Logical and Analytic Truths That Are Not Necessary*

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The notions of logical truth, analytic truth, and necessary truth are extremely important in philosophy. A logically true sentence remains true no matter what the interpretation of the nonlogical constants. An analytically true sentence is true in virtue of the meanings of its words. Necessarily true sentences are true in all possible worlds. The concepts involved in these definitions are central to philosophy, and it is of the utmost importance that philosophers chart their interactions and examine where and how the distinctions among them evolve. Many philosophers agree that logical truths are analytic, since they are true in virtue of the fixed meanings of the logical constants. Examples like "All bachelors are unmarried" demonstrate that not all analytic truths are logically true, for though this sentence is true in virtue of the standard interpretation (meaning) of 'bachelor' and 'unmarried', there may be interpretations of these terms in which this sentence is false. Many philosophers also agree that there are necessary truths which are not logically true. Paradigm cases of such are identity statements of the form '$a = b$' where 'a' and 'b' are both names (or logical constants). But many philosophers regard all logical and analytic truths as paradigm cases of necessary truths. After all, if a sentence is true solely in virtue of its logical form or in virtue of the meanings of its words, it seems that it could not fail to be true.

Saul Kripke just stipulates that analytic truths are necessary in his lectures of 1970:[1]

> Another term used in philosophy is 'analytic'. Here it won't be too important to get any clearer about this in this talk. The common examples, nowadays, are like 'bachelors are unmarried'. . . . At any rate, let's just make it a matter of stipulation that an analytic statement is in some sense true by virtue of its meaning and true in all possible worlds by virtue of its meaning. Then something which is analytically true will be both necessary and *a priori* (p. 264).

Of course, Kripke is entitled to stipulate whatever he wants, but in this paper we plan to show that the following are consequences of the traditional conception of logical truth, analytic truth, and necessity: (1) there are sentences which are logically true but which are not necessary, and (2) there are sentences which are analytically true, true in virtue of the meanings of their words, yet which are not true in all possible worlds.

To argue for these consequences in a philosophicallly neutral way, we shall suppose that interpreted sentences, rather than propositions, are the bearers of truth. In section I, we establish that there are surprisingly simple examples of both logical truths that are not necessary (LTNNs) and analytic truths that are not necessary (ATNNs). In section II, we consider whether there are any ways to undermine the claims in section I. In this section, a certain definition of logical truth for modal languages, which has become established in the literature as an alternative to Kripke's original definition, is discovered to be incorrect. In section III, we investigate whether there are simpler examples of LTNNs and ATNNs than those described in section I. Finally, section IV contains a brief discussion of some important consequences that should be drawn.

I

We can find examples of LTNNs and ATNNs in a first-order modal language in which rigidly designating definite descriptions of the form $(\imath x)\varphi$

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[1] "Naming and Necessity," in Donald Davidson and Gilbert Harman, eds., (Boston: D. Reidel, 1972), pp. 252–355.

are generated as primitive singular terms.[2] The following, simultaneous recursive definition of *term* and *formula* defines such a language:

1. Every constant a_i and variable x_i is a term.

2. If R^n is any n-place predicate, and $\tau_1, \ldots, \tau_n$ are any terms, then $R^n\tau_1 \ldots \tau_n$ is a formula.

3. If φ and ψ are any formulas, and x_i is any variable, then $(\sim\varphi)$, $(\varphi \rightarrow \psi)$, $(\forall x_i)\varphi$, and $(\Box\varphi)$ are formulas.

4. If φ is any formula, and x_i is any variable, then $(\imath x_i)\varphi$ is a term.

Since descriptions are generated as primitive, complex terms, they do not have scope. The notion of scope no more applies to them than it does to constants or variables. Since descriptions are not contextually defined, they cannot be eliminated, and so the question of eliminating them with wide or narrow scope does not arise. Intuitively, the symbol '$\imath x$' expresses the primitive logical notion "the x such that," just as '$\sim$' expresses the primitive logical notion "it is not the case that," and '$\forall x$' expresses the primitive logical notion "every x is such that."

If the primitive descriptions are to designate rigidly, we have to interpret the language so that they denote that object at the distinguished actual world which uniquely satisfies the description (should there be one), even when the description occurs in the scope of a modal operator. This could be done by using the following semantic characterizations for the language. First, we specify an interpretation of the language to

[2]Such a language is of considerable interest to a philosophical logician. It is the principal tool by which one can investigate the claim that certain definite descriptions of English can be adequately represented as rigid designators. Elsewhere, I have shown that a simple modification of this language can handle nicely a recalcitrant group of English descriptions. See *Abstract Objects: An Introduction to Axiomatic Metaphysics* (Boston: D. Reidel, 1983), pp. 99–106. This language is also interesting because it is immune to W. V. Quine's attempt to prove that modal distinctions collapse in ordinary quantified modal logic with complex singular terms (see "Three Grades of Modal Involvement," *Proceedings of the XIth International Congress of Philosophy* (Brussels 1953): pp. 65–81; reprinted in Quine, *The Ways of Paradox and Other Essays* (New York: Random House, 1966), pp. 156–173). Dagfinn Føllesdal has shown that Quine's argument does not apply to modal languages in which all (complex) singular terms are rigid (see "Referential Opacity and Modal Logic," PhD thesis: Harvard University, 1961, p. 116; "Situation Semantics and the 'Slingshot' Argument," *Erkenntnis* XIX, 1 (May 1983): 91–98).

be something like that in Kripke's work.[3] An interpretation **I** is a set $<\mathbf{W},\mathbf{w}_0,\mathbf{D},\mathbf{F}>$, where **W** is a nonempty set (of possible worlds), $\mathbf{w}_0$ is a distinguished element of **W** ($\mathbf{w}_0$ is the base world), **D** is the domain of (possible) objects, and **F** is the function that assigns to each constant a_i of the language an element of **D**, and assigns to each predicate letter R^n_i a function from worlds to sets of n-tuples.[4] An assignment **f** to the variables with respect to an interpretation **I** is then defined in the usual way. For convenience, we always drop the subscript that relativizes **f** to an interpretation, though all **f**'s are to be understood as so relativized. In terms of these two notions, we may define the notions of denotation and satisfaction, which recursively call each other.

The definition of denotation is given first, and in it, we use **o** as a metavariable ranging over the elements of **D**, and we use "$\mathbf{f}' \overset{x}{=} \mathbf{f}$" to mean that $\mathbf{f}'$ is just like **f** except perhaps in what it assigns to x. The *denotation* of term τ *with respect to* interpretation **I** *and* assignment **f** ("$\mathbf{d}_{\mathbf{I},\mathbf{f}}(\tau)$") is defined as follows:

1. where τ is any constant κ, $\mathbf{d}_{\mathbf{I},\mathbf{f}}(\tau) = \mathbf{F}(\kappa)$

2. where τ is any variable ν, $\mathbf{d}_{\mathbf{I},\mathbf{f}}(\tau) = \mathbf{f}(\nu)$

3. Where τ is any description $(\imath x)\psi$, then

$$\mathbf{d}_{\mathbf{I},\mathbf{f}}(\tau) = \begin{cases} \mathbf{o} \text{ iff } (\exists \mathbf{f}')(\mathbf{f}' \overset{x}{=} \mathbf{f} \;\&\; \mathbf{f}'(x) = \mathbf{o} \;\&\; \mathbf{f}' \text{ satisfies}_{\mathbf{I}}\ \psi \\ \quad \text{at } \mathbf{w}_0 \;\&\; (\forall \mathbf{f}'')(\mathbf{f}'' \overset{x}{=} \mathbf{f}' \;\&\; \mathbf{f}'' \text{ satisfies}_{\mathbf{I}}\ \psi \text{ at} \\ \quad \mathbf{w}_0 \rightarrow \mathbf{f}'' = \mathbf{f}')) \\ \text{undefined, otherwise} \end{cases}$$

Clause (3) says that for $(\imath x)\psi$ to denote an object **o**, there must be a certain assignment to the variables which uniquely satisfies ψ at the actual world. Intuitively, this guarantees that the denotation of a description $(\imath x)\psi$ is just the unique object **o** that the description picks out at the actual world, should there be one. This packs Russell's analysis of the

[3]"Semantical Considerations on Modal Logic," reprinted in Leonard Linsky, ed., *Reference and Modality* (New York: Oxford, 1971), pp. 63–72.

[4]It will not be crucial to my argument that the domain of individuals be nonempty. My examples of LTNNs do not depend on the nonlogical claim that there must be at least one individual in the domain of discourse. Also, for the purposes of this paper, predicates are assigned functions from worlds to sets of n-tuples, in the usual way. I prefer the formulation in my [1983], however, in which predicates denote primitive relations, the *extensions* of which are functions from worlds to sets of n-tuples.

description into the semantic conditions that must obtain for the description to denote an object. Note that this definition of denotation defines a unary function on the terms of the language. All the terms receive denotations *simpliciter*; they do not receive denotations relative to each world, for this is unnecessary when all terms rigidly designate.

We need next the notion of satisfaction, and it will suffice to state only the base clause, quantifier clause, and modal clause of this definition, since the others are routine. There is one little twist to the definition of satisfaction, which prevents the base clause from being undefined for formulas that may have nondenoting descriptions. Typically, the base clause for atomic formulas φ of the form $\rho^n\tau_1 \ldots \tau_n$ reads as follows: $\mathbf{f}$ satisfies φ at world $\mathbf{w}$ iff $< \mathbf{d_{I,f}}(\tau_1), \ldots, \mathbf{d_{I,f}}(\tau_n) > \in [\mathbf{F}(\rho^n)](\mathbf{w})$. But this clause would be undefined for formulas containing descriptions that failed to denote. So the clause requires a minor modification, which is accomplished by the following, general definition:

> If given an interpretation $\mathbf{I}$ and assignment $\mathbf{f}$, we define $\mathbf{f}$ *satisfies* φ *at* world $\mathbf{w}$ as follows:
>
> 1. Where φ is an atomic formula of the form $\rho^n\tau_1 \ldots \tau_n$, $\mathbf{f}$ satisfies φ at $\mathbf{w}$ iff $(\exists \mathbf{o}_1) \ldots (\exists \mathbf{o}_n)(\mathbf{o}_1 = \mathbf{d_{I,f}}(\tau_1)\ \&\ \ldots\ \&\ \mathbf{o}_n = \mathbf{d_{I,f}}(\tau_n)\ \&$ $< \mathbf{o}_1, \ldots, \mathbf{o}_n > \in [\mathbf{F}(\rho^n)](\mathbf{w}))$
> 2. Where φ is a quantified formula of the form $\forall x\psi$, $\mathbf{f}$ satisfies φ at $\mathbf{w}$ iff $\forall \mathbf{f}'(\mathbf{f}' \overset{x}{=} \mathbf{f} \rightarrow \mathbf{f}'$ satisfies ψ at $\mathbf{w})$
> 3. Where φ is a modal formula of the form $\Box\psi$, $\mathbf{f}$ satisfies φ with respect to world $\mathbf{w}$ iff $(\forall \mathbf{w}')(\mathbf{f}$ satisfies ψ at $\mathbf{w}')$.

This base clause for satisfaction simply stipulates that, if an assignment $\mathbf{f}$ is to satisfy an atomic formula φ at a world $\mathbf{w}$, every term in φ must have a denotation (*simpliciter*) and the denotations must exhibit the appropriate structure at $\mathbf{w}$. One should check that the definition works as it should by considering a few basic cases. Note that the definition of satisfaction and the definition of denotation recursively call each other, and that the interaction yields the correct satisfaction conditions for formulas containing descriptions. These two definitions are not circular, despite appearances. By fixing an arbitrary interpretation, and working through a simple example like "$\mathbf{f}$ satisfies $P(\imath x)Qx$ at $\mathbf{w}_0$," one is able to derive satisfaction conditions stated solely in terms of set-theoretic membership

(see the Appendix, for the details). In other words, both notions are eliminable, and, consequently, there is no circularity.

The second clause in the definition of satisfaction ensures that the universal quantifier, and the existential quantifier defined in terms of it, are 'possibilist'. This simply means that such quantifiers range over everything whatsoever in the fixed domain of all possible objects.

The modal clause in the definition of satisfaction is perfectly standard: $\mathbf{f}$ satisfies $\Box\varphi$ with respect to world $\mathbf{w}$ iff $(\forall \mathbf{w}')(\mathbf{f}$ satisfies φ at $\mathbf{w}')$. This S5 interpretation without an accessibility relation keeps everything simple.

We are now in a position to state the definitions of truth and logical truth, and it will be seen that they are the ordinary, straightforward concepts with which we are all familiar. First, we say that φ is *true under* interpretation $\mathbf{I}$ *at* world $\mathbf{w}$ iff every assignment $\mathbf{f}$ satisfies φ at $\mathbf{w}$. Next we say, φ is *true under* $\mathbf{I}$ iff φ is true under $\mathbf{I}$ at $\mathbf{w}_0$. Finally, φ is *logically true* iff for every interpretation $\mathbf{I}$, φ is true under $\mathbf{I}$ (compare Kripke [1963], p. 64).

These definitions reveal that there are both LTNNs and ATNNs. We can demonstrate this by considering a particular sentence of our language and establishing several important claims about it. The sentence that will play a central role in the next few pages is φ_1:

$$(\varphi_1)\quad P(\imath x)Qx \rightarrow (\exists y)Qy$$

This is to be read: If the Q-thing is P, then something is Q. It should be relatively easy to see that the following four claims are true (some are straightforward consequences of the foregoing definitions).

Claim 1: φ_1 is logically true.

To see this, pick an arbitrary interpretation $\mathbf{I}_j$ and assignment $\mathbf{f}_k$. Clearly, either $\mathbf{f}_k$ satisfies the antecedent of φ_1 at $\mathbf{w}_0$, or it does not. If $\mathbf{f}_k$ satisfies $P(\imath x)Qx$ at $\mathbf{w}_0$, then by the definition of denotation, the description uniquely picks out an object at $\mathbf{w}_0$. If so, then $\mathbf{f}_k$ satisfies $(\exists y)Qy$ at $\mathbf{w}_0$ (by the recursive clause for quantifiers in the definition of satisfaction). So by the clause in the definition of satisfaction governing conditionals, $\mathbf{f}_k$ satisfies φ_1 at $\mathbf{w}_0$. If $\mathbf{f}_k$ fails to satisfy the antecedent of φ_1 at $\mathbf{w}_0$, then $\mathbf{f}_k$ still satisfies φ_1 at $\mathbf{w}_0$. So φ_1 is logically true.

Claim 2: $\Box\varphi_1$ is not logically true.

To prove this, we have to show that there is an interpretation under which $\Box\varphi_1$ is not true. Consider the interpretation I_1 having the following characteristics: (1) there is an object which not only uniquely exemplifies Q at $\mathbf{w}_0$, but also exemplifies P in every possible world, and (2) there is a world $\mathbf{w}_1$ at which nothing exemplifies Q. Under I_1, $\Box\varphi_1$ is not true. The reason is that there is a world, namely w_1, at which φ_1 is not true. φ_1 fails to be true at w_1 because the antecedent '$P(\imath x)Qx$' is true there (by characteristic (1) of I_1) while the consequent '$(\exists y)Qy$' is false there (by characteristic (2) of I_1). Since there is interpretation under which $\Box\varphi_1$ is not true, $\Box\varphi_1$ is not logically true.

Claim 3: Under $\mathbf{I}_1$, φ_1 is a logical truth that is not necessary.

By Claim 1, φ_1 is a logical truth, and so remains a logical truth no matter what the interpretation. To see that under $\mathbf{I}_1$, φ_1 is not metaphysically necessary, note that $\Box\varphi_1$ is not true under $\mathbf{I}_1$. Therefore, $\sim\Box\varphi_1$ is true under $\mathbf{I}_1$. That is, under $\mathbf{I}_1$, φ_1 is not metaphysically necessary.

Claim 4: Under $\mathbf{I}_1$, φ_1 is an analytic truth that is not necessary.

To see this, focus on what φ_1 says. If one considers just what the logical constants 'if-then', 'the', and 'there is' mean, it seems reasonable to suggest that φ_1 is true solely in virtue of the meanings of these words. Since in this interpretation, φ_1 is not necessary, we have an example of an analytic truth that is not necessary. If one just grants that the logical truths form a subclass of the analytic truths, then this claim follows immediately from the others.

It turns out that we need not have relied on the semantics of rigid descriptions to find sentences for which claims like the above hold. We could have used a simple modal predicate logic that has both nonrigid descriptions and an operator on formulas which, without regard for modal contexts, looks back to the base world to evaluate the truth of the formula. So, for example, in a modal logic with an actuality operator $\mathcal{A}$, we can still generate LTNNs and ATNNs. To verify this, suppose we added such an operator to our language by generating formulas of the form $\mathcal{A}\varphi$ in the clause for complex formulas. To interpret $\mathcal{A}$, we need the following clause for satisfaction of formulas of the form $\mathcal{A}\varphi$: $\mathbf{f}$ satisfies $\mathcal{A}\varphi$ at $\mathbf{w}$ iff $\mathbf{f}$ satisfies φ at $\mathbf{w}_0$.[5] This guarantees that $\mathcal{A}\varphi$ is true a world iff φ is true

[5]See J. Crossley and I. Humberstone, "The Logic of 'Actually'," *Reports on Mathematical Logic* VIII (1977) pp. 11–29.

at the actual world. Furthermore, let descriptions of the form $(\imath x)\psi$ be interpreted non-rigidly. Now consider the following formula:

(φ_2) $P(\imath x)\mathcal{A}Qx \rightarrow (\exists y)Qy$

This is to be read: If the actual Q-thing is P, then something is Q. Such a sentence is a logical truth, since at the actual world of every interpretation, the truth of the antecedent materially implies the truth of the consequent. But, under an interpretation like $\mathbf{I}_1$ (described above), φ_2 is not necessary. The reader should verify that claims similar to Claims 1 – 4 hold with respect to this conditional as well.

In fact, an even simpler example can be constructed in the modal *propositional* calculus with an actuality operator. Consider the following formula schema:

(φ_3) $\mathcal{A}\psi \rightarrow \psi$

Any instance of this schema in the modal propositional calculus will be logically true, since, again, in any interpretation, the antecedent materially implies the consequent. But, under an interpretation where ψ is true at $\mathbf{w}_0$ and false at some other world, such an instance of φ_3 is not necessary. The reader should verify that claims similar to Claims 1 – 4 above are true with respect to instances of φ_3.

So rigid descriptions are not the only means of distinguishing logical truth, analytic truth, and metaphysical necessity. One should beware of the inference from logical truth to metaphysical necessity even in languages without rigid descriptions, for there may be an (implicit) actuality operator somewhere that invalidates the move. It is surprising that, by merely adding expressive power that allows us to talk about what is actually the case, the modal propositional calculus involves distinctions that undermine the traditional view that logical truths are necessary.

II

If correct, these results call for a reassessment of the traditional understanding of the notion of logical truth and its relation to metaphysical necessity. There are, however, ways one might try to challenge the claims in the previous section. One way would be to argue that we have used the wrong definition of logical truth. Recently, an alternative to the definition of logical truth used in section I has found its way into the literature. It is a consequence of this alternative definition that $\varphi_1 - \varphi_3$ are not logical truths and, hence, not LTNNs. But, without begging any questions, it can be shown that this alternative definition of logical truth is incorrect.

Note, first of all, that our definition of logical truth is the standard Tarskian-Kripkean notion of truth-in-all-interpretations as it applies to our particular modal language. Logical truth is defined in terms of truth under an interpretation, and truth under an interpretation is defined as truth under an interpretation with respect to the actual world. This clearly squares with Kripke's original definition of validity. Recall that Kripke defined models φ that recursively assigned each formula a truth value relative to each world on a given model structure. Valid formulas are those which receive the assignment T at the actual world for every model on a model structure (Kripke [1963], p. 64; in Kripke's notation, a formula A is valid iff $\varphi(A, \mathbf{G})$ =T, for every model φ on a model structure ($\mathbf{G}$,$\mathbf{K}$,$\mathbf{R}$), where $\mathbf{G}$ is the real world, $\mathbf{K}$ is the set of all worlds, and $\mathbf{R}$ is the accessibility relation). The only real difference is that, for simplicity, we have ignored the accessibility relation.

Recently, certain modal and intensional logicians have deviated from Kripke's definitions, in a seemingly innocuous way, by dropping the distinguished world from models. Instead of the above series of definitions, they proceed directly from the definition of truth-under-an-interpretation-with-respect-to-a-world to the definition of logical truth, without the intermediate definition of truth under an interpretation. Theorists such as G. E. Hughes and M. J. Cresswell,[6] Richard Montague,[7] E. J. Lemmon and Dana Scott,[8] Brian Chellas,[9] D. Dowty, F. Wall, and S. Peters,[10] and J. F. K. van Benthem,[11] to mention a few, use a definition of "φ is true under $\mathbf{I}$ at world $\mathbf{w}$" similar to the one above, but then directly define "φ is logically true (valid)" as: $(\forall \mathbf{I})(\forall \mathbf{w})(\varphi$ is true under $\mathbf{I}$ at $\mathbf{w})$. These authors never say why they alter Kripke's definition. The reason may have stemmed from their natural zeal to avoid metaphysical questions while doing logic. By eliminating the actual world from the semantics of modal logic, one can avoid the question of what it is for a world to be actual, a question figuring prominently in the literature. Since the alternative definition seems to be equivalent to Kripke's, it therefore seems more attractive. Hughes and Cresswell say that the definition of validity as truth in every model in Kripke's sense is "precisely equivalent" to its definition as truth in every world in every model (351). The idea seems to be that the permutation of the base world with any other world in a Kripkean interpretation produces a perfectly good new Kripkean interpretation, and so, ultimately, every world in every such interpretation will play the role of the base world in some interpretation or other. So it seems that the alternative definition makes the arbitrariness of the choice of the base world explicit. Dowty, Wall, and Peters say as much in their recent text (*op. cit.*):

> Thus, the definitions themselves do not single out any particular world as the actual ones; we may alternatively choose one or the other of them as the "actual" world, all other worlds becoming possible but not actual worlds relative to this choice. (Alternatively, we could change the definition of a model to include the designation of one particular world as the actual one—in fact, Kripke's original treatment followed this procedure. But with his approach there will still be alternative models differing only in the choice of which world is so designated. Thus the definition of validity in our approach as truth in every world in every model is equivalent to the definition of validity in Kripke's approach as truth in every model) (127).

The problem is, however, that the two definitions are equivalent only with respect to modal and intensional languages of the kind these authors describe—languages that do not have rigid descriptions or actuality operators. To see that the alternative definition of validity is not always equivalent to Kripke's, consider our language and note that, on their definition, φ_1 fails to be a logical truth. It fails to be true in $\mathbf{I}_1$, for example, since an arbitrarily chosen assignment $\mathbf{f}$ satisfies $P(\imath x)Qx$ at world $\mathbf{w}_1$ (the object that is Q in the actual world is P in every world), but fails to satisfy $(\exists y)Qy$ at $\mathbf{w}_1$ (since nothing is in the extension of Q at $\mathbf{w}_1$). So there is an interpretation and world where φ_1 fails to be true, and this shows that φ_1 fails to be logically true on the alternative definition. Hence, the two definitions are not equivalent with respect to our

[6] *An Introduction to Modal Logic* (London: Methuen, 1974), pp. 71, 351.

[7] "Universal Grammar," pp. 222–246 in *Formal Philosophy*, Richmond Thomason, ed., (New Haven: Yale, 1970), p. 229.

[8] *An Introduction to Modal Logic*, American Philosophical Quarterly Monograph Series, Number 11 (Oxford: Basil Blackwell, 1977), p. 24.

[9] *Modal Logic: An Introduction* (New York: Cambridge, 1980), p. 4.

[10] *An Introduction to Montague Semantics* (Boston: D. Reidel, 1981), p. 127.

[11] *A Manual of Intensional Logic*, CSLI Lecture Notes (Number 1), Center for the Study of Language and Information, Stanford University, 1985, pp. 13/4.

language. None of the above authors acknowledge or hint that the two definitions are not equivalent for languages with rigid descriptions or actuality operators (but, then, none develop a language that contains such expressions). Nevertheless, a simple enrichment of the modal predicate calculus distinguishes the two definitions.

Which definition of logical truth is correct? If Kripke's is, our work in section I must be taken seriously; if the alternative is, it need not be. Is there an argument that shows the alternative definition to be correct, other than the question-begging argument which so concludes on the grounds that there are no LTNNs? Alternatively, would we be begging the question in the opposite direction if we were to claim that the alternative definition is a conflation of the formal notion of validity (truth-under-all-interpretations) with the metaphysical notion of necessity (truth-at-all-possible-worlds)? The answer to this last question is No!, and the reason is that there are independent grounds for claiming that the alternative definition is incorrect. Those grounds are just the following: (1) the most important semantic definition for a language is the definition of truth under an interpretation, and the alternative method, in which no world is distinguished as the actual world, has no means of defining this notion; and (2) the semantic notion of logical truth is properly defined in terms of the semantic notion of truth, and the alternative definition of logical truth is the wrong one because it fails to do this.

Once we take Tarksi's notion of truth seriously, it becomes immediately clear why Kripke distinguished one of the worlds in each model. The semantic notion of truth-under-an-interpretation for a modal language with Kripke-style semantics requires a distinguished world, *as a matter of logic*! That is, the very logical setup requires a distinguished world. And Tarski's insight is that logical truth is truth-under-all-interpretations. When we define a modal language, we need a definition of truth-under-an-interpretation if we are to define logical truth by applying Tarski's insight. Kripke's models permit the definition of logical truth to follow this pattern (the alternative models do not), and when we enrich modal languages interpreted by such models, by adding rigid descriptions or actuality operators, the definitions of truth and validity remain essentially the same. So we do not beg any questions when we argue that the alternative definition of logical truth is incorrect. It fails to take the notion of truth seriously. A second look at the alternative definition reveals it to be a conflation of the semantic notion of validity and the metaphysical notion of necessity.[12]

A second way to challenge our claims in section I would be to argue that the notions "the" and "actually," represented by the term-forming operator '$\imath$' and the sentence-forming operator '$\mathcal{A}$', respectively, are *non-logical* notions. If this could be established, then φ_1 – φ_3 would not be logical truths and, hence, not LTNNs. Ideally, the question of whether or not these notions are logical should be decided by appeal to a clear definition of "logical notion." Unfortunately, there is no definition that identifies all and only the genuine logical notions. Nor do we plan to try to produce such a definition in the present paper. Consequently, we cannot say with certainty that the notions represented by our operators are logical ones. Nevertheless, on the traditional understanding of what is logical and what is not, our operators and the notions they represent seem to be logical.

Part of the traditional conception of a logical operator is this: the logical operators of a language are the operators evaluated in the recursive clause of the definition of truth. Clearly, our actuality operator is one of these. In addition, in the definition of the language itself, the complex formulas that involve this operator are generated in the same clause that generates the other logically complex formulas. The traditional conception of logic also regards the definite description operator as logical. Traditionally, it has been defined in purely logical terms. Moreover, the

[12]This conclusion is much stronger than those in Martin Davies and Lloyd Humberstone, "Two Notions of Necessity," *Philosophical Studies* XXXVIII, 1 (July 1980): 1–30. These authors construct new technical distinctions based on what they see as just two different notions of validity—they call the alternative definition of validity "general" validity and Kripke's original notion of validity "real world" validity. So φ_1 – φ_3 are real-world valid, though not generally valid. They employ a new operator $\mathcal{F}$ ("fixedly"), where '$\mathcal{F}\varphi$' is true just in case φ is true in all interpretations that differ only with respect to which world is designated as actual. And, following Gareth Evans ["Reference and Contingency," *The Monist* LXII, 2 (April 1979): 161–189], they distinguish "deep" necessity ($\mathcal{F}\mathcal{A}\varphi$) from "superficial" necessity ($\Box\varphi$).

If one accepts the alternative definition of validity, then these distinctions can certainly be drawn, even if they are not supported by any direct evidence. But the notion of *general* validity does not seem to be a proper notion of logical truth, since it is not defined in terms of the notion of truth. It appears to conflate the notions of logical truth and necessity. Moreover, Davies and Humberstone do not make it clear that the most important semantic notion, namely, truth *simpliciter*, cannot be defined in modal languages without a world distinguished as actual. Consequently, there seems to be no reason to distinguish deep necessity and superficial necessity in order to try to preserve the intuition that logical truths are necessary. It seems much more realistic to admit that this intuition is false.

argument which concludes that the notions represented by our operators are nonlogical on the grounds that they are semantically interpreted by reference to the actual world, is not a good argument. We have just seen that as far as the semantics of modality goes, an actual world must be distinguished as a matter of logic. This is essential to the definition of truth. So the fact that our operators exploit this distinction does not imply that they are non-logical.

Consequently, the suggestion that our operators are nonlogical appears to conflict with the traditional understanding of what is logical. Of course, the traditional conception may be mistaken, but, if so, our claims in section I might then become part of the argument to show that the traditional conception rests on a mistake.

III

We have not found in the literature any examples of LTNNs and ATNNs simpler than those described in section I.[13] Even though Kripke stipulates that analytic truths are necessary, one might suspect that somewhere in his discussion in "Naming and Necessity," there are bound to be some simple LTNNs or ATNNs. He does not appear, however, to regard typical English descriptions as rigid designators. Consequently, his examples in which definite descriptions are used to fix the reference of certain terms could not be construed as LTNNs or ATNNs.[14] The other prominent examples of Kripke's lectures cannot be employed in the demonstration that there are LTNNs either. The examples we have in mind here are the following:

(1) The inventor of bifocals invented bifocals.

(2) The inventor of bifocals might not have invented bifocals.

[13]Davies and Humberstone in [1980] informally describe a variant of φ_2. D. Kaplan, in his unpublished monograph "Demonstratives" (1977, Draft #2), describes a variant of φ_3 in footnote 35.2 and in the handwritten addition to Remark 3 in Section XIX. "Demonstratives" is being published in *Themes From Kaplan*, J. Almog, J. Perry, and H. Wettstein (eds.), New York: Oxford University Press, 1989. In the published version, the variant of φ_3 appears in footnote 65 (p. 539), and in Remark 3 (p. 547).

[14]For example, by fixing the reference of 'Neptune' with the description 'the planet causing certain perturbations,' we do not get LTNNs by biconditionalizing "Neptune exists" with "Some unique planet causing certain perturbations exists," unless we replace 'Neptune' in the resulting biconditional with a rigid description; see Kripke, "Naming and Necessity," pp. 347/8, note 33.

(3) The teacher of Alexander taught Alexander.

(4) The teacher of Alexander might not have taught Alexander.

Let us assume that the definite descriptions that constitute the subject terms of these sentences are genuine, ineliminable logical units. For the moment, let us not be concerned with the Russellian analyses in which the descriptions are eliminated, since these do not do justice to their apparent subject-predicate form. The logical form of (1) and (3) seems to be captured by (5); while the form of (2) and (4) is captured by (6):

(5) $P(\imath x)Px$

(6) $[\lambda y \, \Diamond \sim Py](\imath x)Px$

We may read 'Px' as either "x invented bifocals" or "x taught Alexander." Now if (1) and (3) are analyzed as (5), then they seem to be true. If (2) and (4) are analyzed as (6) (where the description is still rigid), then they also seem to be true. Moreover, one might think that (1) and (3) are LTNNs. For it looks as if what (5) says is logically true, and it looks as if we can use λ-conversion on (6) to show that (5) is not necessary (λ-conversion is a valid rule when used with rigid descriptions).

But this is a mistake. Sentences like (5) are not logically true, since there are many interpretations in which the description fails to denote. And when it does, such sentences are not true, by the definition of satisfaction. Consequently, (1) and (3) are not LTNNs, since they are not examples of logical truths. Nor are they ATNNs. They are not analytic, because their truth does not depend solely on the meanings of their words and their arrangement (their truth depends on the fact that something satisfies the description). So these examples cannot be used to establish the claims in this paper. One should note that (2) and (4) are English sentences which are most plausibly analyzed by construing the descriptions as rigid.[15] A nonrigid description would not help us capture the truth expressed by (2). Nor would these examples be of any help if we were to use the Russellian elimination of the descriptions. It is a straightforward exercise to show that none of the various Russellian readings of (1) – (4) are either LTNNs or ATNNs.

[15]See Baruch Brody, "Kripke on Proper Names," pp. 75–80 in *Contemporary Perspectives in the Philosophy of Language*, French, Uehling, and Wettstein, eds. (Minneapolis: Minnesota UP, 1979), p. 79.

There are in the literature some extraordinary examples of LTNNs, but these do not identify the point at which the distinction between logical truth and necessity first evolves. In his work on demonstratives, David Kaplan offers "I am here now"[16] and "I exist"[17] The idea is that these sentences, under any possible interpretation and in any context of utterance, will always be assigned as content a proposition which is in fact true but which might have been false. Thus, we have examples of LTNN's. There are, however, several reasons why these examples do not cut to the heart of the distinctions we are after.

For one thing, Kaplan's examples require a much more powerful logical apparatus than simple (quantified) modal logic. They involve the indexical 'I' and are expressed in the language that forms the basis of the logic of demonstratives. This logic involves indexicals, demonstratives, tense operators, contexts of utterance, agents, positions, times, and the distinction between character and content. Kaplan even suggests that the distinction between character and content is the key to distinguishing logical truth and necessary truth:

> The bearers of logical truth and of contingency are different entities. It is the *character* [his emphasis] (or, the sentence, if you prefer) that is logically true, producing a true content in every context. But it is the content (the proposition, if you will) that is contingent or necessary ("Demonstratives," p. 72).

There is something to what Kaplan says here, but we have now seen examples of this phenomenon in a logic that is far simpler than the logic of demonstratives. The distinction between character and content, and the rest of the apparatus involved in the logic of demonstratives, is not required to construct the definitions that are essential for separating the logically true from the necessary.

There are two other reasons why Kaplan's examples do not illuminate the basic insights about the relevant notions. One is that, if the examples are to work, we must accept a proper thesis of metaphysics. The required thesis is: either there are no nonexistent objects or, if there are,

[16] "On the Logic of Demonstratives," *Journal of Philosophical Logic* VIII, 1 (February 1978): 81–98; also published in French, Uehling, and Wettstein, *op. cit.*, pp. 401–412.

[17] "Demonstratives," p. 73 in the manuscript, Draft # 2; p. 540 in *Themes From Kaplan*, *op. cit.*

they cannot be the agents of real utterances. For if there are nonexistent objects in the domain of individuals and they are allowed to be the agents of utterances, then there will be interpretations and contexts in which the indexical 'I' denotes a nonexistent object. In such interpretations and contexts, the sentences "I am here now" and "I exist" will be false. Hence, they will not be logical truths. Recent work in philosophy reveals that the thesis that there are nonexistent objects is viable.[18] The present point, however, does not depend on one's inclination (not) to believe in nonexistent objects or on the natural hypothesis that they cannot be the agents of ordinary contexts of utterance (they can be agents in fictional contexts). The point simply is that proper metaphysical theses are required for these examples to be LTNNs. Our LTNNs do not require such nonlogical theses.

One final reservation concerning these examples is that it is unclear whether they are ATNNs. Kaplan asserts that one need only understand the meaning of "I am here now" to know that it cannot be uttered falsely.[19] Even if one agrees with Kaplan about this, it is unclear whether this still counts as an analytic truth, since contexts of utterance are now involved. That is, this is an example of an analytic truth only if one uses a somewhat expanded notion of analyticity. We cannot consider the truth of the sentence without appealing to some context, and so we cannot simply say that it has the property that traditional analytic truths have, namely, being true in virtue of the meanings of its words. Rather, it has the property of being true in all contexts in virtue of the meanings of its words relative to such contexts. Although the sentence seems to qualify as analytic in this expanded sense, the examples we have described are analytic in the stricter, traditional sense.

But Kaplan has another example that is more instructive than the previous two. It is expressed with his operator '*dthat*', which turns any term that it prefaces into a rigid designator.[20] In Kaplan's system, a definite description such as '$(\iota x)\varphi$' is not rigid, but '$dthat[(\iota x)\varphi]$' is (we assume some familiarity with the '*dthat*' operator). The example Kaplan offers is: $\tau = dthat[\tau]$, where τ is a definite description ("Demonstratives," p. 71). The idea here is that no matter what interpretation and assignment to

[18] See Terence Parsons, "A Prolegomena to Meinongian Semantics," this JOURNAL LXXI, 16 (Sept. 19, 1974): 561–580; *Nonexistent Objects* (New Haven: Yale, 1980); and Zalta, *op. cit.*

[19] "On the Logic of Demonstratives," *op. cit.*, journal version, p. 82.

[20] See "Dthat," in French, Uehling, and Wettstein, *op. cit.*, pp. 383–400.

the variables one considers, the assignment satisfies this sentence with respect to the actual world because the denotations of the two descriptions are identical at that world. The sentence is not necessary, however, because there will be worlds in which the denotations of the descriptions differ. Although this example is also expressed in the language underlying the logic of demonstratives, only a very small portion of the language and logic of demonstratives is required for its analysis. Even if we resect, from the logic of demonstratives, just the apparatus required for the analysis, our examples in section I get closer to the crux of the distinction between logical truth and necessary truth.

There are two reasons for this. The first is that they do not require a modal logic with identity formulas, and so do not require that we face, as Kaplan must, the host of vexing questions about the interactions of identities and modal contexts. The second, and more important, reason is that our examples do not require any special treatment of nondenoting descriptions. Kaplan's sentence works as an example of a logical truth that is not necessary only because he has an artificial way of treating descriptions that no object satisfies. Here is why.

In Kaplan's system, a description that no object satisfies is assigned an arbitrary, "alien" entity, and so descriptions never fail to denote. Without this device, there would be numerous interpretations in which both $(\iota x)\varphi$ and $dthat[(\iota x)\varphi]$ fail to denote anything whatsoever. In such interpretations, Kaplan's sentence would be false, for his truth definition, like ours, is constructed in accordance with the correspondence theory of truth. It guarantees that atomic and identity sentences are true iff both all of their terms denote and the denotations stand in the right relationship. Hence, such sentences would not be logical truths. Kaplan's technique of having descriptions that no object satisfies denote the alien entity does preserve his sentence as a logical truth—when nothing satisfies $(\iota x)\varphi$, $(\iota x)\varphi = dthat[(\iota x)\varphi]$ is true, because both descriptions denote the alien entity. But this piece of logician's art undermines whatever insight this example might have offered those trying to track down the point at which logical truth and necessity first diverge. Nothing in the logic of demonstratives is needed for identifying LTNN's.

IV

The traditional concepts of logical truth and metaphysical necessity are therefore independent, though not mutually exclusive. Identity statements involving rigid names and descriptions are examples of necessary truths that are not logically true, and the axioms of propositional logic are examples that are both logically true and necessarily true. Since the logical truths form a subclass of the analytic truths, it follows that the concepts of analytic truth and necessity are also independent, though not mutually exclusive. The same examples suffice to show this. One preliminary conclusion we can draw from the foregoing is that there is at least one sense of "logical" for which the locutions "logically possible" and "logically necessary" represent confusions of the formal and material modes. For the statement that φ is logically possible, where φ is some English sentence, may represent a conflation of the formal statement that φ is satisfiable (true in some interpretation) with the material statement that φ, under the interpretation we know it to have, is true in some metaphysically possible world. Typically, when philosophers say that φ is logically possible (and one does not go far in philosophy before encountering such statements), they mean the latter. But, if so, we have seen that this kind of possibility is independent of logic.

Our formal work clearly shows that there are contingent truths known *a priori*, regardless of whether anyone fixes the reference of proper names. The formulas φ_1, φ_2, and φ_3 are all examples of this. Since they are logical truths, they are all true and, hence, possible. Since none are necessary, their negations are possible as well. Given that they are possible and that their negations are possible, they must be contingent. Since they are analytic truths, they can be known *a priori*. The claim that there are ATNNs, then, is stronger than Kripke's claim that there are contingent truths known *a priori*, for the former implies the latter, but not vice versa.

We also ought to be aware that the definition of a valid argument as an argument in which it is impossible that the premises be true and the conclusion false, something we all learn/teach in a first philosophy class, is incorrect. The argument that has the antecedent of φ_2 as a premise and the consequent of φ_2 as a conclusion is surely valid. Yet it is one where it is metaphysically possible for the premise to be true and the conclusion to be false. It is always good to know the exact place where we have bent the truth a little to simplify a concept.

Finally, it may still come as a surprise that there are logical falsehoods that are metaphysically possible. The negations of φ_1, φ_2, and φ_3 are examples of this. Since φ_1, φ_2, and φ_3 are logically true, their negations are logically false. Since φ_1, φ_2, and φ_3 are not necessary, their negations are metaphysically possible. Hence, their negations are logical falsehoods that are metaphysically possible. Contemplation of these examples should rid us of the automatic tendency to regard the self-contradictory quality a modal statement may have as the justification for why it could not be the case.[21]

Appendix

We provide the reader with an example to show that the definitions of denotation and satisfaction work correctly. For simplicity, we ignore the relativization to possible worlds. Suppose $\mathbf{f}$ satisfies $P(\iota x)Qx$. Then $(\exists\mathbf{o})(\mathbf{o} = \mathbf{d}_{\mathbf{I},\mathbf{f}}((\iota x)Qx)\ \&\ \mathbf{o} \in \mathbf{F}(P))$. Call an arbitrary such $\mathbf{o}$, "$\mathbf{o}_1$." So $\mathbf{o}_1 = \mathbf{d}_{\mathbf{I},\mathbf{f}}((\iota x)Qx)\ \&\ \mathbf{o}_1 \in \mathbf{F}(P)$. By the definition of denotation, the left conjunct is equivalent to: $(\exists!\mathbf{f}')(\mathbf{f}' \stackrel{x}{=} \mathbf{f}\ \&\ \mathbf{f}'(x) = \mathbf{o}_1\ \&\ \mathbf{f}'$ satisfies $Qx)$. Call the unique such $\mathbf{f}$, $\mathbf{f}_1$. We now have, $\mathbf{f}_1 = \mathbf{f}\ \&\ \mathbf{f}_1(x) = \mathbf{o}_1\ \&\ \mathbf{f}_1$ satisfies $Qx\ \&\ \mathbf{o}_1 \in \mathbf{F}(P)$. By the definition of satisfaction, the third conjunct is equivalent to: $(\exists\mathbf{o}')(\mathbf{o}' = \mathbf{d}_{\mathbf{I},\mathbf{f}_1}(x)\ \&\ \mathbf{o}' \in \mathbf{F}(Q))$. If we call such an $\mathbf{o}'$, "$\mathbf{o}_2$," we know: $\mathbf{o}_2 = \mathbf{d}_{\mathbf{I},\mathbf{f}_1}(x)\ \&\ \mathbf{o}_2 \in \mathbf{F}(Q)$. But since the denotation of a variable is just what the assignment assigns to it, we have: $\mathbf{o}_2 = \mathbf{o}_1$. Therefore, the third conjunct becomes equivalent to: $\mathbf{o}_1 \in \mathbf{F}(Q)$. Reassembling our four part conjunction, we get: $\mathbf{f}_1 \stackrel{x}{=} \mathbf{f}\ \&\ \mathbf{f}_1(x) = \mathbf{o}_1\ \&\ \mathbf{o}_1 \in \mathbf{F}(Q)\ \&\ \mathbf{o}_1 \in \mathbf{F}(P)$. In other words, $\mathbf{f}$ satisfies $P(\iota x)Qx$ iff $(\exists!\mathbf{f}')(\exists\mathbf{o})(\mathbf{f}' \stackrel{x}{=} \mathbf{f}\ \&\ \mathbf{f}'(x) = \mathbf{o}\ \&\ \mathbf{o} \in \mathbf{F}(Q)\ \&\ \mathbf{o} \in \mathbf{F}(P))$. These conditions are the correct satisfaction conditions, and they are ones in which the notions of denotation and satisfaction do not appear.

[21] Subsequent to the submission of this paper, Joseph Almog's paper, "Naming Without Necessity" was published in this JOURNAL, LXXXIII, 4 (April 1986): 210–242. In the last section of the paper, Almog identifies several contingent propositions that are "structurally true." It is unclear what the relationship is between structural truth and logical truth, since the notion of a structural truth is not defined. But, more importantly, Almog's examples count as contingent structural truths only because he has adopted essentialist and actualist metaphysical assumptions. Examples (3), (4), and (5) (240/1) require essentialist assumptions to be contingent, and example (6) requires actualist assumptions to be structurally true. In contrast to this, our examples require no proper thesis of metaphysics to count as LTNNs and ATNNs—they cut directly to the origin of the distinctions among logical, analytic, and necessary truth.