

THE *TRACTATUS* ON LOGICAL CONSEQUENCE*

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ABSTRACT: I discuss the account of logical consequence advanced in Wittgenstein's *Tractatus*. I argue that the role that elementary propositions are meant to play in this account can be used to explain two remarkable features that Wittgenstein ascribes to them: that they are logically independent from one another and that their components refer to simple objects. I end with a proposal as to how to understand Wittgenstein's claim that all propositions can be analysed as truth functions of elementary propositions.

1. Introduction

One of the most distinctive doctrines of the *Tractatus* (Wittgenstein 1974) is an account of logical consequence exclusively in terms of the resources of truth-functional propositional logic. The account is clearly expressed in proposition 5.11:

If all the truth-grounds that are common to a number of propositions are at the same time truth-grounds of a certain proposition, then we say that the truth of that proposition follows from the truth of the others.

In spite of the conditional form of the sentence, the context makes it clear that it is meant as an equivalence: the truth of a proposition follows from the truth of others if *and only if* the truth-grounds that are common to the latter are also truth-grounds of the former.

The background to this account of logical consequence is well known. Wittgenstein has told us earlier (5) that all propositions are truth functions of elementary propositions, i.e. that every truth-value assignment to the elementary propositions yields a unique truth-value for every proposition. And he has defined the truth-grounds of a proposition as the combinations of truth values for elementary propositions that make the proposition true (5.101). Hence 5.11 can be rephrased as the following account of logical consequence:

A proposition p is a *logical consequence* of a set Γ of propositions just in case no truth-value assignment to the elementary propositions makes every element of Γ true and p false.

This account of logical consequence hasn't received as much attention from commentators as other Tractarian doctrines. One reason for this is that the account seems perfectly clear—it doesn't seem to require substantial exegetical work. Another reason is that the account seems clearly wrong. Of course, truth-functional composition explains many instances of logical consequence along the lines that Wittgenstein contemplates. But there are lots of other cases of a proposition following from other propositions that seem best construed in a different way—in terms of how propositions are put together from common sub-propositional components.

I think that this lack of attention has made us overlook important connections between the Tractarian account of logical consequence and other central doctrines of the book. I think, in particular, that the account can be used to provide an appealing explanation of some of the most challenging features of elementary propositions.

In this paper I want to do three things. First I'd like to make a few remarks about the nature of the Tractarian account of logical consequence. Then I want to explain how it can be used to explain two central features of elementary propositions: their logical independence and the fact that the referents of their components are simple objects. Finally I'd like to make a proposal as to how to understand Wittgenstein's commitment to the existence of elementary propositions and to the analysability of every proposition as a truth-function of these. I will model my reading on Hidé Ishiguro's account of the Tractarian notion of reference.

2. Logical possibility and combinatorial possibility

The claim that a proposition p is a logical consequence of a set of propositions Γ is the claim that a certain combination of truth values—true for the elements of Γ and false for p —is logically impossible. Hence an account of logical consequence is an account of this notion of logical impossibility for truth-value combinations—of what makes it the case that certain combinations of truth values for propositions are impossible as a matter of logic.

Let p , q and r be three propositions, and suppose that the shaded rows in the truth-table below correspond to logically impossible truth-value combinations:

p	q	r
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

Table 1

Contemporary logic recognises several approaches to the task of accounting for the logical impossibility of these truth-value combinations. But according to the *Tractatus*, the approach associated with truth-functional propositional logic suffices on its own for a complete account of the phenomenon.

I want to suggest that the central idea of the truth-functional account of logical impossibility is to reduce logical possibility to combinatorial possibility. In our example, the idea is to pair truth-value combinations for p , q and r with truth-value combinations for some other propositions in such a way that logically impossible truth-value combinations for p , q and r don't get paired with any truth-value combination for these other propositions.

Let's consider how this approach would work for our example. Suppose that there are two propositions, α and β , satisfying the following conditions:

- a. It is logically impossible for p and β to have the same truth value (i.e., as a matter of logical necessity, p is true if and only if β is false).
- b. It is logically impossible for q to be false if α is true or β is true, and it is logically impossible for q to be true if both α and β are false (i.e. as a matter of logical necessity, q is true if and only if either α or β is true).
- c. It is logically impossible for α and r to have different truth values (i.e. as a matter of logical necessity, r and α have the same truth value).

These three facts will yield a unique truth-value combination for p , q and r for every truth-value combination for α and β , as expressed by the next table:

α	β	p	q	r
T	T	F	T	T
T	F	T	T	T
F	T	F	T	F
F	F	T	F	F

Table 2

And this circumstance can now be used to explain the logical impossibility of the shaded rows in Table 1. They are logically impossible because they don't correspond to any of the combinatorially possible truth-value combinations for α and β . We can highlight this fact if we enter each truth-value combination for α and β against the corresponding truth-value combination for p , q and r in Table 1:

α	β	p	q	r
T	F	T	T	T
		T	T	F
		T	F	T
F	F	T	F	F
T	T	F	T	T
F	T	F	T	F
		F	F	T
		F	F	F

I am suggesting that according to the *Tractatus* this strategy provides a universal treatment of logical consequence and the corresponding notion of logically impossible truth-value

combinations. The claim that I am attributing to the *Tractatus* can be formulated in the following terms:

- (1) For every set Γ of propositions, there is a set Δ of propositions and a function f extending every truth-value assignment on Δ to a truth-value assignment on $\Delta \cup \Gamma$, such that for every truth-value assignment v on Γ , v is logically possible just in case there is a truth-value assignment w on Δ with $w \cup v$ as its image under f .

In our example, Γ is $\{p, q, r\}$, Δ is $\{\alpha, \beta\}$, and f is the function represented by Table 2. (1) tells us is that we will always be able to explain the fact that certain combinations of truth values for a set of propositions are logically impossible as arising from the fact that these combinations don't correspond to any of the combinatorially possible truth-value combinations of some other set of propositions.

(1) may seem to be weaker than what is required for the account of logical consequence that the *Tractatus* puts forward at 5.11. For (1) allows Δ and f to be different for each Γ , whereas according to the *Tractatus* there is a set of propositions—the elementary propositions—and a function pairing truth-value assignments on these with universal truth-value assignments that can do the job for every Γ . But what 5.11 calls for is not stronger than (1). (1) has as an instance the case in which Γ is the set of all propositions. Hence (1) is equivalent to:

- (2) There is a set Δ of propositions and a function f extending every truth-value assignment on Δ to a universal truth-value assignment, such that for every universal truth-value assignment v , v is logically possible just in case there is a truth-value assignment on Δ with v as its image under f .

According to (2), we will be able to explain the fact that any given combination of truth values for propositions is logically impossible as arising from the fact that these combinations don't correspond to any of the combinatorially possible truth-value combinations for a single fixed set of propositions.

And (2) entails the claim that sustains the account at 5.11:

- (3) There is a set Δ of propositions and a function f extending every truth-value assignment on Δ to a universal truth-value assignment, such that for every set Γ of propositions, a truth-value assignment v on Γ is logically possible just in case there is a truth-value assignment on Δ whose image under f includes v .

Δ in (3) is a set in terms of which all the logically impossible truth-value combinations can be explained along the lines we are considering. This existential claim is satisfied, according to the *Tractatus*, by the set of elementary propositions and by the function that represents every proposition as a truth function of elementary propositions. I am going to refer to (3) as the *Fundamental Principle* of Tractarian logic.

Now, there is one respect in which the account of logical impossibility that we are considering cannot be fully universal. The point can be easily appreciated in terms of our illustration. We have explained the logical-impossibility facts represented by Table 1, but we have done so with the help of other logical-impossibility facts—those expressed by a, b and c. This is unavoidable—to reduce logical possibility to combinatorial possibility we need to invoke logical-impossibility facts to underwrite the reduction.

The logical-impossibility facts that play this role in (3) are the facts represented by f —facts to the effect that, for every proposition p , it is logically impossible for the elementary propositions to have the combination of truth values that they receive from a truth-value

assignment v while p doesn't have the truth value that it would receive from the image of v under f . These logical-impossibility facts are presupposed by (3), and not explained by it.

I think that Wittgenstein was perfectly aware of the need to provide a separate account of these basic logical-impossibility facts. His approach to this issue was to conceive of these logical-impossibility facts as arising from the identity of molecular propositions. p and $\neg p$ are not just two independent propositions calling for an explanation of the fact that it's logically impossible for them to have the same truth value. $\neg p$ is essentially the proposition that is true when p is false, and false when p is true. The logical impossibility of p and $\neg p$ having the same truth value is explained by the identity of $\neg p$ and the internal relation that it bears to p as a result.

This is, I think, the point that is being made by 5.2341:

The sense of a truth-function of p is a function of the sense of p .

A similar remark in the 'Notes on Logic' is glossed as follows:

In *not-p*, p is exactly the same as if it stands alone; this point is absolutely fundamental. (Wittgenstein 1979: 95)

And the same goes for every molecular proposition: its identity is exhausted by the contrast between the combinations of truth values for elementary propositions that would make them true and those that would make them false—in other words, by how its truth value is made to depend on the truth values of elementary propositions by f . What makes it logically impossible for a proposition to have a truth value that is at odds with what f dictates is that a proposition with such a truth value wouldn't be the same proposition.

3. Logical atomism

In the *Tractatus*, the account of logical consequence comes after the metaphysics and the semantics. The basic units of the metaphysics of the *Tractatus* are states of affairs—

combinations of simple objects that are logically independent of each other. And the central idea of Tractarian semantics is that language makes contact with the world through a correlation between states of affairs and elementary propositions, whose constituents are correlated with the constituents of the states of affairs. Every proposition can be analysed in terms of these.

So by the time elementary propositions are assigned the task of explaining logical consequence, we have already been told that they have to be logically independent of each other and that their constituents have to refer to simple objects. The resulting combination of demands on elementary propositions can be expressed with the following claim:

- (4) There is a set Δ of propositions and a function f extending every truth-value assignment on Δ to a universal truth-value assignment, such that:
- A. The elements of Δ are logically independent of each other.
 - B. The constituents of the elements of Δ refer to simple items.
 - C. For every set Γ of propositions, a truth-value assignment v on Γ is logically possible just in case there is a truth-value assignment on Δ whose image under f includes v .

Finding support in the *Tractatus* for the claim that elementary propositions satisfy conditions A and B is one of the most daunting tasks faced by interpreters of the book—a task that, in my view, is still outstanding. Here I want to make a proposal as to how to discharge this task. I want to argue that conditions A and B could be motivated in terms of condition C. I want to argue that, subject to some ancillary Tractarian assumptions, a set Δ and a function f won't satisfy condition C unless Δ satisfies conditions A and B. Subject to these assumptions, (4) is

not stronger than (3). The basic tenets of the logical atomism of the *Tractatus* follow from its account of logical consequence.

4. Logical independence

Let me consider first the logical independence of elementary propositions. It is unquestionable that the *Tractatus* is committed to this claim.¹ It is also obvious that Wittgenstein attached great importance to it, as the realisation that elementary propositions couldn't satisfy it in general appears to have been the main catalyst for the disintegration of the Tractarian system.²

But why did Wittgenstein think that elementary propositions had to be logically independent of each other, and why did he attach so much importance to this issue? These questions can be immediately answered if we consider the role that elementary propositions have to play in the explanation of logical consequence. A set Δ and a function f won't satisfy the Fundamental Principle unless the elements of Δ are logically independent of each other.

To see this, notice that for the elements of Δ not to be logically independent means that some truth-value assignments on Δ are logically impossible. Let v^* be such an assignment. (3) entails as a special case that a truth-value assignment v on Δ is logically possible just in case there is a truth-value assignment on Δ whose image under f includes v . But v^* is a counterexample to this—a logically impossible truth-value assignment included in the image under f of a truth-value assignment on Δ , namely v^* itself.

What this argument shows is that logical possibility for truth-value combinations of arbitrary sets of propositions can be reduced to combinatorial possibility for truth-value combinations of elementary propositions only if elementary propositions are logically independent. Hence Wittgenstein's commitment to the reduction of logical possibility to

combinatorial possibility for elementary propositions would suffice to explain his commitment to the logical independence of elementary propositions.³

5. Simples

Let me turn now to the claim that the constituents of elementary propositions refer to simple objects.⁴ I want to argue that, in the presence of an assumption to which the *Tractatus* is independently committed, a set Δ and a function f won't satisfy the Fundamental Principle unless the constituents of the elements of Δ refer to simple objects.

The additional assumption that we need is the claim that there are logical links between propositions about complexes and propositions about the constituents of these complexes.⁵

Wittgenstein's commitment to this principle is, I think, uncontroversial. It is clearly expressed at 3.24:

A proposition about a complex stands in an internal relation to a proposition about a constituent of the complex.

Earlier on he gives a slightly more detailed account of the relation he has in mind:

2.0201 Every statement about complexes can be resolved into a statement about their constituents and into the propositions that describe the complexes completely.

And in the *Notebooks* we get an illustration of the kind of connection that Wittgenstein has in mind:

$\phi(a). \phi(b). aRb = \text{Def } \phi[aRb]$ (Wittgenstein 1979: 4)

This is the conception of statements about complexes that is mocked in §60 of the

Philosophical Investigations:

[...] does someone who says that the broom is in the corner really mean: the broomstick is there, and so is the brush, and the broomstick is fixed to the brush? (Wittgenstein 2001)

Some passages in the *Notebooks* clearly highlight the importance that Wittgenstein attaches to the logical nature of the link:

But suppose that a simple name denotes an infinitely complex object? For example, perhaps we assert of a patch in our visual field that it is to the right of a line, and we assume that every patch in our visual field is infinitely complex. Then if we say that a point in that patch is to the right of the line, this proposition follows from the previous one, and if there are infinitely many points in the patch *then infinitely many propositions of different content follow LOGICALLY from that first one.* (Wittgenstein 1979: 64)⁶

These passages seem to presuppose that a statement about a complex is logically linked with statements that assert about the constituents of the complex what the statement about the complex asserts about it—e.g., $\phi(x)$, that it is in the corner, or that it is to the right of the line. This aspect of the approach is much more plausible for the examples that Wittgenstein uses than for other cases, and it doesn't play any role in the argument that we are considering. I propose to work with a version of the thought that is not committed to this:

- (5) If $\phi(C)$ is a proposition about a complex C , and c_1, \dots, c_n are the constituents of C , there are propositions $\psi_1(c_1), \dots, \psi_n(c_n)$, each about a constituent of C , and a proposition $\gamma(c_1, \dots, c_n)$ about all the constituents of C , such that $\phi(C)$ is logically equivalent to the conjunction of these $n+1$ propositions.

My claim is that from (5) and (3) we can mount a powerful argument for the existence of simple objects. The argument focuses on the logical impossibilities entailed by (5). (5) dictates that it is logically impossible for $\phi(C)$ to be false if $\psi_1(c_1), \dots, \psi_n(c_n)$, and $\gamma(c_1, \dots, c_n)$ are all true and that it is logically impossible for $\phi(C)$ to be true if $\psi_1(c_1), \dots, \psi_n(c_n)$, and $\gamma(c_1, \dots, c_n)$ are not all true. According to the Fundamental Principle, these logical-impossibility facts have to be reduced to combinatorial impossibility. They have to be construed as arising from the fact that no truth-value assignment on the elementary

propositions has an image under f including any of the assignments to $\phi(C)$, $\psi_1(c_1), \dots, \psi_n(c_n)$ and $\gamma(c_1, \dots, c_n)$ deemed logically impossible by (5). And this cannot be done if $\phi(C)$ is an elementary proposition.

We can present the argument as a reductio:

- i. Assume, towards a contradiction that there are no simple objects.
- ii. Then the elements of Δ are about complexes.
- iii. Let $\phi(C)$ be an element of Δ about a complex C , let c_1, \dots, c_n be the constituents of C . There are propositions $\psi_1(c_1), \dots, \psi_n(c_n)$, $\gamma(c_1, \dots, c_n)$ such that it is it is logically impossible for $\phi(C)$ to be false if $\psi_1(c_1), \dots, \psi_n(c_n)$, and $\gamma(c_1, \dots, c_n)$ are all true and it is logically impossible for $\phi(C)$ to be true if $\psi_1(c_1), \dots, \psi_n(c_n)$, and $\gamma(c_1, \dots, c_n)$ are not all true. (from 5)
- iv. Every truth-value assignment that is the image under f of a truth-value assignment on Δ yields the value True for $\phi(C)$ just in case it yields the value True for all of $\psi_1(c_1), \dots, \psi_n(c_n)$, and $\gamma(c_1, \dots, c_n)$ (from 3 and iii).
- v. But iv is incompatible with the hypothesis that $\phi(C)$ is an element of Δ .
- vi. Therefore there are simple objects.

If this argument is correct, then the metaphysical claim that there are simple objects, and the semantic claim that language makes contact with reality in propositions that refer to simples, can be seen as necessitated by the demands of the Tractarian account of logical consequence, together with the commitment to the logical nature of links between propositions about complexes and propositions about their components.

6. A gap in the argument?

The argument for simples that I have presented in the previous section rests on the assumption (v) that the logical impossibilities entailed by (5) cannot be explained truth-functionally if propositions about complexes are elementary. What the argument assumes is that the logical connections between a proposition about a complex and propositions about its constituents can be represented truth-functionally only by treating the proposition about the complex as a truth-function of the propositions about the constituents of the complex. I think that this is in fact assumed without argument in the *Tractatus*, and the thought is perhaps too obvious to need an argument in its support. Nevertheless, it is interesting to consider why it holds—why if a proposition about a complex were elementary, we wouldn't be able to represent truth-functionally the logical links postulated by (5).

Suppose, then, that $\phi(C)$ is elementary. Then none of $\psi_1(c_1), \dots, \psi_n(c_n)$, and $\gamma(c_1, \dots, c_n)$ can be elementary, since they are not logically independent of $\phi(C)$. Hence we need to consider how f makes the truth value of $\psi_1(c_1), \dots, \psi_n(c_n)$, and $\gamma(c_1, \dots, c_n)$ depend on the truth values of elementary propositions. Clearly, if a truth-value assignment on Δ yields the value True for $\phi(C)$, its image under f will have to yield the value True for all of $\psi_1(c_1), \dots, \psi_n(c_n)$, and $\gamma(c_1, \dots, c_n)$. And every other truth-value assignment on Δ will have an image under f that yields the value False for at least one of $\psi_1(c_1), \dots, \psi_n(c_n)$, and $\gamma(c_1, \dots, c_n)$. But each of these propositions will have to receive the value True from the images under f of some truth-value assignments on Δ that yield the value False for $\phi(C)$. So among the truth-value assignments on Δ that yield the value False for $\phi(C)$, some will have images under f that yield the value True for $\psi_i(c_i)$ (for any i no greater than n), and some will have images that yield the value False for this proposition.

We can see the problem in terms of what an elementary assignment v would have to be like in order for its image under f to yield the value true for $\psi_i(c_i)$. $v(\phi(C)) = T$ is a sufficient but unnecessary condition for this. What would be required is a collection of other sufficient and, together with $v(\phi(C)) = T$, disjunctively necessary conditions on v for its image under f to make $\psi_i(c_i)$ true. The reason why premise v is so compelling is that it is hard to see how these individually sufficient and disjunctively necessary conditions could be provided. A formal argument for this conclusion cannot be expected, since we don't know which other propositions, besides $\phi(C)$, will be elementary. What we do know is that they will have to be logically independent of $\phi(C)$, and this rules out both $\psi_i(c_i)$ and any propositions about constituents of c_i with which $\psi_i(c_i)$ might be logically linked.

An example might make the challenge more vivid. Suppose that the proposition that the broom is in the corner were elementary. The truth of the proposition that the broom is in the corner is a sufficient but unnecessary condition for the truth of the proposition that the broomstick is in the corner. The challenge is to find the remaining individually sufficient and disjunctively necessary conditions for the truth of the proposition that the broomstick is in the corner in terms of the truth values of elementary propositions—keeping in mind that every elementary proposition will have to be logically independent of the proposition that the broom is in the corner. Other sufficient conditions might include propositions of the form *x is in the corner*, where x is a complex with the broomstick as a constituent. But these won't be disjunctively necessary for the truth of the proposition that the broomstick is in the corner, since, presumably, the broomstick can be in the corner by itself—it's certainly not impossible *as a matter of logic* that the broomstick is there by itself. If the broomstick is a complex, then, by (5), the proposition that the broomstick is in the corner will be logically equivalent to a conjunction of propositions about the constituents of the broomstick. But each of these propositions will be a logical consequence of the proposition that the broom is in the corner.

Hence they won't be among the elementary propositions, and any propositions that figure further down the ensuing chain of analysis will be similarly ruled out as elementary.

7. The existential claims of the Fundamental Principle

As the Fundamental Principle makes clear, the Tractarian account of logical consequence carries an extraordinary existential commitment. For the account to work, there has to be a set of propositions such that every other proposition is a truth-function of propositions in this set in such a way as to reduce precisely the logically possible combinations to combinatorially possible combinations for this set. And this requires, as we have seen, that the propositions in the set are logically independent of each other, and, if (5) is right, that they are about simple objects.

This commitment raises at least three urgent questions. The first is—how could Wittgenstein be so certain that these propositions exist? To be sure, insofar as the account of logical consequence is independently motivated, it should lend support to the existential claim. But the connection works both ways, and the apparent implausibility of the existential claim should raise suspicion concerning the account of logical consequence.

The second question concerns Wittgenstein's confidence that the logical relations between everyday propositions that result from their relationship to elementary propositions will be largely in line with our intuitions as to what follows from what. If logical relations between everyday propositions arise from as yet unknown links to as yet unknown elementary propositions, it seems that we should be open to the discovery of unsuspected logical links or of the contingent nature of links that we regarded as logical. But such discoveries seem to be ruled out by Wittgenstein's pronouncements against the possibility of mistakes or surprises in logic:

5.473 Logic must look after itself.

[...] Whatever is possible in logic is also permitted. [...] In a certain sense, we cannot make mistakes in logic.

5.4731 Self-evidence, which Russell talked about so much, can become dispensable in logic, only because language itself prevents every logical mistake.—What makes logic a priori is the *impossibility* of illogical thought.

Another way to raise this second question is to reflect on the importance that Wittgenstein attached to the colour-exclusion problem. If logical links between propositions are determined by facts about truth-functional composition of which we are ignorant, but which analysis is supposed to reveal, then when Wittgenstein came to the conclusion that ‘this is red’ and ‘this is green’ couldn’t be analysed in such a way as to represent their logical incompatibility truth-functionally, one would expect him to contemplate the possibility that, contrary to initial impressions, the propositions are not logically incompatible after all.

The third question concerns the nature of the system of elementary propositions, and truth-functions of these, underlying everyday propositions, according to the Fundamental Principle. We have a fairly clear sense of what it is for the identity of a proposition to consist in being a truth function of other propositions when this fact can be grounded in grammatical structure. But ‘surface’ grammar gives out long before we have reached a level of propositions that stand any chance of satisfying the demands imposed by the Fundamental Principle on the elements of Δ . This raises the question of the nature of the facts about the identity of propositions that generate the truth functional links between everyday propositions and their elementary counterparts.

In the remainder I want to sketch a construal of the existential claim expressed by the Fundamental Principle that sustains satisfactory answers to these questions.

8. Ishiguro on reference

I want to introduce this account by looking first at the account of the reference relation in the *Tractatus* that Hidé Ishiguro presented in her 1969 paper ‘Use and Reference of Names’. In that paper, Ishiguro took issue with the prevailing interpretation of the Tractarian account of how language is connected with reality, according to which propositions obtain their senses as a result of a more fundamental pairing of the names that figure in them with the objects in the world that serve as their referents.⁷ She presents the dichotomy between this reading and the alternative reading that she wants to put forward in the following terms:

The interesting question, I think, is whether the meaning of a name can be secured independently of its use in propositions by some method which links it to an object, as many, including Russell, have thought, or whether the identity of the object referred to is only settled by the use of the name in a set of propositions. (Ishiguro 1969: 20-21)

Ishiguro’s answer to this question set the foundations for a reading of the *Tractatus* fundamentally different from the atomistic reading that treats the reference of names as more basic than the sense of propositions:⁸

Contrary to widespread belief, Wittgenstein rejected the former view throughout his writings and tried to work out various versions of the second. (Ishiguro 1969: 21)

Ishiguro’s proposal was to take seriously Wittgenstein’s acceptance of Frege’s context principle:

[...] in the *Tractatus* Wittgenstein is anxious to stress that we cannot see how the name refers to an object except by understanding the rôle it plays in propositions. (Ishiguro 1969: 23)

And again:

It is a fundamental difference between Wittgenstein’s and Russell’s position that Wittgenstein holds that no expression, not even a name that cannot be further analysed, can be said to have reference out of the context of propositions. It is *not* a part of the *Tractatus* theory that if a symbol is logically simple and cannot be further analysed then it can be secured a

reference independently of and prior to its occurrence in a proposition [...] (Ishiguro 1969: 24)

So, on Ishiguro's reading of the *Tractatus*, the reference of a name is only settled by our use of propositions in which the name figures. The specific aspect of our use of propositions that gives rise to reference consists in the fact that we treat certain names and not others as interchangeable in propositions without affecting their truth value. This determines, on Ishiguro's reading, whether or not the names refer to the same object:

In the *Tractatus* one does not decide that one can substitute one expression for another *because* they refer to the same object. If two names are used in such a way that one can be substituted for the other, *then* the names *do* refer to the same object. (Ishiguro 1969: 30-31)

The account of reference that this passage ascribes to the *Tractatus* can be characterised as involving two components, one substantive and one formal.

The substantive component is its account of co-referentiality in terms of use: two names refer to the same object, on this account, when we use them as interchangeable *salva veritate*.

The formal component is the claim that an account of co-referentiality is all that's needed for a fully satisfactory account of reference. Once we have specified when two terms have the same reference, our account of reference is complete. We don't need to provide, in addition, an independent identification of the items that play the role of referents:

[...] the simple objects whose existence was posited were not so much a kind of metaphysical entity conjured up to support a logical theory as something whose existence adds no extra content to the logical theory. (Ishiguro 1969: 40)

It is illuminating to see the parallels between the formal aspect of the Tractarian account of reference, as interpreted by Ishiguro, and the contextual definition of number that Frege considers, and rejects, in the *Grundlagen* (Frege 1980: §§63-66). The proposal there is to define number using Hume's Principle: The number of F's equals the number of G's just in case there is a one-to-one correspondence between the F's and the G's. Hume's Principle

gives a complete specification of the truth conditions of sentences of the form ‘The number of F’s equals the number of G’s’. But Frege famously rejected the view that Hume’s Principle can be treated as an adequate definition of number on the grounds that it doesn’t provide an independent identification of the entities that play the role of numbers—it doesn’t even tell us whether Julius Caesar is one of them.

My suggestion is that Ishiguro’s reading of the Tractarian notion of reference is akin to the definition of number in terms of Hume’s Principle that Frege rejects. It treats a specification of the truth conditions for sentences of the form ‘name A refers to the same object as name B’ as a satisfactory definition of reference, even though it doesn’t provide an independent identification of the items that are supposed to play the role of referents.

9. Logical consequence

My goal in the remainder is to sketch a reading of the existential commitments expressed by the Fundamental Principle that mirrors the two features of Ishiguro’s reading of the Tractarian notion of reference that I presented in the preceding section.

My proposal will mirror the formal aspect of Ishiguro’s reading of the Tractarian notion of reference by treating Δ and f as contextually defined. They won’t be defined by providing an independent identification of elementary propositions or of the facts about everyday propositions that make them truth functions of elementary propositions. They will be defined, instead, by specifying the truth conditions of sentences of the form: ‘The truth-value assignments on Δ that make every element of a set Γ of propositions true also make p true’—i.e. ‘The truth-grounds shared by the elements of Γ are truth-grounds of p ’. On my proposal, once you’ve specified the truth conditions of sentences of this form, you’ve defined the set Δ of elementary propositions and the function f pairing each elementary assignment with a universal assignment. We don’t need to provide, in addition, an independent identification of

the items that play the role of elementary propositions or of the relation between an everyday proposition and the elementary propositions in terms of which it can be analysed.

The substantive aspect of Ishiguro's reading of the Tractarian notion of reference will be mirrored by the specification of the truth conditions of sentences of the form 'The truth-grounds shared by the elements of Γ are truth grounds of p '. My proposal is to stipulate that a sentence of this form is true just in case we treat p as a logical consequence of Γ .

In support of this proposal, I'd like to mention the role that the *Tractatus* ascribes to use in the constitution of symbols:

3.262 What signs fail to express, their application shows. What signs slur over, their application says clearly.

3.326 In order to recognize a symbol by its sign we must observe how it is used with a sense.

3.327 A sign does not determine a logical form unless it is taken together with its logico-syntactical employment.

The truth-functional structure of propositions will be present at the level of symbols, and clearly the aspect of use that would be most relevant for these purposes is whether or not we take a proposition to follow from another.⁹

While a definition along these lines doesn't make the existential commitments of the Fundamental Principle hostage to metaphysical fortune, there are certain formal adequacy conditions that would have to be satisfied for the existential claims to be vindicated.

The first requirement is consistency. In order for the definition to succeed, it has to be possible to assert without contradiction the existence of the defined items—of a set of elementary propositions and a function from truth-value assignments on these to universal truth-value assignments that makes logical consequence agree with use—i.e. with when we treat a proposition as a logical consequence of a set.

In order for this to be possible, our use will have to be coherent. The definition will fail if, e.g., we treat all the elements of Λ as a logical consequences of Γ and p as a logical consequence of Λ but not as a logical consequence of Γ , or if we treat p as a logical consequence of Γ but not of some subset of Γ . In general, our use will be coherent just in case it defines a set of universal truth-value assignments as logically possible.

Now, assuming that our use is coherent in this sense, does it follow that our contextual definition of Δ and f is consistent? If κ is the cardinality of the set of everyday propositions, a set of truth-value assignments treated as logically possible by our use will be represented by a subset Σ of $\{T, F\}^\kappa$. Then the consistency of our definition will depend on whether, for some cardinality λ , there is a function from $\{T, F\}^\lambda$ to $\{T, F\}^\kappa$ that pairs with elements of $\{T, F\}^\lambda$ precisely the elements of Σ .

Hence the consistency of our contextual definition will follow from the following result:

- (6) For every nonempty $\Sigma \subseteq \{T, F\}^\kappa$, there is a function f from $\{T, F\}^\kappa$ to $\{T, F\}^\kappa$ such that, for every $v \in \{T, F\}^\kappa$, there is a $v' \in \{T, F\}^\kappa$ such that $f(v') = v$ if and only if $v \in \Sigma$.

To see that (6) holds, let Σ be a nonempty subset of $\{T, F\}^\kappa$, and let $v^* \in \Sigma$. We define f as follows: For every $v \in \{T, F\}^\kappa$, $f(v) = v$ if $v \in \Sigma$, and $f(v) = v^*$ otherwise. It is obvious that, for every $v \in \{T, F\}^\kappa$, there is a $v' \in \{T, F\}^\kappa$ such that $f(v') = v$ if and only if $v \in \Sigma$.

The second requirement that a contextual definition of Δ and f would have to fulfil is uniqueness—insofar as it can be expected from any contextual definition. Suppose that Δ and f , and Δ^* and f^* both satisfy (3), with logical possibility for everyday propositions contextually defined by our use. Then, clearly, for every truth-value assignment v' to the everyday propositions that our use defines as logically possible, there will be truth-value

assignments on Δ whose image under f includes v' and truth-value assignments on Δ^* whose image under f^* includes v' . Uniqueness, for our purposes, will be a matter of how similar these truth-value assignments on Δ and these truth-value assignments on Δ^* would have to be before we can say that our contextual definition singles out a unique set of elementary propositions and truth functions from these. Here we face a range of options as to how demanding we want to be, and this is not the place to explore the issue in detail. The only point I want to make in this connection is that on this point the contextual definition of elementary propositions and of the analysis of everyday propositions in terms of these might well fail by its own standards. The Δ/f -pairs that could accommodate our use might simply be too disparate for the idea that we have defined a set of propositions and truth functions to have much plausibility.¹⁰

In any case, my main contention is not that this account of logical consequence in terms of contextually defined elementary propositions and truth functions is correct. My claim is that it offers a reading of the Tractarian account of logical consequence that is in line with well established readings of other aspects of the book, and sustains satisfactory answers to some important questions that other readings can't handle adequately. Notice, first, that Wittgenstein's confidence that elementary propositions exist would no longer seem as reckless as on the standard reading. For on the reading that I am presenting, the Tractarian account of logical consequence would no longer turn on the independent availability of suitable items to play the role of elementary propositions, but only on the formal constraints on the contextual definition that I've just outlined. Second, Wittgenstein's certainty that logical-possibility facts are in line with our intuitions would appear entirely justified, since the facts about elementary propositions and truth functions, on which logical possibility depends, are construed, precisely, in terms of these intuitions. It is not possible to infer a proposition p from a proposition q if the truth-grounds of q are not truth-grounds of p simply because the

fact that we infer p from q is what makes it the case that the truth-grounds of q are truth-grounds of p . Finally, the ontological status of the system of elementary propositions and truth functions would no longer appear mysterious. For, on this account, we can apply to them the words that Ishiguro used for simple objects, with which I'd like to close. They would be:

not so much a kind of metaphysical entity conjured up to support a logical theory as something whose existence adds no extra content to the logical theory.

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NOTES

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¹ See, e.g., *Tractatus* 1.21, 2.061, 2.062, 4.211, 5.134.

² See (Wittgenstein 1929).

³ I believe that this account of Wittgenstein's reasons for holding that elementary propositions are logically independent is broadly in line with Anscombe's account. See (Anscombe 1971: 31-33).

⁴ See *Tractatus* 2.02, 3.203, 3.22.

⁵ Wittgenstein's commitment to this claim has been diagnosed by Hidé Ishiguro as arising from '[...] a false assimilation in the *Tractatus* of the relation of propositions and the facts they express and the relation between an expression and a complex object which it signifies'. (Ishiguro 1969: 37). Since facts are essentially linked to the propositions that describe them, the assimilation of complexes to facts leads Wittgenstein to assume that the same goes for complexes. But this, according to Ishiguro, is a mistake:

The Wittgenstein of the *Tractatus* (like Russell) is, however, wrong to talk about all complex objects in the same way as he does about facts. For although the identity of a fact cannot be settled except by settling the identity of the proposition which describes it, the identity of complex objects such as General de Gaulle does not depend on our articulating any one particular description. [...] Therefore 3.24 is wrong when it says 'A proposition about a complex stands in an internal relation to a proposition about a constituent of the complex'. (Ishiguro 1969: 39)

⁶ See also (Wittgenstein 1979: 62): '[...] the complexity of spatial objects is a logical complexity, for to say that one thing is part of another is always a tautology'.

⁷ One of the leading advocates of the reading opposed by Ishiguro is David Pears. See (Pears 1987: Chapter 4).

⁸ For the contest between the reading that Ishiguro attacked and the reading that her article brought into existence, see (Kremer 1997).

⁹ Section 3.20103 of the *Prototractatus* lends support to the idea that the syntactical employment of a proposition includes what propositions follow from it:

The requirement of determinateness could also be formulated in the following way: if a proposition is to have sense, the syntactical employment of each of its parts must have been established in advance. For example, it cannot occur to one only subsequently that a certain proposition follows from it. Before a proposition can have a sense, it must be completely settled what propositions follow from it. ((Wittgenstein 1971))

Michael Kremer has also endorsed this conception of syntactical employment. See (Kremer 1997: 98).

¹⁰ An additional formal constraint on Δ and f is introduced at 5.32: ‘All truth-functions are results of successive applications to elementary propositions of a finite number of truth-operations’. Consideration of this constraint will be left for another occasion.

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