Thinking through Sets: Exploring How Chinese Pictographic Language Shapes Chinese Logic

By Jinmei Yuan[*]

The author of this paper argues that ancient Chinese thinkers practiced an alternative logic that is significantly different from Aristotelian logic. The paper has two objectives: 1) to clarify what Chinese logic looks like; 2) to re-evaluate the wisdom in classic texts with a clear understanding of Chinese logic. The author uses two major approaches in her reasoning: an etymological approach and logic of sets. An etymological study shows that Chinese pictographic characters were created according to sets - the collections of characters. The author examines how Chinese pictographic language shapes ancient Chinese thinkers' thinking patterns and provides evidence that the habit of thinking in sets could naturally shape a practice of a primary logic of sets. Borrowing some expressions and symbols from modern Set Theory, the author demonstrates how the logic of sets was practiced by Gongsun Longzi, Confucius, Zhuangzi, as well as Chinese mathematicians in their teachings and reasoning. A theory of sets provides help with a much better understanding about reasoning methods in ancient Chinese thinkers' minds, so that one can evaluate their wisdom fairly. Understanding Chinese logic as it is provides a fruitful opening for new research in Comparative Philosophy.

Keywords: *set, etymology, membership, Chinese logic, and comparative philosophy*

Introduction

In this paper, I argue that there is an alternative logic that was widely practiced among ancient Chinese thinkers. This logic is significantly different from Aristotelian logic, for Law of Identity and Law of Non-contradiction fail to work in such an alternative logic. Membership relationships in hierarchical classes of genius and spices can hardly be found, and propositions of facts are not simply verifiable as either true or false. I do not think that it is fair to assume that ancient Chinese people did not have their own logic but only linguistic clarification[1]. Nor do I agree to simply bend Chinese ways of reasoning to match Aristotelian propositional logic. Logic should be a plural term, logics.

This paper carries two tasks: 1) to clarify what Chinese logic looks like; 2) to re-evaluate the wisdom in classic texts with a clear understanding of Chinese logic.

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[1]Chad Hansen claims that there was nothing that could be called logic in the classical Chinese philosophical tradition. He says, "Chinese linguistic theory focused on the question of what term to assign to things rather than on the propositional units so central to western theory of language and logic. The dominant conception was that a word had a scope or range of application, rather than referring to individuals or objects" (Hansen 1998).

An etymological approach and the logic of sets are two major approaches involved in my reasoning.

An etymological study shows that Chinese pictographic characters were created according to sets - the collections of characters. These collections consist of 364 radicals, which function as primary sets or units. All Chinese characters were created and sorted according to these 364 different radicals, which are simple sketches of the images for 364 common objects, things, or activities in human experiences. Normally, a Chinese character is a member in one of those primary sets of radicals. It associates with the radical one way or another in meaning by having the radical as a part of the character itself. The meaning of the primary radical is represented by the totality of its members. For example, the water radical,

shui (simplified as " " in modern Chinese) includes objects or elements which associate with water, such as, river , ocean , creak , fulfill ,[2] etc., and the radical is interpreted or defined by all the members in the set of water radical.

Both Chinese radicals and characters are pictographic. This unique feature preserves not only many hidden moral codes in ancient Chinese culture, but also shapes traditional ways of reasoning in Chinese logic, such as categorizing or sorting, giving definitions, and constructing arguments. This uniqueness provides trustworthy clues for us to investigate ancient Chinese thinkers' minds. Fortunately, ancient Chinese scholars preserved the classical Chinese language, in both written forms and pronunciations, carefully. The early Chinese etymology dictionary, *Shuo Wen Jie Zi* or *Shuo Wen*《說文解字》, by 許慎 Xu Shen (c. 58 – c. 147 CE) will be used as an etymological reference when I examine how Chinese pictographic language shapes Chinese people's thinking patterns. In *Shuo Wen*, Xu Shen not only summarized the patterns of sorting Chinese characters, but also demonstrates how Chinese characters were logically created according to the sets of radicals. More than that, *Shuo Wen* recorded the primal meanings of every Chinese character understood and used by ancient Chinese people during the pre-Qin and pre-Han eras.

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[2]I only listed a few members in the set of water radicals as examples. These members are also pictographic. The meaning of the characters can be interpreted by looking at the images of these characters. According to *Shou Wen*, 'River is the water which passes around Kunlun Mountains and then goes into the ocean. ，水。出焞煌塞外昆侖山，發原注海。'" "Ocean is a pond as large as the sky. It collects hundreds of rivers and creeks. ，天池也。以納百川者。" "Creaks are ditches which are four feet wide and four feet deep. ，水瀆。廣四尺、深四尺。" "Fulfill means overflow. ，盈溢也。"（*Shuo We*n, Book 11,Water Unit)
The translations of sentences from *Shuo Wen* in this paper are mine.

In the introduction to *Shou Wen*, Xu Shen discusses the history before his time and notes that since the first Chinese emperor of Qin (221–206 B.C.) ordered the burning of all classic books by Confucius and other pre-Qin scholars, except farming books, a few hundred years later, people could not understand the classic texts accurately. Many interpretations of classic texts reflect this misunderstanding. Fortunately, during early Han Dynasty (about 157 B.C.-141B.C.), some scholars found many hidden pre-Qin texts between the walls of Confucius' old house. They submitted the texts to Emperor Wu of Han. Later, more hidden classic texts were found in many other places in China. These discoveries made the *Shuo Wen* project necessary. The task of Xu Shen and his followers to write *Shou Wen* was to get rid of misunderstandings of the concepts, examine the sketched images of every character, and record the original meanings used in the pre-Qin classic texts. As an authoritative ancient text, *Shuo Wen* represents some important characteristics of Chinese associative thinking during the pre-Qin era (Paleolithic age–221 B.C.) to Han Dynasty (206 B.C.–220 A.D.) from an etymological point of view. It has been serving as a rich resource for contemporary comparative philosophers to translate and understand the classic texts written by ancient philosophers such as Confucius, Laozi, and Zhuangzi. For example, Roger Ames' etymological approach to finding unmatched meanings between classical Chinese and English has sparked an active discussion in the study of Chinese Philosophy. I share Ames' concern and will conduct my etymological studies carefully when reconstructing ancient Chinese scholars' arguments to reduce the risk of misrepresenting ancient wisdom. My etymological studies in this paper will also offer clear evidence of how a language shapes one's thoughts and ways of reasoning.

Turning to the discussion of the relationship of language and thought, according to their studies on Navajo Native American languages, American linguists Edward Sapir and Benjamin Lee Whorf advanced a hypothesis of linguistic relativity, also known as the Sapir-Whorf hypothesis. They claim that the language one speaks determines how one thinks. People's perceptions are relative to their spoken language. Edward Sapir points out, "The fact of matter is that the 'real world' is to a large extent unconsciously built up on the language habits of the group... We see and bear and otherwise experience very largely as we do because the language habits of our community predispose certain choices of interpretation" (Sapir 1929, pp. 209–210). The founding of Sapir-Whorf research[3] provides me with a useful base to ask a further question: If both sounds and images of a language are symbols that play roles necessarily in human communications, and if logics are also languages that map the thought movements in human communications, then is it possible that the language, such as ancient Chinese pictographic language, shapes the logical patterns when Chinese philosophers think and communicate? If the Chinese pictographic language provides a different way of naming and sorting, then learning and thinking in this language will name and organize thoughts and reasoning in different ways.

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[3] For detailed findings, see Whorf (1952). Also, in "Reference Module in Neuroscience and Biobehavioral Psychology, 2018," John F. Kihlstrom and Lillian Park summarize: "The Sapir-Whorf hypothesis takes two forms: that language determines thought or that language influences thought" (an online source).

I argue that there clearly is a logic of sets that was practiced naturally and widely among the pre-Qin and Han Dynasty thinkers. I will present what this logic looks like and explain the evidence on which I base my claim that there is a Chinese logic of sets.

I will begin by briefly introducing the meaning of sets and giving a few examples of sets that, I think, were recognized by ancient Chinese thinkers. According to Set Theory, sets are a concept of sorting members or elements as collective units. The objects that make up the set are called its elements or members. The elements of a set may be any objects whatsoever. In the West, Set Theory, as a separate mathematical discipline, begins with the work of Georg Cantor (1845-1918). The fundamental relation in Cantor's Set Theory is membership. Cantor's definition of sets holds two important meanings: 1) the criterion for membership in the set, by which any individual object can be recognized as a member of the set; 2) every member is separated from any other members in the set, so that every member can be counted only once. Cantor introduces his original concept of a set as follows: "A set S is any collection of definite, distinguishable objects of our intuition or of our intellect to be conceived as a whole. The objects are called the **elements** or *members* of S…with regard to the objects which may be allowed in a set, the phrase 'objects of our intuition' gives considerable freedom" (Stoll 1963, pp. 2–3). Cantor's Set Theory discovers a rich field of transfinite sets, in which totality and individual objects within the whole could be plurally thought of as a unit.

I shall borrow some symbolic expressions from modern Set Theory when discussing the logic of sets in ancient Chinese texts, such as Finite Set {a, b, c}, Infinity Set {a, b, c…}; Subset < >, Universal Set $\mathbb{U}$, and Empty Set $\emptyset$ or { }. I shall explain these expressions accordingly in the following four sections.

In Section 1, I will prove that a "plurality thought," thought of as a unit is required when learning Chinese pictographic language, which is very similar to learning sets taught by modern Set Theory. Borrowing some expressions and symbols from modern Set Theory, I will demonstrate how an alternative logic, the logic of sets, has roots in the system of Chinese pictographic language and was widely practiced among ancient Chinese thinkers in their teachings and reasoning.

In Section 2, I shall examine the efforts to clarify memberships made by ancient Chinese logicians, such as Gongsun Longzi in the School of Name and Mohists. When these logicians discuss set identity, they clarify concepts by sorting them into different sets. In his book, *Language, Truth and Logic*, Alred Jules Ayer says, "Philosophy is not a search for first principles," instead, "Philosophizing is an activity of analysis." (Ayer 2022, p. 27). This definition of philosophy fits well in doing Chinese philosophy. Ancient Chinese thinkers, including mathematicians, showed almost no effort to search for the first principles, but they were warmly engaged in many activities in the analysis of the relationship between a general set and the members or elements in the set.

In Section 3, I shall further explore the reasoning done by other thinkers from different schools during the pre-Qin era to the Han Dynasty. Evidence shows that not only early logicians, but also Confucius and Daoists viewed the relationship between totality and individual elements according to the logic of sets. They

employed a comment logic rule, Pointing Out, to indicate the elements in sets. Their activities of analysis were recorded in their written texts and the stories they taught. These activities clearly went beyond linguistic discussions but were guided by commonly accepted logical rules.

In Section 4, I argue that due to a good understanding of the logic of sets, ancient Chinese thinkers had higher achievements in understanding the order of elements in a set and the equivalence of two number sets. Chinese mathematicians were able to use the concept of sets to explain negative and positive integers, as well as the concept of zero creatively and effectively.

The conclusion of this paper is that it is not unusual for a practice to appear much earlier than a theory is formed. The practice of sets, with roots in Chinese pictographic language, is an example of this. To understand why and how the logic of sets was practiced during the pre-Qin and Han Dynasty, one can better understand the classic texts written by Confucius, Daoists, Mohists, and even ancient Chinese mathematicians. We want to not only appreciate the ancient wisdom, but also the beauty of the movements in ancient thinkers' reasoning.

A Plurality thought of as a Unit: Learning Chinese Pictographic Characters vs Learning Sets

A set is broadly defined as a collection of objects. According to Mary Tiles, "[T]he 'definition' of 'set' is less a definition than an attempt at explication of something which is being given the status of primitive, undefined, term" (Tiles 2004, p. 99). She then introduces Hausdorff's explication to the term "set":

> A set is formed by grouping together of single objects into a whole. A set is a plurality thought of as a unit (Hausdorff 1957, p. 11).

The first and most important feature of a set is that it is a plurality thought of as a unit. To correctly understand such a unit or a set, it is necessary to shift from the classification of membership with a single order of genius and species to a plurality thought of as a unit.

Philosopher Alfed J. Ayer says, "In particular, it is worth remarking that the process of defining *per genus et differentiam*, to which, Aristotelian logicians devote so much attention, always yields definitions which are explicit in the foregoing sense" (Ayer 2022, p. 60). Set Theory was brought in as a new way to understand the relationship between totality and individual objects, and this new way overcomes some of the Aristotelian logicians' difficulties in defining memberships.

Aristotelian logic does not allow "a plurality thought of as a unit" because of the Law of Identity ("A is A")[4] and the Law of Non-contradiction ($\sim (p\ \&\ \sim p)$)[5]. A

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[4]Aristotle says, "When A belongs to the whole of B and to C and is affirmed of nothing else, and B also belongs to all C, it is necessary that A and B should be convertible: for since A is said of B and C only, and B is *affirmed both of itself* and of C, it is clear that B will be said of everything of which

term in Aristotelian logic can be read as essences or membership. For creating an objective language, which can be studied by anyone objectively, classical propositional logic focuses only on the membership readings. A proposition is required as to be either true or false. A sequence of such propositions forms an argument. As long as the patterns (rules) are correctly followed, one will be able to reach a valid argument. A proposition is made by a subject term and a predicate. The distributive relations between terms are determined based on a presupposed order of the "membership."

For example, "all birds are animals" is a proposition, which can be either true or false in Aristotelian propositional logic. To confirm that it is true, its subject term must distribute to the predicate. In other words, all the members of birds distribute to the genus, "animal." The truth value of a proposition is pre-determined by the relations of terms in the hierarchical system of genus and species. "Hierarchical" system and "genus and species" relationship are both required for deciding the truth value of a proposition. The following is an example of a hierarchical tree of genus and species in the case of defining birds:

Figure 1. *Tree of Animals*
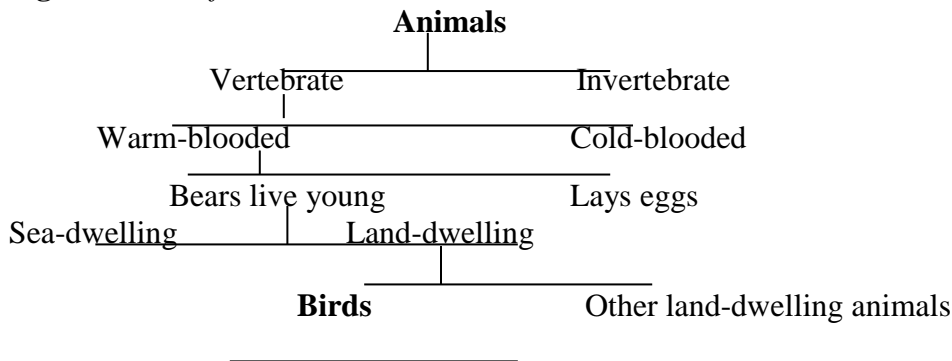
The tree has two characteristics: hierarchical and distributive. A pre-existing order guarantees a stable position to locate a member. The prefixed relations with other members can be found in this tree of genus and species.

However, understanding the above sentence, "All birds are animals", in terms of sets, the membership relation would be different. If all animals are in Set Animal, or Set A, and if all birds are in Set Bird, or Set B, we can say that Set B is an element in Set A, and at the same time, it is an independent set of all birds as well. We can, first of all, have the following set:

A is said, except A itself." (Aristotle, *Prior Analytics,* Book II, Part 22, 68a). Following Aristotelian tradition, modern philosopher, Wilhelm Wundt comes with the symbolic formulation, "A is A,"

[5]This law was practiced by ancient Greek philosophers, and Aristotle summarizes it as "No one can believe that the same thing can (at the same time) be and not be." "The most certain of all basic principles is that contradictory propositions are not true simultaneously."(Whitaker, CWA Aristotle's *De Interpretatione: Contradiction and Dialectic,* p. 184). After Fargo, philosophers, such as Russell symbolizes it as "~ (p & ~p)."

Set A {*α*, *β*, *ε*, …*ω*} (definition: A: Animals; *α*: dogs, *β*: birds, *ε*: cats, *ω*: tigers)
"β" (birds) is one of elements in the Set A. It contributes to the definition of Set A.

However, as for all birds, "β" can also be a subset itself as well, if we call it Set Bird or Set B:

Set B {$α_1$, $β_2$, $ε_3$, …$ω_x$} (definition: B: Birds; $α_1$: magpie, $β_2$: blue jay, $ε_3$: crow, $ω_x$: swallow)

Curly brackets, "{ }", are used as the symbol of a set in Set Theory. A set can be either a finite set or infinite set. Three dots, "…", are used to represent a large or countless numbers of elements within the set. An infinite set, for example, can be written as Set N {1, 2, 3, 4…} (definition: N: Number). The order differences of elements within a set does not change the value of the set, for example, Set N_1{3, 4} = Set N_2{4, 3}. Of course, a set can have only one member, such as Set S{Socrates}, or have no member, such as an Empty Set {} or ∅. These are basic concepts borrowed from Set Theory that I will use to re-evaluate ancient Chinese thinkers' reasoning processes in the later parts of this paper, in addition to my current discussion on plurality of thoughts for understanding sets.

The above sets, Set A (Animals) and Set B (Birds), are both finite sets. According to the relationship between totality and individuals, a set reading of "birds" can be both a member in the Set Animal and an individual set itself. In this kind of plural reading, Aristotelian hierarchical and distributive classes of genus and spices no longer serve as a presupposed order of the "membership." The meaning of membership here is replaced by a plurality thought of as a unit.

Turning to a discussion on the logical rational in the language system of Chinese pictographic characters, as well as its basic grammars, I want to, first of all, call an attention to the Chinese pictographic character *ji* 雧 , a set ([Han 漢], XU Shen, 許慎 221 bc, Book 4, Flock of Birds Unit). The character is a sketch on which a few birds are sitting on a tree. If the tree is a domain that includes a few individual members, then the image of character *ji* represents a fundamental understanding of sets: A set is about membership. The image of *ji* shows a finite set. In the light of Set Theory, this character could be viewed as a primal way of writing a set:

A Set is {$α_1$, $β_2$, $ε_3$} (definition: $α_1$: bird1, $β_2$: bird2, $ε_3$: bird3}

The character *ji* 雧 has no restriction on what kind of birds are on the tree, but simply represents a collection of members. Three birds in the sketch could represent more than one member within the domain of a tree. If all of them are sparrows, the birds in the set could be sparrow1, sparrow2, and sparrow3. They are the elements that represent the meaning of a set. If the birds are magpie, blue jay, and crow, the Chinese primitive understanding of sets could be like the modern concept of Set Bird or Set B, which I discussed above:

Set B {α₁, β₂, ε₃, …ωₓ} (definition: B: Birds; α₁: magpie, β₂: blue jay, ε₃: crow, ωₓ: swallow)

In addition, many Chinese characters in classical Chinese can be used as either nouns or verbs. *Ji* is one of them. As a noun, *ji* means a set. As a verb, it means collecting elements or members of the set. In fact, Zhuangzi used *Ji* in both senses when he talked about knowing *Dao* in his inner chapter four, which I will discuss later.

With the above understanding of a set, I think that it would not be difficult to see the obvious fact that learning Chinese characters requires training in understanding a plurality thought of as a unit, because the genius ancestors of Chinese people created their language in sets thousands of years before the theory of sets was born. The following are a few examples which show that thinking in sets or a plurality thought of as a unit is rooted in the Chinese language:

The example of "water" radical, *shui* and its elements, which I mentioned in the introduction of this paper, is a set, Set W or Set and it includes a list of elements which all associate with the water radical one way or the other. The ancient *Shuo Wen* dictionary by Xu Shen records them in water unit, one of radical sets ([Han 漢], XU Shen, 許慎 221 bc, Book 11, Water Unit):

Set {river , ocean , creak … fulfill } or, borrowing the symbol sings of elements from Set Theory:
Set W {α, β, ε, …ω} (Definition: W: water, α: river, β: ocean, ε: creak, ω: fulfill)

In the above set or water radical unit, element α, river, for instance, is a member of the radical unit. However, it is also a subset itself, Set R, which includes many rivers in the Set River or Set R:

Set R {α₁, β₂, ε₃, …ωₓ} (definition: R: Rivers; α₁: yangzi river, β₂: yellow river, ε₃:luo river, ωₓ: huai river)

Understanding Set W and Set R requires so called "a plurality thought of as a unit." To be clear, the character river is an element in the Set Water and at the same time, it could be a set for many different rivers in China itself, which is called a "subset" in Set Theory.

Every Chinese character has a position in one of the radical units or is a member or an element within its primary radical set. This means that the relationship between any radical and its associated elements follows the above pattern of set and members, like the water radical and its elements. All Chinese characters, according to philologists, are divided into six categories (*liushu* 六書). Two of the six are the most important ones. They are pictographs and ideographs. I will pick up one example from each of these two major categories to explain:

The first category, pictographs, is also called as imitative drafts. According to L. Wieger, they are "rough sketches representing the object; 畫成其物, 隨體詰詘…" (Yuan 2021, p. 252). *Shuo Wen* contains 364 of these primary radicals (Wieger and Davrout 1965, p. 10). These basic pictographic radicals are building blocks or indicators for the 5,000-7,000 most useful Chinese characters in classical Chinese language. Like the water radical that I discussed above, all other radicals and their associated characters clearly share the common nature of a plurality thought of as a finite set, which gives each radical its domain like a general set, and each character has its position in the general set as one of the elements. The following is another example in *Shuo Wen* ([Han 漢], XU Shen, 許慎 221 bc, Book 10, Horse Unit):

Character horse, *ma* 馬 is a radical with a finite domain, which holds many members, such as, "black horse 驪 *li* (α)," "one-year-old horse" " 駒 *ju* (β)." "Examination" is written as " 驗 *yan* (ε). It refers to the act of checking the mouth and legs of a horse." "Stopping at a place" is written as " 駐 *zhu* (ω)," which represents a horse tethered to a tree trunk.

Categorizing words within the domain of a radical is already close to a kind of set-oriented thinking. If we borrow some expressions from set theory again, we can have Set H (H: Horse Radical)[6]:

Set H $\{\alpha, \beta, \varepsilon, \ldots\omega\}$

"Black horse," "one year old horse," "an examination," "stopping at a place"…are unrelated concepts from the perspective of Aristotelian categorical propositions; they cannot be put in one category or treated as the members within a class at all. However, it is so natural for the Chinese people to treat them in one category or within a more general set, "horse, *ma* 馬 ." The rationale of doing so has to be viewed pictographically. Every member in the Set H contains a horse radical 馬 and associates with the horse one way or the other in human experiences. One of the attractive aspects according to Cantor's Set Theory is that sets can be designed subjectively. Human consciousness cannot be eliminated completely when learning sets. In other words, human experiences can be involved in the process of understanding different sets. Creating Chinese pictographic characters

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[6]I discussed the horse radical in my paper, "On an Alternative Logic of Knowing (知 *Zhi*) in Zhuang Zi: A Comparison of Knowing Sets to Knowing the Way" (Yuan 2021).

heavily depends on human experiences. This feature is also found in the second category of creating and learning Chinese characters.

The second category is called "ideographs," which includes the characters that their meanings can be inferred from a combination of a few primary radicals. Many characters that hold more abstract meanings fall in this category. If 364 primary radicals are sketches of tangible things in human experiences, such as water, mountain, the Moon, the Sun, horse, sheep, human, mouth, eyes…and etc., then learning ideographs involves more subjective imagination. A simple example in the category of ideographs is: the primary radical of the Sun is written as ⊟ ,

and the primary radical of the Moon is written as ⊖ . Putting these two primary

radicals together，⊙⟩ , it creates a new character, "brightness." The new meaning is given by both radicals, the Sun and the Moon. When the lights of the Sun and the Moon are combined, one can of course imagine how bright it would be.[7]

In her recent book, *Ten Lectures on Chinese Character and Chinese Culture*, Ning Wang confirms, "Chinese characters are ideographs. They were created according to original meanings. By putting two or more radicals that already have accepted meanings together, ancient Chinese people created new words…Not only are the parts pictographic, but also the rationale of creating new words refers to ideographs" (Wang 2019, p. 39).

Using ideographs, Xu Shen demonstrated more complicated functions of set-oriented thinking when Chinese ancestors created and practiced with Chinese pictographic language. Their practice of sets was long before Set Theory was born. Yet, with the principles and discovery of modern Set Theory, one can re-evaluate the ancient thinkers' contributions and appreciate their wisdom objectively. According to Set Theory, the fundamental relation is about membership. In many cases, an object or an element can be a member of (or occur in) both Set A and Set B. Such a relation is called "intersection" and is written as A∩B (it read as "the intersection of A and B"). It is defined as the set composed of all elements that belong to both A and B. For example, a white horse can be a member in the Set H (Horse) and Set W (White), for a white horse can be a member of horse and a member of color white simultaneously, therefore, we can say Set H ∩ Set W (there is an intersection of Set Horse and Set White). During the pre-Qin era (Paleolithic age–221 B.C.), Chinese logicians were profoundly interested in the discussion of A∩B issue, such as Gongsun Longzi's famous claim: "A white horse is not a horse," which I will discuss further in the next section. At this point, I am maintaining that the strong interests those logicians had at that time in discussing the intersection of two different sets benefited from the advanced thinking of sets when studying communication with Chinese pictographic language and are beneficial to current discussions in Set Theory too.

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[7]Henry Rosemont says, "[T]extbook examples of this category of ideograms are *ming,* which combines the pictures of the sun 日 and moon 月 together to signify the Chinese word for 'bright', 明" (Rosemont 2019).

Let's look at two characters, *mei*, beauty and *shan*, goodness.

They both are ideographs with abstract meanings. *Shou Wen* says, "Beauty means having a delicious taste. It goes with the radical sheep and the radical big. Among six different animals, sheep are the main source of formal meals. Beauty and goodness share the same meaning. 美，甘也。从羊从大。羊在六畜主給膳也。美與善同意。" ([Han 漢], XU Shen, 許慎 221 bc, Book 4, Sheep Unit).

Shuo Wen also says, "Goodness means auspicious. It goes with the radical sheep and the radical praising loudly. Goodness and beauty share the same meaning. 善，吉也。从誩从羊。此與義美同意。" ([Han 漢], XU Shen, 許慎 221 bc, Book 3, Competitive Speech Unit).

Beauty is an aesthetic concept. Goodness is an ethical concept. When Greek philosopher Plato teaches that the good is the beautiful, he says clearly, "Virtue is a kind of health, beauty and good habit of the soul." (Plato 2010, p. 13). The Form of beauty and the Form of goodness share their connotation in the soul. It is interesting that when ancient Chinese people talk about beauty and goodness, these two concepts do not associate with the soul but a tangible object, sheep. A big and fat sheep represents beauty and having a sheep and praising it loudly means goodness. In ancient China, sheep is the main contribution when worshiping ancestors. So that both characters, *mei*, beauty and *shan*, goodness associate with sheep pictographically. In addition to the elements beauty (*b*) and goodness (*g*), as a primary radical of set, Set or Set S (sheep) also includes other elements, such as " *gao,* baby lamb (*l*) , *qun*, a herd of sheep (*h*), *qiang*, a herdsman in the western border (*m*).[8] Therefore, the Set or Set S (Sheep) is:

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[8]Shuo Wen says, " *gao,* baby lamb, going with the sheet radical. 羔， 羊子也。从羊."

" qun, a herd of sheep, go with the sheep radical and the pronunciation goes with *jun*. 群，輩也。从羊君聲。" " *qiang,*a herd man in the western border. 羌，西戎牧羊人也。" （*Shuo We*n, Book 4, Sheep Unit）

Set 羊 {lamb 羔, herd 羣, herdsman 羌…beauty 羑, goodness 譱} or Set S {*l. h, m,…b, g*} .

Every element in the Set 羊 or Set S has a part associating with the radical sheep, which is a sketched image of a sheep. None would overlook this feature.

However, the character goodness 譱 is also an element in the unit or set 誩, which is a sketch of two persons with big mouths, talking competitively. *Shuo Wen* says:

誩, talking competitively. It goes with two speech radicals, *yan* 言. Everything associated with competitive speeches falls into this unit or primary radical set. The pronunciation of the character 誩 is *jin*, as same as the character competition, 競, 競言也。从二言。凡誩之屬皆从誩。讀若競。” ([Han 漢], XU Shen, 許慎 221 bc, Book 3, Competitive Speech Unit).

Hence, we have to look at another set, Set 誩 or Set C (Competitive Speech). Set C also includes goodness (*g*) as one of elements, as well as other members, such as 競 *jin*, competition (*c*) and 讙 *yuan*, accusing (*a*)[9]:

Set 誩 {competition 競, accusing 讙 … goodness 譱} or Set C {*c, a…g*}

In the above radical set, every element contains the radical 誩 *jin.* Since character 譱 *san*, goodness is a member in both Set S (Sheep) and Set C

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[9]Shuo Wen says, “ 競 *jin*, local language. It means chasing each other, going with the radical of competitive speech. It is an image of two persons running. 競, 彊語也。一曰逐也。从誩，从二人。” “ 讙 *yuan*, accusing with hatery. 讟，痛怨也.”

(Competitive Speech), there is an intersection, *g*, existing in both sets. Borrowing the expression from Set Theory, we can say that Set S ∩ Set C. Character goodness (*g*) is not the only character which exists in more than one radical set. Many ideographs share this feature since they are made of more than one radical.

To sum up, both characters, beauty and goodness, are associated with the radical sheep by having the primary radical as a part of these characters. And in the case of *san*, it associates with two primary radicals and shows an intersection of two sets. This fact proves that a plurality thought of membership is not an unfamiliar field for ancient Chinese thinkers.

Set Identity: Exploring Ancient Chinese Logicians' Efforts to Clarify Memberships in Different Sets

Scholars in the field of Comparative Philosophy have many discussions on how ancient Chinese thinkers tried to find the proper associations among the myriad things (*wanwu* 万物), and their relationships with *Dao*. Finding a proper association seems to be one of the main tasks in a Chinese traditional way of reasoning. Some philosophers argue that the associations can be found according to ancient Chinese thinkers' personal experiences, while others think that the associations relate to the presupposition that everything is changing in Chinese Philosophy. In this paper, I also discuss associations. I shall focus on the membership of sets, which is rooted in the special nature and structure of Chinese pictographic language. When I suggest viewing how objects, things, relations, and *Dao* are associated with one and the other from the perspective of sets, I want to avoid an existing problem in reading ancient texts: if we simply use the concepts in our own languages to translate classic texts, it could end up interpreting ancient Chinese thinkers' thoughts with our own, and their original wisdom is lost.

This problem was pointed out by comparative philosopher Roger Ames in the 1990s. If it is the fact that the above two major philosophical concepts, beauty and goodness, in the tradition of ancient China could be understood so differently from that in the Platonic tradition, if even modern Chinese people do not associate concepts of beauty and goodness with sheep necessarily anymore [10] as their ancestors did, then we encounter the problem that Ames points out. Some Chinese key concepts have no matching words in English. Ames' concern affects many other philosophers who study unfamiliar texts from Chinese culture. In his paper, "Philosophy as Hermeneutics Reflections on Roger Ames, Translation, and Comparative Methodology," comparative philosopher Steve Coutinho summaries Ames' claim and impact as follow:

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[10]Although when writing these two characters, Chinese people still need to write the sheep radical, hardly any learners of the language would ask why.

> Roger Ames has a radical claim: The ancient Chinese cultural and philosophical traditions do not share what some might take to be our universal concepts and methodologies. If we insist on imposing them, we misread the texts altogether. We find only our familiar presuppositions reflected in them and fail to allow the texts to philosophize in their own distinctive voices.
>
> The problems become most pressing when attempting to engage with ancient texts through the medium of the English language. Even within a single language, synonyms are rarely, if ever, exact counterparts, so we should not expect to find terms that have identical meanings in languages and cultures that are historically, geographically, and linguistically distant. (Coutinho 2021, p. 69)

Knowing the difficulties involved in engaging with ancient texts, my effort in this paper is to present a new interpretation of ancient Chinese thinkers' wisdom. Fortunately, Chinese pictographic characters preserve a rich cultural heritage. And Xu Shen's *Shou Wen* preserved these cultural codes systematically. It turns out that an etymological study is necessary when engaging with ancient texts in the pre-Qin times when discussing Chinese philosophy and logic. By doing so, one reduces his or her dependence on the medium of either English or modern Chinese, for the sake of avoiding the initial misunderstanding that traditional Western philosophy or modern simplified Chinese characters project onto ancient Chinese philosophy.

What I want to explore further is that if learning Chinese pictographic characters in the pre-Qin era involves a practice of a plurality thought of as a unit, or in other words, if typically, learning and writing Chinese pictographic characters in classical Chinese involves a necessary practice of thinking in sets, which is formed while one learns these characters at a young age, then with such a thinking habit, a logical question to ask is: is it possible that ancient Chinese thinkers used their logic of sets to structure their reasoning process and arguments effectively?

My efforts in this and the next sections include three steps. Firstly, I will analyze what kind of relationships ancient Chinese logicians tried to clarify during the pre-Qin era. Secondly, I will explore the original meanings that a key Chinese character or concept carries. I will explain how and why pre-Qin philosophers, such as Confucius, followed the logic of sets when giving definitions. Thirdly, I will follow the possible logical patterns that ancient Chinese thinkers in the pre-Qin era might have used and let the classic texts themselves reveal such an alternative logic of sets, so that an appreciation of the profundity of Chinese philosophy can be reached.

Firstly, I start with a discussion on one of the major tasks that ancient Chinese logicians carried out. Both Gongsun Longzi (320-250 BCE) in the School of Names and Mohists are interested in clarifying logical relations. They are viewed as ancient logicians in China for that reason. Let us look at what kind of relationships or memberships they try to clarify in their teachings. The following is Gongsun Longzi's famous argument: "A white horse is not a horse." (*Bai ma fei ma ye*. 白馬非也).

"Horse" is that by means of which one names the shape. "White" is that by means of which one names the color. What names the color is not what names the shape. Hence, I say that a white horse is not a horse.

…

If one wants a horse, that extends to a yellow or black horse. But if one wants a white horse, that does not extend to a yellow or black horse. Suppose that a white horse were a horse. Then what one wants [in the two cases] would be the same. If what one wants were the same, then a white [horse] would not differ from a horse. If what one wants does not differ, then how is it that a yellow or black horse is sometimes acceptable and sometimes unacceptable? …Thus, it is evident that white horses are not horses. (Ivanhoe and Van Norden 2005)

Considering that plural thinking of membership and understanding of an intersection of two sets were familiar skills that those pre-Qin thinkers gained from learning Chinese pictographic language, the impacts of Chinese classical language in shaping Chinese logic could fit in the situation, as anthropologist-linguists Edward Sapir and Michel Breal have discovered:

Language is forced to select…In every country, ancient and modern, language has supplied the instrument and the prime material of instruction. This universal agreement is natural…Language is a transition of reality, a transition in which objects figures already generalized and classified by the labor of thought (Sapir and Breal 2023, pp. 102–109).

With respect to linguists' discoveries, one should be able to see the fact that the primary method of thought in doing reasoning among ancient Chinese thinkers was via a logical thinking of sets. What Gongsun Longzi was talking about, in fact, is how to separate two different sets. One is the Set Shape of Horse, or Set H. The other is the Set Color of White, or Set W. He made it very clear that in Set H, it can include the horses with different colors as members or elements, including a white horse. However, in Set W, only a white horse can be included as one of the members, not the horses of other colors. Gongsun Longzi presents two sets:

Set H $\{w, y, b…x\}$ (Definition: H: the Shape of Horse; w: white horse, y: yellow horse; b: black horse; x: any other color of horse)
Set W $\{w…x\}$ (Definition: W: the Color of White; w: white horse, x: any other object with color white)

The above two sets, Set H and Set W, share intersection, namely, Set H ∩ Set W. The intersection is "white horse." "White horse" can be both a member of Set H and a member of Set W.

The ancient Chinese logicians had strong interests in separating different sets. They were not sophists, but the thinkers who loved to play their own language game according to their own rules. More examples can be found in the Mohists' teaching. Taking the following three statements as examples, in the Mohist logic text *Xiao Qu*, Mohists say:

a) 獲之亲人也, 獲事其亲非事人也.

Jill's parents are *ren* (people), but Jill's serving her parents is not serving *ren* (people). (*Xiao Qu*, 11) (Tiles and Yuan 2015)

Understanding the above statement from a perspective of sets. The statement is evidently talking about that there are two sets, and they are not identical sets, although they have an intersection:

Set J {*f, m*} (Definition: J: Jill's parents, f: Jill's farther; m: Jill's mother)
Set P {*a, b*, c, *f, j, m*,,,..*x*…}(Definition: P: People; *a*: Amy; *b*: Bob; c: Cathy; f: Jill's farther; J: Jill; m: Jill's mother; *x*: anyone else)
Therefore, Set J ≠ Set P, although Set J ∩ Set P.

Similarly, the following two statements from Mohists represent the same logic pattern:

b) 其弟美人也, 愛弟非愛美人也.

Her younger brother is a handsome man, but loving her younger brother is not loving handsome men. (*Xiao Qu*, 11) (Tiles and Yuan, 2015)

Representing the above statement in sets:

Set H {*b*, c, d…x…} (Definition: H: Handsome Men; b: younger brother; c: Charles; d: Danny; x: any handsome man)
Set L {*b*} (Definition: L: Love; *b*: younger brother)
Therefore, Set H ≠ Set L, although Set H ∩ Set L.

c) 盗人也, 杀盗非杀人也.

Robbers are people, but killing robbers is not killing people. (*Xiao Qu*, 11) (Tiles and Yuan 2015)

Representing the above statement in sets:

Set P {*a, b*, c, *d, r*…*x*…} (Definition: P: People, *a*, Aaron; *b*: brother; c: Charles; *d*: Danny; *r*: Robbers x: anyone else)
Set K {*r*} (Definition: K: killing; *r*: Robbers)

Therefore, Set P ≠ Set K, although Set P ∩ Set K. In addition, robbers can be a subset itself. It can be treated as an element in the Set P (people), at the same time, it can have its own members in the Set R (Robbers) if there are more than one robber. The membership involved in the above three statements are not propositional but about how to distinguish different sets.

Before discussing how widely the logic of sets was practiced among the pre-Qin thinkers, I must clarify one of the problems that falls into Ames's category of misleading due to translation. If one reads the affirmative and negative statements, such as "a white horse is a horse" or "a white horse is not a horse" in classical

Chinese, there is hardly a possibility to confuse them with Aristotelian propositions. However, Classical Chinese does not have a "to be" structure.[11] Using the copula "to be" to serves as the connecting link between subject (S) and predicate (P) of a proposition is added in when translating these Chinese statements into English sentences, according to English grammar. The add-in copula "to be" is necessary, however it often gives readers a fake hope that one might be able to find similar propositional patterns and rules in Chinese logic. The original affirmative and negative statements in classical Chinese in fact follow the following patterns:

SP **ye** (affirmative pattern) and S *fei* P (negative pattern)
"SP **ye**" is a pattern for the statements like:

"*Bei ma, ma ye.* " (A white horse is a horse).

Confirmative character, *ye* is an image of having intercourse, according to *Shuo Wen*. A white horse is a member in Set Horse. A member and a general set are associated. The image of *Ye* confirms that this member falls into the domain of the general set. There is no need for a copula "to be" for the Law of Identity. The membership relation between subject (S) and predicate (P) is not genus and species, but a member or element and a general set.
"S *fei* P" is a pattern for the statements like:

"*Bei ma fei ma ye.* (A white horse is not a horse.)

Negative character, *fei* is an image of fences and a ditch that block the Set of White Horses mixing with the Set of Horses. *Ye* at the end confirms that they are different sets because the set identity of two sets is not equivalent. One is a subset (White Horses) and the other is a general set (Horses). *Fei* negates that one subset equals a general set. There are different members in each of these two sets. Therefore, Set W ≠ Set H, even though a subset "while horses" does exist in Set H (Horse).

Totality and Individual Elements: The Practices of Logic of Sets among Ancient Chinese Philosophers

While learning Chinese radicals and characters, language learners must have a logical understanding of the totality and individual objects to master this pictographic language. If language shapes one's way of thinking, then thinking in

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[11]Even *shi*, 是 (to be) and *bu* 不 (not so) in modern Chinese are not necessary. A. C. Graham says, "In Classic Chinese, there is a radical difference between the nominal sentence with the final *yeh* negated by *fei* and the verb sentence negated by *pu* [*bu*]. This difference by no means coincides with that between sentences with or without copulative 'is' in English, since we use the copula with predicative adjectives, which in Chinese are translatable by verbs." (Graham 1978, p. 26)

Chinese pictographic language could naturally shape the practice of the primary logic of sets in reasoning. Pre-Qin logicians did summarize a few rules for grasping the totality and individual elements in a set. This practice can be found in other schools too. Solid evidence of practicing the logic of sets was found in many pre-Qin thinkers' writings. Due to limited length of this paper, I shall mainly focus on demonstrating how Confucius and the Daoists practice logic of sets. They are two major schools during the pre-Qin era.

In his "On Pointing at Things" (指物論; *Zhiwu Lun*), Gongsun Longzi introduces one of popular rules in reasoning, pointing out, *zhi*, which is a hand radical (left part) pointing out the core meaning. The right part of 指 *zhi* is "旨," means "meaning." *Zhi* serves as both a verb and a noun in classical Chinese. As a verb, *zhi* means pointing, as a noun, *zhi* mean core meaning.[12] A. C. Graham says:

> We have plenty of evidence as to how pre-Han philosophers used the word *chih* [*zhi*], both as a verb ("point out") and as a noun ("what is being pointed out"); they applied it not only to the gesture of pointing but to the meaning of discourse and the meanings of word. ("universal," "quality," "logical class") is wrong-headed in principle (Graham 1978, p. 458).

A. C. Graham translates *zhi* as pointing. However, he immediately says, "One is convinced that if only one could identify what the writer meant by the word *zhi*, everything he says would fall into place" (Graham 1955, p. 282). In Chinese logic, the rule, *zhi* 指 was emphasized in the *Later Mohist Canons* and used by many other philosophers in different schools during the pre-Qin and pre-Han eras. such as Confucius (ca. 551-479 BCE).

Gongsun Longzi claims, "Every object is in nothing but sets. However, a set cannot identify the set itself…The identity of a set is not a tangible existence in the world. The tangible elements in the set are objective existences in the world. 物莫非指，而指非指……指也者，天下之所无也；物也者，天下之所有也."[13] If, as I suggest, looking at Gongsun Longzi's discussion on logical rule *zhi* from the perspective of sets, what he says could convincingly fall into place. Intuitively, a set is a collection of objects, and the set identity can only be defined by every member in the set. *Zhi* as a noun ("what is being pointed out") is the core meaning of the set or is about the set identity. *Zhi* as a verb is an action of pointing out the members which associate with the set as elements within the set. Plurality of thought as of a unit needs both because the meaning of a set is demonstrative in the process of time. Pointing out the members or elements in a set is the way to pursue a clear understanding of the general set and its relationship with the elements.

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[12]A. C. Graham summarized that the word 指 *zhi* has three main functions: 1) Noun, "finger"; 2) Verb, "point out one form another"; 3) Noun, "meaning," the direction in which discourse points, its meaning or drift, the main point in contrast with details or side issues (Graham 1978, p. 458).

[13]The translation is mine. There are different translations on these claims. My translation is based on an understanding of Gongsun Longzi's logic of sets.

Taking Confucius' way of teaching his disciples virtues as an example, he rarely gave definitions to his ethical concepts. Instead, he pointed out one after the other examples of virtuous conduct to demonstrate what he meant. In this sense, I claim that Confucius taught his disciples with the set-oriented thinking. A full investigation of Confucius is beyond the topic of this paper. Within this limited space, I will use one example from the *Analects* to illustrate that thinking in sets was a common practice in the pre-Qin era. When Confucius teaches his disciples the practice of being filial, he uses the classical character of *xiao* 㦔 which shows an image that a son supports the elder from the bottom while an aged man sits at the top. Ames correctly translates *xiao* as: filial conduct (Confucius et al. 1999), because the primary image of *xiao* is about a type of conduct. *Xiao* is one of the core virtues in Confucius' *Analects*. When Confucius teaches this core virtue, instead of giving a universal definition, he points out several examples of performing filial conduct to illuminate the concept of being filial:

> Meng Yizi asked about filial conduct (*xiao*). The Master replied: "Do not act contrary." (*Analects* 2.5).
> Meng Wubo asked about filial conduct (*xiao*). The Master replied: "Give your mother and father nothing to worry about besides your physical well-being." (*Analects* 2.6).
> Ziyou asked about filial conduct (*xiao*). The Master replied: "Those today who are filial are considered so because they are able to provide for their parents. But even dogs and horses are given that much care. If you do not respect your parents, what is the difference?" (*Analects* 2.7).
> Zixia asked about filial conduct (*xiao*). The Master replied: "It all lies showing the proper countenance. As for the young, contributing their energies when there is work to be done, and deferring to their elder when there is wine and food to be had-how can merely doing this be considered being filial?" (*Analects* 2.8).
> …

Reading the above quotes in *Analects* from the perspective of sets*,* one can see that when Confucius did his logical reasoning, he pointed out one after the other filial conduct (element) in *xiao* (a set of collection of "filial conducts") to help his disciples identify the virtuous *xiao* or the way of being filial. No definition is given to what is *xiao*. The repeated applications of filial conducts reveal the set identity of *xiao*. The way of being filial is understood as following:

> Set X   Pointing Out Filial Conducts in the Domain X
> *Xaio*, A son supports the elder (X),   X is $\{\alpha, \beta, \varepsilon, \omega, ...\}$
> (Definition: X: *Xiao*; α: Do not act contrary; β: Give your mother and father nothing to worry about besides your physical well-being; ε: respect your parents in addition to providing food to them; ω: young works, parents drink...) (Yuan 2017)

It seems that Confucius' disciples grasped what the meaning of being filial was in the real world where they lived in without any difficulties, and they learned the virtue via the repeated applications or exemplary filial conduct. Confucius also taught other virtuous codes, such as, *Ren* authoritative contact, *yi* sense of appropriateness*, he* harmony… with the similar logical approach.

Since character *xiao* 肅 holds a clear demonstrative meaning, like beauty, *mei* and goodness, *san*, which I discussed in the section one, although *xiao* is a more abstractive notion than horse or sheep, Confucius successfully demonstrates the meaning of *xiao* by pointing out a list of filial conduct. It is like understanding Set Horse by pointing out every individual horse in the set.

During the pre-Qin era, more metaphysical concepts, such as *Dao* 道, the world 天下 *tianxia*, myriad things 萬物 *wanwu*, nothingness 無物 *wuwu*, etc. were actively discussed by Daoists and other thinkers. A. C. Graham points out that Chinese philosophers used the rule "Pointing Out" again and again in their dialogues and debates[14]. They used this rule to separate one element from the other and one relation from the other. It is a popular rule in Chinese logical reasoning. Engelfriet also points out: "A Chinese term for "demonstration": The term, *zhilun* 指论，"discussing by pointing out," was meant in a literal way: arguing something with the help of a model or diagram" (Engelfriet 1998, p. 150).

Philosopher Jean-Paul Reding suggests understanding Gongsun Longzi's discussion on *zhi* together with his notions of the world, *tianxia* 天下 and *fei zhi* 非指. Reding suggests translating *fei zhi* as "points at things that do not exist in the world" or "the pointing is at nothing." He says, "[I]f even the pointing at nothing is a pointing at something, then every pointing is indeed at something. Nothing is also something, otherwise we could not say anything about it." (Reding 2002, pp. 200–201). Although I disagree with Reding to label Gongsun Longzi's statements as propositions, I highly appreciate Reding's approach and believe that it can be fruitful if one takes the pre-Qin notion, the world *tianxia*, as a universal set and "the pointing is at nothing" as an empty set, when evaluate how Daoists practiced the logic of sets. Borrowing the expressions from Set Theory again:

Universal Set is written as: 𝕌: A set of all possible values.
Empty Set is written as: {} or ∅: A set can be within any other sets without changing values.

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[14]In my paper, "The Role of Time in the Structure of Chinese Logic" (*Philosophy East and West*, January, 2006), I list the following examples of the discussion about "*zhi*, Pointing Out" among Chinese scholars:

A. C. Graham says, "Gongsun Long in Name School, Zhuang Zi in Daosim and other Chinese philosophers all have discussions on the rule, 'Pointing Out.' Gongsun Long says: '且指天下之所兼。天下无指者， 物不可谓无指也。' Moreover [the pointed out] meanings are collected together by the world. That nothing within the world is the [pointed out] meaning is in the case of things; it being inadmissible to pronounce that nothing is the meaning……" (Graham 1978, 462).

Sun Zi, a pre-Qin philosopher in the field of the philosophy of war, says: "故知者为之分别，制名以指 实。Therefore wise men made divisions and distinctions on behalf of them and instituted names in order to point out objects" (Graham 1978, p. 458). Zhuangzi says: "周，偏，咸三者，异名同实，其指一也。These three, *zhou, pian,* and *xian*, are different names for the same object; their pointed-out meaning is one." Yan Shigu (581-645), a linguist of the Sui Dynasty says: "指谓义之所趋，若人以手指物也。Meaning refers to what the sense runs to words, like a man pointing out a thing with his hand" (Graham 1978, p. 459).

Taking Zhuangzi as example, Zhuangzi's famous writing, inner chapter two, "The Sorting Which Evens Things Out" is a poetic and philosophical essay which is full of interesting stories without deductive steps to lead a conclusion. Having read the essay, most likely, one must agree that Zhuangzi did find out the sorting which can even things out in his own way. Zhuangzi talks about different sounds, the difference between subjective words and objective meanings, between *zhi* and *fei zhi* (pointing at something and pointing is at nothing), relativity of time (today and yesterday), the gap between death and life, variance in sizes of large and small (mountain and the tip of an autumn hair), this and that, beginning and ending, *you* 有 and *wu* 無 (existence and no-existence), relative standards, transformation between Zhuangzi and a butterfly…etc.

Reading this essay from the perspective of logic of set, Zhuangzi's "The Sorting Which Evens Things Out" is a logical reasoning based on the framework of thinking in sets. Robert Stroll says, "The essential point of Cantor's concept [of set] is that a collection of objects is to be regarded as a single entity (to be convince as a whole)…With regard to the objects which may be allowed in a set, the phrase 'objects of our intuition or our intellect' gives considerable freedom." (Stoll 1963, pp. 2–3). This freedom allows sets to be designed freely, and a subset can be either an individual object or a relationship. For any such a pair of objects to appear as elements of a particular set, one can treat the pairs as entities. Or, in other words, one can treat these relationships as individual elements. This is what Zhuangzi does. A. C. Graham says, "Chuang-tzu never does say that everything is one…always speaks subjectively of the sage treating as one" (Graham 2001, p. 56). Zhuangzi lists several stories about opposite relationships and treats them as subsets in *Dao*.

Utilizing an expression of an ordered pair set from Set Theory: $<x, y>$, the above list of relationships can be written as following: $<t_1, p>$ (t_1: tubes of men, p: pipes of Heaven); $<s, o>$ (*s*: subjective words, *o*: objective meanings); $<z, f>$ (*z: zhi, f: fei zhi*); $<t_2, y_2>$ (t_2: today, *y*: yesterday); $<d, l>$ (*d*: death, *l*: life); $<b, e>$ (*b*: beginning, e: ending); $<m, h>$ (m: mountain, *h*: hair); $<y, w>$ (y*: you,* w*: wu*); $<c, b>$ (*c*: Chuang-tzu, *b*: butterfly)

Zhuangzi intends to demonstrate that "Heaven and Earth are the one meaning, the myriad things are the one horse. 天地，一指也；万物，一马也。" (Graham 2001, p. 53). How could it be possible that myriad things are the one horse? A legitimate explanation is from the perspective of sets. Heaven and Earth, as *Tianxi,* have one meaning, which is *Dao*. Objects, actions, relations, and so on, between Heaven and Earth are members of the Universal Set *D* (*Dao*). It does not matter whether it is an individual horse, a subset of autumn hairs, or many subsets of different relations, they are all elements within the Universal Set *D* (*Dao*). Elements can be different, but as an element, it simply functions as one of the members in the set. A subset of relationship or an individual horse, or myriad things, in the sense of a member in the set, they have no difference. They are simply one and another element. All these members or elements represent the identity of Universal Set *Dao* together. Pointing to a horse is pointing at a member in the Set *D*, and is pointing at *Dao* simultaneously. Pointing at myriad objects, things, or relations is pointing at one and another member in the Set *D*. Pointing

out these elements, we also point at the Set *Dao*. It does not matter whether it is a finite or an infinite set, "a set is completely determined by its members" (Stoll, 1963, p.3). So does the Universal Set *Dao*. In the sense of sets, Zhuangzi's above claim stands solidly.

$\mathbb{U}$ Set *Dao* can be expressed as:

$\mathbb{U}$ Set *D* {*α, β, γ, δ, ε…*}, "*α, β, γ, δ, ε…*" are myriad things or relations which associate with *Dao* as elements or members. They are parts of *Dao*, and represent *Dao* in a way that elements represents the general sets.[15] As elements of the Set *Dao*, myriad things weight indifferently from another element, horse. In "The Sorting which Evens Things Out", Zhuangzi presents *Dao* as:

$\mathbb{U}$ Set *D* {<t_1, p>, <s, o>, <z, f>, <t_2, y>, <d, l>, <b, e>, <m, h>, <y, w>, <c, b>… *H*} (Definition: *H*: horse)

Zhuangzi does not only discuss *Dao* and myriads things, but also teaches people how to be with *Dao*. Zhuangzi employs the concept, 集虚 *jixu,* an empty set, which he uses when he talks about knowing *dao* in his inner chapter four. Evidence shows that, in the pre-Qin era, ancient thinkers understood that "Every pointing is at something, and yet the pointing is at nothing" (Reding 2002, pp. 201–202). "[T]he pointing is at nothing" implies that an empty set can be pointed out as an element. On one hand, it is a fact that Zhuangzi discussed the empty set much earlier than Set Theory was born. However, his concept of *jixu* empty set did not gain enough attention. On the other hand, because of the guidance of Set Theory, we can study Zhuangzi's wisdom with a better understanding by borrowing the symbol of Empty Set { } or ∅ from Set Theory.

According to Set Theory, an empty set can be a member of any sets. When Zhuangzi teaches about knowing *Dao* and myriad things, the first practice in his "Sorting which Even Things Out" is to practice "loss I" or make "I" an empty set. This is an effective way to explain how one disappears in the Universal Set *Dao* when he/she is with myriad things indifferently in *Dao* as an element of an empty set. If an empty set can be within any sets without changing the values of those sets, then the Empty Set I: ∅ can be in either a subset, a general set, or the Universal Set *Dao*:

Set H {h_1, h_2,..h_x…} ≡ Set H {h_1, h_2,..h_x…∅} (Definition: H: horses; h:an individual horse)
$\mathbb{U}$ Set *D* {α, β, γ, δ, ε…} ≡ $\mathbb{U}$ Set *D* {α, β, γ, δ, ε…∅}

Therefore, for Zhuang Zi, losing the self "I" is a way to be with *Dao* and to understand *Dao* together with myriad things indifferently. In his own words, let

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[15]Starting from Laozi, Daoists never give *Dao* a definition as the most general concept in an hierarchical structure of classes of genies and species. All Laozi said was "The nameless was the beginning of heaven and earth" (Lao Tzu, et al. 1963, p.5). *Dao* is great!

"the frame made to be like withered wood, the heart like dead ashes." "This time I had lost my own self" (Graham 2001, p. 48).

Logic of Sets and Etymological Approach: Discovering the Achievements of Ancient Chinese Mathematics

To discover the minds of ancient thinkers is like exploring an unknown universe. Logic of sets is a modern device that I use to analyze and evaluate ancient texts. This etymological study is similar to archaeological research, which is based on discovered artifacts. Cultural codes preserved in Chinese characters help me investigate how the ancient language shaped the minds of pre-Qin and pre-Han thinkers. In concluding this investigation, I will not only summarize the significant functions of these two approaches in doing Comparative Philosophy but also reveal new discoveries from ancient wisdom with the assistance of Set Theory and etymological studies.

First, it is a fact that language is one of the few resources that can be called upon as a means of preserving cultural heritages and bringing ancient wisdom to us. However, as the Chinese language has undergone significant changes over the past several thousand years, so has the culture. Chinese people today no longer think in classical Chinese. Hence, misunderstanding can easily result when translating classical texts into modern Chinese or other languages, such as English. Some ancient wisdom could easily be lost in time. My set-logical approach helps my efforts to uncover this wisdom. Like many other theories, Set Theory appeared much later after people in China and the West have employed sets in their lives and thinking. Yet, with the assistance of the theory, when re-evaluating ancient Chinese thinkers' teachings, new ways of understanding them are made available.

For example, when reading Zhuangzi, modern readers can easily sense the beauty of his writings, but hardly understand the logic that supports the flow of his thoughts. Zhuangzi has a famous story about monkeys and a monkey keeper in his work, "The Sorting Which Evens Things Out":

> A monkey keeper handing out nuts said, "Three every morning and four every evening." The monkeys were all in a rage. "All right then," He said, "four every morning and three every evening." The monkeys were all delighted. Without anything being missed out either in name or in substance, their pleasure and anger were put to use (Graham 2001, p. 54).

This is a fun story to read, and I read it myself when I was a child. However, why did Zhuangzi write this story? How does it help to prove his main argument that there is a way of sorting which can even things out? One might remain unclear on these sorts of questions until interpreting with the aid of sets.

If there are two sets, Set M (three nuts in the morning and four in the evening) and Set E (three nuts in the evening and four in the morning), this story in fact is about two number sets. It says:

The Set M {3, 4} ≡ Set E {4, 3} (Definition: M: morning; E: evening) (Yuan 2021)

Set Theory tells us that changing the order of elements in a set does not change the value of the set. In fact, Zhuangzi could have been the first person in the world to employ, unknowingly, the equivalence of two number sets. He was using present mathematical understanding that the different order of elements in a set has nothing to do with the value change of the set. By switching the order of two numbers, one can obtain the equivalence of two number sets.

Zhuangzi's story implies that viewing the world from the perspective of *Dao* or Universal Set *D*, many differences might be caused simply by subjective feelings, such as how monkeys reacted on the number sets, Set M and Set E. The monkey story interpreted in this way fits in and supports Zhuangzi' argument in the whole essay. If the Daoist way of sorting is to sort objects, things, or relations in sets, then this sorting can even things out by viewing objects, things, or relations as elements in *Dao* or Set *D* indifferently. "[T]hings however peculiar or incongruous, the Way interchanges them and deems them one…Only the man who sees right through knows how to interchange and deem them one" (Graham 2001, p. 53). For man can lose himself and be an Empty Set I, as a feelingless element (ash or wood), to be with myriad things in the Universal Set *Dao*. I believe that even today, Zhuangzi's Monkey story can still serve as a good example when teaching the order of elements in a set and the equivalence of sets in today's classroom. The image of painting Zhuangzi as a romantic Daoist is no longer accurate. Zhuangzi also displays his ability and wisdom in understanding the order of numbers in sets mathematically.

Secondly, I suggest the importance of including etymological studies when interpreting pre-Qin and pre-Han thinkers' writings. Logic is a language which can display the core structure of thought. However, thought also needs to be expressed via natural languages in human communications. The task of studying ancient Chinese thinkers' minds overlaps with both ancient logic and classical language. Fortunately, Chinese pictographic language carries rich historical information and practical meanings in the characters, which were carefully recorded in *Shuo Wen* for the purpose of keeping the correct meanings used by ancient sages and ancestors in classic texts. *Shuo Wen* is thus one of the most important sources for etymological studies.

Each language has its own history. The changes and reformations from classical Chinese into modern Chinese during the past three thousand years have resulted in the fact that that classical Chinese is no longer a living language. The original meanings of many key concepts or characters have either changed or lost in modern Chinese. Neither modern Chinese nor any other language can trustfully convey the meanings in classical Chinese texts. Using the concepts or words in modern Chinese to interpret ancient texts, even if what we say makes sense to us, doesn't follow that this is what was intended by these ancient thinkers. This is the danger of losing ancient wisdom, as Roger Ames warns us. Hopefully, an etymologic study such as this can help represent ancient thinkers' wisdom as it

was intended or at least more accurately. The following example illustrates my point of view in this regard:

Western Set Theory is an approach employed by mathematicians when dealing with the infinity of numbers. An integer is a whole number (not a fractional number) that can be positive, negative, or zero. According to Set Theory, the set of integers is represented by the letter Z. An integer is any number in the infinite Set Z:

Z {<all negative numbers>, 0, <all positive numbers>}
For instance:
Z {…-3, -2, -1, 0, 1, 2, 3…}

Before to conclude this paper, I would like to present in more detail how ancient Chinese mathematicians explained the continuity of numbers. If classical Chinese language shaped the way ancient thinkers thought, then it would be natural for ancient Chinese mathematicians to have achieved a high-level understanding of integers in sets without even realizing it.

As I discussed earlier in this paper, some Chinese characters can function as both nouns and verbs. Confucius' *Xiao* is a noun and a verb. As a noun, it refers to a set, and as a verb, it denotes the actions of filial conduct. Similarly, Zhuangzi's *Ji* in his concept *jixu* is both a noun and a verb. As a noun, it refers to a set, and as a verb, it denotes to the actions of accumulating emptiness (*xu*). Chinese thinkers' understanding of sets is linguistically associates with the actions of pointing at elements in the sets. It is also the case that the ancient Chinese mathematicians' notion of number 數 *shu* is both a noun and a verb. As a noun, it means numbers or the set of numbers. As a verb, it denotes the action of counting numbers.

The character of number, *shu* in its classical written form is 數 . The right radical 攴 *pu* is an image of a hand holding a stick. The left radical of *shu* is *luo* 婁 . According to *Shuo Wen*, *luo* is an image that a woman holds a curtain. A woman radical 女 is at the bottom of the sketch of a curtain. In ancient China, most curtains were made of bamboo strips. The sunlight can go through the slots between strips and create bright spots, which is called "麗廔 *lilou*, bright slots" according to the explanation on *shu* in *Shou Wen*. *Lilou* can be seen. They are created by many little holes on the curtain and seen as bright specks. There are also dark slots at the place bamboo strips block the sunlight. The dark slots can be seen too. They are many dark specks. Both bright and dark specks on the curtain can be counted, assume that *shu* is about counting integers. The hand radical, *Pu* 攴 , as a matter of fact, is an image of a hand holding a stick to count those bright and dark spots on the curtain. So that *shu* is an action. Counting numbers is to point out objects or elements in a set of integers. According to *Shuo Wen*, the left radical *luo* 婁 holds the meanings of numbers, counting numbers and emptiness.

The image of the curtain makes emptiness one of the meanings or elements that are rooted in Chinese concept of *shu* 數 .[16]

Understanding the concept of numbers in modern Chinese language does not differ that much from that in the West. However, an etymological study shows that there are significant differences in understanding the concept of numbers between ancient and modern times. The ancient understanding of number shaped how ancient Chinese mathematicians formed their approaches to mathematics. The core meanings held in the character *shu* can explain why difficult concepts such as negative members and zero did not trouble ancient Chinese mathematicians as much as they did the mathematicians in the West.

It might surprise many mathematicians and philosophers that one of ancient mathematicians, Liu Hui 劉徽 (fl. 3rd century CE), introduces positive and negative numbers in his *The Nine Chapters on Mathematical Art, Jiuzhang Suanshu* 《九章算術》 by employing colors red and black. *The Nine Chapters* (the 10th–2nd century BCE) is one of the earliest applied mathematics books that summarized the contributions in the pre-Qin and pre-Han eras. When explaining integers, Liu Hui took advantage of set-oriented thinking, a habit built through learning Chinese pictographic language. He did something unfamiliar to western mathematicians by offering two different colors to introduce positive and negative numbers. With these two colors, Liu Hui clearly distinguished positive numbers (or natural numbers) from negative numbers. He says:

> 正算赤，负算黑。 *Zheng suan chi,fu suan hei.*
> Using red rods to count positive numbers. Using black rods to count negative numbers (Liu 1993, p. 420).[17]

According to the original pictographic definition of *shu,* numbers, both bright and dark specks on the curtains are what one can count with a counting stick. So, if we go with the Chinese way of thinking of numbers, colors red and black can be associated with numbers smoothly. They are simply a different way to say brightness and darkness.

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[16]*Shuo Wen* says, "攴, 小擊也。[注]手部曰。擊、攴也。*Pu, Xiaoji ye. [Note]Shou Bu yue:ji,pu ye.*

凡攴之屬皆从攴。*Fan pu zhishu jiecong pu. Pu*, hitting with a stick softly. [Note] it represents the unit of hand radical. Hitting is a hand activity. Characters associate with hands all fall in the unit of hand radical. (Xu Shen & Duan, 2003, Book 3, p. 122)

Shuo Wen says, "婁, 空也。从母。从中女。*Lou, Kong ye.Chong mu,cong zhong lu.*[注] 凡中空曰婁……凡一實一虛、層見疊出曰婁。……故婁之義又爲數也。此正如窗牖。麗廔之多孔也。*Lou* means emptiness. It has a female radical at the lower part, a standing woman in the middle. [Note] Whenever there is an emptiness in the middle, it is *lou*…Whenever solid materials and emptiness are next to one and the other, between the overlapped parts will have an empty spot shown in the middle, that is *lou…Therefore, the another meaning of lou is number.* [16] This is exactly like a curtain, when the sunlight goes through many empty holes and create many bright spots, it is called as *lilou* 麗廔 (Shen 2003, p. 624).

[17]Translation is mine.

To do counting, counting sticks are basic mathematical tools. Chinese people have been using rods to count numbers since the oracle bone period. What Liu Hui suggested was to dye counting rods into two different colors. He assigned the color red to Set Red and let it be the set of positive numbers. For example, when one gains something, he or she picks up red rods to count. Therefore, Set R is a collection of all the red rods, which one uses to count positive numbers. Red rods can be added one after the other endlessly, and they are all members of Set R. Similarly, he assigned color black to Set Black or Set B, and let it be the set of negative numbers. For example, when one owes others something, he or she picks up black rods to count. Therefore, Set B is a collection of all the black rods, which one can use to count negative numbers. Black rods can be added one after the other endlessly, and they are all members of the Set B.

Now we have:

Set R {all positive numbers} = Set R {all red rods}
Set B {all negative numbers} = Set B {all black rods}

Liu Hui's ideas are clear and simple. Colors help to visualize abstract concepts, such as positive and negative numbers, and make them more concrete. Although Liu Hui did not specifically discuss the number zero in *The Nine Chapters*, when he introduced positive and negative numbers, the existence of zero seems too obvious to be overlooked or avoided in Chinese history. The part of meanings in *shu* number comes from the radical *lou*, which means emptiness. With little empty holes on the curtain, bright specks are shown. Since ancient time, Chinese mathematicians have represented zero by leaving an empty space, a blank, a gap to represent zero, or by drawing a small empty circle to represent number zero. Zero clearly has a position in the line of integers.[18]

To summarize Liu Hui's understanding of integer as following:

Z {<the color for all negative numbers>, <∅>, <The Color for all positive numbers>}
Or, for instance,
Z {…-3, -2, -1, 0, 1, 2, 3…}

Without counting out all rods infinitely, Liu Hui's color sets and the empty set indeed help us understand integers one way or the other from a unique perspective.

Conclusion

To sum up, language is a creation, but it is not a random one. Chinese pictographic language was created according to sets evidently, and the creators of such a language followed a primitive logic of sets without addressing it in the

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[18]In his book, *A History of Chinese Mathematics*, Jean-Claude Martzloff discusses the evidence of zero in Chinese Mathematics and reports that empty spaces, gaps, and circles were used to represent zero in Chinese history. (Martzloff 1997, pp. 104-108)

format of a theory. One's language shapes one's ways of thinking. Using Chinese pictographic language to think opens a unique path to pursue truth, goodness, and beauty. For instance, as discussed in this paper, pursing truth or understanding *Dao* in Chinese *Daoist* tradition can be a process of losing oneself in the nature and becoming an empty set within *Dao*, together with myriad things between heaven and earth. Being a part of *Dao* or an element in Set *Dao* turns out to be a possible way to understand *Dao*. Goodness in Confucian tradition is not defined by words but by activities or actions of performing a certain type of ethical conduct. Pointing out different types of good actions required by different virtues is like assigning different individual elements into different general sets. This is Confucius' way of teaching moral behaviors to help his disciples go through the process of person-making. Beauty as an aesthetic feeling overlaps with the nature and goodness. The characters of beauty *mei* and goodness *shan* were created with an intersection, which is the core meaning shared by both beauty and goodness. This intersection is the radical sheep. Empirical judgement of beauty and goodness allows that eating and food play significant ethical and aesthetical roles in Chinese culture and tradition. Subjective imaginations and plurality thoughts are inseparable from the understanding of beauty and goodness. Even the mathematical beauty of numeracy is not the product of purely abstract thinking, instead, it can be associated with colors empirically. These unique features of knowing the world are attractive and deserve further research.

Findings from my investigation disapprove the clam which denies the existence of Chinese logic. Evidence demonstrated in this paper shows that ancient Chinese thinkers understood memberships, totality, and individual elements in sets, and they practiced the logic of sets in their reasoning. The wisdom they offered to us is not only poetic but also logical. Findings from my investigation prove that although the notion of set is simple, thinking in sets is very different from the Aristotelian understanding of memberships in a totality. Chinese pictographic characters provide linguistic sources for studying such an alternative logic in the minds of ancient Chinese thinkers.

An etymological study can help clarify ancient wisdom more accurately than interpreting ancient thoughts with our own concepts. When we understand Chinese logic as it is, fruitful discoveries from ancient wisdoms can be found. Understanding Chinese logic as the logic of sets provides a fruitful opening for new research among Chinese and the Western philosophers. Logic of sets could serve as a base to open a healthy discourse for a mutual understanding between Chinese and Aristotelian traditions.

Acknowledgments

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