What is maths differance?

Zhicheng Yu
Free mathematician Strongart in Suzhou, China
Email: Strongart@qq.com

Abstract: The paper presents a new framework for the maths development which is called maths differance. There are three typical maths differaence: prove, axiom and shift, and also some others.

1. Differance and maths difference

What is the essential part of maths? Maybe non-maths students will say calculation,and some maths students will say prove, but the mathematician G.H.Hardy says that strictly speaking, there is no proof, in the final analysis, we can only point.

I think Hardy is right, but we can go further. In fact, Derrida's differance is more suitable for this job. Jacques Derrida is a philosopher, but not a mathematician. He wants to express a special kind of difference, not only a difference with A and B, but a difference with A and A', which contains a significance of the chain, an extension of meaning, so he modifies the word difference and creates a new word differance. Maybe this is not the original meaning of Derrida, but it is a clear explanation of the differance and also enough for this paper, the differance of differance.

Many things contains meaning, so differance can be widely used. But I think maths is the best stages for differance because it contains the pure and holonomy meaning. When the maths develops, you can imagine the crystal growth, which is a vivid picture of differance. Theoretically, you can grow from every point of the maths theory,, but some points have more priority than others and some great mathematician can change the value of the point in the whole maths theory. We can say that there is no fixed point in the maths.

2. Three typical maths differaence

At least, there are three typical maths differaence with the different lever in the mathematics, prove, axiom and shift. Now I will show an outline of them.

2.1 Prove

Prove is the basic part of maths, the typical example is the Euclidean geometry， you can prove a lot of propositions from some definitions with a little axioms. I think the main reasons for someone who like the prove is that prove is exact and powerful, which makes the maths total different from the science. Some persons treat the maths as the model of truth and treat the prove as a model of the model.

But some mathematicians dislike the prove, they think prove is too stuffy and do not as exact as we have seen. Some of them emphasize the maths experience and some of them emphasize the maths intuition. I think prove is only the basic rule, if you play a chess, the wonderful thing is not the rule, but how do we use the rule. In my opinion, experience is the base of prove and prove is the base of intuition. Here the base is only material, but not the soul. I think the soul of maths is something as the genius of art and the most wonderful maths is created by intuition, maybe that is because it is hard for us to say how do we create it.

2.2 Axiom

Axiom is also an important part in the maths, specially in the modern maths. The basic philosophy of axiom is upgrading the typical properties into a law which we call it axiom, then we can get some high level concepts which can do more representation. For example, the distance has three typical properties: nonnegative, symmetric and triangle inequality, we make them as axioms, we can get the metric space. According to this method, we can make the properties of absolute value into normed space and so on.

Sometimes, the result of the axioms maybe not a whole space, but only a concept. Give any group G, we can ask that whether we have a CW-complex whose fundamental group is G. The answer is yes and we call this CW-complex Eilenberg-MacLane space, many modern maths are relate with it such as Eilenberg-Ganea problem (see [1]) and higher algebraic K-theory (see [2]).

Now look at an similar story, Give any group G, can you find an extension field F over K, such that the Galois group Gal(F/K)=G. In fact, Galois groups are all profinite groups, even for the finite groups, there is a famous inverse Galois problem asking that whether every finite group appears as the Galois group of some Galois extension of the rational numbers Q. It is unsolved (see [3]), so we can not get an axiom definition.

In fact, the axiom definition and the inverse problem are two sides of a coin. The request of a properties is an inverse problem, if an inverse problem has a complete solutions, we can get an axiom definition from it, if not, we only have a unsolved inverse question. If there are some partial solutions, we can do this program locally.

2.3 Shift

The last differance is called shift by myself. Let's imagine them vivid first. Prove is run, you can catch a runner by your eyes. Axiom is jump, you must raise your head to see the jumper. But shift is such a special skill, disappearing here then appearing there, you can not see the middle traces.

Traceless is an important character of the shift. Let's take an example to explain it. The group theory is starting with solution question of the high algebraic equations. As we know, the equation of degree 5 and higher degrees are not always solvable with radicals, we can prove it with the group and Galois theory. But the group and Galois theory are totally different with the algebraic equation, we can imagine such a person who is an expert of group theory, but he can not solve the three or four degree algebraic equations.

For the same example, we also can compare the axiom and shift. If we do the axiom for the multiplication, then we can get the groups. If we follow the trace of the axiom, only require the abstract form, then we can catch the the theme, but if we do same thing for the shift, studying many algebraic equations, we will lost the target and trap into the details.

Traceless brings the non-historical of the shift. We can ask such an fundamental inevitable question: if there is the parallel earth whose civilization level is similar with us, comparing our maths history, which parts or which order of maths will be the same? We can imagine that some mathematicians of the parallel earth finding the groups from the multiplication directly, we even can imagine that they get groups from the vector spaces just as we get the metric space from the Euclidean spaces, but it is hard for us to imagine that they get the root formula of three degree algebraic equations before than they can solve quadratic equations. The general law is the shift has the less minimum inevitability and the prove is the maximum.

3. Other maths difference

Of course, there are some other kinds of differance in the maths. We have the relative differance, one example is to define the relative open or closed set in the topology, the other is to modify the homological group to the relative homological groups, then we have the long exact sequence. We also have the local differance, the stand example is local rings in the commutative algebras and algebraic geometry, we also can modify the theory locally, from the Banach space to the local Banach space with the holomorphic functional calculus (see [4]).

Some differance has no system name, but is also valuable. For example, I find that many modern maths branches related to change the coefficient into p-adic numbers, then we have many p-adic maths object such as p-adic Lie algebra, p-adic Banach space, p-adic cohomology, p-adic Hodge theory and even p-adic quantum mechanics. Why does people like p-adic numbers? It is complete as the real numbers. It related the prime numbers, so we do the factor job from it. At last, it is the balance point from abstract and concrete, if you change the coefficient into a field or a commutative ring directly, the theory will be poor, you can choose the p-adic numbers as your first step.

Remark: The paper is a summary of my original Chinese blog articles, you can see them at: http://blog.sina.com.cn/strongart.

Reference:

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