Are synthetic *a priori* propositions informative?

Yongfeng YUAN[1]

(Department of Philosophy (Zhuhai), Sun Yat-Sen University)

Abstract According to rationalists, synthetic *a priori* propositions convey new knowledge, whereas analytic propositions are non-informative or vacuous conceptual truths. However, as we argue in this article, each *a priori* proposition is necessarily true because of its semantic constituents and the way they are combined, and hence can be transformed into its equivalent analytic form. So each synthetic *a priori* proposition conveys only non-informative conceptual truths like analytic propositions.

Keywords *A priori*, analytic, synthetic, necessary truth, informative, knowledge

1.Introduction

In the literature, there are three distinctions proposed for propositions. The first one is a metaphysical distinction, namely necessary/contingent distinction. A proposition is necessarily true (false) if it is true (false) in all possible worlds. For example, the tautology "$\varphi \vee \sim\varphi$" is necessarily true, and "$\sim(\varphi \vee \sim\varphi)$" is necessarily false. By contrast, a proposition is contingent if it is true in at least one possible world and false in at least one possible world. For example, "all swans are white" is a contingent proposition.

The second one is an epistemological distinction, namely *a priori*/*a posteriori* distinction. A proposition is *a priori* proposition if it can be known independent of any experience. For example, "all bachelors are unmarried", "$\varphi \vee \sim\varphi$", and "5+7=12". A proposition is *a posteriori* proposition if it cannot be known independent of experience. For example, "all bachelors are rich", and "it is raining outside the window". They are true or false because of confirmation/disconfirmation, or satisfaction/dissatisfaction, by empirical evidence.

The third one is a linguistic distinction, namely analytic/synthetic distinction, introduced by Kant (1781, A6-A7). A proposition is analytic if and only if its predicate B belongs to the subject A as something that is (covertly) contained in this concept A. And a proposition is synthetic if and only if its predicate B is not (covertly) contained in the subject A but adds something new to the subject. For example, "all women are female" is an analytic proposition, but "all women are beautiful" is a synthetic proposition. Since analytic propositions don't give us any knowledge or information about the world, they are non-informative or vacuous, and convey only *conceptual truths*. However, for each synthetic proposition its predicate is not contained in the subject but adds something new to the subject. So synthetic propositions seem to be informative or non-vacuous.

[1] Yongfeng YUAN, Department of Philosophy (Zhuhai), Sun Yat-Sen University, Tangjiawan, 519082, Xiangzhou District, Zhuhai, Guangdong, China.

Combining synthetic proposition with *a priori* proposition, Kant proposes one kind of propositions, namely *synthetic a priori* propositions, that may begin with experience but do not arise from experience. For example, "5+7=12" seems to be a synthetic *a priori* proposition, because at the first glance the concept '12' doesn't seem to be already contained in the concept '5+7'. Besides, some philosophers also accept "the shortest distance between two points is a straight line" as a synthetic *a priori* proposition. Since for each synthetic *a priori* proposition its predicate cannot be contained in the subject but adds something new to the subject, it seems to be informative or non-vacuous, and conveys new knowledge.

In the literature, there are many disputes on the possibility of synthetic *a priori* propositions. According to rationalists, in metaphysics and mathematics there are some propositions that are necessarily true and convey new knowledge. They are not empirical like *a posteriori* propositions, but informative like synthetic *a priori* propositions. However, all forms of empiricism reject the existence of synthetic *a priori* propositions (Feigl 1947). For example, M. Schilick argues that "all propositions are either synthetic *a posteriori* or tautologous; synthetic *a priori* propositions seem to it to be a logical impossibility." **(**Schlick 2012) G. Frege and A. J. Ayer also argues that arithmetic propositions like "5+7=12" are analytic *a priori*, rather than synthetic *a priori*, propositions (Katz 2000; Ayer 2001). For more discussions, see (Coleman 1979; Copi 1949,1950; Felch 1950; Glassen 1958; Hintikka 1968; Langford 1949; Johnson 1960; Krishna 1961; Morawetz 1974; Pap 1950; Sellars 1953; Turquette 1950). If empiricism is correct, then it seems to eliminate metaphysics. So the possibility of synthetic *a priori* propositions is vital for metaphysics.

As we know, there are already so many debates on this problem. However, many of them are quite misleading because of a wrong presupposition, namely *an a priori proposition is synthetic if and only if it is informative or conveys new knowledge*}. Hence, we will argue as follows that: there are some synthetic *a priori* propositions, but all *a priori* propositions are non-informative and convey only conceptual truths as analytic propositions.

2.All *a priori* propositions are non-informative

For the possibility of synthetic *a priori* propositions, we need only to find some *a priori* propositions whose predicates cannot be contained in their subjects. For example, "$1\in\{1,2,3\}$" is a synthetic *a priori* proposition. Firstly, it is obvious that "$1 \in \{1,2,3\}$" is an *a priori* proposition. Secondly, "$1 \in \{1,2,3\}$" is a synthetic proposition. Because the predicate "$\{1,2,3\}$" cannot be contained in the natural number "1"; otherwise, there would be infinite number of predicates, such as "$\{1,2,4\}$", "$\{1,3,4\}$", "$\{1,3,4,5\}$", etc., contained in the single number "1". And that will be absurd! Hence, there are some synthetic *a priori* propositions.

Is "$1\in\{1,2,3\}$" informative, or does it convey new knowledge? Since "$1\in\{1,2,3\}$" is equivalent to "$\{1,2,3\}$ contains 1", if one of them is informative, then so is the other. And, clearly, "$\{1,2,3\}$ contains 1" is analytic or non-informative. Hence, "$1\in\{1,2,3\}$" is only one conceptual truth, rather than conveying new knowledge. From the example, we know that some synthetic a priori propositions have some kind of equivalent analytic form, and hence are non-informative. Now we have a question: are there any synthetic *a priori*, or *a priori*, propositions which are informative or convey new knowledge? We will argue in the following that: each *a priori* proposition is equivalent to an analytic proposition, and hence non-informative.

Let φ be an *a priori* proposition. Then φ can be known independent of any possible experience. It means that φ can be derived from some *a priori* epistemic reasons which can also be known independent of any possible experience. Then there is, *in principle*, an ideal set (denoted by O) of *all a priori* epistemic reasons for φ such that the conjunction of all O's members, denoted by Ω, is equivalent to φ. Because there is no need of other *unnecessary* reasons for φ's justification. In other words, if φ can be derived from Ω, but not vice versa, then O must contain some members which cannot be derived from φ. And these members must contain some information which is not contained in φ. So this information is irrelevant and unnecessary for φ's justification, and it should be cut off. Hence, to justify φ, we only need to find this kind of ideal set of all *a priori* epistemic reasons for φ. Since every O's member is *a priori* epistemic reason for φ, Ω's truth value is also independent of any possible experience. Now suppose that: there is $\psi \in O$ such that

(1). τ is one of ψ's *a priori* epistemic reasons,

(2). τ doesn't contain any information which is unnecessary for ψ's justification,

(3). $\tau \notin O$.

From $\psi \in O$ and the clause (1), we know that τ is also one of φ's *a priori* epistemic reasons. Then from O's definition and the clause (2), we have $\tau \in O$. Contradiction. So the supposition is false. No O's member can have any *a priori* epistemic reason, which is not in O but contains only the information necessary for the justification of the member. So Ω's truth value doesn't depend on any other *a priori* epistemic reason which is not in O. Since Ω's truth value is also independent of any possible experience, its truth value depends only on the proposition Ω itself, namely *its semantic constituents and the way they are combined*. Hence, φ's truth value depends only on the proposition φ itself. It means that φ's semantic constituents and the way they are combined are enough to determine its truth. Therefore, the essence for a proposition to be *a priori* or independent of any possible experience is that its truth value depends only on the proposition itself.

Since φ's truth depends only upon the proposition itself, no matter which situation or possible world it is in, it is true *by itself*, namely it is necessarily true by itself. From φ's necessary truth, we know that ∼φ is false in all logically possible worlds. Since the only difference between φ and ∼φ is that ∼φ has one more negation (a truth function), ∼φ's universal falsity is also determined by itself. In other words, its semantic constituents and the way they are combined are also enough to determine its falsity, or in other words, prevent it from being true in any logically possible worlds. What is the *intrinsic* reason in ∼φ for its necessary/universal falsity? The only reason is that ∼φ itself must be inconsistent. By *reductio ad absurdum*, suppose that ∼φ is consistent or doesn't contain any conceptual contradiction. Then it can be true for several reasons as follows.

(1). In logic, every consistent proposition is satisfiable in some assignments of truth value, interpretations, or logically possible worlds.
(2). If a proposition doesn't contain any kind of *conceptual* contradiction, then it is logically *conceivable* to be true in some possible situations.
(3). If $\sim\varphi$ is consistent, then some parts of the universe are, in principle, possible to evolve into a future world where $\sim\varphi$ is true in it.
(4). If $\sim\varphi$ is consistent, then God can, in principle, design a possible world or future world where $\sim\varphi$ is true, provided that God exists.

In a word, there is no reason which can prevent a consistent proposition $\sim\varphi$ from being true in some logically possible worlds. So, to know whether φ is true, we need to know first which situation or possible/future world it is in. Then φ cannot be necessarily true, and is not *a priori*. Contradiction. So the supposition is false, $\sim\varphi$ itself must be inconsistent.

Since $\sim\varphi$ is inconsistent, there is a proposition ψ such that $\sim\varphi$ entails $\psi\wedge\sim\psi$. As $\sim\varphi$ may contain more information or semantic contents other than $\psi\wedge\sim\psi$, let $\sim\varphi\leftrightarrow\psi\wedge\sim\psi\wedge\tau$, where τ denotes $\sim\varphi$'s remaining information except $\psi\wedge\sim\psi$. Hence, we have

$$\begin{aligned}\varphi &\leftrightarrow\sim(\psi\wedge\sim\psi\wedge\tau)\\ &\leftrightarrow\sim((\psi\wedge\tau)\wedge\sim\psi)\\ &\leftrightarrow(\psi\wedge\tau)\rightarrow\psi\end{aligned}$$

Obviously, $(\psi\wedge\tau)\rightarrow\psi$ is an analytic proposition. Therefore, every *a priori* proposition is equivalent to an analytic proposition. Since the formal transformation above *only* changes the logical form of *a priori* proposition, each synthetic *a priori* proposition can be different from its analytic form *only* in the aspect of its logical form. Hence, each synthetic *a priori* proposition cannot be more informative than its analytic form. Therefore, each *a priori* proposition cannot be informative or convey new knowledge, but is only one conceptual truth like analytic propositions.

Take "5+7=12" for example. Its semantic constituents and the way they are combined are enough to determine its truth. So it is necessarily true by itself. And then its negation "5+7≠12" is necessarily false by itself. So "5+7≠12" must contain a conceptual contradiction; otherwise, it is consistent, and then satisfiable in some logically possible worlds. What is the contradiction in "5+7≠12"? From the concepts '5', '+', '7', and '12', we know that the left side of '≠' outputs 12. However, the proposition claims that the result is not 12. That is the conceptual contradiction. So "5+7=12" is only a negation of a conceptual contradiction, and it is non-informative but only one conceptual truth. Moreover, "5+7=12" means the same as "|||||+|||||||=||||||||||||". Since the symbol '+' just means to put two numbers together, "|||||+|||||||" is the same as "||||| |||||||". So '+' is a redundant symbol in this terminology. And hence the predicate "||||||||||||" contains in the subject "||||| |||||||". The abbreviation symbols in "5+7=12" conceal its analytic essence.

But why do some *a priori* propositions in mathematics and metaphysics seem to be informative? We think the main reasons are as follows. Firstly, since those propositions may involve various abstract or complex concepts and logical forms, we may not aware of all their essential details during our understanding process.

Secondly, even if we can do that, we cannot know their *a priori* relations through simple conceptual analysis. So we are apt to think of them as conveying new knowledge that cannot be known by conceptual analysis. Take "the shortest distance between two points is a straight line" for example. At the first glance, we are not aware of the conceptual relation between the concepts "distance" and "straight line". Even if we know that the concept of "distance" is defined on the basis of the concept of "straight line", we cannot know their *a priori* relations through simple conceptual analysis. That is the reason why we consider it as informative or conveying new knowledge that cannot be known by conceptual analysis.

In contrast, if the concepts and logical forms involving in an *a priori* proposition are simple or not abstract, then we may aware of their essential details, and hence know their simple *a priori* relations through simple conceptual analysis. After that, we are apt to regard the proposition as vacuous, or non-informative, conceptual truth. For example, "a cube has 12 edges" and "a triangle has 3 edges". During our understanding process of the two propositions, we are aware of the essential details of the concepts of cube, triangle, and edge. From these concepts we conclude that if an object doesn't have 12 (or 3) edges, then it can't be a cube (or triangle). So, obviously and trivially, a cube (or triangle) has 12 (or 3) edges. That is why we regard them as vacuous or non-informative.

Some rationalists may reply that: the two simple propositions in last paragraph are non-informative because they are analytic; if they were synthetic propositions, then they would be informative. Obviously, the above example involving "$1 \in \{1,2,3\}$" and "{1,2,3} contains 1" is a counter example of these points of view. Besides, suppose that they are right. As we know, "$\sqrt{144}$ edges" and "$\sqrt[3]{27}$ edges" cannot be contained in the subjects "a cube" and "a triangle" respectively; otherwise, "$\sqrt[3]{1728}$", "$\sqrt[4]{20736}$", etc., and "$\sqrt{9}$", "$\sqrt[4]{81}$", etc., are contained in the subjects "a cube" and "a triangle" respectively. That is absurd! So "a cube has $\sqrt{144}$ edges" and "a triangle has $\sqrt[3]{27}$ edges" are both synthetic. Then from the supposition above we know that they are both informative. However, they are, in fact, equivalent to "a cube has 12 edges" and "a triangle has 3 edges" respectively. It follows that the former two cannot be more informative than the latter two respectively.

In fact, no matter simple or complex *a priori* propositions, the *a priori* relations among the concepts involving in them, in essence, are of the same kind. They are just *conceptual truths*, namely negations of some conceptual contradictions, determined by the concepts involving in them and the way they are combined. How can one kind of *conceptual truths* (like "$1 \in \{1,2,3\}$") be more informative than the other kind (like "{1,2,3} contains 1")?

3. Conclusion

As we show above, there are some synthetic *a priori* propositions. However, as we argue, they don't convey any new knowledge. The main reason is that: each *a priori* proposition is necessarily true because of its semantic constituents and the way they are combined, and hence can be transformed into its equivalent analytic form. So each *a priori* proposition is just a conceptual truth which is non-informative or vacuous like analytic propositions. Besides, the reason for regarding synthetic *a priori* propositions as informative is that the concepts and logical forms involving in them are too abstract or complex for us to reveal their conceptual essence through simple conceptual analysis.

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