

# A Common Ground and Some Surprising Connections\*

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There are various strategies one might follow in preparing and delivering a lecture of this kind. Since this conference is entitled *Origins: The Common Sources of the Analytic and Phenomenological Traditions*, I decided that the best approach would be to provide you with a kind of field guide to certain passages in the literature which bear upon the foundational theory of objects I have developed over the years. I hope this proves to be appropriate, for I believe that this foundational theory assimilates ideas from key philosophers in both the analytical and phenomenological traditions.

Many of you will already know that the theory I've developed has its roots in the work of Alexius Meinong and Ernst Mally, and that it is grounded in a distinction between two kinds of predication. For those of you who don't know the theory, let me say briefly that the theory postulates special abstract objects that *encode* the properties by which they are conceived and which constitute their nature. These abstract objects *exemplify* properties as well; indeed, they are complete with respect to the properties they exemplify, though they may be incomplete with respect to the properties they encode. Moreover, the theory asserts that ordinary ob-

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jects stand in contrast to abstract objects in part by the fact that they *only* exemplify their properties. (This will all become clearer later in the talk.)

The following field guide will not only document the passages in which the distinction between two kinds of predication originates, but also document the other surprising, and often unrelated, contexts where the distinction reappears in the work of others. It will also document ways in which the theory can be used to represent precisely the ideas of certain important philosophers. The resulting guide will bring together the works of many different authors, including some clearly within the analytic tradition, some clearly within the phenomenological tradition, and some who straddle the divide. It seems to me that fundamental problems in metaphysics (concerning abstract objects), in the philosophy of language (concerning intensionality), and in the philosophy of mind (concerning intentionality) are inextricably linked and will require a joint solution. Moreover, I think that any such solution will preserve many of the insights from the people who work(ed) at the intersection of these two traditions.

## 1. Brentano, Meinong, Husserl, and Mally

Although I will offer evidence later in the talk that the salient distinction between two kinds of predication goes back to Plato, it is better, for the purposes of this conference, to begin by setting the stage *in medias res*, namely, with Brentano.

The following passage (Volume I, Book II, Chapter 1, §5) from Brentano's seminal work, *Psychologie vom empirischen Standpunkte*, is where he suggests that the distinguishing mark of mental states is that they are directed towards objects:

Jedes psychische Phänomen ist durch das charakterisirt [*sic*] was die Scholastiker des Mittelalters die intentionale (auch wohl mentale) Inexistenz eines Gegenstandes genannt haben, und was wir, obwohl mit nicht ganz unzweideutigen Ausdrücken, die Beziehung auf einen Inhalt, die Richtung auf ein Object [*sic*] (worunter hier nicht eine Realität zu verstehen ist), oder die immanente Gegenständlichkeit nennen würden. Jedes enthält etwas als Object in sich, obwohl nicht jedes in gleicher Weise. In der Vorstellung ist etwas vorgestellt, in dem Urtheile ist etwas anerkannt oder verworfen, in der Liebe geliebt, in dem Hasse gehasst, in dem Begehren begehrt, u.s.w.

Diese intentionale Inexistenz ist den psychischen Phänomenen  
ausschliesslich eigenthümlich. (1874, 115-116)

The translation of this part of Brentano 1874 by D.B. Terrell appears in  
Chisholm 1960 (39-61) and goes as follows:

Every mental phenomena is characterised by what the scholastics of the Middle Ages called the intentional (and also mental) inexistence of an object, and what we would call, although not in entirely unambiguous terms, the reference to a content, a direction upon an object (by which we are not to understand a reality in this case), or an immanent objectivity. Each one includes something as object within itself, although not always in the same way. In presentation, something is presented, in judgment something is affirmed or denied, in love [something is] loved, in hate [something] hated, in desire [something] desired, etc.

This intentional inexistence is exclusively characteristic of mental phenomena. (Chisholm 1960, 50)

This passage suggests that even when we entertain states of affairs which don't obtain, or have attitudes towards fictional objects (such as worshipping Zeus, fearing Grendel, searching for Atlantis, loathing Iago, or being inspired by Sherlock Holmes), our mental states are directed upon objects of some kind.

Alexius Meinong accepted this idea and tried to construct a theory about the objects upon which our mental states are directed. In his famous essay of 1904 'Über Gegenstandstheorie', Meinong says:

Daß man nicht erkennen kann, ohne etwas zu erkennen, allgemeiner: daß man nicht urteilen, ja auch nicht vorstellen kann, ohne über etwas zu urteilen, etwas vorzustellen, gehört zum Selbstverständlichsten, das bereits eine ganz elementare Betrachtung dieser Erlebnisse ergibt. (1904, 1)

The English translation by I. Levi, D. Terrell, and R. Chisholm, in Chisholm 1960 (76-117), goes as follows:

That knowing is impossible without something being known, and more generally, that judgments and ideas or presentations are impossible without being judgments about and presentations of something, is revealed to be self-evident by a quite elementary examination of these experiences. (1960, 76)

Meinong then suggests that when we think about such fictional objects as the golden mountain and the round square, the objects of our thought have the properties by which these objects are conceived. He says:

Nicht nur der vielberufene goldene Berg ist von Gold, sondern auch das runde Viereck ist so gewiß rund als es viereckig ist. (1904, 8)

In the English translation just cited, this reads:

Not only is the much heralded gold mountain made of gold, but the round square is as surely round as it is square. (1960, 82)

Most students of philosophy are trained on the famous counterexamples Bertrand Russell developed to this last claim. Russell reasoned that the round square violated not only certain logical laws but the contingent facts as well. If we assume the geometrical law that whatever is round fails to be square, then Meinong's view implies that the round square is both square and fails to be square, thus violating the law of noncontradiction. Moreover, on Meinong's principles, we would have to say that the existing golden mountain is golden, is a mountain, and exists, contrary to fact.

It is often supposed that Edmund Husserl took a different route in explaining how our mental states are directed in the cases of fictional objects. He denied that there are round squares, golden mountains, etc., of any kind. Instead, using the method of phenomenological reduction, he 'bracketed off' the external world and focused on the world of mental phenomena, noticing that these phenomena have a certain content, the *noematic sense*, which is not only responsible for directing our thoughts toward the world but also responsible for those thoughts being *as if* they were of trees, people, round squares, golden mountains, etc. Here is a classic passage from Husserl's *Ideen*, in which he discusses the mental state of perceiving a real tree in the natural world, and distinguishes the tree in nature from the *sense* or significance of the perception, which is something abstract and which involves something like the concept of a tree:

"In" der reduzierten Wahrnehmung (im phänomenologisch reinen Erlebnis) finden wir, als zu ihrem Wesen unaufhebbar gehörig, das Wahrgenommene als solches, auszudrücken als "materielles Ding", "Pflanze", "Baum", "blühend" usw. Die *Anführungszeichen* sind offenbar bedeutsam, sie drücken jene Vorzeichenänderung, die entsprechende radikale Bedeutungsmodifikation der Worte aus. Der

Baum *schlechthin*, das Ding in der Natur, ist nichts weniger als dieses *Baumwahrgenommene als solches*, das als Wahrnehmungssinn zur Wahrnehmung und unabtrennbar gehört. Der Baum schlechthin kann abbrennen, sich in seine chemischen Elemente auflösen usw. Der Sinn aber—Sinn *dieser* Wahrnehmung, ein notwendig zu ihrem Wesen Gehöriges—kann nicht abbrennen, er hat keine chemischen Elemente, keine Kräfte, keine realen Eigenschaften. (1913, 184)

Here is the English translation by F. Kersten:

“In” the reduced perception (in the phenomenologically pure mental process), we find, as indefeasibly belonging to its essence, the perceived as perceived, to be expressed as “material thing,” “plant,” “tree,” “blossoming,” and so forth. Obviously, the *inverted commas* are significant in that they express that change in sign, the corresponding radical signification modification of the words. The *tree simpliciter*, the physical thing belonging to Nature, is nothing less than this *perceived tree as perceived* which, as perceptual sense, inseparably belongs to the perception. The tree simpliciter can burn up, be resolved into its chemical elements, etc. But the sense—the sense of *this* perception, something belonging necessarily to its essence—cannot burn up; it has no chemical elements, no forces, no real properties. (Husserl 1913/1982, 216)

Now, if we mistakenly think we see a tree in the distance, or correctly think that the round square is impossible, or search for the fountain of youth, Husserl would say that although the world doesn’t contain the tree, the round square or the fountain of youth, our mental states can be characterized as having a sense to which the concepts tree, or round and square, or fountain conferring everlasting life, respectively, apply. Moreover, these concepts apply to the sense or content of our mental state in some special way. Although it is tempting to think that Husserl’s special inverted commas (quote marks) change the meaning of the words to which they are applied, this causes a problem. For if the word ‘tree’ changes its meaning when Husserl both places it in quote marks and uses the result to describe the sense of our mental state, then it would be a mystery how the mental state could direct us towards *trees*. But it turns out that there is a better interpretation of Husserl’s use of quote marks.

This interpretation will emerge once we consider the contributions of Ernst Mally, Meinong’s student and successor to his Chair at the University of Graz. Mally writes, in his 1912 book, *Gegenstandstheoretische Grundlagen der Logik und Logistik*:

... Im Gedanken “geschlossene ebene Kurve, deren Punkte von *einem* Punkte gleichen Abstand haben” ist etwas gemeint, das die angenommenen Objektive erfüllt, irgendein Individuum oder Ding aus der Klasse der Kreise ... Was aber im Begriffe unmittelbar gedacht ist, das ist der Gegenstand “geschlossene ebene Kurve, u.s.w.” Dieses begriffliche Abstraktum ist im Begriffe bloß gedacht, nicht auch gemeint. Von ihm ist die Erfüllung der konstitutiven Objektive nicht vorausgesetzt, ... “*der Kreis*” (in abstracto) *erfüllt* die im Kreisbegriffe angenommenen Objektive *nicht*, ... er ist nicht ein Kreis; er fällt deshalb auch nicht unter den Umfang des Kreisbegriffes, gehört der Klasse der Kreise nicht an, ...

... Wir sagen: der (abstrakte) Gegenstand “Kreis” ist definiert oder determiniert durch die Objektive “eine geschlossene Linie zu sein”, “in der Ebene zu liegen”, und “nur Punkte zu enthalten, die von *einem* Punkte gleichen Abstand haben”; (1912, 63-64)

Here is an English translation which Alfons Süßbauer and I put together and which appears in my paper of 1998:

... In the thought “closed plane curve, every point of which lies equidistant from *a single* point,” something is meant which satisfies these hypothesized objectives, some individual or thing from the class of circles ... But what is directly conceived in this concept is the object “closed plane curve, etc.” This conceptual abstractum is only conceived in this concept but not meant. That it satisfies the constitutive objectives is not presupposed ... “*the circle*” (in abstraction) *does not satisfy* the hypothesized objectives in the circle-concept, ... it is not a circle; therefore it isn’t in the extension of the circle-concept, it doesn’t belong to the class of circles...

... We say: the (abstract) object “circle” is defined or determined by the objectives “to be a closed line”, “to lie in a plane”, and “to contain only points which are equidistant from *a single* point”; (Zalta 1998, 11-12)

It seems clear here that Mally would say that the round square is *determined* by the properties of being round and being square, but that it

does not *satisfy* these properties. As some of you know, I have suggested that we replace Mally's terminology by saying: the round square *encodes* roundness and squareness but it does not *exemplify* them. Exemplification and encoding are to be understood as two modes of predication. Note that Mally's view is immune to Russell's objections: if the round square simply encodes roundness and squareness, then it is consistent with the law that whatever *exemplifies* roundness fails to *exemplify* squareness; if the existing golden mountain only encodes these three properties, then it is consistent with the contingent fact that nothing exemplifies these three properties.

In Zalta 1998, the above passage and other passages from Mally 1912 (76) and Mally 1971 (58, note 14), were compared with the above and other passages from Husserl 1913 (184, 187, 270-271). The conclusions I drew from the comparison were as follows. Mally agrees with Husserl that nothing exemplifies the properties of being round and square, or of being a fountain of youth, etc., and they would agree that our mental states about the round square, the fountain of youth, etc., involve some intermediate object. Mally would call that intermediate object an abstract determinate, while Husserl would call it a noematic sense, or maybe an essence. A logic for (or theory of) Mally's objects might go some way towards giving us a logic for (or theory of) Husserl's noematic senses.

What helps to confirm this idea is the fact that Mally's second mode of predication could be used to explain Husserl's inverted commas. When Husserl says that "tree" (i.e., the word 'tree' in inverted commas) characterizes the noematic sense of our perception of a tree, instead of thinking that the inverted commas change the meaning of the word 'tree', we can suppose that the inverted commas change the mode of predication. The very same property of being a tree is not only exemplified by the tree in nature, but is also encoded by the noematic sense in such a way as to give it a direction towards things in the world which exemplify this property. In other words, whenever Husserl correctly uses words in inverted commas to characterize the noematic sense, this is equivalent to asserting that the noematic sense encodes the property expressed by those words. In the case of thinking of the round square or of the fountain of youth, the noematic sense of our mental state encodes the relevant properties, though no object exemplifies these properties.

Although this connection between Mally and Husserl suggests that their views differ from Meinong, there is a way to preserve Meinong's

insights within this picture. In cases where the object fails to exist, we can use Mally's objects not only as noematic senses (contents) of mental states, but also both as the *object* of those states and as the *denotations* of the terms like 'Zeus', 'the fountain of youth', 'the round square', etc. Meinong said, in natural language, that the round square *is* [my emphasis] round and square. But if natural language predication is ambiguous, so that '*x is F*' is structurally ambiguous between '*x encodes F*' and '*x exemplifies F*', then there is a reading of Meinong's words on which they turn out true: The round and square (conceived as Mally conceives of it) *is* round and square for it encodes roundness and encodes squareness.

## 2. Findlay, Castañeda, Rapaport

Mally's work was described for English speakers by his student J.N. Findlay in 1933. In that year, Findlay published *Meinong's Theory of Objects*, and in it we find the following (quoted from the second edition of 1963):

On the view of Mally every determination determines an object, but not every determination is satisfied (*erfüllt*) by an object. The determination 'being two-legged and featherless' determines the abstract determinate 'featherless biped', which is usually called a 'concept', but it is *satisfied* by nearly every human being. On the other hand, the determination 'being round and square' determines the abstract determinate 'round square', but it is not satisfied by any object. ... the determinate of a certain determination need not really *possess* that determination. The round square is not really round, nor is it a square at all... (1963, 111)

Findlay later says:

On the theory of Mally, the object 'something that is blue' is merely the determinate of the determination 'being blue'; it does not *satisfy* this determination. The only objects which satisfy the determination of being blue are concrete blue existents. ...

In apprehending concrete existents, we do so by means of the determinates of certain determinations. We grasp *through* the determinate *at* the object which satisfies a set of determinations. The determinate is not referred to specially, it is apprehended in being grasped through (*wir erfassen es durchgreifend*), while the object which *has* the determinations is actually referred to... (1963, 183)

Findlay's discussion of Mally lay pretty much dormant for almost the next 50 years.

In 1974, H.-N. Castañeda independently developed a theory of 'guises' for the analysis of thought and language. He supposed that there were several kinds of predication which were relevant to guises. As far as I can discover, Castañeda doesn't mention any connection between his guises and Mally's abstract determinates. However, his student W. Rapaport also developed a theory of Meinongian objects on the basis of two kinds of predication. Rapaport developed this 'dual-copula' view in his 1976 dissertation, but we quote from his paper of 1978:

While the notion that there is more than one way for a subject to possess a property is most likely traceable back to Aristotle's *Categories*, the first fully developed theory embodying two copulas is that of Castañeda. His 'internal' predication corresponds roughly to what I shall call 'constituency' below, and his 'external' predication serves to associate pairs of 'guises' (which correspond *very* roughly to Meinongian objects) with 'sameness relations' such as identity or 'consubstantiation'.

...

For various reasons, among them the historical precedence of Castañeda's theory, I employ two modes of predication in the revision of Meinong's theory. (1978, 160)

But, it seems to me, non-existing golden mountains cannot be made of gold in the same way that existing golden rings are. (1978, 161)

... *whatever* the Meinongian object, *my gold ring*, may exemplify, it *doesn't* exemplify the property of being gold, as we saw above. My actual gold ring, on the other hand, *does* exemplify this property. (1978, 162)

Although Rapaport doesn't mention Mally in his discussion of the 'two-copula' theory, these passages suggest that the theory Rapaport developed is a version of Mally's theory.

### 3. Formal Object Theory

I was led to Mally's work by reading Findlay 1933/1963, Castañeda 1974, and Rapaport 1978 as I was studying the manuscript which T. Parsons eventually published as his 1980 book *Nonexistent Objects*. Parsons

showed how to rigorously formulate a Meinongian theory which systematized the distinction between nuclear and extranuclear properties.<sup>1</sup> It seemed natural to adopt Parsons' method and develop an alternative, formal Meinongian theory based on a regimentation of the distinction between two kinds of predication.

After elaborating the formal system in a series of unpublished papers (which were assimilated into my dissertation) and publishing an early piece with A. McMichael on nonexistent objects (McMichael and Zalta 1980), I eventually produced a more polished version of the formalism and axioms in my book of 1983.<sup>2</sup> In that work, I represented the two kinds of atomic formulas as follows:

$$F^n x_1 \dots x_n \text{ ('} x_1, \dots, x_n \text{ exemplify the relation } F^n \text{' )}$$

$$xF^1 \text{ ('} x \text{ encodes property } F^1 \text{' )}$$

Using these kinds of formulas as a basis, it is a relatively straightforward matter to construct a modal predicate calculus for expressing and asserting simple and complex claims involving the two kinds of predication. Using an existence predicate '*E!*', I defined ordinary objects ('*O!x*') to be objects which possibly exist (' $\diamond E!x$ '), and abstract objects ('*A!x*') to be objects which couldn't possibly exist (' $\neg \diamond E!x$ '). This allows one to formulate quantified claims like ' $\exists x A!x$ ', which assert (given a non-Quinean reading of the quantifier) that *there are* abstract objects (with no implication that these objects exist), i.e., there are objects which couldn't possibly exist. However, it is important to note that one may alternatively read the predicate '*E!*' as denoting the property of being concrete, in which case ordinary objects become defined as possibly concrete objects and abstract objects become defined as objects which couldn't be concrete.<sup>3</sup> Then, using the Quinean reading of the quantifier ' $\exists$ ' as *there*

<sup>1</sup>This is a distinction that Meinong borrowed from Mally's distinction between formal and extraformal properties. (See Meinong 1915, 176.) As Parsons 1980 shows, the distinction can be put to use in solving Russell's objections to Meinong's naive theory of objects, in developing a theory of fiction, and in analyzing other problems from the history of philosophy. Parsons 1980 had a deep and lasting impact on my philosophical understanding.

<sup>2</sup>Alan McMichael's contributions to the development of the theory were critical. He helped formulate the final, consistent axiom needed to complete the theory (relation comprehension) and introduced me to the techniques of algebraic semantics needed to interpret this axiom.

<sup>3</sup>I emphasized the importance of this alternative reading in Zalta 1988, 102-104, and noted something similar in Zalta 1983, 50-52.

*exists*, the claim ‘ $\exists x A!x$ ’ would assert the existence of abstract objects, not simply that they have being. I tend to use this alternative reading today, since it turns the theory into a kind of Platonism, but for our current purposes, the interpretation on which ‘*E!*’ means ‘exists’ brings us more in line with Meinong’s views.

Now it is relatively straightforward to represent claims about ordinary and abstract objects in our language. To assert that ordinary objects obey Leibniz’s Law, we can write:

$$O!x \ \& \ O!y \ \& \ \forall F(Fx \equiv Fy) \rightarrow x=y$$

In other words, if ordinary objects  $x$  and  $y$  exemplify the same properties, they are identical. Similarly, we can assert that abstract objects obey a parallel principle of identity:

$$A!x \ \& \ A!y \ \& \ \forall F(xF \equiv yF) \rightarrow x=y$$

In other words, if abstract objects  $x$  and  $y$  encode the same properties, they are identical. We can assert that ordinary objects necessarily fail to encode properties as follows:

$$O!x \rightarrow \Box \neg \exists F xF$$

Finally, we can assert the fundamental axiom (‘comprehension’) schema of the theory, namely, that for any condition  $\phi$  on properties expressible in the language, there is an abstract object which encodes exactly the properties which meet the condition:

$$\exists x(A!x \ \& \ \forall F(xF \equiv \phi)), \text{ where } \phi \text{ has no free } xs$$

The following instance of this comprehension schema asserts that there is an abstract object which encodes just two properties, roundness (‘*R*’) and squareness (‘*S*’):

$$\exists x(A!x \ \& \ \forall F(xF \equiv F=R \vee F=S))$$

If we let ‘ $G \Rightarrow F$ ’ abbreviate the claim that the property  $G$  necessarily implies the property  $F$  (i.e.,  $\Box \forall y(Gy \rightarrow Fy)$ ), then we can also assert that there is an abstract object which encodes all of the properties implied by the properties of being round and being square:

$$\exists x(A!x \ \& \ \forall F(xF \equiv R \Rightarrow F \vee S \Rightarrow F))$$

The two objects just discussed prove to be useful for analyzing various natural language contexts involving the ordinary description ‘the round square’. Note also that these objects are consistent with the law that whatever exemplifies roundness fails to exemplify squareness ( $\forall x(Rx \rightarrow \neg Sx)$ ).

## 4. Leibniz and Plato

We turn now to examine the ways in which the theory can be applied to the work of philosophers outside the tradition we have been outlining. In order to apply the theory, it will be useful to have certain canonical descriptions for abstract objects at our disposal. Note that it follows from our comprehension schema and the principle governing identity for abstract objects that for any formula  $\phi$  we choose, there is a unique abstract object that encodes just the properties meeting the condition  $\phi$  (for there couldn’t be two *distinct* abstract objects which encode exactly the properties meeting the condition  $\phi$ , since distinct abstract objects must differ by one of their encoded properties). So, no matter which formula  $\phi$  we use, the following definite description is always well-defined:

$$\iota x(A!x \ \& \ \forall F(xF \equiv \phi))$$

This description reads: the abstract object which encodes exactly the properties meeting the condition  $\phi$ .

### 4.1 Leibniz

In a recent paper (2000a), I tried to show how the above theory integrates ideas expressed in Leibniz’s logical papers with those expressed in his modal metaphysics. To represent the latter, we may define (1) the (complete individual) concept of ordinary object  $u$  (‘ $c_u$ ’), (2) the concept  $G$  (‘ $c_G$ ’), and (3) concept  $c$  contains concept  $c'$  (‘ $c \succeq c'$ ’), as follows:

$$c_u =_{df} \iota x(A!x \ \& \ \forall F(xF \equiv Fu))$$

$$c_G =_{df} \iota x(A!x \ \& \ \forall F(xF \equiv G \Rightarrow F))$$

$$c \succeq c' =_{df} \forall F(c'F \rightarrow cF)$$

From these definitions and the fact that Alexander exemplifies the property of being a king ( $'Ka'$ ), it follows that the (complete individual) concept of Alexander ( $'c_a'$ ) contains the concept of being a king ( $'c_K'$ ). For suppose  $c_K$  encodes a property, say  $P$ . Then, by definition of  $c_K$ ,  $K \Rightarrow P$ , i.e.,  $\Box \forall x(Kx \rightarrow Px)$ . But since  $Ka$  by hypothesis, it follows that  $Pa$ . So by definition of  $c_a$ , it follows that the concept of Alexander encodes the property  $P$ ; i.e., that  $c_a P$ . Since we showed, for an arbitrary property  $P$ , that if  $c_K$  encodes  $P$  then  $c_a$  encodes  $P$ , we have established that every property  $c_K$  encodes is encoded by  $c_a$ . So, by definition, the concept of Alexander contains the concept of being a king ( $'c_a \succeq c_K'$ ).

One can extend these ideas about complete individual concepts and concept containment by defining the notion of possible world and investigating its properties. In previous work (1983, 1993), I noted that abstract objects could be said to encode a proposition  $p$  by encoding the propositional property *being such that*  $p$  ( $'[\lambda y p]'$ ).<sup>4</sup> An object  $x$  was then defined to be a possible world just in case  $x$  might have encoded all and only the true propositions (i.e., iff  $\Diamond \forall p(x[\lambda y p] \equiv p)$ ). This definition and its resulting consequences about worlds, were employed in Zalta 2000a so as to link the 'calculus of concepts' which Leibniz described in various unpublished logical papers with his modal metaphysics of complete individual concepts. The fundamental theorem from the calculus of concepts and the fundamental theorem of Leibniz's modal metaphysics were thereby shown to be consequences of a single theory of concepts. The fundamental theorem of the calculus of concepts is the claim:

The concept  $F$  contains the concept  $G$  iff the concept  $F$  is identical to the sum of the concept  $F$  and the concept  $G$

<sup>4</sup>These propositional properties are logically well-behaved, and are governed by  $\lambda$ -conversion:

$$[\lambda y \phi(y)]x \equiv \phi(x/y)$$

This asserts: an object  $x$  exemplifies the property *being a  $y$  such that  $\phi$*  iff  $x$  is such that  $\phi$ . (The notation  $'\phi(y)'$  indicates that  $y$  may or may not be free in  $\phi$ , and the notation  $'\phi(x/y)'$  indicates the result of substituting  $x$  for  $y$  in  $\phi$ , where this is defined in the usual way.) As a particular instance applying to cases where  $y$  is not free in  $\phi$ , consider the  $\lambda$ -expression  $'[\lambda y Pb]'$  which might represent the property *being such that Bush is president*:

$$[\lambda y Pb]x \equiv Pb$$

(Note that when  $\phi(y)$  is the formula  $'Pb'$ , then  $\phi(x/y)$  is also the formula  $'Pb'$ .) So, something exemplifies the property of being such that Bush is president if, and only if, Bush is president.

$$c_F \succeq c_G \equiv c_F = c_F \oplus c_G$$

(Here, the sum of concepts  $x$  and  $y$ ,  $x \oplus y$ , is defined to be the object that encodes the properties encoded in both  $x$  and  $y$ .)

To state what may be the fundamental theorem of Leibniz's modal metaphysics, we defined a 'counterpart' of the concept Adam, say, to be any of the concepts of the 'many possible Adams' that Leibniz sometimes refers to. (This can be made formally precise, but we need not pause here to do so.) Further let us say that an individual concept  $c_u$  of an ordinary object  $u$  *appears at* a world  $w$  just in case there is an ordinary individual who exemplifies at  $w$  all and only the properties  $c_u$  encodes. Then the fundamental theorem of Leibniz's modal metaphysics seems to be captured by something like the following claim:

If ordinary object  $u$  exemplifies a property  $F$  but might not have exemplified that property, then (the concept  $u$  contains the concept  $F$  and) there is a counterpart to the concept  $u$  which doesn't contain  $F$  but which *appears* at some (non-actual) possible world.

$$(Fu \ \& \ \Diamond \neg Fu) \rightarrow [(c_u \succeq c_F) \ \& \ \exists x(\text{Counterpart}(x, c_u) \ \& \ x \not\succeq c_F \ \& \ \exists w(w \neq w_\alpha \ \& \ \text{Appears}(x, w)))]$$

As far as I can tell, few philosophical systems have integrated Leibniz's logical calculi and his modal metaphysics in such a way that the fundamental theorems of both can be derived.

## 4.2 Plato

We turn next to an interesting discovery by Constance Meinwald (in 1991 and 1992) which suggests that Plato was implicitly developing a theory involving two modes of predication in his *Parmenides*, and that such a distinction in predication can solve the Third Man problem. In Meinwald 1992, we find:

It is now time to turn to the second part of the dialogue. I believe that Plato so composed that exercise as to lead us to recognize a distinction between two kinds of predication, marked in the *Parmenides* by the phrases 'in relation to itself' (*pros heauto*) and 'in relation to others' (*pros ta alla*). . . . A predication of a subject in relation to itself holds in virtue of a relation internal to the subject's own nature, and can so be employed to reveal the structure

of that nature. A predication in relation to the others by contrast concerns its subject's display of some feature. . . (1992, 378)

Can we represent this distinction between two kinds of predication in Plato's work in terms of our distinction between ' $x$  encodes  $F$ ' (' $x$  is  $F$  in relation to itself') and ' $x$  exemplifies  $F$ ' (' $x$  is  $F$  in relation to others')? Well, later in Meinwald 1992 we find:

We noted before that, while the [Third Man] argument is seriously underspecified, it relies on some version of the crucial claim

The Large is large

in order to reach the threatening conclusion

The Large and the other large things now require to have something new in common, by which all of them will appear large

. . . But we are now clear that that [first] predication does not claim that the Large itself is large in the same way that the original groups of large things is. [*sic*] It therefore does not force on us a new group of large things whose display of a common feature requires us to crank up our machinery again and produce a new Form.

(1992, 385-6)

I think that the distinction between ' $xF$ ' and ' $Fx$ ' is in play here. In my book of 1983 (Chapter II, Section 1), I tried to undermine the Third Man argument in just the way that Meinwald suggests. At the time, I conceived of the Form of  $F$  as the abstract object which encodes just the property  $F$ . I argued that the Form of  $F$  'is'  $F$  only in the sense that it encodes  $F$ , not in the sense that it exemplifies  $F$ . Although I tried to show that the resulting theory of Forms captured ideas in Plato's texts, my work wasn't based on the kind of scholarship that Meinwald produced in support of her thesis. But, apparently, a Plato scholar has found textual support for the idea.

Later, F.J. Pelletier and I came to revise my original view somewhat, so as to conceive of the Form of  $F$  as the abstract object which encodes all of the properties necessarily implied by  $F$ . In our paper Pelletier & Zalta 2000, we argued:

One of our goals is to show that there is a logically coherent position involving two modes of predication which both (1) allows for a

precise statement of the theory of Forms, and (2) removes the threat that the Third Man argument poses. (Pelletier & Zalta, 166)

Using the distinction between encoding and exemplifying a property, we noted that there are two kinds of *participation* that can be defined. In the paper, we showed that if one really wants to put the Third Man argument to rest on the basis of a distinction in modes of predication, one has to consider whether and how the argument applies to both modes of predication.<sup>5</sup>

## 5. Frege and Russell

### 5.1 Frege

Another paper which suggests that two kinds of predication are central to the view of a well-known philosopher is Boolos 1987. In discussing Frege's *Foundations of Arithmetic*, Boolos says:

Thus, although a division into two types of entity, concepts and objects, can be found in the *Foundations*, it is plain that Frege uses not one but two instantiation relations, 'falling under' (relating some objects to some concepts), and 'being in' (relating some concepts to some objects), and that both relations sometimes obtain reciprocally: the number 1 is an object that falls under 'identical with 1', a concept that is in the number 1. (1987, 3)

Are the two instantiation relations Boolos mentions captured by our two modes of predication? Well, in Zalta 1999, I was able to define (1)  $F$  and  $G$  are in 1-1 correspondence (with respect to the ordinary objects) (' $F \approx_E G$ '), (2)  $x$  numbers (the ordinary)  $G$ s (' $Numbers(x, G)$ '), and (3) the number of (ordinary)  $G$ s (' $\#_G$ '), as follows:

$$F \approx_E G =_{df} \exists R[\forall u(Fu \rightarrow \exists!v(Gv \& Ruv)) \& \forall u(Gu \rightarrow \exists!v(Fv \& Rvu))]$$

$$Numbers(x, G) =_{df} \exists!x \& \forall F(xF \equiv F \approx_E G)$$

<sup>5</sup>It may come as a surprise to those who read the papers developing the two applications described in this section that the definition of the Platonic Form of  $F$  which Pelletier and I used is equivalent to (and indeed, is the same as) the definition of the Leibnizian concept of  $F$ . This becomes rather interesting as one works through the theorems that can be proved regarding these philosophical objects.



$$\#_G =_{df} \iota x \text{Numbers}(x, G)$$

In other words, (1)  $F$  and  $G$  are in 1-1 correspondence on the ordinary objects just in case there is a relation  $R$  which pairs each object exemplifying  $F$  with a unique object exemplifying  $G$ , and vice versa, (2)  $x$  numbers the ordinary  $G$ s just in case  $x$  encodes all and only the properties  $F$  which are in 1-1 correspondence with  $G$  on the ordinary objects, and (3) the number of  $G$ s is the abstract object which encodes all and only those concepts (i.e., properties) which are in 1-1 correspondence with  $G$  (on the ordinary objects). From these and other definitions, and two independently plausible assumptions, one can derive the Dedekind-Peano axioms for number theory (Zalta 1999).

Moreover, other Fregean logical objects can be defined using our second mode of predication. For example, we may define *the truth-value of*  $p$  ( $p^\circ$ ) as follows:

$$p^\circ =_{df} \iota x (A!x \ \& \ \forall F(xF \equiv \exists q(q \equiv p \ \& \ F = [\lambda y q])))$$

In other words, the truth-value of proposition  $p$  is the abstract object that encodes all and only the propositions  $q$  which are materially equivalent to  $p$ . From definitions such as this, one can define the Fregean objects The True and The False, and prove that they are truth-values, and indeed, prove that there are exactly two truth-values.<sup>6</sup>

I hope you will also find it interesting that our abstract objects may serve to unify Frege's philosophy of mathematics with his philosophy of language. For it seems natural to suppose that the sense of a term  $\tau$  for person  $x$  can be identified as an abstract object which encodes properties, namely, the properties which strike  $x$  as characteristic of the object denoted by  $\tau$  when  $x$  learns  $\tau$ . I tried to spell out the advantages of this conception of Fregean senses in Zalta 1988 and 2001, though it is important to recognize that I did not strictly follow Frege's conception of senses. On the theory I proposed, the sense of a term does not have to

<sup>6</sup>This work has not yet been published, but the idea is simple enough. The True ( $\top$ ) is the object that encodes all of the true propositions, and The False ( $\perp$ ) is the object that encodes all of the false propositions. (Recall that we defined above the sense in which an abstract object can encode a proposition.) An object  $x$  is a *truth-value* just in case there is some proposition  $p$  such that  $x = p^\circ$ . It now follows that both  $\top$  and  $\perp$  are truth-values and that any object which satisfies the definition of *truth-value* is identical to either  $\top$  or  $\perp$ . [Note appended after publication: The work referenced in this footnote has now appeared in D. Anderson and E. Zalta, "Frege, Boolos, and Logical Objects," *Journal of Philosophical Logic*, 33/1 (February 2004): 1–26. 2004.]

'determine' the denotation of the term, for senses may encode misinformation, i.e., properties that the denotation of the term fails to exemplify. I also assumed that the sense of a term  $\tau$  can vary from time to time as the person learns new information about the denotation of  $\tau$  in contexts in which  $\tau$  is used.

## 5.2 Russell

Let me now describe how an insight of Bertrand Russell can be preserved in the theory of objects. In his paper of 1908, Russell showed how we could safely predicate properties of both properties and relations by using a typing scheme which prevented properties from being predicated of themselves. It is an interesting exercise to reformulate object theory in terms of a simple version of Russell's theory of types. By building a type-theoretic version of object theory and a type-theoretic version of the comprehension schema for abstract objects, we can not only assert that there are abstract individuals, but also assert that there are abstract properties, abstract relations, abstract properties of properties, etc. At every type  $t$ , there are both ordinary and abstract objects of that type. The abstract objects of type  $t$  can encode properties which things of type  $t$  typically exemplify. So, when  $t$  is the logical type for properties of individuals, the theory will assert that there are abstract properties which encode properties of properties of individuals; and when  $t$  is the type of relations among individuals, then the theory will assert that there are abstract relations which encode properties of relations among individuals. Abstract properties and relations prove useful for the analysis of fictional properties and relations (phlogiston, absolute simultaneity, being a unicorn, etc.) as well as for the analysis of mathematical properties and relations (being a number, being a set, successor, membership, etc.).

To see this in practice, consider the following simple definition of *type*:

1.  $i$  is a type  
(The type for individuals)
2. whenever,  $t_1, \dots, t_n$  are types,  $\langle t_1, \dots, t_n \rangle$  is a type  
(The type for  $n$ -place relations having arguments of types  $t_1, \dots, t_n$ )

Now one can build a language around typed atomic formulas of the form ' $F^{\langle t_1, \dots, t_n \rangle} x^{t_1} \dots x^{t_n}$ ' and ' $x^t F^{(t)}$ '. Then the general formulation of the

comprehension schema for abstract objects is:

$$\exists x^t(A!^{\langle t \rangle}x \ \& \ \forall F^{\langle t \rangle}(xF \equiv \phi)), \text{ where } \phi \text{ has no free } xs$$

To see a more specific version of this principle, let  $t$  be  $\langle i \rangle$ , i.e., the type for properties of individuals. The following then asserts that for every condition  $\phi$  on properties of properties of individuals, there is an abstract property which encodes just the properties of properties satisfying the condition:

$$\exists x^{\langle i \rangle}(A!^{\langle \langle i \rangle \rangle}x \ \& \ \forall F^{\langle \langle i \rangle \rangle}(xF \equiv \phi)), \text{ where } \phi \text{ has no free } xs$$

We will see, in the final two sections, why such abstract properties are needed in philosophy.

## 6. Kripke

It will come as a surprise to most people, I think, to discover that the dual modes of predication view appears in some unpublished work by Kripke. But before I get to that, let me first discuss how the type-theoretic ideas just presented help us to capture a certain view that Kripke expressed in his 1980 book. He says:

Similarly, I hold the metaphysical view that, granted that there is no Sherlock Holmes, one cannot say of any possible person that he *would have been* Sherlock Holmes, had he existed. Several distinct possible people, and even actual ones . . . might have performed the exploits of Holmes, but there is none of whom we can say that he would have *been* Holmes had he performed these exploits. For if so, which one? (1980, 158)

Just prior to this passage, Kripke argued something similar with respect to fictional species, such as unicorns:

If we suppose, as I do, that the unicorns of the myth were supposed to be a particular species, but that the myth provides insufficient information about their internal structure to determine a unique species, then there is no actual or possible species of which we can say that it would have been the species of unicorns. (1980, 157)

These claims by Kripke become provable as theorems in the type-theoretic version of object theory. If we think of stories as abstract objects which

encode propositions (in the sense defined above), then we can say that  $x$  is a character of story  $s$  just in case for some property  $F$ ,  $s$  encodes the proposition that  $x$  exemplifies  $F$ . This allows both ordinary and abstract objects alike to be characters of fictions. But we only want to identify the objects that *originate* in a fiction in the *a priori* portion of our ontology. So we may say that an object  $x$  originates in  $s$  just in case  $x$  is abstract,  $x$  a character of  $s$ , and  $x$  is not a character of any earlier fiction. Thus, a *fictional* object may be defined as any object which originates in some story. From these definitions, it follows that fictional objects are not identical with any ordinary (possible) object:

$$Fictional(x) \rightarrow \neg \exists y(\diamond E!y \ \& \ y = x)$$

Note that if the variables  $x, y$  in the above are of type  $i$  (and ‘ $E!$ ’ and ‘ $=$ ’ are of type  $\langle i \rangle$  and  $\langle i, i \rangle$ , respectively), then the resulting claim asserts that no fictional individual (such as Holmes) is identical with an ordinary, possible individual. If the variable is of type  $\langle i \rangle$  (and again ‘ $E!$ ’ and ‘ $=$ ’ are of the corresponding higher type), then the resulting claim asserts that no fictional property (such as being a unicorn) is identical with any ordinary (possible) property.<sup>7</sup>

It is now relevant to mention that our distinction in modes of predication has turned up in Kripke’s unpublished lectures of 1973. In Lecture 3, Kripke discusses a ‘confusing double usage of predication’, and says that sentences like ‘Hamlet has been discussed by some critics’ and ‘Hamlet was melancholy’ are examples of the ‘two types of predication’ that can be made about this fictional character. Kripke goes on to say that if we don’t ‘get the two different kinds of predication straight’ then we ‘will get quite confused’. The distinction in kinds of predication is mentioned again in Lecture 4 and Lecture 5, and in general, plays an important role in Kripke’s lectures.

Now it seems clear that the distinction Kripke has drawn either is, or is captured neatly by, our distinction between encoding and exemplification. We can represent the claims Kripke discusses as:

<sup>7</sup>This work is spelled out in more technical detail in an unpublished paper ‘How to Prove Important Kripkean Claims (and Validate Other Such Claims)’. This paper was presented at the conference ‘Naming, Necessity, and More’, held in Kripke’s honor at the University of Haifa in 1999. It is available online at <<https://mally.stanford.edu/publications.html#kripke>>. [Note appended after publication: The paper referenced in this footnote has now been published under the title “Deriving and Validating Kripkean Claims Using the Theory of Abstract Objects”, *Noûs*, 40/4 (December 2006): 591–622.]

Hamlet was melancholy.  
 $hM$

Hamlet has been discussed by some critics.  
 $[\lambda y \exists x(Cx \ \& \ Dxy)]h$

The first is an encoding claim while the second is an exemplification claim. This kind of logical representation of discourse within, and about, fiction is the subject of Zalta 2000b, so let me refer you there for the details.

## 7. Gödel

So far, we have seen how the seminal ideas underlying the theory of objects can be traced to the work of Brentano's students and their students. We've also seen how, on the one hand, the theory might offer a logic of Husserl's noematic senses (or maybe a logic of essences) and might thereby be of interest to phenomenologists, while on the other hand, the work of various analytic philosophers either invokes (explicitly or implicitly) something like this distinction in modes of predication or else can be systematized in terms of a theory that makes this distinction. I would like to conclude this talk by discussing a case of a well-known analytic philosopher who explicitly appeals to phenomenological ideas, to see whether the present theory can help us strengthen the appeal.

Kurt Gödel expressed some rather enigmatic ideas in various unpublished work and in conversation with Hao Wang. Recent papers by Føllesdal (1995) and by Tieszen (1992, 1998) have remarked upon these ideas. For example, in a paper referred to as \*1961/?, Gödel says:

... there exists today the beginnings of a science which claims to possess a systematic method for such clarification of meaning, and that is the phenomenology founded by Husserl. Here clarification of meaning consists in concentrating more intensely on the concepts in question by directing our attention in a certain way, namely, onto our own acts in the use of those concepts, onto our own powers in carrying out those acts, etc. In so doing, one must keep clearly in mind that this phenomenology is not a science in the same sense as the other sciences. Rather it is [or in any case should be] a procedure or technique that should produce in us a new state of consciousness in which we describe in detail the basic

concepts we use in our thought, or grasp other, hitherto, unknown, basic concepts. (\*1961/?, 383)

I am not sure what Gödel had in mind here, so before we venture any guesses, let us consider some of the other remarks he made that may be relevant to the present study. Reporting on the tenor of their conversations in general, Wang observed:

In discussions with me, Gödel stressed the central importance of the axiomatic method for philosophy. (Wang 1996, 244)

Wang also recorded the following remarks by Gödel in conversation:

Philosophy as an exact science should do for metaphysics as much as Newton did for physics. (quoted in Wang 1974, 85)

5.3.11 The beginning of physics was Newton's work of 1687, which needs only very simple primitives: force, mass, law. I look for a similar theory for philosophy or metaphysics. Metaphysicians believe it possible to find out what the objective reality is; there are only a few primitive entities causing the existence of other entities. Form (*So-Sein*) should be distinguished from existence (*Da-Sein*): the forms—though not the existence—of the objects were, in the middle ages, thought to be within us.

(quoted in Wang 1996, 167)

5.3.17 The basis of everything is meaningful predication, such as  $Px$ ,  $x$  belongs to  $A$ ,  $xRy$ , and so on. Husserl had this.

(quoted in Wang 1996, 168)

9.3.20 Philosophy is more general than science. Already the theory of concepts is more general than mathematics.

(quoted in Wang 1996, 308)

In addition to these conversational remarks, consider the following, which was transcribed from Gödel's Gabelsberger shorthand, with the heading 'My Philosophical Viewpoint':

12. There is a scientific (exact) philosophy and theology, which deals with concepts of the highest abstractness; and this is also most highly fruitful for science. (Gödel \*1960/?, 316)

Clearly, these passages are only vague and suggestive. But they document that Gödel believed that some part of philosophy could be formulated as a rigorous discipline, that the axiomatic method should be used if possible, and that predication serves as the ‘basis for everything’. Moreover, he was aware of the distinction between *being* and *so-being*, and was influenced by Husserl’s method of phenomenological reduction.

Though it is obvious that Gödel didn’t have the present theory in mind in the above remarks, our theory does nevertheless satisfy his desiderata for a ‘scientific (exact) philosophy’ and it does so in a way that has a straightforward connection to Husserl’s views. If the appeal to Mallyan abstract objects forms part of our best understanding of the intentionality and meaningfulness of both our mental states and expressions of language, then the present theory goes some way towards satisfying the constraints Gödel seems to be imposing.<sup>8</sup>

Gödel was interested in Husserl’s ideas, at least in part, because of the insight they might provide about basic questions in the philosophy of mathematics. Let me distinguish the fundamental questions in the *philosophy* of mathematics (e.g., do mathematical objects exist, what is mathematical language about, how do we account for the apparent truth and meaningfulness of mathematical language, etc.) with the fundamental questions in the *foundations* of mathematics (e.g., what version of set theory is the most fruitful, what is the most powerful mathematical theory, does mathematics need new axioms, etc.). If this distinction is appropriate, then the present theory has an answer to basic philosophical questions about mathematics. The answers to these questions were first sketched in Linsky and Zalta 1995, and I provided numerous details in Zalta 2000c. The idea is basically this.

If we think of the basic data from mathematics as truths of the form ‘In theory  $T$ ,  $p$ ’ or ‘ $p$  is true in  $T$ ’, then we can treat mathematical theories as abstract objects that encode propositions (in the sense defined above). Let us use ‘ $T \models p$ ’ to represent the claim that proposition  $p$  is true in theory  $T$ , where this is defined as the claim that  $T$  encodes the propositional property *being such that*  $p$ . Now suppose that  $\kappa$  is a term of theory  $T$ .

<sup>8</sup>In support of the idea that the present metaphysical theory forms the basis of an exact science, let me recommend at least a brief perusal of a manuscript that is still in preparation, but which is available online. The unpublished monograph *Principia Metaphysica* contains all the formal consequences of object theory that I have proved in various publications over the years. Those interested may examine a version of the document at <<https://mally.stanford.edu/principia1.pdf>>.

Then we can identify a particular abstract object,  $\kappa_T$ , as follows:

$$\kappa_T = \iota x(A!x \ \& \ \forall F(xF \equiv T \models F\kappa))$$

This asserts that the object  $\kappa$  of theory  $T$  is the abstract object that encodes just the properties  $F$  such that the proposition that  $F\kappa$  is true in theory  $T$ .<sup>9</sup> In other words, the object  $\kappa$  of theory  $T$  encodes exactly the properties that  $\kappa$  exemplifies in  $T$ . So, for example, the null set  $\emptyset$  of ZF (Zermelo-Fraenkel set theory) is now identified as the abstract object that encodes exactly the properties  $F$  such that the proposition *that*  $\emptyset$  *exemplifies*  $F$  is true in ZF.

The type-theoretic version of object theory allows us to generalize, so that we can identify the properties and relations of mathematical theories as well. For consider the type theoretic version of the above identification principle:

$$\kappa_T^t = \iota x^t(A!(^t)x \ \& \ \forall F^{(t)}(xF \equiv T \models F\kappa))$$

To see an instance of this principle, let’s identify the membership relation  $\in$  of ZF. This is a relation of type  $\langle i, i \rangle$ , since it is a relation between individuals (sets). Then where ‘ $\in$ ’ and the variable ‘ $x$ ’ are of type  $\langle i, i \rangle$ , and ‘ $A!$ ’ and the variable ‘ $F$ ’ are of type  $\langle \langle i, i \rangle \rangle$  (i.e., they denote, or range over, properties of relations between individuals), we have the following instance of our identification principle:

$$\in_{ZF} = \iota x(A!x \ \& \ \forall F(xF \equiv T \models F\in))$$

This tells us that the membership relation  $\in$  of ZF is the abstract relation which encodes exactly the properties of relations which  $\in$  exemplifies in ZF. It is important to remember here that this is not a *definition* of the symbol ‘ $\in$ ’, but serves to theoretically identify the membership relation of ZF in terms of the truths of ZF.

It will be immediately obvious to anyone familiar with Gödel’s writings on the philosophy and foundations of mathematics that important elements of the position he takes in print run counter to the view being described here. The present view takes each mathematical theory to be (consist of truths) about its own domain of objects and relations. It takes the membership relation of ZF to be different from the membership relation of ZF + Axiom of Choice, and rejects the idea that there is only

<sup>9</sup>Strictly speaking, all of this has to be done in the language of object theory, and the details of how this is done can be found in Zalta 2000c.

one correct set theory. This allows us to analyze the meaningfulness of the language used in arbitrary mathematical theories. By contrast, Gödel apparently believed that there is only one correct set theory, and that we simply have to keep searching until we find the axioms which best characterize the domain of sets (all other axioms simply being false). Moreover, Gödel conceived of the mind-independence and objectivity of mathematical objects (and in particular, sets) on the model of physical objects. He thought there was “something like a perception . . . of the objects of set theory” (1964, 271), and claimed that the question of the objective existence of the objects of mathematical intuition ‘is an exact replica of the question of the objective existence of the outer world’ (1964, 272).

But Gödel’s views were not informed by a precise *philosophical* theory of mathematical objects and concepts, such as the one we now have before us. Gödel’s use of the terms ‘mathematical concept’ and ‘abstract concept’ was philosophically naive. If the best theory of mathematical objects and concepts is true and implies that each mathematical theory is about its own domain, then Gödel’s view would need to be revised. As Linsky and I argued in our 1995 paper, it is a mistake to model the mind-independence and objectivity of abstract objects by analogy with physical objects. Abstract objects and concepts are not mind-independent and objective in the same way physical objects are; the former, not the latter, can and should be systematized by comprehension principles. These comprehension principles ground the mind-independence and objectivity of abstract objects. Linsky and I developed the basic metaphysical and epistemological principles which are appropriate to this kind of mind-independence and objectivity. I believe that if Gödel had encountered a powerful, axiomatic metaphysics which could explain the meaningfulness of mathematical language and which offered a subject matter for arbitrary mathematical theories, he would have taken the view seriously. This is suggested by the following remark:

By abstract concepts, in this context, are meant concepts which are essentially of the second or higher level, i.e., which do not have as their content properties or relations of concrete objects (such as combinations of symbols), but rather of thought structures or thought contents (e.g., proofs, meaningful propositions, and so on), where in the proofs of propositions about these mental objects insights are needed which are not derived from a reflection upon the combinatorial (space-time) properties of the symbols representing

them, but rather from a reflection upon the meanings involved.

(1972, 271-2)

The study of the common origins of analytic philosophy and phenomenology shows us that meaning is grounded in intentions and intentionality, and so I suggest that intentional objects such as Mallyan abstract objects are the place to look if we are to find what Gödel calls the ‘meanings involved’.

## Conclusion

I have tried to be thorough in developing this field guide to the literature, though I have no doubt overlooked other places where a two modes of predication view of the kind presented here has surfaced. Still, I hope that those of you already familiar with some of my previous work will have found something surprising by this particular juxtaposition of sources relating to the foundations and application of the theory of abstract objects.

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