R. L. EPSTEIN and W. A. CARNIELLI. *Computability. Computable Functions, Logic, and the Foundations of Mathematics,* 2nd Edition, with 'Computability and Undecidability—A Timeline. The Story of the Development of Computable Functions and the Undecidability of Arithmetic to 1970', by R. L. Epstein. Belmont, Calif.: Wadsworth/Thomson Learning, 2000. xiv + 299 + 38 pp., includes poster. $29.95. £16.99. ISBN 0-534-54644-7.

Reviewed by

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Epstein and Carnielli's fine textbook on logic and computability is now in its second edition. The readers of this journal might be particularly interested in the timeline 'Computability and Undecidability' added in this edition, and the included wall-poster of the same title. The text itself, however, has some aspects which are worth commenting on.

Possibly the most distinguishing feature of *Computability* is the emphasis it puts on the historical and philosophical dimensions of logic and computability theory. The text is divided into four parts, 'Fundamentals', 'Computable Functions', 'Logic and Arithmetic', and 'Church's Thesis and Constructive Mathematics'. The inclusion of an entire part, not just a section or footnote, on the philosophical aspects of computability and the foundations of mathematics make the book not only unique among advanced logic textbooks, but, in particular, suitable for courses aimed at philosophy students. But already in the first part, 'The Fundamentals', we find sections on paradoxes and the nature of mathematical proofs as well as an entire chapter containing a long excerpt from Hilbert's 'On the infinite'. In fact, throughout the text, theoretical discussion and exercises are interspersed with selections from some important papers on the subject. This includes, aside from ‘On the infinite’, sections from Turing’s ‘On computable numbers’ (1936), Post’s ‘Finite combinatory processes’ (1936) in the part on computable functions, from Gödel’s ‘On formally undecidable propositions’ (in a section discussing the relevance of the incompleteness theorems to Hilbert’s programme in Chapter 24), and selections from Church, Turing, Gödel, and Kalmár in Chapter 25 (‘Church’s Thesis’). The final chapter (Chapter 26), ‘Constructivist Views of Mathematics’ gives the book a nice ending by tying the discussion of computability and undecidability to the general philosophical issues of the nature of mathematics. Selections from Brouwer’s ‘Intuitionism and formalism’ (1913), Bishop’s *Foundations of Constructive Analysis*’ (1967), and a section on strict finitism (with readings from van Dantzig and David Isles) provide a basis for an informed discussion of these issues.

The technical material is well-organised and covers almost all the bases: Turing machines, recursive functions, the halting problem, Kleene’s normal form, s–m–n and fixed point theorems, recursive enumerability, and the equivalence of Turing computable and partial recursivity. Part III, ‘Logic and arithmetic’ deals with propositional logic, the language of arithmetic, and the first and second incompleteness theorems. Notably absent is a presentation of first-order logic separate from that of arithmetic, in particular, the completeness theorem is not proved. This is perhaps the most important fault of the text. Advanced logic courses leading up to the incompleteness theorem, in the reviewer’s experience in any case, always include a discussion of the important metatheorems of first-order logic (completeness, compactness, Löwenheim-Skolem). Any textbook with that target audience in mind should thus include a discussion of these topics. The omission of a separate section on first-order logic is all the more surprising given the overall historical and philosophical emphasis of the text. Surely the completeness theorem, and perhaps the Löwenheim-Skolem theorem, rank with the incompleteness theorem both in historical importance and in relevance to the foundations and philosophy of mathematics (higher, in any case, then the Grzegorczyk hierarchy of computable functions, which gets its own chapter). Furthermore, without a discussion of first-order logic in general it is hardly possible to discuss the decision problem for first-order logic (both Church's negative solution—which is relegated to an exercise—and the various positive results for fragments of first-order logic). Since one of the motivations for the material that frame Epstein and Carnielli's discussions is Hilbert's program, it should be expected that one of the main aims of Hilbert, viz., the solution of the decision problem, should be given adequate treatment.

The second edition comes with a 38-page timeline of computability and undecidability by Epstein, which includes a separate bibliography, as well as a poster roughly 45cm by 71cm ($18''$ by $28''$) in size. The timeline starts with Babbage's 1834 Analytical Engine and ends with Matiyasevich's final contributions to the solution of Hilbert's Tenth Problem. This is a very useful supplement to the book, and certainly of interest to mathematicians and philosophers with a general interest in logic and foundations. The entries themselves are well-researched and combine factual information with discussion which puts the results discussed in context. Some entries are perhaps a bit too short, some too long. I am unsure, for instance, if the average student would have an idea of what it was that Boone and Novikov proved when they showed that 'the word problem for groups is undecidable' (1954B). On the other hand, what purpose is served by reporting Hilbert's misgivings about Ackermann's decision to get married (in the 1928 entry)?

Given the focus of the book, the timeline is reasonably complete, yet surely many will find some of their favourite results missing. My personal choices for inclusion in the next edition are Trakhtenbrot's 1950 theorem on the nonaxiomatizability of finite models, and Rabin's Tree Theorem on the decidability of the monadic second-order theory of two successors (1969). Certainly the wealth of results published in the last 50 years makes a selection for such a chronology a hard task indeed. Two omissions which I feel are not of the character of 'pet theorems', however, bear remarking on. One is the notable lack of emphasis on the decision problem for first-order logic, already noted above in regards to the main text. Only Löwenheim and Skolem's result on the decidability of the monadic fragment is included, but no mention of Behmann's proof of the decidability of monadic second-order logic (1922), of Bernays and Schönfinkel's, and Ackermann's 1928 decidability results, or of the decidability of the Gödel-Kalmár-Schütte class. (The interested reader may find an extensive bibliography and historical remarks in Börger *et al.* 1997). The other important lacuna is the complete absence of results on complexity theory. Arguably, certainly in terms of 'practical' importance but perhaps also in foundational significance, the development of complexity theory in the 1960s and 1970s deserves a place in any 'timeline of computability'. Nowadays computability is as much the domain of the theoretical computer scientists as it is that of the mathematician, and the pioneering work of Hartmanis and Stearns should not be missing from a history of the subject, even if it is only a poster. I can find no fault with choosing Matiyasevich as the end-point of the timeline, yet I wonder whether Cook's Theorem on the NP-completeness of SAT (1971) might not have been a better choice. Still, the timeline is a welcome addition to the textbook, and the poster a beautiful addition to any logician's office decor.

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