

If I give someone a stone with the words: Find the weight of this, I have given him

> precisely the object he is to investigate. But if I place a pile of playing cards in his hands with the word: Find the Number of these, this does not tell him whether I wish to know the number of cards, or of complete pack of cards . . . . I must add some further word—cards, or packs . . . . (1980, 28f)

Many take this to show that the number question is incomplete because numerical predicates must be complemented by sortal terms: 'card', 'pack', etc. I disagree.

Adding sortals does not resolve the problem. If someone asks "What is the number of these *squares*?" while giving a large square (A) divided into four small squares (B, C, D, E), the correct answer depends on which squares the question is about: the one large square, the four small squares, all the five squares, or the large square and some of the small squares (e.g., A and B). The problem arises because the demonstrative phrase 'these *squares*', even with the sortal term, fails to have determinate reference. Similarly, the problem with Frege's number question arises because it involves a demonstrative without determinate reference. In the situation envisaged, it might be used to refer to the cards in the pile, the packs composed of cards in the file, etc. Adding a sortal might sometimes help to specify what the demonstrative is used to refer to, but it is not necessarily sufficient. Nor it is it essential. One might simply ask, e.g., "What is the number of B, C, D, and E (or A and B)?" using the letters to refer to the squares. Or what the question is about might simply be understood. Once the question is made specific by determining what it is about, whether or not with a sortal term, the answer is no less determinate in the card case than in the square case.

We can reach the same conclusion by considering a case that does not concern numbers.

Suppose someone gives a bronze statue with the question "How old is *this*?" This falls short of specifying whether the question is about the material of the statue (a lump of bronze) or the statue made of the material, and the correct answer might be different depending on what it is about (the statue might be 10 days old while the lump is 10 years old). Surely, this does not mean that 'is 10 days old', unlike 'weighs 10 pounds', is an incomplete predicate to be complemented by a sortal, nor does it mean that one and the same thing can be both 10 days and 10 years old. What it means is that the question is indeterminate because it involves an improper use of the demonstrative 'this'. Similarly, Frege's number question is indeterminate because it involves an improper use of the demonstrative 'these'. Once we reject the plurality approach to plurals, we have no reason to take one to make a proper use of the demonstrative simply because one utters it while giving a pile of cards. The cards in the pile are not identical with the packs in it as the *Illiad* is not identical with its books.

While rejecting the plurality approach, Dummett argues that "a plural noun phrase, even when it is preceded by the definite article, cannot be functioning analogously to a singular term" because such a phrase "under a correct analysis . . . is seen to figure predicatively" (1991, 93).[44] To illustrate this analysis, he mentions Frege's analysis of "the King's carriage is drawn by four horses." Frege takes this to "assign the number four to the concept 'horse that draws the King's carriage'" (1980, 59), and Dummett takes it to have the form "There are four objects each of which *is a horse that draws the Kaiser's carriage*" (1991, 93; original italics). Although this might capture one reading of the sentence, the sentence has a more natural reading: 'The King's

[44]Whitehead & Russell (1962) also reject the plurality approach and take a sophisticated predicate approach. See my (2013).

carriage **is drawn by** four horses (taken together)' (where the boldfaced is a plural predicate). The predicate approach cannot capture the sentence on this reading, nor can it explain the use of the numeral 'four' in it. In plural languages, by contrast, it has a straightforward counterpart, which amounts to 'The King's carriage **is drawn by** some things that **are four** and each of which is a horse' (where the boldfaced are plural predicates). Similarly, the predicate approach cannot give a correct analysis of 'Those who **wrote** *PM* **are two**', which is not equivalent to 'There are exactly two objects each of which wrote *PM*' ('Russell wrote *PM*' is false). But it is straightforward to analyze the former in plural languages; it can be paraphrased by '$\mathbf{Two}((\mathbf{I}xs)\boldsymbol{W}(xs, p))$', where '$(\mathbf{I}xs)\boldsymbol{W}(xs, p)$' is the counterpart of 'those who wrote *PM*' (see §2.1).

In plural language analysis, plural terms (e.g., 'Venus and Serena', 'Bunsen and Kirchoff', 'the residents of London', 'those who wrote *PM*', 'the four horses drawing the King's carriage') do not figure as predicates. Like singular terms, they figure as arguments of (first-order) predicates although they fill plural argument places. And they are referential. Both 'Venus and Serena' and 'those who wrote *PM*' refer to two humans though not to either of them, and 'the residents of London' to 10 million or so humans. So 'Venus and Serena are two' is true because the two humans that 'Venus and Serena' refers to (together) have the property signified by '*to* be two'.

Plural terms, then, function analogously to singular terms. A typical plural term refers to some things. Surely, it is wrong to conclude from this that a plural term, if it refers at all, must refer to something (viz., some one thing). This is a logical error, the same error as inferring 'Something is many' from 'Some things are many.' The error leads to pluralitism, which we have seen is logically false. By attending to the logic of plurals, we can avoid the error, and give a proper account of plurals and reach a suitable view of number. Spelling out the account and view is beyond the scope of this work. Hopefully the sketches drawn above suffice for its aim.[45]

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[45]See my (2002; 2005; 2006; forthcoming) for my account of plurals, and my (1998; 1999a; 2002, Chapter 4) for my account of number.

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