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Point Equation, Line Equation, Plane Equation etc and Point Solution, Line Solution, Plane Solution etc —Expanding Concepts of Equation and Solution with Neutrosophy and Quad-stage Method

Fu Yuhua

CNOOC Research Institute, China. E-mail: fuyh1945@sina.com

Abstract. The concepts of equations and solutions are constantly developed and expanded. With Neutrosophy and Quad-stage method, this paper attempts to expand the concepts of equations and solutions in the way of referring to the concepts of domain of function, the geometry elements included in domain of function, and the like; and discusses point equation, line equation, plane equation, solid equation, sub-domain equation, whole-domain equation, and the like; as well as point solution, line solution, plane solution, solid solution, sub-domain solution, whole-domain solution, and the like. Where: the point solutions may be the solutions of point equation, line equation, plane equation, and the like; similarly, the line solutions may be the solutions of point equation, line equation, plane equation, and the like; and so on. This paper focuses on discussing the single point method to determine "point solution".

Keywords: Neutrosophy, Quad-stage, point equation, line equation, plane equation, point solution, line solution, plane solution, single point method

1 Introduction

As well-known, equations are equalities that contain unknown.

Also, the concepts of equations and solutions are constantly developed and expanded. From the historical perspective, these developments and expansions are mainly processed for the complexity of variables, functional relationships, operation methods, and the like. For example, from elementary mathematical equations develop and expand into secondary mathematical equations, and advanced mathematical equations. Again, from algebra equations develop and expand into geometry equations, trigonometric equations, differential equations, integral equations, and the like.

With Neutrosophy and Quad-stage method, this paper considers another thought, and attempts to expand the concepts of equations and solutions in the way of referring to the concepts of domain of function, the geometry elements included in domain of function, and the like; and discusses point equation, line equation, plane equation, solid equation, sub-domain equation, whole-domain equation, and the like; as well as point solution, line solution, plane solution, solid solution, sub-domain solution, whole-domain solution, and the like. Where: the point solutions may be the solutions of point equation, line equation, plane equation, and the like; similarly, the line solutions may be the solutions of point equation, line equation, plane equation, and the like; and so on.

2 Basic Contents of Neutrosophy

Neutrosophy is proposed by Prof. Florentin Smarandache in 1995.

Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.

This theory considers every notion or idea <A> together with its opposite or negation <Anti-A> and the spectrum of "neutralities" <Neut-A> (i.e. notions or ideas located between the two extremes, supporting neither <A> nor <Anti-A>). The <Neut-A> and <Anti-A> ideas together are referred to as <Non-A>.

Neutrosophy is the base of neutrosophic logic, neutrosophic set, neutrosophic probability and statistics used in engineering applications (especially for software and information fusion), medicine, military, cybernetics, and physics.

Neutrosophic Logic is a general framework for unification of many existing logics, such as fuzzy logic (especially intuitionistic fuzzy logic), paraconsistent logic, intuitionistic logic, etc. The main idea of NL is to characterize each logical statement in a 3D Neutrosophic Space, where each dimension of the space represents respectively the truth (T), the falsehood (F), and the indeterminacy (I) of the statement under consideration, where T, I, F are standard or non-standard real subsets of]-0, 1+[without necessarily connection between them.

More information about Neutrosophy can be found in references [1，2].

3 Basic Contents of Quad-stage

The first kind of “four stages” is presented in reference [3], and is named as “Quad-stage”. It is the expansion of Hegel’s triad-stage (triad thesis, antithesis, synthesis of development). The four stages are "general theses", "general antitheses", "the most important and the most complicated universal relations", and "general syntheses". They can be stated as follows.

The first stage, for the beginning of development (thesis), the thesis should be widely, deeply, carefully and repeatedly contacted, explored, analyzed, perfected and so on; this is the stage of general theses. It should be noted that, here the thesis will be evolved into two or three, even more theses step by step. In addition, if in other stage we find that the first stage’s work is not yet completed, then we may come back to do some additional work for the first stage.

The second stage, for the appearance of opposite (antithesis), the antithesis should be also widely, deeply, carefully and repeatedly contacted, explored, analyzed, perfected and so on; this is the stage of general antitheses. It should be also noted that, here the antithesis will be evolved into two or three, even more antitheses step by step.

The third stage is the one that the most important and the most complicated universal relations, namely the seedtime inherited from the past and carried on for the future. Its purpose is to establish the universal relations in the widest scope. This widest scope contains all the regions related and non-related to the "general theses", "general antitheses", and the like. This stage's foundational works are to contact, grasp, discover, dig, and even create the opportunities, pieces of information, and so on as many as possible. The degree of the universal relations may be different, theoretically its upper limit is to connect all the existences, pieces of information and so on related to matters, spirits and so on in the universe; for the cases such as to create science fiction, even may connect all the existences, pieces of information and so on in the virtual world. Obviously, this stage provides all possibilities to fully use the complete achievements of nature and society, as well as all the humanity's wisdoms in the past, present and future. Therefore this stage is shortened as "universal relations" (for other stages, the universal relations are also existed, but their importance and complexity cannot be compared with the ones in this stage).

The fourth stage, to carry on the unification and synthesis regarding various opposites and the suitable pieces of information, factors, and so on; and reach one or more results which are the best or agreed with some conditions; this is the stage of "general syntheses". The results of this stage are called "synthesized second generation theses", all or partial of them may become the beginning of the next quad-stage.

4 Expanding concepts of equations and solutions with Neutrosophy and Quad-stage method

For realizing the innovations in the areas such as science and technology, literature and art, and the like, it is a very useful tool to combine neutrosophy with quad-stage method. For example, in reference [4], expanding Newton mechanics with neutrosophy and quad-stage method, and establishing New Newton Mechanics taking law of conservation of energy as unique source law; in reference [5], negating four color theorem with neutrosophy and quad-stage method, and "the two color theorem" and "the five color theorem" are derived to replace "the four color theorem"; in reference [6], expanding Hegelian triad thesis, antithesis, synthesis with Neutrosophy and Quad-stage Method; in reference [7], interpretating and expanding Laozi’s governing a large country is like cooking a small fish with Neutrosophy and Quad-stage Method; in reference [8], interpretating and expanding the meaning of “Yi” with Neutrosophy and Quad-stage Method; and in reference [9], creating generalized and hybrid set and library with Neutrosophy and Quad-stage Method.

Now we briefly describe the general application of neutrosophy to quad-stage method.

In quad-stage method, "general theses" may be considered as the notion or idea <A>; "general antitheses" may be considered as the notion or idea <Anti-A>; "the most important and the most complicated universal relations" may be considered as the notion or idea <Neut-A>; and "general syntheses" are the final results.

The different kinds of results in the above mentioned four stages can also be classified and induced with the viewpoints of neutrosophy. Thus, the theory and achievement of neutrosophy can be applied as many as possible, and the method of quad-stage will be more effective.

The process of expanding concepts of equations and solutions can be divided into four stages.

The first stage (stage of "general theses"), for the beginning of development, the thesis (namely "traditional concepts of equations and solutions") should be widely, deeply, carefully and repeatedly contacted, explored, analyzed, perfected and so on.

The concepts of equations and solutions have been continuously developed and expanded. From the historical perspective, in this process of development and expansion, for equations, the linear equation, dual linear equation, quadratic equation, multiple equation, geometry equation, trigonometric equation, ordinary differential equation, partial differential equation, integral equation, and the like are appeared step by step; for solutions, the approximate solution, accurate solution, analytical solution, numerical solution, and the like are also appeared step by step. Obviously, these developments and expansions are mainly processed for the complexity of variables, functional relationships, operation methods, and the like.

In the second stage (the stage of "general antitheses"), the opposites (antitheses) should be discussed carefully. Obviously, there are more than one opposites (antitheses) here.

For example, according to the viewpoint of Neutrosophy, if "traditional concepts of equations and solutions" are considered as the concept <A>, the opposite <Anti-A> may be: "non-traditional concepts of equations and solutions"; while the neutral (middle state) fields <Neut-A> including: "undetermined concepts of equations and solutions" (neither "traditional concepts of equations and solutions", nor "non-traditional concepts of equations and solutions"; or, sometimes they are "traditional concepts of equations and solutions", and sometimes they are "non-traditional concepts of equations and solutions"; and the like).

In the third stage, considering the most important and the most complicated universal relations to link with "concepts of equations and solutions". The purpose of this provision stage is to establish the universal relations in the widest scope.

Here, differ with traditional thought, we consider a new thought, and attempt to expand the concepts of equations and solutions in the way of referring to the concepts of domain of function, the geometry elements included in domain of function, and the like.

Obviously, considering other thought, different result may be reached; but this situation will not be discussed in this paper.

In the fourth stage, we will carry on the unification and synthesis regarding various opposites and the suitable pieces of information, factors, and the like that are related to the concepts of equations and solutions; and reach one or more results for expanding the concepts of equations and solutions, which are the best or agreed with some conditions.

It should be noted that, in this stage, various methods can also be applied. Here, we will seek the results according to Neutrosophy and Quad-stage method.

Firstly, analyzing the concept of "domain of function". According to the viewpoint of Neutrosophy, the two extreme elements of "domain of function" are "point domain" and "whole-domain", and in the middle there are: "line domain", "plane domain", "solid domain", "sub-domain", and the like; therefore, we can discuss the concepts of point equation, line equation, plane equation, solid equation, sub-domain equation, whole-domain equation, and the like; as well as the concepts of point solution, line solution, plane solution, solid solution, sub-domain solution, whole-domain solution, and the like.

4.1 Point equation and point solution, line equation and line solutiom, and the like

We already know that, "point equation" is the one suitable for a certain solitary point only. For example, when considering the gravity between the Sun (coordinates: 0,0,0) and a planet located at a certain solitary point (coordinates: x_0,y_0,z_0), then according to the law of gravity, the following "point equation" can be reached.

$$F = -\frac{GM_{\text{sun}}m}{x_0^2 + y_0^2 + z_0^2} \qquad (1)$$

where, M_{sun} is the mass of the Sun; the unknown in the equation is the mass of the planet only.

When considering the gravity between the Sun and a planet located at its elliptical orbit, substituting the polar equation of the ellipse into the law of gravity, then the following "line equation" can be reached, and it is suitable for the entire elliptical orbit.

$$F = -\frac{GM_{\text{sun}}m(1+e\cos\varphi)^2}{a^2(1-e^2)^2} \qquad (2)$$

When considering the gravity between the Sun and a planet located at the inner surface of the sphere ($r = r_0$), substituting $r = r_0$ into the law of gravity, then the following "plane (inner surface) equation" can be reached, and it is suitable for the entire inner surface of the sphere.

$$F = -\frac{GM_{\text{sun}}m}{r_0^2} \qquad (3)$$

When considering the gravity between the Sun and a point located in a hollow ball ($r_1 \le r \le r_2$), substituting $r_1 \le r \le r_2$ into the law of gravity, then the following "solid equation" can be reached, and it is suitable for the entire hollow ball.

$$F = -\frac{GM_{\text{sun}}m}{r^2}, \quad r_1 \le r \le r_2 \qquad (4)$$

When considering the gravity between the Sun and a point located in the sub-domain ($x \ge x_0$), substituting $x \ge x_0$ into the law of gravity, then the following "sub-domain equation" can be reached, and it is suitable for the entire sub-domain.

$$F = -\frac{GM_{\text{sun}} m}{x^2 + y^2 + z^2}, \quad x \ge x_0 \qquad (5)$$

When considering the gravity between any two objects, according to the law of gravity, the following "whole-domain equation" can be reached, it is suitable for the entire three-dimensional space, and the two objects may not include the Sun.

$$F = -\frac{GMm}{x^2 + y^2 + z^2} \qquad (6)$$

Accordingly, when considering the gravity between the Sun (coordinates: 0,0,0) and a planet located at a certain solitary point (coordinates: x_0, y_0, z_0), and if the mass of the planet is given (equals to m_0), then according to the law of gravity, the following "point solution" can be reached.

$$F = -\frac{GM_{\text{sun}} m_0}{x_0^2 + y_0^2 + z_0^2} \qquad (7)$$

where, M_{sun} is the mass of the Sun; m_0 is the mass of the planet.

When considering the gravity between the Sun and a planet located at its elliptical orbit, substituting the polar equation of the ellipse into the law of gravity, if the planet's parameters are given (equal to e_0 and a_0), and the mass of the planet is also given (equals to m_0), then the following "line solution" can be reached, and it is suitable for the entire elliptical orbit.

$$F = -\frac{GM_{\text{sun}} m_0 (1 + e_0 \cos\varphi)^2}{a_0^2 (1 - e_0^2)^2} \qquad (8)$$

When considering the gravity between the Sun and a planet located at the inner surface of the sphere ($r = r_0$), substituting $r = r_0$ into the law of gravity, and if the mass of the planet is given (equals to m_0), then the following "plane （inner surface） solution" can be reached, and it is suitable for the entire inner surface of the sphere.

$$F = -\frac{GM_{\text{sun}} m_0}{r_0^2} \qquad (9)$$

When considering the gravity between the Sun and a point located in a hollow ball ($r_1 \le r \le r_2$), substituting $r_1 \le r \le r_2$ into the law of gravity, and if the mass of the point is given (equals to m_0), then the following "solid solution" can be reached, and it is suitable for the entire hollow ball.

$$F = -\frac{GM_{\text{sun}} m_0}{r^2}, \quad r_1 \le r \le r_2 \qquad (10)$$

When considering the gravity between the Sun and a point located in the sub-domain ($x \ge x_0$), substituting $x \ge x_0$ into the law of gravity, and if the mass of the point is given (equals to m_0), then the following "sub-domain solution" can be reached, and it is suitable for the entire sub-domain.

$$F = -\frac{GM_{\text{sun}} m_0}{x^2 + y^2 + z^2}, \quad x \ge x_0 \qquad (11)$$

When considering the gravity between any two objects, if both the masses of the two objects are given (equal to M_0 and m_0), then according to the law of gravity, the following "whole-domain solution" can be reached, it is suitable for the entire three-dimensional space, and the two objects may not include the Sun.

$$F = -\frac{GM_0 m_0}{x^2 + y^2 + z^2} \qquad (12)$$

4.2 Determining point solution with single point method

In the existing methods for solving ordinary differential equations, there are already the examples for seeking the solution (point solution) suitable for one solitary point.

For example, consider the following differential equation

$$y' = y, \quad y(0) = 1 \qquad (13)$$

It gives

$$y'(0) = y''(0) = y'''(0) = y^{(n)}(0) = 1$$

According to the power series formula for $x = x_0$

$$y = y(x_0) + y'(x_0)x/1! + y''(x_0)x^2/2! + \cdots$$

It gives the “point solution” for $x_0 = 0$ as follows

$$y = 1 + x/1! + x^2/2! + \cdots$$

However, this “point solution” is applicable to the "whole-domain", while in this paper we will consider the “point solution” suitable for one solitary point only.

For example, the single point method can be used to find the “point solution” of hydraulic problem that is suitable for one solitary point only. This kind of “point solution” is finding independently, namely the effect of other points may not be considered. As finding “point solution” for a certain point, the point collocation method should be used; that means that the “point solution” will satisfy the boundary condition on some selected boundary points; and on this certain point satisfy the hydraulic equation and the derived equations that are formed by running the derivitive operations to the hydraulic equation. Finally all the undetermined constants for the “point solution” will be determined by solving the equations that are formed by above mentioned point collocation method.

In reference [10], the single point method was used to determine the “point solution” on a certain solitary point for the problem of potential flow around a cylinder between two parallel plates.

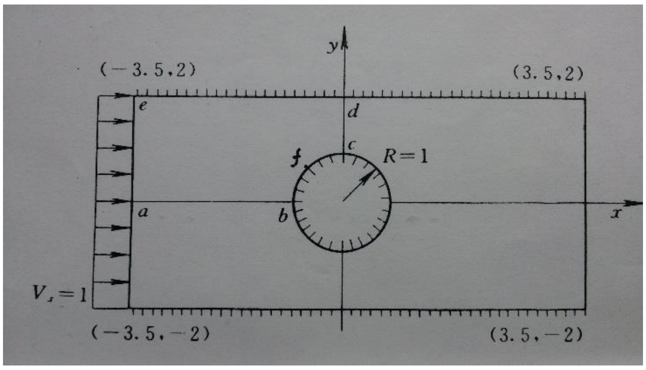

Fig. 1. Potential flow around a cylinder between two parallel plates

As shown in Figure 1, due to symmetry, one-fourth flow field in the second quadrant can be considered only.

The differential equation is as follows

$$F=\partial^2\varphi/\partial x^2+\partial^2\varphi/\partial y^2=0 \qquad (14)$$

On boundary ab

$$\varphi=0,\quad v_y=0$$

On cylinder boundary bc

$$\varphi=0,\quad v_r=0$$

On boundary cd

$$v_y=0$$

On plate boundary ed

$$\varphi=2,\quad v_y=0$$

On entrance boundary ae

$$\varphi=y,\quad v_x=1$$

Taking “point solution” as the following form containing n undetermined constants

$$\varphi=y+y(x^2-12.25)(y^2-4)(K_1+K_2x^2+K_3y^2+K_4x^4+K_5y^4+K_6x^2y^2+\cdots+K_nx^py^q) \qquad (15)$$

Other 4 boundary equations are as follows

On point b

$$v_r(-1,0)=0 \qquad (16)$$

On point c

$$\varphi(0,1)=0 \qquad (17)$$

On point f

$$\varphi(-0.7071,0.7071)=0 \qquad (18)$$

$$v_r(-0.7071,0.7071)=0 \qquad (19)$$

For a certain solitary point (x_0,y_0), as $n=6$, only 2 boundary equations Eq.(16) and Eq.(17) are considered; and the following 4 single point equations are considered.

The first single point equation is reached by Eq.(14)

$$F(x_0,y_0)=0 \qquad (20)$$

Other 3 single point equations are reached as follows by running the derivitive operations to Eq.(14).

$$\partial F(x_0,y_0)/\partial x=0 \qquad (21)$$

$$\partial F(x_0,y_0)/\partial y=0 \qquad (22)$$

$$\partial^2 F(x_0,y_0)/\partial x\partial y=0 \qquad (23)$$

Substituting the coordinates values (x_0,y_0) into Eq.(16) and Eq.(17), and Eq.(20) to Eq.(23); after solving these 6 equations, the 6 undetermined constants K_1 to K_6 can be determined, namely the “point solution” for $n=6$ is reached.

As $n>8$, the 4 boundary equations Eq.(16) to Eq.(19) are considered; and besides the 4 single point equations Eq.(20) to Eq.(23), the following single point equations derived by running the derivitive operations to Eq.(14) are also considered.

$$\partial^2 F(x_0,y_0)/\partial x^2=0 \qquad (24)$$

$$\partial^2 F(x_0,y_0)/\partial y^2=0 \qquad (25)$$

$$\partial^3 F(x_0,y_0)/\partial x^3=0 \qquad (26)$$

$$\partial^3 F(x_0,y_0)/\partial x^2\partial y=0 \qquad (27)$$

$$\partial^3 F(x_0,y_0)/\partial x\partial y^2=0 \qquad (28)$$

$$\partial^3 F(x_0,y_0)/\partial y^3=0 \qquad (29)$$

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Substituting the coordinates values (x_0,y_0) into Eq.(16) to Eq.(19), as well as Eq.(20) to Eq.(24), and the like; after solving these n equations, the n undetermined constants K_1 to K_n can be determined, namely the "point solution" as the form of Eq.(15) is reached.

For 8 solitary points, the comparisons between accurate analytical solution (AS) and point solution (PS) for the values of φ are shown in table 1.

Table 1. Comparisons between accurate analytical solution (AS) and point solution (PS) for the values of φ

x_0	y_0	AS	$n=6$	$n=10$	$n=14$	$n=19$
-3.4	1.75	1.747	1.744	1.743	1.746	1.746
-3.0	1.75	1.744	1.729	1.735	1.736	1.738
-2.5	1.75	1.736	1.694	1.751	1.713	1.732
-2.0	1.75	1.721	1.609	1.782	1.631	1.766
-3.4	1.50	1.494	1.489	1.483	1.492	1.493
-3.0	1.50	1.488	1.459	1.452	1.473	1.474
-2.5	1.50	1.474	1.397	1.450	1.439	1.460
-2.0	1.50	1.445	1.248	1.518	1.272	1.563

For more information about single point method, see references [11-13].

The single point method can also be used for prediction.

For example, the sea surface temperature distribution of a given region, is a special two-dimensional problem influenced by many factors, and it is very difficult to be changed into 2 one-dimensional problems. However, this problem can be predicted for a certain solitary point by single point method.

The following example is predicting the monthly average sea surface temperature.

Based on sectional variable dimension fractals, the concept of weighted fractals is presented, i.e., for the data points in an interval, their r coordinates multiply by different weighted coefficients, and making these data points locate at a straight-line in the double logarithmic coordinates. By using weighted fractals, the monthly average sea surface temperature (MASST) data on the point 30ºN, 125ºE of Northwest Pacific Ocean are analyzed. According to the MASST from January to August in a certain year (eight-point-method), the MASST from September to December of that year has been predicted. Also, according to the MASST of August merely in a certain year (one-point-method), the MASST from September to December of that year has been predicted.

The MASST prediction results are as follows.

Table 2. MASST prediction results (unit: ℃) by using eight-point-method (8PM) and one-point-method(1PM)

Year	Notes	Sep.	Oct.	Nov.	Dec.
1958	8PM	28.21	25.51	22.67	20.17
	1PM	28.24	25.55	22.72	20.22
	Real value	27.7	25.5	21.2	20
1959	8PM	28.20	25.56	22.75	20.28
	1PM	28.19	25.54	22.73	20.26
	Real value	27.6	24.7	22.9	20
1960	8PM	27.95	25.36	22.60	20.16
	1PM	28.05	25.51	22.78	20.36
	Real value	28	26	21.8	20
1961	8PM	28.70	26.14	23.37	20.91
	1PM	28.34	25.57	22.69	20.16
	Real value	28.4	26.2	22.8	22
1962	8PM	28.30	26.00	23.46	21.17
	1PM	27.90	25.48	22.83	20.47
	Real value	28	25	21	20
1963	8PM	29.36	27.86	25.78	23.80
	1PM	27.86	25.47	22.85	20.50
	Real value	27.5	24.5	21	18
1964	8PM	28.04	25.83	23.32	21.05
	1PM	27.80	25.46	22.86	20.54
	Real value	28	24.5	22	19

In addition, according to the phenomenon of fractal interrelation and the fractal coefficients of this point's MASST and the monthly average air temperature of August of some points, the monthly average air temperatures of these points from September to December have also been predicted. For detailed information, see reference [14].

4.3 Relationship between various equations and various solutions

According to Neutrosophy and Quad-stage method; and contacting the concepts of domain of function, the geometry elements included in domain of function, and the like; the concept of equation can be expanded into the concepts of point equation, line equation, plane equation, solid equation, sub-domain equation, whole-domain equation, and the like; and the concept of solution can be expanded into the concepts of point solution, line solution, plane solution, solid solution, sub-domain solution, whole-domain solution, and the like. However, the relationships between them are not the one by one corresponding relationships. Where: the point solutions may be the solutions of point equation, line equation, plane equation, and the like; similarly, the line solutions may be the solutions of point equation, line equation, plane equation, and the like; and so on.

5 Conclusions

The combination of neutrosophy and quad-stage method can be applied to effectively reliaze the expansion of "traditional concepts of equations and solutions". The results of expansion are not fixed and immutable, but the results are changeable depending on the times, places and specific conditions. This paper deals only with a limited number of situations and instances as an initial attempt, and we hope that it will play a valuable role.

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Received: Jan 12, 2016. Accepted: Feb. 28, 2016.