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**Scientific and Aesthetic Understanding:
The Case of Musical Exemplification**

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Candidate

Ivano Zanzarella
108019233449

Supervisor

Prof. Dr. Helmut Pulte

Advisor

Dr. Alfredo Vernazzani

Ruhr-Universität Bochum

Fakultät für Philosophie und Erziehungswissenschaft
„History, Philosophy and Culture of Science“ (HPS+)

**Scientific and Aesthetic Understanding:
The Case of Musical Exemplification**

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vorgelegt von:

Ivano Zanzarella
Widumestraße 24, 44787 Bochum
108019233449

Referenten

Prof. Dr. Helmut Pulte
Dr. Alfredo Vernazzani

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Abstract

The Greek composer and architect Iannis Xenakis has shown in *Formalized Music* (1963) how it is possible to compose or describe music and sound by means of probabilistic laws from mathematics, information theory and statistical mechanics. In his theory, scientific concepts and properties such as entropy take on a musical meaning in that they become also properties structurally instantiable by music. Philosophically speaking, this raises many important questions about the relation between science and the arts. One of these questions concerns in particular the possibility for aesthetic symbols (like musical compositions) to convey scientific understanding, and understanding in general. In the present work, I claim that this question can be answered positively. In general, understanding does not necessarily depend on truth, explanation and propositionality (non-factualism, non-reductivism). Understanding can be conveyed also in non-propositional domains, in particular by means of exemplification. Since aesthetic and musical symbols are non-propositional, they can advance understanding possibly by exemplification, and in particular scientific understanding as long as they exemplify scientific concepts and properties. I moreover substantiate my claim by taking a case study: the concept of entropy in music. On the basis of Xenakis' stochastic theory of music, I show how by exemplifying this concept, music can advance understanding of it.

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Introduction

In 1963, Greek composer, architect and engineer Iannis Xenakis (1922–2001) published for the first time *Formalized Music*, one of the most groundbreaking works in history of music theory¹. There, Xenakis intended to explain how music can be described by mathematical concepts and how it is possible to develop new techniques for music composition by bringing mathematics and informatics into music [Chapter 3].

Perhaps because too difficult for the musician and ultimately irrelevant for the scientist and the mathematician, Xenakis' formal theory of music has brought about no significant impact in their respective disciplines, which is also confirmed by a remarkable scarcity of publications about it, even in recent times. This is actually not exactly what Xenakis dreamed of. In fact, he wanted his work not only to provide a touchpoint between different forms of art (music, architecture, painting, etc. [Xenakis 1992, 39-42]), but also to promote a fruitful exchange between art and science [Xenakis 1992, 261].

Yet, I believe that Xenakis' work can really fulfill such a task, and I think that a way in which it can do so is by stimulating the philosophical reflection about the relationship between art and science. Indeed, carrying out a reflection of this kind is what I shall do in the present work. However, the way in which I propose to philosophically problematize Xenakis' experience is not that of the traditional and controversial problem of the

¹ Originally published in Paris in 1963 with the title of *Musiques formelles: nouveaux principes formels de composition musicale*, the work was republished in an expanded edition by Stock Musique in 1981. The first English edition of the work was published with three additional chapters by Indiana University Press in 1971, and then republished with the title of *Formalized Music: Thought and Mathematics in Composition* by Pendragon Press in 1992 and 2001. In this work, we will always refer to [Xenakis 1992].

demarcation between art and science. The perspective from which I will in fact accomplish such reflection is that of another important issue in philosophy and epistemology, namely that of *understanding*.

As an autonomous cognitive achievement different, for example, from knowledge, understanding has been in recent times at the center of the epistemological debate [Chapter 1]. Defining the nature of understanding, the necessary and sufficient conditions for someone to understand something, are some of the tasks epistemologists and philosophers have been engaged with at least for the last forty years.

One of the most relevant questions about understanding has been that concerning the existence of many possible kinds of understanding: scientific understanding, aesthetic understanding, mathematical understanding, etc. Yet, no agreement has been reached about how many kinds of understanding there are, what they are different in, whether they can be reduced to a single, general kind of understanding, etc.

A way in which questions like these have been answered is that of referring to the object of understanding, which in turn defines the kind of understanding [Section 1.1]. So, one who understands *The Origin of Species* possesses (theoretical) scientific understanding. One who understands the *Elements* possesses mathematical understanding. One who understands the *Air on the G String* possesses aesthetic or musical understanding, and so on.

Exactly in this regard, Xenakis' case raises important philosophical questions. What kind of object is a *musical* work composed after stochastic laws and instantiating, say, an entropy development conceptually similar to that of a thermodynamic system? What kind of understanding possesses one who understands this object? Is this understanding purely musical, purely scientific-mathematical, or both of them? And, more importantly, can a work like that aesthetically conveying an understanding of the scientific or mathematical concepts it refers to? In other words, can it advance, as an aesthetic object, scientific understanding?

Giving an answer to these questions and tackling especially the more general problem of the aesthetic transmission of scientific understanding is the main purpose of this work, which I will in particular address in

[Chapter 3] by taking Xenakis' theory of music and musical production as case study.

Before coming to this, however, an articulate investigation of the concept of understanding is necessary. Before speaking about the possibilities music has to convey scientific understanding, in fact, one has firstly to examine whether music can in general convey some kind of understanding at all, and how it can do this.

In order to answer these questions, I will firstly show in [Chapter 1], after a general overview of the concept of understanding, that understanding is non-explanatory, non-factual and objective, i.e. that it does not have explanation, truth and propositionality as necessary conditions. If understanding does not necessarily depend on truth and is therefore not necessarily propositional, this means that we can gain understanding not only of or through false propositions, but also of or through symbols which are not truth-apt either because non-propositional – only what is propositional can be namely truth-apt.

In order to explain how non-propositional understanding works, I will take in [Chapter 2] the particular case of music. I will firstly demonstrate that music is a non-propositional and non-truth-apt symbolic structure and then claim that, as such, it can be legitimately admitted as an object or vehicle of understanding. I will then introduce the concept of exemplification as one of the epistemic mechanisms by which music can advance understanding, both of music itself and of other things depending on what it exemplifies.

If music can advance understanding by means of exemplification, it can also advance scientific understanding when it exemplifies scientific concepts. In [Chapter 3], after explaining how aesthetic symbols in general can be legitimately considered as capable of advancing scientific understanding, I will substantiate this claim by taking a case study: the musical exemplification of the concept of entropy. I will briefly explain what entropy is both in science and in music by referring to Boltzmann's statistical mechanics, Shannon's information theory and Xenakis' stochastic music theory. Then, I will show how exactly music can exemplify this concept and thus advance understanding of it. In this

way, the main concern of this work will be answered.

Finally, in the [Conclusion] I will briefly recapitulate my argumentation and express some thoughts about the extent to which the philosophical reflection in this work may be not only scientifically, but also didactically relevant.

Chapter 1

The Nature of Understanding

In this chapter I shall deal with some important open questions from present-day epistemology of understanding. In [Section 1.1] I give a general overview of the concept of understanding, of its different forms and of its general characteristics. I will then introduce explanationism and objectuality, i.e. the two main rival positions about understanding trying to define its general epistemological nature.

In this respect, I will defend an objectual, non-reductivist and non-factual conception of understanding. In [Section 1.2], I will claim, against explanationism and explanatory reductivism, that understanding does not necessarily depend on explanation and cannot be reduced to an *understanding-why*. In [Section 1.3] and [Section 1.4], I will then show that understanding is not necessarily propositional and therefore not reducible to a form of propositional knowledge.

As to the latter issue, my argumentative starting point will be, in [Section 1.3], the renunciation of factualism about understanding, i.e. the epistemological stance making understanding necessarily dependent on truth. I will substantiate non-factualism about understanding by referring principally to idealizations and falsehoods in science on the one hand, and cases from history of science on the other.

In [Section 1.4], I will then show that non-factualism implies non-propositionality of understanding, and that understanding cannot be therefore reduced to a form of propositional knowledge. If so, not only falsehoods are to be considered as epistemically valuable for understanding,

but also symbols which are not true in the sense of being non-truth-apt because non-propositional.

In this chapter, I furthermore claim that a renunciation of strict factualism for non-factualism allows epistemology to build a better account of understanding. A non-factualist account of understanding, in fact, is epistemologically more accurate in that it acknowledges the cognitive function falsehoods *actually* perform within propositional contexts like science – that is, what factualism tends instead to deny or disregard. Moreover, it is also broader and more comprehensive in that it is able to epistemologically accommodate non-propositional, and hence non-truth-apt, symbols as well as genuine objects or means of understanding – which are instead excluded from the realm of epistemology by factualism because of its truth requirement on understanding.

However, exactly the accommodation of non-propositional symbols as genuine objects or means of understanding – which is indeed one of the main concerns of this work – presents not a few theoretical difficulties to epistemology. For example, it is not immediately clear how what non-propositional symbols exactly are, how non-propositional understanding actually works, etc. In [Section 1.4], I will discuss some of these problems and open the way to the concept of exemplification as an insightful solution to them, which will be the object of our inquiry in [Chapter 2] and [Chapter 3].

1.1 What is Understanding?

All of us are more or less familiar with the concept of *understanding*. We learn to understand verbal and non-verbal languages. We understand what was wrong with throwing marmalade on the living room's wall. At some point, we have to show that we understood correctly Newton's First Law or the traffic code. In the same way, we understand people, situation and tasks.

Understanding is steadily a part of our cognitive life, and as such it has become, especially from the 1970s onwards, an important subject of philosophical research. Especially in the earlier phases of the debate

about understanding, scholars in epistemology and philosophy have tried to account for it largely by comparison with knowledge [Hannon 2021]. This is indeed unsurprising, since also in our everyday life understanding and knowledge are often used as interchangeable concepts. Yet, at a closer look, it seems that, albeit related to each other, understanding and knowledge are independent cognitive achievements. So, what the difference between understanding and knowledge is, what the nature, the conditions, the object of understanding are, what possibly different kinds of understanding exist have become some of the questions around which the debate about understanding has mostly developed.

To begin with, most philosophers and epistemologists agree on the fact that, depending on the nature of the understood object, various forms of understanding are distinguishable (see e.g. [Hannon 2021, 281-284], [Baumberger and Brun 2017, 4-6], [Elgin 2017b, 33], [Kvanvig 2003, 191]). One can understand p , whereby p , the object of understanding, is a scientific phenomenon, a scientific theory, a mathematical expression, an artwork, a decision, etc. Correspondingly, one who understand p would have scientific understanding, mathematical understanding, aesthetic understanding, moral understanding, etc.

Depending on the nature of p , some take as a possible component of understanding also skills and *know-how* abilities¹ (see e.g. [Grimm 2014], [Elgin 2017b]). In some cases, if one understands p , one also *knows how* to wield p to further cognitive, epistemic and practical ends, one is also able to complicate, simplify, reformulate p , etc. The understanding of p is, so to say, partly the result and expression of the ability of manipulating p in various ways.

As it has moreover been highlighted, another important characteristic of understanding seems to be the fact that it comes in *degrees* (see e.g. [Kvanvig 2003], [Kelp 2017], [Le Bihan 2017], [Elgin 2017b]), which eventually makes it different from knowledge. In fact, you either know p or not. There are no further options in between. Conversely, you can

¹ It is however still object of debate whether understanding *necessarily* involves *know-how* abilities, and it is also open to question whether such abilities are just epistemic or practical as well (see [Baumberger and Brun 2017], [Hannon 2021], [Footnote 1.5]).

understand p to different extents.

Understanding seems to be also highly contextual, as well as dependent on the background assumptions of the potential understander (see e.g. [de Regt and Dieks 2005], [Elgin 2017b], [Section 1.3.2] and [Section 2.2]). Suppose, for example, p is the atomic structure. There are many kinds of representation of p , which may lead you to gain, at different degrees, an understanding of p . It is however very improbable that just by drawing a nicely colored picture of Bohr's atomic model you can show in a quantum mechanics exam that you successfully understood atomic structure. Equally, it is very improbable that a five-year-old child can understand something of the atomic structure if confronted just with a bunch of equations from quantum mechanics. Still, it is very plausible that the child who today understand the picture-based account of atomic structure will tomorrow more easily understand also the equation-based one, differently, for instance, from another subject who did never hear the word "atom".

Despite these traits, that most instances of understanding usually shares with each other, there is no agreement on the general epistemological nature of understanding [Grimm et al. 2017]. It is for example still not clear whether it always involves a *know-how* component, and whether a single, archetypical, epistemological model of understanding can be built. With regard to these issues two positions in particular are distinguishable in the literature (see e.g. [Khalifa 2013]): objectualism and explanationism. Whereas authors defending the former claim that understanding is basically *objectual* (see e.g. [Kvanvig 2003], [Elgin 2017b]), authors defending the latter claim that understanding is instead basically *explanatory* (see e.g. [Grimm 2014], [Hills 2015]). Let us see what this exactly means.

For explanationists, to understand p necessarily depends on having an explanation for p . To understand a scientific phenomenon or a scientific theory is nothing but having an explanation for why the former occurs and the latter holds (see also [Hempel 1966], [Friedman 1974], [Strevens 2013a]). To understand an artwork, is nothing but having an explanation for why it is aesthetically valuable (see e.g. [Hills 2017]), and so on.

For objectualists, instead, in order to understand p , it is not always necessary, nor sometimes sufficient, to have an explanation for p . On the contrary, understanding p depends on one's endorsing a complex cognitive *account* about p , which delivers *as a whole* an understanding of that object. An account is a complex body of information, a comprehensive, interconnected system of cognitive commitments about p , which may not necessarily have explanatory form, i.e. which may not only contain the single facts explaining p . In an account, namely, also non-explanatory (and non-propositional) elements such as moral norms, rules of thumbs, epistemic abilities, practical skills, etc. may be contained, or even symbols like diagrams, three-dimensional models and graphs which, arguably, are not strictly speaking explanatory². Elements like these may in fact still play a cognitive role and so contribute, in a non-explanatory way, to one's understanding of p . Sometimes, they could be even sufficient to this purpose. And, the more extended and interconnected one's account about p is, the more one knows how to manipulate p for further ends, the more numerous and various these ends are, the larger the understanding of p is one will be said to possess.

Since objectual understanding depends not only on grasping just the *single* explanation of an object p , but on endorsing a *whole*, complex, account about p , in grasping the relation between p and its context, between p and other objects, objectual understanding is often said to be in this sense also *holistic*³ (see e.g. [Kvanvig 2003], [Elgin 2017b], [Hannon 2021], [Section 1.3], [Section 1.4]).

Thus, for objectualists, understanding is not always *reducible* to an *understanding-why*, to explanatory understanding. Conversely, this is exactly the claim made by explanationists, known in the literature namely as *explanatory reductivism* about understanding. Moreover, inasmuch one considers explanations to be largely propositional⁴, this reductivist claim

² The explanatory or non-explanatory nature of symbols like these is however something still controversial. Authors like e.g. [Abrahamsen and Bechtel 2015] and [Abrahamsen et al. 2017] namely claim that they may be explanatory (see also [Footnote 1.4], [Footnote 1.8]).

³ Especially in relation to authors like Elgin, the presence of a Quinean legacy in the objectual and holistic conception of understanding is very clear.

⁴ It is indeed to point out that whether explanations have *always* propositional form

takes on also a further meaning. In this case, understanding may namely become reducible also to propositional knowledge⁵.

On the common epistemological account, in fact, knowledge is always propositional⁶ in that it always takes the form of “*S* knows that *p*”, whereby *S* is the knowing subject and *p* necessarily a proposition⁷. Thus, if the understanding of *p* depends on having an explanation of *p*, whereby this explanation is propositional, one is in the same epistemic state of *knowing* propositions that *p* as well as that *r*, *q*, *s*, etc., i.e. the explanatory facts in light of which *p* has to be expected (nomologically, causally, mechanistically, etc. depending on which model of explanation one endorses). Understanding, in other words, becomes a form of knowledge, and more precisely, a knowledge of causes⁸ (see also e.g. [Salmon 1984], [Lipton 2004], [Khalifa 2011], [Greco 2014], [Kelp 2014], [Baumberger and Brun 2017]).

Finally, as long as one accepts understanding as reducible to propositions, i.e. to propositional explanations or propositional knowledge, an implicit commitment to *factualism* about understanding is generally made. As *S* cannot be said to know that *p* if *p* turns out to be false, *S* fails to

is something still very debated (see e.g. [Pritchard 2010], [Abrahamsen and Bechtel 2015], [Footnote 1.2], [Footnote 1.8]).

⁵ Yet, if an explanationist were to acknowledge also skills and *know-how* abilities as essential components of understanding, then understanding would be in this case not entirely reducible to propositionality (and propositional knowledge). As pointed out, it is however still disputed whether *know-how* really counts as a necessary component of understanding (see [Footnote 1.1]), and moreover whether it can be reduced to propositionality. In this respect, [Sullivan 2018] for example claims that the abilities involved in understanding are not the practical ones commonly identified with *know-how*, but cognitive abilities not different from those involved in cases of *know-that*, to which thus understanding may be unproblematically reduced.

⁶ Some authors (see e.g. [Ichikawa and Steup 2018], [Stanley 2011], [Williamson 2001]) point however out that also forms of knowledge exists whose propositional character is still debated.

⁷ Propositions do not necessarily coincide with the sentences of a natural language. Conversely, they are «mind-independent extra-linguistic abstract objects» [Bealer 1998, 1] that can bear different properties among which primarily truth and falsity, and that as such can be also expressed by symbolic and representational systems different from language.

⁸ This topic is indeed still very controversial. Authors such as [Grimm 2014], for example, claim that understanding is explanatory, even if non-propositional. Others, such as [Kelp 2016] claim that, albeit basically propositional, understanding is not necessarily explanatory.

understand p , if p and each proposition figuring in the explanation of p are not true (see e.g. [Baumberger and Brun 2017, 6], and [Hannon 2021, 271]). In other words, understanding should derive from explanations (or descriptions, representations, etc.) of p containing *necessarily true* propositions⁹.

Indeed, factualism and propositionality are two notions closely related to each other. Accepting factualism about understanding namely presupposes accepting propositionality about it as well. In fact, all what is true can be such because of the property of truth-aptness it possesses, and in epistemology, truth-aptness is generally considered a co-intensional property with propositionality, to the extent that what is truth-apt is necessarily propositional and what is propositional is necessarily truth-apt. Hence, if understanding is necessarily factual, it must be also necessarily propositional, since only propositions possess truth-aptness, can bear truth values and thus possibly be true.

Until now, I exposed some general characteristics of understanding. We saw that understanding is a cognitive achievement related to, but ultimately different from, knowledge – understanding for example comes in degrees and is sometimes connected with skills. Then, I introduced the main positions in present-day epistemology trying to define the most fundamental epistemological nature of understanding.

In the following, I will defend, especially on the basis of [Elgin 2017b], an objectual, non-reductivist and non-factual conception of understanding. In particular, in [Section 1.2], I will claim, against explanatory reductivism, that explanation is in general neither necessary nor sufficient for understanding. In [Section 1.3] and [Section 1.4], I will then claim against propositional reductivism that understanding is not necessarily propositional either – and thus consequently not necessarily a form of propositional knowledge. The way I will do that in particular is primarily by resisting factualism about understanding. Namely, if truth is not a necessary condition for understanding, it becomes theoretically possible

⁹ It is indeed still not very clear if also false explanation can deliver understanding (see e.g. [Baumberger and Brun 2017]).

not only to admit strictly speaking *false* explanations and representations as capable of providing genuine understanding, but also symbols which are *non-truth-apt* because inherently *non-propositional*. This ultimately amounts to a preparatory theoretical work for what we shall do in the next two chapters, namely admit a non-explanatory, non-propositional and non-truth-apt symbolic structure like music as epistemologically legitimate vehicle of (scientific) understanding.

1.2 Non-reductivist Understanding: Explanation and Understanding

As a first argument against explanatory reductivism, consider the consequences of denying truth and truth-aptness as necessary conditions for understanding, a claim which I will support in [Section 1.3]. From this claim namely results that understanding is not necessarily propositional [Section 1.4]. If one however takes explanations arguably as being mostly propositional [Section 1.1], it then follows that understanding is not necessarily explanatory.

Of course, it may be disputed whether non-propositional explanations are possible as well (see e.g. [Grimm 2014] and [Footnote 1.2], [Footnote 1.4], [Footnote 1.8]), which would indeed invalidate the argument above. Yet, no minimal agreement is there on what in general an explanation is, what general epistemological form an explanation has, and how a good explanation should look like [de Regt 2009, Ch. 3].

Moreover, there are cases in which understanding may be gained also without explanations (see e.g. [de Regt 2009], [Elgin 2017b]). For example, one may understand Kepler's First Law without having any explanation for why it holds. One can understand a subway system without being able to explain why it works how it works. One may understand a musical composition without being able to explain why structural elements in it are arranged in that and not in other ways, and so on. In all these cases, understanding would be irreducibly objectual in the sense explained above, would depend not on explanation, but on one's endorsing a complex account of various cognitive commitments bearing

on those objects.

[Lipton 2009a] has for example stressed that even scientific understanding may in some cases not depend on explanation, and thus be in this sense objectual. One could for instance understand (or make understand) the retrograde motion of the planets visually by an orrery (or a digital graphic simulation). Similarly, one could understand (or make understand) why all terrestrial bodies fall with the same acceleration independently of their masses, without explanations, but simply by referring to Galileo’s thought experiment demonstrating the absurdity of the opposite thesis: that independence remains understandable even in absence of an explanation of why it holds. Further, as for example [Dellsén 2020] points out, there are fundamental phenomena – “explanatory brute” «facts that are not merely unexplained, but for which there is no (correct) explanation» – scientists (and ordinary people) still understand even in absence of effective explanations¹⁰.

At this point, however, an explanationist may object that in cases like these we should indeed not speak of *genuine* understanding. A child who understands the retrograde motion of Mars by *seeing* it on the orrery traveling around the sun more slowly than Earth, would indeed have no understanding of the phenomenon: she lacks in fact an explanation for it. Scientists who uses fundamental phenomena and brute facts for explaining further things would indeed have no real understanding of them, and so on. However, this seems just implausible. The understanding conveyed by non-explanatory means indeed counts as genuine understanding. It is surely plausible that for example the child does not have an understanding of retrograde planetary motion *equal in degree* to that possessed by an astronomer who can also explain it. Yet, denying that the child has no

¹⁰ In epistemology of science, this also leads to the very controversial question about the distinction between *phenomenal* and *theoretical* scientific understanding, namely the question about the difference and the relation between understanding a phenomenon and understanding the theory which explains it. It is for example still debated whether phenomenon and theory should be regarded, from the point of view of understanding, as two independent entities, as two independent objects of understanding (see e.g. [Baumberger and Brun 2017], [Hannon 2021]). And it is still open to question the extent to which the epistemological distinction between phenomenon and theory derives from, or reflects, also an ontological one – which is a central topic in the debate about scientific realism.

understanding at all would be actually unfair. Understanding can come in degrees, and to minimally understand corresponds namely not to not understanding at all [Section 1.1].

Similar considerations can be made in the case of explanatory aesthetic understanding – an indeed not so popular view defended in particular by Alison Hills. [Hills 2017], for example, claims that also aesthetic understanding is basically an *understanding-why*. We understand a musical composition like a piano sonata by Mozart, inasmuch we have (and can give) an explanation for why the composer made determined aesthetic choices, for why he used here a major chord, there a deceptive cadence, etc.

Yet, even if it seems plausible that an aesthetic explanation may be *in some cases* sufficient for aesthetic understanding, it is actually hard to see how aesthetic understanding should depend, *in general*, necessarily on explanations. Above all, in fact, differently from the case of science, there are no general, rigorous models for (good) aesthetic explanation, and even generalizing the traditional ones (nomological, mechanist, causal, etc.) to aesthetics seems actually a hard task. Moreover, even if we had such a model, the fact would remain that art works are mostly understandable even in lack of explanations. One could understand the sonata by Mozart even without having an explanation for all the aesthetic choices made by the composer, for *why* the work is a sonata, *why* it has determined formal characteristics, etc. The sonata could be understood for instance just by recognizing what it exemplifies [Section 2.2]. Of course, having an aesthetic explanation of it would enlarge one's aesthetic understanding, which comes namely in degrees as well. Yet, this understanding would not necessarily depend on having that explanation. It would therefore be objectual.

But not only does explanation seem to be not necessary for understanding. In some cases, it turns out to be also *not sufficient* for it. For example, there may be (and actually there are) scientific phenomena or art works which are neither completely understood nor completely understandable even if backed by good explanations. As pointed out in [Section 1.1], understanding is moreover contextual and depends on the under-

stander's background assumptions. Thus, not everywhere, and not for all subjects, a formal explanation of Kepler's Laws or of Mozart's sonata would be *sufficient* for gaining scientific or aesthetic understanding¹¹.

The aforementioned cases thus show that not always understanding can be reduced to *understanding-why*, that not always explanation is necessary or sufficient for understanding (see also [Hannon 2021, 283]). If so, understanding should be regarded as basically objectual, which does however not exclude that depending on object, context and understander it may be partly or totally explanatory. Simply, understanding *need not* derive *always* and *necessarily* from explanations.

Let us turn now to the second kind of reductivism about understanding, an epistemological stance related to, but different from, explanatory reductivism: namely that considering understanding reducible to propositions and propositional knowledge. The way I will resist this kind of reductivism is, as pointed out in [Section 1.1], by criticizing factualism, which takes truth as a necessary requirement for understanding. By denying this, in fact, we theoretically admit the possibility that also non-propositional, and therefore *neither true* nor false, symbols may advance understanding, and hence that forms of understanding exist which are *not* reducible to propositional knowledge either.

¹¹ A similar, but controversial case in which explanation would turn out to be insufficient for understanding is that concerning epistemic and practical tasks, for which namely epistemic and practical *know-how* abilities are needed (see also [Section 2.3]). If one regards, objectivistically, such *know-how* as integral part of understanding (see [Footnote 1.1], [Footnote 1.5]), as components of the objectual account through which the understanding of a given object is conveyed, then explanation is probably insufficient for understanding. Jerry may for example even be able to explain why the Navier–Stokes equations hold, what they are for, etc. Yet, he would fail to really understand them until he is able to solve physics exercises with them, apply them to specific theoretical or experimental cases, etc. Or, similarly, one could even explain just verbally to Jerry musical rhythm. He may have read every possible book about the topic. Yet, he will not really understand it and be consequently able to rhythmically beat a pencil on the desk before experiencing someone doing this and practicing this himself.

1.3 Non-reductivist Understanding: Truth and Understanding

The relation between truth and understanding is one of the most striking problems in present-day epistemology of understanding. On the one hand, reductivists claim that, being reducible to knowledge (and possibly to knowledge of causes), understanding must answer to the facts exactly as knowledge does (see e.g. [Lipton 2009b], [Grimm 2010], [Pritchard 2010], [Strevens 2013b], [Frigg and Nguyen 2019a]). Reductivism, in fact, generally implies factualism about understanding. As pointed out before, it is namely impossible to know something that turns out to be false. If one knows a fact, if one knows that p , then p must be true. Otherwise, we would not have knowledge, in the same sense in which Jerry would *not* know how long a day is if he were absolutely convinced of living not on Earth but on Jupiter. In this case, in fact, the content of his alleged knowledge would be false, and thus not amount to knowledge at all. Correspondingly, reductivists and factualists claim that it is impossible to understand p , if p is false or one relies upon an (objectual, representative or explanatory) account about p that turns out even minimally to be false. Factualism indeed requires such an account to be *utterly* true, *as a whole* – whereby *utterly* or *absolutely* true means here, in a specifically *logical* sense, that, if the account is seen as a long *conjunction* of elements¹² bearing on p , then all of them have to be true, since only one falsehood in it would falsify the entire account.

Against this version of strict factualism it has been however argued in recent literature that, even in relation to largely propositional and truth-apt contexts like for example science, such a conception of understanding based on a necessary truth requirement turns out to be too narrow. Sometimes, in fact, an account which is only *partly* true containing just few falsehoods at its periphery¹³ (quasi-factualism), or

¹² These elements must obviously have propositional form, since only propositions can be truth-apt and appear in a logical conjunction.

¹³ As also [Baumberger and Brun 2017, 10] points out, what counts as “core” and “periphery” of an account (or theory) is still something unclear. This distinction should be indeed object of definition for all those epistemological positions referring

indifferently everywhere and to all extents in it (non-factualism), still continues to be epistemically valuable for the sake of the understanding of the object it is about. In some cases, such falsehoods are in an account even what primarily fosters understanding. Further, the narrowness of strict factualism appears even more evident if one considers that we are commonly able to understand not only (by) false propositions, but even (by) symbols that, being non-propositional, cannot be neither true nor false of anything either, objects like maps, three-dimensional models, musical compositions, dances or abstract paintings¹⁴. Thus the options here are two: either we keep truth as necessary condition for understanding and refuse to speak about understanding in these latter cases – which makes in turn reductivism about understanding possible and legitimate, or we renounce to truth in order to build a more comprehensive epistemological account of understanding.

In this section I argue that the best option for epistemology is the second one. I will analyze in detail the relationship between truth and understanding, and defend against reductivism and factualism the claim that *absolute* truth is not a necessary condition for understanding. An account, namely, has just to be *true enough* [Elgin 2017b], that is, to contain that p -dependent amount of true (and propositional) elements sufficient for understanding p – whereby p may even be a falsehoods or a non-propositional symbol. In this way, a more accurate and broader notion of understanding, suitable for various and different domains from science to the arts, will be achieved. I will defend in particular *non-factualism* about understanding, and this by two arguments: in [Section 1.3.1], by considering the role of idealizations and falsehood in advancing understanding in largely propositional contexts like science; and in [Section 1.3.2], referring to history of science, by showing how theories such as Newtonian Mechanics, today superseded and therefore considered as no longer true, can still afford us understanding of the phenomena they

to it (like namely quasi-factualism). A general characterization of both notions may however be given for example in the terms of [Lakatos 1978].

¹⁴It is however to be noticed that some authors claim that maps, diagrams and other related symbols may be propositional or at least propositionalizable (see e.g. [Rescorla 2009], [Kulvicki 2015]).

purport to describe and explain. Respectively at the end of each section, some possible objections to the exposed arguments, with relative replies, will be considered.

1.3.1 The Role of Idealizations in Science

Idealizations and models are largely used in science. Some of them are not strictly speaking true in that they misrepresent their target phenomena for some specific epistemic or practical reasons. Yet, such falsehoods are accepted by scientists and contribute to understanding the phenomena they refer to. Now, with the words of [Elgin 2017b, 15]:

«if truth is mandatory, much of our best science turns out to be epistemologically unacceptable and perhaps intellectually dishonest. [...] We can retain the truth requirement and construe science either as cognitively defective or as non-cognitive, or we can reject, revise, or relax the truth requirement and remain cognitivists about and devotees of science».

I argue for the second option, and claim that strictly speaking false scientific models and idealizations do not fail to advance our understanding of the phenomena they purport to represent or describe (see also [Elgin 2007], [Elgin 2009], [de Regt and Gijssbers 2017], [de Regt 2017], [Strevens 2017], [Potochnik 2020], etc.). In that, they namely work as *felicitous falsehoods*, namely by fostering our understanding of the world. Let us see in which sense they do this by considering an example: the ideal gas law

$$pV = nRT,$$

which describes the relation between pressure, volume and temperature of a gas. Among the conditions that make the law work there are important theoretical assumptions such as that of the particles of the gas having negligible volume and being perfectly spherical, and that of the collisions between the particles and the walls of the container and between the particles themselves being perfectly elastic. Now, accepting the law along with its conditions indeed amounts to accepting a falsehood, a misrepresentation of the actual behavior of a gas, for no such a gas really exists with perfectly spherical particles, for instance. Yet, for all those

who want to *understand* the behavior of a gas, relying upon the ideal gas law is something essential.

The fact that this law is strictly speaking false does not hinder its efficacy in delivering an understanding of the phenomenon it purports to describe. Conversely, the falsity of the description is even what primarily fosters the understanding of the phenomenon. Such aspects like the dimension of the particles are deliberately disregarded, approximated or misrepresented in order to highlight, make salient the very object of understanding, namely the relation obtaining between pressure, volume and temperature of a gas.

Now, it appears unquestionable that science allows us to understand the world. If, as shown, this understanding can be conveyed also by means of such felicitous falsehoods, we have to conclude that truth, even in a largely propositional and truth-apt context like science, is generally not a necessary requirement for understanding.

Objections and Replies

Against the view of falsehoods in science as felicitous for the sake of understanding many objections have been raised. Authors like [Alexandrova 2008] and [Sullivan and Khalifa 2019], for example, claim that the presence of strictly speaking false idealizations and models in science do not represent a threat for factualism. Such idealizations play just a practical, heuristic role in science, facilitating for example calculations. Indeed they could be in principle eliminated, or better, *deidealized*: however formally complicated and practically inconvenient, a model for describing the behavior of a gas could be constructed which does not deviate even minimally from reality.

Others, like [Frigg and Nguyen 2019b], instead claim that idealizations and models should not be interpreted literally. Taken literally, in fact, they turn out to be false. Interpreted as figuratively representing their target object, they can be instead considered as true [Frigg and Nguyen 2019b, 2440, 2443]:

«Just because the idealized pendulum exemplifies being subject to no air resistance or friction, it needn't be taken to represent the actual pendulum in the grandfather clock in your office as being subject to no

air resistance or friction. Rather, it represents the clock mechanism as being such that the effect of those aspects is negligible for ordinary time-keeping. In this way one can salvage veritism at the expense of literalism [and] retain much of the philosophy of science (and epistemology) based on veritism».

As to the first objection, it has to be highlighted that idealizations perform their epistemic function in relation to understanding exactly in virtue of the fact that they deviate from reality. This deviation has not to be assessed as an unfortunate epistemic shortage of our way of representing phenomena, but rather as a true epistemic resource, which depends on an active and deliberate epistemic choice regarding how to build scientific representations. A representation or a “deidealized idealization” mirroring a phenomenon even in the most minimal details might make it epistemically “murky”, might render none of its properties and features salient, clear and discernible. By them, in other words, the phenomenon would remain epistemically inaccessible and therefore mostly not understandable [Elgin 2017b, Ch. 3].

As to the second objection, it has indeed to be shown firstly how exactly figurative interpretation of models works, what the epistemic criteria for an adequate figurative interpretation are – things which are indeed not well defined by [Frigg and Nguyen 2019b]. Secondly, it has to be shown whether figurative truth of models and idealizations can perform the same epistemic function for understanding as felicitous falsehoods. Moreover, on such an account even theories like astrology or fictions like *Winnie the Pooh*, if figuratively interpreted as true, may be epistemically valuable for the sake of understanding, which appears tenable unless one does not deviate from factualism (and scientific realism) [Elgin 2020, 3-4].

1.3.2 A Case from History of Science

In order to understand the relationship between understanding and truth, the case of science is interesting also for another reason. There are in fact scientific theories that, albeit superseded and therefore considered as strictly speaking false, can still afford us an understanding of the phenomena they purport to describe [Baumberger and Brun 2017, 8]. Now, if we were to accept with factualism that only absolutely true

theories can convey understanding, we should consider the understanding we (undoubtedly) gain from false or not absolutely true theories as epistemologically not valuable¹⁵. Thus again, either we regard the appeal to such theories as epistemologically unacceptable and dishonest, or we refuse factualism about understanding.

One of such theories is for example Newtonian Mechanics (NM). Basically unchallenged for more than two centuries, NW has been superseded only in 1915 by Einstein's General Theory of Relativity (GTR), the theory we currently accept as the "true" representation of the Universe. Thus, strictly speaking, for present science NM is considered as a false theory. It refers to concepts and notions which are from the point of view of GTR practically empty.

Consider for example the case of the concept of force of gravity. The NM-based claim that Earth stays in orbit around the Sun as an effect of the force of gravity is strictly speaking false. For GTR, in fact, there exist no such thing as a force acting at a distance between celestial bodies. The phenomenon should be rather described in terms of Earth's following a geodesic path around the Sun as an effect of the Sun's mass curving spacetime. The concept of force of gravity from NM, in other words, is for GTR empty in that it stays for nothing we know really existing in the world. For this and many other reasons, from today's scientific perspective, NM has thus to be regarded as a false theory.

If we accept factualism with its requirement of absolute truth on

¹⁵ According to [Laudan 1981], we cannot regard even our current scientific theories as "the true ones", since they too, as it can be *pessimistically induced* from the history of science, may be one day superseded and may therefore turn out to be false. If we were to accept factualism, then even our claim to gain understanding through the current "true" theories would be epistemologically untenable. After all, as the New Philosophy of Science with [Kuhn 1962], [Hanson 1958] among others, and positions like constructive empiricism, scientific skepticism and scientific anti-realism (see e.g. [Dummett 1978], [Cartwright 1983], [van Fraassen 2010], [Bernecker and Pritchard 2010, Ch. 44], [Chakravartty 2017]) thought us, there may even not exist such a thing as an utterly objective, mind-independent reality. This would eventually make impossible to judge our theories and representations after the epistemic categories of absolute, objective truth and falsity. Consequences deriving from the acceptance of such philosophical stances would surely count as powerful arguments against factualism about understanding. It cannot be however in the scope of the present work taking a position in the debate around scientific realism and scientific truth.

understanding, we should deny the fact that still today we gain understanding at least of “gravitational” phenomena by means of NM. Similarly, we should admit that for over two centuries scientist did not even minimally understand them, or worst, that only misunderstood them. Yet, this seems to be impossible and unfair [de Regt and Gijsbers 2017, 66-69]. NM is still the theory by which a child at the school can gain understanding of why an apple falls from a tree and why Earth stays in orbit around the Sun. NM is still the theory by which a scientist can successfully predict things about phenomena of this kind. NM is still the theory by which engineers can build safe skyscrapers.

NM is thus still today not only just practically or didactically valuable. It continues to perform an important *epistemic* function by affording us understanding of at least part of the world, the same way idealizations and models do.

Consider now we were not to abandon factualism. We would hence take GTR as the only epistemically valuable theory for the sake of the understanding of the world, i.e. also of terrestrial “gravitational” phenomena such as a falling apple or tides. Like in the case of models deidealization, constructing an GTR-based description of such phenomena is theoretically possible. However, depending on the context and on the background knowledge of the potential understander [Section 1.1], this GTR-based description might run the risk of making the phenomenon described rather epistemically “murky” than definite and clear. Think, for example, of an average skilled elementary school student confronted with an GTR-based explanation of why an apple falls from a tree. Such explanation would make the phenomenon to her epistemically inaccessible and eventually ununderstandable. On the contrary, a NM-based description of the phenomenon would be in this case more adequate in that it would result for her more intuitive, as well as technically easier to build and to handle. Thus, considered that for terrestrial gravitational phenomena, both theories are essentially equivalent and yield the same results, there would be no point in this case in preferring GTR to NM. In fact, only the latter leads to the understanding of the phenomenon, even if it is false. Once more, we hence see that the equation tying understanding and truth does

not work¹⁶.

Objections and Replies

So far, we have seen how partly or totally false statements in a propositional and truth-apt context like science *actually* perform an active epistemic function in advancing understanding, which eventually makes factualism an untenable epistemological position, in that it fails to acknowledge that. We do not need every element in the representation one has of something and through which one understands that thing be necessarily true. It is not a problem for understanding if the long conjunction this representation consists of turns out as a whole to be false, and this independently of where the representation contains falsehoods, if in the periphery or at the center, and of what and how many the falsehoods are. The epistemic capability of a given representation to advance understanding must be in fact assessed *holistically*, i.e. independently of the the epistemic condition of the *single* elements figuring in it (see e.g. [Elgin 2007], [Elgin 2017b] and [Section 1.1]).

Against these non-factualist conclusions about the relationship between truth and understanding a lot of objections have been raised (see e.g. [Baumberger and Brun 2017, 10-11], [Hannon 2021]). One of the most fundamental and controversial characterizes the non-factualist negation of truth as a necessary condition for understanding as a denial of an important epistemic criterion for assessing theories and representations. For example, both Aristotelian and Newtonian physics offer a way for genuinely understanding physical phenomena. Both theories are strictly speaking false because they contain false statements. As we know, for non-facualists, it is not a concern which and how many statements are false, since theories and representations are assessed holistically. Yet, the question arises: why should we prefer Newton to Aristotles for understanding nature, if truth has not to be considered a criterion for taking

¹⁶ As an objection to this particular example, it has been pointed out that NM can be still regarded as true or approximately true in that it follows from GTR as a limiting case. Therefore, it could not be used, as we did, as a proof for the non-factualist thesis that false theories too can advance understanding. However, as [de Regt and Gijssbers 2017, 66] also highlights, NM and GTR ascribe to the world two radically different ontologies, of which only one can be considered as true.

this decision?

To this problem different solutions have been given. [de Regt and Gijssbers 2017, 50-51], for instance, point out that not truth, but *effectiveness* constitutes the epistemic criterion for the assessment of theories and representations. Effectiveness means here the epistemic tendency of a theory «to produce useful scientific outcomes [such as] correct predictions, successful practical applications and fruitful ideas for further research». A theory is preferable to another – also in terms of understanding advancements – not because it is more veridical, but because it is more effective. Thus, for example, even if Aristotelian and Newtonian physics are both false, the latter is preferable to the former, because, differently from this, it still shows for some epistemic subjects in given epistemic contexts great effectiveness.

Other authors try to answer the problem in a different way, referring for example to notions such as *correctness* (see e.g. [Elgin 1996] and [Elgin 2017b, 75-76]) or *truth-likeness* (see e.g. [Oddie 2016]), among others. In particular, for [Elgin 2017b], the epistemic acceptability of an account or representation depends on the mutual coherence and intelligibility of all “atomic” elements constituting it, as well as on the fact that the community of cognitive agents holding it accepts and develops it reflectively following specific epistemic and normative standards. This is the concept of *reflective equilibrium*, that is for Elgin the condition of epistemic acceptability for an account (and thus for understanding) instead of truth¹⁷. Independently of the epistemic condition of the single commitments of two rival accounts, it should be accepted and further developed the one which conforms with the epistemic and normative standards *reflectively* endorsed by a given community of cognitive agents at a given time, and which is internally in *equilibrium*, i.e. presenting all commitments in it in a state of mutual coherency¹⁸. The epistemic acceptability of the single statements then just follows from the relative

¹⁷ The concept of *reflective equilibrium* has been firstly introduced by [Goodman 1954].

¹⁸ To notice is also that even what counts as objective, as a fact, reflectively depends on the communities of cognitive agents and on the epistemic and normative standards they endorse. Objectivity is namely *procedurally* defined (see e.g. [Fine 1998], [Douglas 2004], [Elgin 2017b, Ch. 7 in pt.] and [Section 3.1]).

account's being in reflective equilibrium, regardless of whether they are true or false.

Another objection raised against non-factualism is that it indeed excludes truth as a condition for epistemic acceptability and understanding just nominally, since by notions like “correctness” or “truth-likeness”, etc. truth or a certain approximation to truth does still remain a necessary requirement for epistemic acceptability and understanding (see e.g. [Le Bihan 2019]). This is the reason why, it is claimed, positions such as that by Elgin do not really distance themselves from factualism at all, be this also its more moderate version, quasi-factualism.

Similar cases are those in which understanding is regarded as objectual and/or involving *know-how*, but even requiring an additional amount of knowledge, which, as we know, requires in turn truth. Take for example the Pythagorean theorem. It seems awkward to say that one (objectually) *understands* the Pythagorean theorem – and is accordingly able to answer questions about it, solve problems with it, apply it in different ways, etc. – and yet *does not know* the Pythagorean theorem. Knowledge, and thus truth, seem to be in cases like these implicitly necessary for understanding.

This objection can be however quite simply overcome if one more closely considers what non-factualism about understanding indeed affirms. Non-factualism is not the thesis that grants any role at all to truth (and propositional knowledge) in understanding, that underestimates truth and its (positive) epistemic value for understanding. Nor is it a thesis affirming that understanding can *only* be gained through falsehoods. Conversely, non-factualism just refuses the factualist idea that understanding can be conveyed only, and without exceptions, through *absolutely* true accounts – i.e. true conjunctions of (propositional) statements about a given object. If this conjunction, as non-factualists claim, has not necessarily to be true, this means that just one, some, or even all elements in the account may possibly be false, or not true because non-propositional and non-truth-apt, and this regardless of where they lie in it. Such an account would still be epistemically acceptably and valuable for the sake of understanding, since the ability of an account to provide understanding *holistically* depends

on the account *as a whole*, and not on the epistemic status of the single elements figuring in it [Elgin 2017b].

The fact that, depending on the object of understanding – like in the case of the Pythagorean theorem – a given amount of truth and knowledge may be required for understanding, does not make non-factualism inconsistent: what is namely denied is not truth in itself, but *absolute* truth as a condition of understanding. For an account, in other words, suffices to be just *true enough* for providing understanding [Elgin 2017b], i.e. to contain that amount of true elements necessary for providing understanding of the target object – an amount which depends on the nature of this object and that, for non-factualism, may possibly be, in a theoretically legitimate way, also *null*. So is for example the ideal gas law, which contains that right amount of truth for providing an understanding of the phenomenon it is about.

For these reasons, non-factualism cannot be considered theoretically and logically equivalent to any form of factualism – or also quasi-factualism, since, as pointed out, it has no importance where the non-true elements lie in the account. Moreover, accepting non-factualism seems to be the only possible way to build a more “realistic” epistemology and to draw a more *accurate* picture of how understanding and disciplines like for example science work: given the cognitive limits intrinsic to our nature as human beings, partial truths and felicitous falsehood are in fact the best we can strive for. Conversely, factualism is apparently incapable of doing that. With its necessary truth requirement on understanding, it is namely incapable of epistemologically accounting for those instances of understanding – like those mentioned above – which are undeniably genuine, but derive from (partial) falsehoods like idealizations, models, etc. or are conveyed – as we shall see in the following – of or through symbols which happens to be non-propositional and therefore not truth-apt either.

1.4 Non-propositional Understanding

1.4.1 Some General Remarks

In the previous section we showed that in largely propositional and truth-apt contexts like science, truth is not a necessary condition for understanding. Also falsehoods, idealizations, false theories, etc. can in fact advance understanding. We also stressed how the acceptance of non-factualism allows epistemology to build a more accurate account of understanding, in that the epistemic function such falsehoods *undeniably* perform in understanding is taken into account and not simply disregarded or underestimated as within a factualist epistemology. Now I claim that the acceptance of a non-factualist conception of understanding allows epistemology to build an account of understanding which is not only more accurate, but also *broader* and *more comprehensive*, in that it is not restricted just to propositionality and truth-aptness.

Let us briefly return to the main thesis of factualism, namely that understanding necessarily requires truth. As pointed out in [Section 1.1], since only propositions can be literally speaking true (or false), factualism needs understanding to be basically propositional too: we can only gain understanding of p , whereby p is a proposition, or of a certain object or phenomenon p through a propositional account about p – regardless of whether it is an explanatory or objectual account¹⁹.

This also means that understanding may be completely reduced to a *know-that*, i.e. to a form of propositional knowledge. Understanding p – where p could be even a practical task or an intrinsically non-propositional object (say, an art work) – would amount to nothing but knowing a bunch of more or less interrelated propositions about p (see e.g. [Williamson 2001] and [Stanley 2011]).

Yet, in [Section 1.3] we saw that factualism is not quite a tenable (and desirable) epistemological stance. If we deny it and accept non-factualism, it follows that truth is no longer a necessary condition for understanding. This means that not only falsehoods and false propositions

¹⁹ It is however still debated whether explanatory and objectual understanding have propositionality as necessary condition (see e.g. [Hannon 2021, 282]).

may be legitimately considered as epistemically valuable for the sake of advancing understanding [Section 1.3], but also symbols which are not true in the sense that, not being propositional, they are non-truth-apt either, and hence lack, strictly speaking, the possibility itself of being true (see also [Section 1.4.2], [Chapter 2], [Chapter 3]). These are symbols like for example diagrams, maps, graphs, abstract geometrical shapes, three-dimensional models, sounds, or visual artworks, dance performances, musical compositions, etc.²⁰.

Now, if a factualist and reductivist epistemology would grant such symbols no value at all [Elgin 1996, 170] or just an instrumental value²¹ [Riggs 2003, 218-219], a non-factualist and non-reductivist one instead grants them a true epistemic function in advancing understanding. Non-propositional symbols are not just (sometimes useful) embellishments *outside* an epistemic account delivering (propositionally) understanding of a given object. They instead play a true epistemic function in advancing understanding as *inner* part of that account. And if these elements can be legitimately part of an epistemic account delivering understanding of something, understanding should be considered as not necessarily propositional, nor entirely reducible to propositional knowledge (see e.g. [Riggs 2003], [Elgin 2007], [Grimm 2014], [Pritchard 2014], [Bengson 2017]).

An account containing one, some, or only non-propositional (and non-truth-apt) elements²², is, in fact, still epistemically valuable for under-

²⁰ The propositional or non-propositional character of such symbols is however still debated. As pointed out before (see [Footnote 1.14]), in fact, some believe that they are propositional or at least propositionalizable. It is impossible now to review what has been said about each of these symbol, and indeed I believe that no generalized answer about their propositionality or non-propositionality can be provided. Each of them namely requires a specific analysis. In this work, we will treat only the case of musical symbols and try to demonstrate their non-propositionality (see [Section 2.1.3]).

²¹ Non-propositional and non-truth-apt symbols may be taken, for example, just as facilitating the process of conveying understanding of a given object or phenomenon. The major cognitive role in this process would be however still played by propositions, and consequently, the conveyed understanding too would still remain entirely expressible in propositional form.

²² As long as one considers *know-how* as a component of understanding, skills and epistemic abilities – if taken, against [Sullivan 2018], as not propositionalizable – would also belong to this kind of elements in an account.

standing. And, as in the case of falsehoods, it is indeed not contradictory to accept that, depending on the nature of the object of understanding, a given amount of propositionality and knowledge may be sometimes required in that account. What is in fact affirmed here, is that understanding *must not always* and *only* be propositional, not that it *must always* and *only* be non-propositional.

If understanding is in this sense not reducible to propositionality and propositional knowledge (of causes), this means, again, that it should be regarded basically as *objectual*. Understanding derives from endorsing an account, a complex, «fairly comprehensive, interconnected constellation of cognitive commitments» [Elgin 2017b, 45] bearing on the object of understanding, which may contain propositional and true, propositional and false, as well as non-propositional and non-truth-apt elements, and which has *as a whole* the capacity to advance understanding of that object, regardless of the epistemic state (and position) of the single elements in it.

Now, an account of understanding based on non-factualism, non-reductivism and objectuality, would be, as anticipated, broader and more comprehensive in scope than one based on the opposite positions. It would be namely able to recognize the value of non-propositional symbols for understanding and to accommodate non-propositional and non-truth-apt instances of understanding within epistemology as *genuine* instances of understanding.

In itself, and if successful, this accommodation would moreover count as a further argument in defense of non-factualism and non-reductivism. If it can be shown that non-propositional and non-truth-apt symbols like a map by Mercator, a fugue by Bach, a choreography by Cunningham, a sculpture by Giacometti, a subway system, a surrealist painting or perhaps even a baby's crying have a *non-propositional cognitive content*, and can be hence legitimate objects or means of understanding, if it can be shown that they provide epistemic access in terms of understanding to the things they may possibly denote, represent, express or exemplify in determined reference contexts, then this would constitute a further proof of the fact that symbols and representations which are *not true*

can advance understanding or even be object of understanding, which is namely the core thesis of non-factualism.

Such epistemological accommodation is however not quite a simple task. Showing that such symbols can have a cognitive content, and specifically a non-propositional one, showing that they can function cognitively, is today for many authors (see e.g. [Elgin 2002], [Carroll 2006], [Galison 2014], [Elgin 2017b]) one of the most important points on the philosophical agenda. As such, this task will be also our main concern in [Chapter 2] and [Chapter 3], in which we will discuss the nature and the functioning of non-propositional understanding by considering the specific case of music²³.

Before coming to the specific case of music and musical understanding, however, it is useful to discuss in general the possible ways in which non-propositional and non-truth-apt understanding instances could be epistemologically accommodated. This is what I will do in the following section.

1.4.2 Two Strategies for an Epistemology of Non-Propositional Understanding

For explaining how symbols like maps, sculptures, musical compositions, etc. advance understanding or can be even object of understanding, two strategies can be conceivably adopted: the one (i) trying to reduce the cognitive contribution of these symbols to propositions; and the other (ii), rejecting the possibility of this reduction, and treating this contribution specifically in non-propositional terms, which obviously presupposes the abandonment of factualism with its commitment to truth-aptness and propositionality.

Strategy (i): Reduction to Propositionality

In [Strategy (i)] the cognitive contribution of symbols or representations we defined as non-propositional is considered as being indeed reducible

²³It is namely reasonable to think that the way in which non-propositional symbols cognitively work and advance understanding depend on what specific kind of symbols they are, so that – at least in the present work – no general account of non-propositional understanding can be provided (see also [Footnote 1.20]).

to propositional and truth-apt content. When we claim to understand p , where p is an object like a map or an abstract painting, we are doing nothing but endorsing a complex system of propositions about p which afford us understanding of p . [Strategy (i)] thus implicitly requires the acceptance of reductivism about understanding: understanding necessarily depends on propositionality, and p is understandable exactly in virtue of the fact that its cognitive content can be reduced to propositions. It does however not necessarily imply factualism: some or all propositions of that system, in fact, may also be false, and the system still considered as epistemically valuable for the sake of the understanding of p .

Yet, the idea of reducing the cognitive contribution of p to propositions in order to explain how p can be understood seems awkward to many authors (see e.g. [Riggs 2003], [Elgin 2017b]), and to me as well. Propositions, in fact, are “atomic” entities which reflect single facts. Propositionalizing an object like p would thus amount to extract just single, limited and perspectival aspects of it. However accurate the system of propositions about p would be, only hardly could it reflect the cognitive content of p and the understanding a certain subject gains of it.

Moreover, by propositionalizing aspects of p , these would be removed from the context in which they originally perform their epistemic function, allowing us to epistemically access (i.e. to understand) p . Indeed, the propositionalization of aspects of p , if desirable at all, would be only possible *a posteriori*, i.e. *just after* we grasped the epistemic content of p as a whole, in a way which is clearly not that of grasping the meaning of propositional sentences – objects like p are namely not able to make statements. In other words, only once we *understood* a sculpture might we start to express in propositions what we understood, whereby it is doubtful that the discrete cluster of more or less interconnected propositional pieces of information we would collect about the sculpture will entirely reflect our understanding of it.

If our understanding of p cannot be expressed in discrete propositions, it should be considered *objectual* and, as such, as reflecting an epistemic content that, presenting itself as a unitary epistemic whole and being therefore not propositionalizable, has to be *holistically* assessed (see also

[Elgin 2017b] and [Section 1.1]).

A further problem with [Strategy (i)] is that, if p is considered to be an implicitly propositional symbol, one should then explain how p “states” something that possesses a truth value, since all what is propositional must be also truth-apt²⁴. To this respect, it is indeed difficult to see how an architectural work can state something, and even something which may be true or false of facts obtaining in the world. It is difficult also to see what would be the truth condition, say, of the negation of a major sixth or of an implication between a *croisée* and a *brisé*. If it is impossible to attribute to symbols of this kind truth values, one should conclude that they cannot be propositional either. After all, it is even not important for such symbols to possess a specific truth-value for functioning epistemically. A Greek temple or a Baroque palace function epistemically as objects of understanding not by saying anything, but rather by manifesting themselves to the understander [Elgin 2017b].

Strategy (ii): Rejecting Propositional Reductivism

[Strategy (i)], hence, does only hardly work, for it is difficult to think that the cognitive content of symbols like maps or musical compositions – as well as the understanding one gains of or through them – could be reduced to propositions. It is for this reason that it seems more reasonable to directly defend the thesis that, since understanding is objectual and does not necessarily depend on propositionality and truth, such symbols, as *truly* non-propositional, can be legitimately considered as capable of advancing understanding. This is exactly the thesis at the core of

²⁴ A (very controversial) solution to this problem may be provided by the notion of *hyperintensionality* (see e.g. [Cresswell 1975], [Berto and Nolan 2021]). In so defined hyperintensional contexts, the truth value of propositional attitudes (see e.g. [Kripke 1979], [Nolan 2019]) or propositions themselves (see e.g. [Bealer 1998]) cannot be definitively established. Since, if one were to consider propositions to be basically hyperintensional, no direct and absolute association could be set between a proposition and its content with a determined truth value, truth-aptness itself as a necessary character of propositionality would fall away. Symbols like p would be thus still considered propositional but non-truth-apt, and therefore epistemically valuable for understanding, taking this too to be essentially propositional. This argument exhibits however many logical shortcomings and it has to be noticed that no hyperintensionalist would really stretch propositionality so far as to admit the possibility of something like non-truth-apt propositions [Elgin 2017b, 207-208].

[Strategy (ii)].

[Strategy (ii)] maintains that understanding has not necessarily to be propositional, i.e. that it can genuinely have as object also non-propositional instances. In order to be accommodated into epistemology of understanding, objects like models, graphs or musical compositions do not need to have a propositional character. On the contrary, they could be just directly and simply accepted as non-propositional, because it is understanding itself, as claimed, that does not require propositionality as a necessary condition (see [Baumberger and Brun 2017], [Riggs 2003] and [Elgin 1996]).

About [Strategy (ii)] as well, however, several points are still in need of an explanation. The following issues are particularly compelling. Firstly, it should be examined how objects like maps, models, musical compositions, etc. can have a cognitive content and be possibly representative or expressive of anything. In current terminology, the notion of “cognitive content” is generally connected to that of proposition. Yet, [Strategy (ii)] should also explain in which sense this content is non-propositional. Secondly, the concrete functioning of non-propositional understanding should be explained.

Nevertheless, the difficulties [Strategy (ii)] presents seem however to be more easily superable than those of [Strategy (i)], and this is the reason why [Strategy (ii)] is the most plausible strategy to follow in order to legitimately accommodate non-truth-apt and non-propositional understanding instances within an epistemological account of understanding that aims to be not only accurate, but also comprehensive.

Conclusion

Let us briefly sum things up. In this chapter I tried to defend an objectual, non-reductivist and non-factualist conception of understanding. Understanding does not necessarily depend on explanation and cannot be always reduced to an *understanding-why*. Understanding does not have truth as a necessary condition. This means that also falsehoods in propositional contexts can deliver understanding, and that also non-true,

because non-propositional and hence non-truth-apt, symbols can do the same. If so, understanding does not have propositionality as a necessary condition and cannot be always reduced to a form of propositional knowledge either.

Atomic propositions are in fact just some of the possible objects or means of understanding. Conversely, most objects of understanding, even in largely propositional contexts like science, are only objectually understandable by means of an account consisting of complex body of information about those objects in which also false, non-propositional and non-truth-apt, or even non-propositionalizable *know-how*-related elements may be contained. Such an objectual account advances understanding *as a whole*, independently of the epistemic state of the elements in it. And the more broader and developed this account is, the higher the *degree* of the conveyed understanding will be.

Endorsing an objectual, non-reductivist and non-factualist conception of understanding moreover allows epistemology to build a more realistic and accurate, as well as broader and more comprehensive account of understanding, in that the cognitive function *actually* and genuinely performed in understanding by falsehoods and non-propositional symbols is not denied or disregarded.

With these last conclusive remarks we should have a satisfying answer to our initial question about the nature of understanding. More importantly for what concerns the next two chapters, a theoretical justification of *why* non-propositional and non-truth-apt instances of understanding are possible and worthy of epistemological attention has been provided. What we shall do now, is to explore, relying on the specific case of music, *how* such instances of understanding actually work, and how they can be accommodated into epistemology, which, in other words, amounts to providing a solution to the issues encountered in discussing [Strategy (ii)]. To this aim we will introduce in the next chapter the notion of *exemplification*.

Chapter 2

Musical Exemplification and Understanding

In [Chapter 1] I argued for a non-factualist and non-reductivist notion of understanding. Especially in [Section 1.3.1] we noticed that the refusal of truth as a necessary condition for understanding leads epistemology to the theoretical possibility of accounting for instances of understanding which depend on non-propositional and non-truth-apt symbols.

However, the epistemological accommodation of such particular understanding instances is, as we pointed out, not quite a simple task. If it is impossible and ultimately undesirable to propositionalize them [Strategy (i)], one should refer to them as being inherently non-propositional and try to epistemologically account for them as such [Strategy (ii)]. Although [Strategy (ii)] seems to be the most reasonable one to follow, it presents some compelling problems.

In this chapter I am going to give a solution to the two main issues highlighted about [Strategy (ii)], primarily by discussing the case of music (see [Footnote 1.20] and [Footnote 1.23]). As to the first issue, I will try in [Section 2.1.1] to give a general characterization of music as a symbolic system. Then, in [Section 2.1.2] I will discuss whether music has a cognitive content and what this cognitive content consists in. In [Section 2.1.3] I will finally show that the cognitive content of music is non-propositional.

As to the second issue, I will examine in [Section 2.2] how music, as a non-propositional symbolic system, works cognitively in advancing

understanding. To this aim, I will introduce the notion of exemplification. After explaining in general terms what the epistemic mechanism of exemplification consists in [Section 2.2.1], I will show how it explains the way non-propositional symbolic systems – thus music in particular [Section 2.2.2] – function cognitively. Moreover, in [Section 2.2.3], I expose some ideas of how the notion of exemplification may answer a further possible difficulty of [Strategy (ii)], namely that concerning the transmission of understanding in non-propositional, *know-how*-related domains.

In this way, we will be able in [Chapter 3] to answer the central question of this work as we formulated it in the [Introduction], namely whether an aesthetic symbolic system as music, when exemplifies scientific concepts, can convey an understanding of them and, if so, of which kind and to what degree.

2.1 Music as a Non-propositional Symbolic System

2.1.1 What is Music?

Specifying all necessary and sufficient conditions for something to count as music is a fairly impossible task. Yet, an at least partial characterization of music is possible in that one considers, paraphrasing [Goodman 1968], those “symptoms” of the musical without which it would be utterly impossible to speak of something as music. Then, whether something exhibiting such symptoms also counts as an *artwork*, whether it is beautiful, why and how, are different questions we need not to address here.

It is firstly clear that what is musical is actually heard or potentially audible. Sound as a theoretical entity or heard phenomenon is in other words one of the most fundamental symptoms of the musical.

Furthermore, in music sound is *intentionally* organized within a structure whose syntax, like that of natural languages, is not immutable and depends on factors of various nature (historical, cultural, stylistic, ethnic, etc.) [Patel 2010]. One of the most essential characters of this structure is its *temporality*. What distinguishes random sound phenomena from a musical work is the sound’s occurring in the latter with a given

temporal order within a given finite interval of time – i.e. the duration of the musical composition.

Music furthermore works also as a symbolic structure [Goodman 1968, 179-192], since the elements belonging to it – i.e. the sounds as theoretical or physical objects, and combinations thereof – can act as symbols capable of reference. As a complex symbolic structure, music can instantiate, and possibly refer to, a wide range of properties. As happens for aesthetic symbols in general (see e.g. [Carroll 2012] and [Vernazzani 2021]), these properties can be ordered into three large categories¹. Firstly there are *formal* or *design* properties, i.e. specifically musical or “intra-musical” properties referring to the syntax, the structure, the form of music and of musical symbols: a chord’s being major or minor, Mozart’s K. 545 being a sonata, Conlon Nancarrow’s *Studies for Player Piano* being polytempic, etc. Then, there are *expressive* properties, i.e. “extra-musical” properties referring to emotional states: the diminished seventh chord’s being tense, the first movement’s of the fifth symphony by Beethoven being grave, serious or melancholic, etc. And finally, there are aesthetic properties like Vivaldi’s *Gloria* being beautiful, grandiose, etc. Formal, expressive and aesthetic properties “make” a musical composition, and *disjunctively* constitute the content of the aesthetic and cognitive experience of it [Vernazzani 2021, 4].

Some of these properties, like the formal ones, can be *literally* instantiated by music. Others, like the expressive ones, can be instead possessed just *metaphorically*. The C major chord, for example, literally instantiates the property of being a major chord. Yet, it possesses cheerfulness just metaphorically, since musical chords cannot be literally cheerful (see also [Goodman 1968, 45 and ff.]). In general, all the extra-musical properties music may refer to are metaphorically instantiated².

Music can moreover also express other things than just the emotions

¹ On where the demarcation line is exactly to be drawn between each category is however, both in music and generally in aesthetics, a still debated metaphysical question, on which we can remain here relatively neutral.

² Yet, there may be cases in which a strictly speaking extra-musical property is however literally instantiated – this is for example the case of symmetry, a property that, albeit not specifically musical, music may literally instantiate (see [Section 2.2.2]).

to which the expressive properties refer. There is namely a difference between the two notions of what I call *musical expressiveness* – which namely depends on music’s possessing expressive properties – and *musical expression* (see also [Matravers 2007]). If the expressive properties are related just to emotions (cheerfulness, sadness, etc.), music can also refer to other objects, processes, states of affairs, etc. outside the emotional realm, and so express them. So, for example, Honegger’s *Pacific 231* expresses the movement of a locomotive’s starting its race, Šostakovič’s seventh symphony expresses the resistance of people to fascisms and totalitarianisms, etc. Yet, musical expressiveness and musical expression are related in that they are both metaphorical: the former depends on music’s namely metaphorically instantiating expressive properties, and the latter on music’s *metaphorically denoting* extra-musical objects, processes, states of affairs, etc. (see also [Goodman 1968, 45-52, 85-95], [Section 2.1.2]).

With no pretense of exhaustiveness, those displayed above (music as sound organized in a temporal symbolic structure instantiating formal, expressive and aesthetic properties and capable of expression) seem thus to be the most fundamental “symptoms” of the musical, the necessary, but perhaps not sufficient, conditions for something to count as music.

Our task is now that of examining whether music, as I described it above, can have a cognitive content [Section 2.1.2], and whether this content is propositional or not [Section 2.1.3].

2.1.2 The Cognitive Content of Music

Whether music has a cognitive content that can be object of understanding, and what this content consists in, are questions philosophers have been discussing fairly since Antiquity (see e.g. [Langer 1943], [Meyer 1956], [Cone 1974], [Davies 1994], [Robinson 1997], [Kivy 2002], [Kramer 2002], [Alperson 2004]). In philosophy and aesthetics of music these questions have been taken as a part of the more general problem of musical meaning. About it basically two solutions have been given, from which an antithesis has developed between positions which namely refute categorically that music can have some kind of meaning the way, say, the words of a language

have, and positions which instead admit in various ways such possibility for it.

On the one hand, positions of the second kind – we label *musical representationalism* – generally take the extra-musical content of musical expressiveness and musical expression (*specific* emotions or other *specific* objects and facts) as the true *meaning* of music, as well as the primary cognitive content of it, i.e. what in music is there primarily to *understand* (see e.g. [Cone 1974], [Robinson 1997]).

On the other hand, positions of the first kind – traditionally labeled *musical formalism*³ – instead assert that music cannot represent anything in this sense and hence have an extra-musical meaning. Indeed, the only meaning of music lies in its structure. What music may denote in terms of expressiveness or expression (a specific emotion or a specific object or fact), is not part of the semantic content of music.

Now, it is indeed impossible to affirm that music is able to *represent* in the sense of musical representationalism. It is impossible to say, for example, that the C major chord represents cheerfulness, or that the continuous sixteenth-notes flow of the left-hand piano part of Schubert's *Gretchen am Spinnrade* represents the movement of a spinning wheel.

Above all, in fact, the relation between a representing symbol and its semantic content is in general not so subjective and accidental as it seems to be in the case of musical symbols and their alleged extra-musical semantic content. In other words, even if musical symbols *always* (metaphorically) instantiate expressive properties or (metaphorically) denote extra-musical objects, processes, situations, etc., what expressive property they *exactly* instantiate and what they exactly denote is not as semantically definite as a representation requires. This seems instead to depend largely on the *context* in which a musical symbol occurs and on the *subjects* dealing with it.

Take again the C major chord. Should it occur in the context of a musical system different from the tonal one, it would still instantiate some

³ Formalism in philosophy and aesthetics of music is a theoretical stance traceable back at least to [Kant 1781] (see also [Meyer 1956, 3], [Fubini 1964, Ch. 12, 13], [Kivy 2002, Ch. 2, 4]). It was later systematized especially by [Hanslick 1854] and [Gurney 1880].

expressive property, yet surely not cheerfulness as in a tonal environment. Still, should one not know that *Pacific 231* is not only the title of a musical work, but also the name of a steam locomotive from the 20s, only hardly would she take Honegger's composition as representing the movement of a locomotive. Or, should one hypothetically change the title of Schubert's work to *Gretchen auf der Autobahn*, it would perhaps no longer "represent" the monotonous spinning of a spinner, but rather the hypnotic flowing of highway stripes while driving.

Similarly, depending on the cultural background of the subjects (see e.g. [Becker 1986] and [Patel 2010, 300-301]) and on their individual emotional state at the time of listening (see e.g. [Kivy 2002, Ch. 7]), the alleged extra-musical semantic content of music may variate. For instance, for a listener who happens to be not acquainted with tonal system and Western music in general it would be impossible to recognize, say, the cheerfulness of a major chord.

But let us admit that musical symbols could represent the way musical representationalism says. Yet, the the fact would remain that a symbol that represents, generally, not only merely denotes something, but also characterizes it in some interesting way and *predicates* something about it (see e.g. [Scruton 1976] and [Kivy 2002, 100-101]). Now, as said in [Section 2.1.1], music (metaphorically) instantiates expressive properties and (metaphorically) expresses things. Hence, music always *refers* (or is recognized as referring) to emotions or other objects and facts (regardless of which ones). Yet, beyond such reference, music lacks any means for characterizing them in some semantically interesting way. Therefore, it cannot be said to be able to represent them, but namely at most to express them – for a symbol to express something, in fact, only reference is required, without predication (see e.g. [Scruton 1976]).

Thus, if there is no stable semantic relation between a musical symbol and a definite extra-musical content, if the symbol cannot predicate anything about the extra-musical properties, objects or facts it refers to, it cannot be said to represent them. If so, music is devoid of extra-musical meanings and this brings us close to formalism, which regards music meaningful primarily in virtue of its structure and of the properties this

structure instantiates.

Yet, I do not properly endorse a strict formalism position about the problem of musical meaning. Firstly, although I take the formal properties of music as the semantically prior ones, I do not exclude that also other non specifically musical properties literally or metaphorically instantiable by music may perform a structural function and thus be semantically significant – as pointed out before ([Section 2.1.1], [Footnote 2.1]), in fact, I remain relatively neutral about the ontology of the formal properties of music. Furthermore, differently from a strict formalist, I do not deny the fact that music instantiates expressive properties and is capable of expression. Yet, not like a strict musical representationalist, I admit musical expressiveness and expression *only* in a *metaphorical* sense.

The problem of musical meaning is a difficult and hotly debated one, and penetrating it more deeply is a task we do not indeed need to undertake here (see e.g. [Dowling 2021]). Yet, what has been said until now can help us in answering our initial epistemological question about the cognitive content of music: what is there in music primarily to understand, if something at all?

As pointed out before, a musical representationalist would answer this question by identifying in what music represents (generally, specific emotional states, objects, situations, etc.) its primary cognitive content. Disregarding or not recognizing it would amount to not understanding, or only to misunderstanding, music. Yet, we saw that music cannot represent, but only express, and that its meaning lies essentially in its structure. If so, the most plausible answer to the question about the cognitive content of music is a quasi-formalist one. Above all, music *does* have a cognitive content, something that can be object of understanding, and this content cannot be but primarily the musical structure.

Musical understanding has one of its most important sufficient and necessary conditions in grasping the structure of music and the properties this structure instantiates. One who does not even minimally grasp the structure of a musical piece (this is namely also something coming in degrees [Section 1.1]) indeed fails to understand it – and very probably also what the music expresses. Still, music remains understandable indepen-

dently of the emotions, facts, etc. it is expressive of. The expressiveness and expression of a piece may also change with the context, be not recognized or even intentionally disregarded. Yet, the piece would remain understandable, and this principally in virtue of its structure (see also [Scruton 1976], [Patel 2010, 301]). Grasping what music (metaphorically) expresses is, in other words, not necessary (nor sufficient) for musical understanding. Nevertheless, I do not claim that musical expressiveness and expression are cognitively completely irrelevant. As for other kinds of understanding, also musical understanding comes in degree [Section 1.1], and grasping what music expresses, *in addition* to its structure, corresponds to having a musical understanding of higher degree. One who does not only grasp the musical structure of the piano part of Schubert's *Gretchen am Spinnrade*, but also knows that it metaphorically expresses the movement of a spinning wheel, can be surely said to have an understanding of the piece of higher degree than that possessed by one who instead just grasp how the piece structurally works.

2.1.3 Is Music propositional?

In [Section 2.1.2] I defended a quasi-formalist position about music by claiming that the meaning of a given musical composition primarily lies in its structural and formal properties. I denied any representational character of music and claimed that music is just able of metaphorical expression. I finally endorsed the thesis that both the structure of music and what it expresses constitute, albeit with different relevance, the cognitive content of music, i.e. the object of musical understanding. What we have to do now is to examine whether the cognitive content of music has a propositional nature or can be at least propositionalized⁴.

In [Chapter 1] we referred to propositions principally considering following aspects of them. Propositions generally represent the *meaning*, i.e. the semantic content, of the sentences of a natural language. However, as extra-linguistic abstract entities, they can also be expressed by other

⁴ Unless otherwise stated, our focus in this section – and in general in this work – will be exclusively on *instrumental* or *absolute* music, i.e. music associated with any verbal and linguistic elements.

kinds of symbolic and representational systems. Propositions are moreover the primary bearers of truth values to the extent that, when true, they are generally regarded as reflecting a state of affairs or fact actually obtaining in the world. Propositions constitute the object (or content) of beliefs, desires, judgments, knowledge and other *propositional attitudes*. As such, propositions are finally sharable and transmissible among different subjects for example by testimony.

At first glance, it seems impossible to think that music is propositional in the sense just described above. Take for example a musical progression or a cadenza. A structural element of music like that makes no statements, and can signify, on a (quasi-)formalist account, nothing but itself: its meaning is an intrinsically musical one. This meaning cannot be true or false of anything. A conjunction or implication between it and another progression or cadenza (or between it and a proposition) has no truth value. That element cannot be backed by something like “musical reasons” which may convince you to accept it justifiedly as the content of your beliefs. It cannot be transmitted to another person by testimony. In other words, it is not propositional and, if so, musical understanding is not propositional either, in that it depends principally and necessarily on a content – musical structure – which is namely non-propositional.

As pointed out, however, music also (metaphorically) refers to emotions and other extra-musical objects, facts, etc. If so, one may think, what music refers to in this sense may be propositional. Now, several authors highlight that propositions not only denote, are about, objects and states of affairs, but also *predicate* something about them (see e.g. [Hermerén 1988], [Grzankowski 2013]). For example, the proposition expressed by the sentence “The grass is green” not only is about the grass and the color green. It also predicates the greenness of the grass [Grzankowski 2013, 1123-1124]. Yet, as we know from [Section 2.1.2], music can only express. Therefore, it cannot predicate anything about what it refers to. If so, it lacks a necessary characteristic of propositionality and cannot consequently be propositional.

Should we however not accept predication as a necessary requirement for propositionality, and therefore regard what music expresses as possibly

propositional, the fact would still remain that, even if expressiveness and expression are also cognitively relevant, musical understanding primarily depends on grasping the structure of music [Section 2.1.2], which is, as said, only hardly considerable as propositional.

One may however object that the cognitive content of music, both in its structural and expressive component, is not propositional, but can be indeed *propositionalized*. After all, this is what you are required to do for example in the final exam of a music analysis course. You cannot simply answer the question “What structural function performs that succession of chords at measure 10?” simply by playing it on the piano in front of the commission, or by notating it on a sheet, all this without saying a single word. In so doing, you would surely fail the exam. Instead, you are required to assert facts about the succession of chords, to characterize it, etc., which amounts to nothing but expressing propositions. However, as pointed out already in [Section 1.1.2] with [Elgin 2017b], such an operation of capturing musical meaning into propositions would not quite correspond to a propositionalization, but rather to simply embedding non-propositional items into propositions. The non-propositional meaning that succession of chords expresses would continue «to do [its] cognitive work independently of such embedding, and [is] only worth embedding because of [its] prior cognitive status» [Elgin 2017b, 207].

The problem of musical propositionalization brings about another important issues for our discussion about the propositional (or non-propositional) character of music, namely that of the *untranslatability* of music (see e.g. [Patel 2010] and [Arbib 2013]).

Not only is music untranslatable in a specifically musical sense. Imagine, you operate following musical “translations”: playing a Bach’s harpsichord fugue on a piano, arranging the *Macarena* theme in Wagnerian style, orchestrating a Chopin’s prelude, transcribing a Rossini’s aria for cello. Since the primary meaning of music is a specifically musical one, such musical translations would have as a result a musical composition which, having a different structural form, would also have different meaning. In music, «the sign is not absorbed by the signifying process, it is not transparent to the signified» [Treitler 1997, 34]. In music the sign *is*

the signified, the meaning. And transforming the sign, for example by a musical translation, corresponds to transforming the meaning too.

Moreover, music is untranslatable also in a more common, linguistic, sense. [Hanslick 1854, 72-73] for example writes: «In music there is both meaning and logical sequence, but in a musical sense; it is a language we speak and understand, but which we are unable to translate» into any other language. The other way, this correspondingly means that in music, a meaning «cannot be expressed in words and subsequently translated into sounds» and musical symbols. It is in other words impossible to express the *musical meaning* in a way, in a language different from music itself. Nor it is possible to formulate musically meanings we commonly express in a natural language.

The latter sense in which music is untranslatable is the most interesting for us. In fact, if we take language, with [Wittgenstein 1922, 22]⁵, to essentially express propositional meanings, it follows from the mutual untranslatability between language and music, i.e. from the lack of a shared semantic content between the two, that music cannot in any case possess a propositional semantic nature. Within a formalist (and quasi-formalist) framework, musical meanings should be definitely regarded as non-propositional.

But consider we were to admit for language, against Wittgenstein, the possibility of expressing also non-propositional meanings⁶. Or similarly, consider the possible mutual translatability between music and other symbolic systems which are different from natural language and mostly regarded as non-propositional⁷. If we decide not to deviate from the formalist thesis of the absolute untranslatability of music and of the absolute specificity of musical language, logically, it would become also impossible to affirm that music possesses a non-propositional semantic nature either. Being absolutely untranslatable, music would share with any other language or symbolic system different from itself neither propositional, nor non-propositional semantic contents. Its meanings would

⁵ See in particular *Proposition 4*, «The totality of propositions is the language».

⁶ See e.g. [Russell 1940] and non-cognitivism in ethics.

⁷ See for example the case of dance (described by [Elgin 2017b, Ch. 10]) or, arguably, of abstract art.

really count as specifically musical and as such not subsumable under the categories of propositionality and non-propositionality.

Indeed, such extremization of the formalist untranslatability of music, which would properly have as a logical result the denial of the non-propositional character of music, seems to me too controversial and unreasonable. After all, as [Levinson 2003] drawing on Wittgenstein also stresses, music is evidently and undeniably a form of thought. Hence, stating that music is subsumable neither under propositionality nor non-propositionality corresponds to denying this simple evidence, since it is difficult to imagine some “third kind” of thought, beyond the propositional and non-propositional one, music might be the expression of.

Rather, it appears more reasonable to restrict the untranslatability of music to the cases of propositional symbolic systems. Since music cannot represent anything extra-musical and cannot convey propositionalizable thoughts about what it may refer to or metaphorically express, music is not propositional and cannot be “translated” into propositions. The untranslatability of music does not however apply in those cases in which non-propositional forms of thoughts and symbolic systems are involved. If music is not propositional, it can be fairly regarded as non-propositional, *tertium non datur*. Therefore, it is plausible to think that some form of “translation” of music into non-propositional languages or into other non-propositional symbolic systems like abstract art or dance may be also possible⁸.

Yet, investigating this possibility for acquiring a further proof of the non-propositionality of music is something that cannot be undertaken here. Such an investigation would in fact firstly require us to perform an analysis of the cognitive nature of the target languages and target

⁸ [Kivy 2002, 91-92] and [Levinson 2003, Par. 2.11] seem for example to admit such possibility. As informal hint to the reader, we mention Anne Teresa De Keersmaeker’s choreographic adaptations of Steve Reich’s phasing music as a (successful) example of non-propositional translation of music – see in particular *Come Out from Fase, Four Movements to the Music of Steve Reich* (1982). Another example of this kind might be that of Iannis Xenakis’ translation of architectural into musical work ad vice versa – see in particular *Philips Pavilion* at Brussels World’s Fair of 1958, whose design has been «pin-pointed» [Xenakis 1992, 10] from *Metastasis*, a musical work – perhaps his masterwork – premiered in 1955 (see also [Xenakis 1971]).

symbolic systems similar to that just performed here in relation to music. Only once the non-propositionality of such languages and systems will be ascertained as well, the concrete modes of non-propositional musical translation could be described.

Thus, rather than deepening the problem of music translatability we choose to carry on to the central question of this chapter. Now that we know from [Chapter 1] that also non-propositional instances of understanding are possible, and now that we are also sufficiently convinced of the non-propositional cognitive nature music, we can eventually examine in the following section how music in particular can be object or vehicle of understanding, and that by means of the concept of exemplification.

2.2 Musical Exemplification and Understanding

As pointed out in [Chapter 1], non-factualism accepts both propositional and truth-apt, and non-propositional and non-truth-apt symbols as epistemologically adequate and valuable for the sake of the understanding of the objects they refer to.

In this section I will examine in particular *how* non-propositional and non-truth-apt symbols can advance understanding, and I will claim that they can do this by exemplification. In this way a possible solution to the second problem of [Strategy (ii)] will be given.

After introducing the concept of exemplification in general terms in [Section 2.2.1], I will refer to the specific case of music as a non-propositional and non-truth-apt symbolic system capable of advancing understanding by exemplification [Section 2.2.2]. The mechanism of musical exemplification and musical understanding will be thus described in details.

2.2.1 The Mechanism of Exemplification

It is worth introducing the concept of exemplification by starting from a related but distinct mode of reference, namely that of representation [Goodman 1968, 52], some aspects of which we already discussed previ-

ously.

What is generally meant by the word “representation” is, on the one hand, the propositional or non-propositional symbol – or system of symbols – staying in a denotative relation to a certain object, and on the other, the relation itself existing between representing symbol and represented object⁹. Thus a drawing of a cat represents a cat, the Marshall’s diagrams represents the relationship between supply and demand in economic systems and the wave function represents the physical state of a quantum system. Drawings, diagrams, equations, etc. are representation *of* the objects they denote [Elgin 2008, 2].

Now the question may be raised, what is necessary for a given symbol to serve as a representation of something and how representation concretely works. As observed, the symbol must not be necessarily propositional and truth-apt. But even more importantly, it must not necessarily resemble the objects it denotes (see [Goodman 1968], [Frigg 2002], [Suárez 2003], [Elgin 2008]). Thus, everything can virtually represent everything, and what is therefore really needed for symbols to serve as representations is, according to one of the most widespread (and debated) theses, just a form of conventional *stipulation* (see e.g. [Goodman 1968], [Callender and Cohen 2005], [Elgin 2008]). For x to be a *representation of y*, what is needed is a stipulation setting up a (stable) relation between x and y .

Moreover, if a symbol has to serve as a representation, it is important that it exhibits the ability to *denote*, regardless of whether it has actual denotata or not. In fact, there is the possibility of symbols without denotatum [Russell 1905]. But having a denotatum is in general not a necessary requirement for symbols to be representations. If y happens to be not real, and therefore x to have no actual denotata, under the given stipulation, x is said to be not a *representation-of y*, but a *y-representation*. Since our Jerry is (hopefully) a fictional character, a portrait of him has no actual denotata, it is a Jerry-representation. But

⁹ We disregard other non-epistemic meanings of the word “representation” (such as the political one) and accept description beside denotation as a possible mode of representation: in literature, for example, we gain a representation of a given object only through a (verbal) description of it (see e.g. [Pitkin 1967], [Goodman 1968], [Elgin 1983], [Elgin 1993], [Elgin 2008], [van Fraassen 2008]).

the portrait does not fail to represent him, since we know that it belongs to a category of symbols, humans portraits, which are known to exhibit, in most cases, a denotational ability. After all, if you never heard of Jerry, only hardly would you discern whether the portrait posses an actual denotatum or not. Hence, as a denoting symbol, the portrait performs its representative function independently of the actuality of its denotatum.

But things can become even more complicated. As happens for caricatures or even scientific models (see e.g. [Hughes 1997], [Frigg 2008], [Elgin 2009], [Frigg 2010], [Frigg and Nguyen 2017],[Elgin 2017b], [Frigg and Nguyen 2020]), for example, a symbol can represent something as something else. So statistical mechanics represents the the possible microstates of a thermodynamic system as points of a mathematical phase space, and for some reason did Jerry decide to represent his grandma as a platypus on her birthday card. When a denotative symbol x *represents* – via stipulation – y *as* z – whereby y and z can be actual or not – we speak of *representation-as*, or of x as a z -*representation* of y – i.e. a representation that as such possesses no actual denotatum (nothing *similar* to a mathematical phase space of thermodynamical microstates or to a platypus-looking elderly woman really exists).

If we make, by stipulation, a symbol without actual denotatum a representation of a certain object (representation-as), we do this for a precise implicit reason. The choice of taking x for representing y *as* z gives us in fact the possibility of highlighting, by means of z , some features of y which are particularly interesting or relevant, or that would be otherwise difficult or impossible to discern. In other words, by representing y *as* z , i.e. by shedding through z light on such peculiar features of y , we implicitly gain an epistemic access to them, which in turn corresponds to find a way for acquiring a broader *understanding* of y as a whole. So, for example, Jerry's representation of his grandma as a platypus is implicitly aimed at highlighting the fact that, like a platypus, Jerry's grandma is a semiaquatic mammal with excellent swimming skills, or that, like a platypus, she was born indeed from an egg. By his representation we epistemically access those features the grandma possesses and we understand better the person we are going to meet tonight as birthday

guests – which is perhaps particularly important for taking our precautions before interacting with such a strange individual.

Now, a to some extent similar mechanism is also at the basis of another mode of reference: namely that of *exemplification*¹⁰. In representation-as what is required from a symbol is simply *reference* to an object as having certain features (features the symbols imputes to it). Instead, in exemplification the symbol (termed *exemplar*) must also itself *instantiate*, i.e. *possess* the features it imputes to an object. It is not enough for x simply to refer to y as having feature z . What is additionally required for x to serve as an *example* or *sample* of y is *instantiating* itself the feature z it imputes to y . More formally, a symbol x works as an exemplar for y iff

- (a) x refers to z of y , and
- (b) x instantiates z as well.

So a drawing representing a cat as a cat or as something else is not a case of exemplification, since notoriously drawings are not cats themselves, and in any case they possess the features represented of their object. On the contrary, the set \mathbb{N} of the natural numbers is an example of mathematical entity since it possesses itself as exemplar the features it imputes to the object it refers to – namely mathematical entities. More in general, this hence means that when something refers to a feature it itself possesses, it can exemplify it.

Reference and instantiation are thus necessary requirements for exemplification. This is also the reason why exemplification does not depend on a conventional stipulation. You cannot conventionally take a garden gnome as an example of a red dwarf. Exemplification in this case works if and only if you take a red dwarf like Proxima Centauri as an example of red dwarf. If not stipulation, what makes an example is simply and directly the *use one intentionally* makes of something as an exemplar in an *exemplificative context*. When something is used as an

¹⁰ Initially introduced by [Goodman 1968], [Goodman 1978], [Goodman and Elgin 1988], the concept has been further articulated especially by [Elgin 1993], [Elgin 1996], [Elgin 2008], [Elgin 2017a], [Elgin 2017b].

exemplar for exemplifying something, we directly speak of that as a case of exemplification. So the teacher exemplifies the notion of mathematical entity by simply taking *a* mathematical entity, the set \mathbb{N} of the natural numbers, and using it as an example of mathematical entity, whereby the set \mathbb{N} can properly work as an exemplar, since it both instantiates and refers to features of the exemplified object.

An exemplar can theoretically exemplify all features it instantiates. The set \mathbb{N} of the natural numbers may thus exemplify the features of being a mathematical entity, an infinite set, a set with cardinality less than 5, the number set more easily explainable to a child, etc. What it cannot however do is exemplifying all its features *at once*. In fact, within a single exemplificative context, an exemplar can exemplify only one or some of the features it instantiates. If we were to give an example of infinite set, we would take \mathbb{N} just as an exemplar of infinite set disregarding its feature of being easily explainable to a child. If we were conversely to give an example of easily accessible pedagogical content, we would take \mathbb{N} as an exemplar of such content downplaying its feature of being an infinite set or of having cardinality \aleph_0 . This means that exemplification is *selective*. An example works only if some of the features instantiated by the associated exemplar are overlooked in favor of others which in that given exemplificative context are considered more salient, relevant or appropriate for the sake of construing an effective, working example.

And exactly selectivity is, from the epistemological point of view, the key point of exemplification. By selecting aspects and features of the object of reference, examples make them in and through the exemplars evident and clearly discernible. In so doing, they provide epistemic access to them, which can in turn lead also to a deeper understanding of the object itself as a whole.

Exemplars, as seen above for all referential symbols, need not necessarily to be propositional and truth-apt. An exemplar can exemplify something and serve its epistemic function for understanding also without “saying a word”. If Jerry is convinced of the fact that no mammals lay eggs, showing him a platypus laying one is enough for allowing him to improve his understanding of zoology. The platypus constitutes an exam-

ple of oviparous mammal and as such it works epistemically for Jerry in a non-propositional and therefore non-truth-apt way. It *shows* him that its understanding of zoology is partly wrong, it indicate him the way to improve it, and it does that by merely manifesting itself, without verbally asserting truths and without giving explanations (see [Wittgenstein 1922], [Elgin 2017b]).

One may also think to propositionalize the epistemic contribution exemplars of this kind can offer, in the way: “This exemplar falsifies Jerry’s zoological convictions: [insert platypus]”. Yet, as pointed out already before, this does not quite amount to propositionalizing the epistemic content of the exemplar, but rather to embedding it into a proposition, whereby the exemplar continues to work epistemically independently of such embedment [Elgin 2017b, 207].

It is not necessary for an exemplar to instantiate *literally* the features it exemplifies. An exemplar can in fact also *metaphorically* possess the features it exemplifies. If so, exemplification too becomes metaphorical, as for instance in the case of Picasso’s *Guernica* exemplifying sorrow, Euler’s identity exemplifying the prominence of human intellect or Henry Purcell’s *Music for the funeral of Queen Mary* exemplifying sadness (see [Section 2.2.2]).

It is also possible for an abstract exemplar to exemplify features of concrete, physical objects. Since the instantiation of the exemplified features by the exemplar is a necessary condition for exemplification, it seems quite counterintuitive that something abstract can instantiate physical features. How then does this kind of exemplification work? Let us explain this briefly. Consider for instance a Fourier’s trigonometric series as the abstract exemplar and a complex tone in acoustics as the exemplified object. In order for the mathematical model to work as an example for physical phenomenon, the former must instantiate itself the features it highlights of the latter, say, a given sound frequency. Yet, being an abstract entity, the model fails to do so, and thus to exemplify as well. At a closer look, however, what the model exemplifies is not a physical property of the target object, but an abstract, mathematical one, namely the frequency value correspondent to that sound. As such it

can be instantiated both by an abstract mathematical model, and by a physical system. In this way, the former can exemplify the latter.

Being symbols, exemplars require interpretation. And since, as pointed out before, the same symbol can exemplify different features among those it instantiates depending on the different exemplificative contexts in which it is used as an exemplar, the interpretation depends a lot on the context of the exemplification. The more the context is regimented, the easier the interpretation is, as the connection between exemplifying symbol and exemplified object or property is more evident or more easily detectable. It is clear that interpreting a platypus as an example of oviparous mammal is easier in the context of a zoology class than in that of a naturalistic walk in which one luckily bumps into it for a while. The range of objects for which the platypus can serve as an exemplar for is considerably more restricted in the first case than in the second, where it could exemplify many of its actually or potentially possessed features, from being an animal, a carnivore, to being cute, shy.

Deliberate context restrictions are obviously always possible if someone or something is ostensibly showing something as an exemplar for a given object. This clearly leads to a facilitation of the exemplar interpretation work. So a qualified guide speaking about the fauna populating the place you are walking in and describing how a platypus lives may help you in interpreting it rightly as an example of oviparous mammal. A similar facilitating role in interpretation is played also by one's background assumptions and knowledge. If one happens to be a zoologist or a person who just read several books about the platypus, one may be able to interpret the exemplar that way quite directly by any hint. A lack of appropriate background assumptions and knowledge may conversely lead not only to wrong interpretations, but also to the impossibility of interpreting or even recognizing an example, as happens say for a person approaching medieval religious art without an even minimal knowledge of saint symbolism.

On context and interpretation eventually depends the ability of an exemplar to function cognitively in the appropriate way. One who fails to recognize or rightly interpret an exemplar has obviously no possibility

to access epistemically the features that exemplar exemplifies.

Similarly, one who use an exemplar or set up an exemplificative context in the wrong way fails to make an example cognitively valuable as a source of understanding. If your aim is that of making your zoology students understand what an oviparous mammal is, how it behaves, etc., you would obviously fail to do so if you bring a horse as an example for that.

Exemplars, in fact, must be chosen by the subject intentionally with the aim of providing epistemic access to the objects they exemplify (see [Textor 2008], [Vernazzani 2021, 13-15]). In order to work epistemically in a successful way and provide epistemic access to the exemplified features of an object, a subject must intentionally build the exemplar in the proper way, for instance by downplaying other irrelevant or confounding features of the exemplar which would hinder the recognition of the exemplified ones. As in the case of the platypus, the intentions of the subject may possibly be implicit: zoology students have enough background knowledge for taking the platypus as an example of oviparous mammal; moreover the exemplificative context is restricted, since the platypus can serve there as an example only of relatively few things. In some other exemplificative contexts – like for instance that of the walk – however, an explicitation of the intentions grounding an exemplification may be crucial for it to work epistemically in the right way.

These considerations eventually seem to answer in a theoretically quite satisfying way the second problem of [Strategy (ii)], namely that of how non-propositional understanding works. Non-propositional symbols can advance understanding by exemplifying. For instance, pointing at a platypus for building an example of oviparous mammals may enhance one's understanding of zoology, whereby this exemplar is neither propositional nor, as seen, propositionalizable. Exemplification is that cognitive mechanism by which it is possible to epistemically access features of a given object which are normally not sufficiently evident or not easily detectable. In so doing, exemplification afford understanding of that object. Since even non-propositional and non-truth-apt symbols can exemplify, by means of exemplification, non-propositional and non-truth-apt symbols

as well can afford understanding.

In the next section we will deepen our understanding of non-propositional understanding by analyzing in particular how a non-propositional symbolic system like music can advance understanding by means of exemplification.

2.2.2 Exemplification in Music

In [Section 2.1] I showed that music is non-propositional and therefore non-truth-apt. If so, according to what we said in [Section 2.2.1], music as a non-propositional and non-truth-apt symbolic structure, is theoretically capable of advancing understanding by means of exemplification. In the following, I will show how music can concretely exemplify and so advance understanding.

In the previous section we observed that for a symbol to serve any exemplificative function two requirements are primarily needed: the symbol must *refer to* the features of the object it purports to exemplify; moreover, it must also itself *possess* or *instantiate* those features. Only by fulfilling both requirements at once a symbol can be properly said to exemplify the features of the object, to serve as an example for them. We also noticed that exemplification is *contextual* and *intentional*: what a symbol serves as an example for, which of its instantiated features it refers to, what epistemic function it performs, basically depends on the context in which one intentionally uses that symbol as an exemplar. The same symbol x may be used in context c_1 as an exemplar for feature α , and in context c_2 as an exemplar for feature β . Or still, within the same context c_1 , symbol x may be used as an exemplar for different features α , δ , γ , etc. [Vernazzani 2021, 14].

Now, all that of course applies to music as well. Musical symbols – from single notes, to chords, themes and entire compositions – can serve, in a given context, according to implicit or explicit intentions, an exemplificative function, can be used as exemplars for something. But *what* exactly can music exemplify? The answer appears simple: of course, all those features it both instantiates and can refer to.

In [Section 2.1], we noticed that music instantiates formal, expressive,

and aesthetics properties. Whereas the formal properties are literally instantiated, the expressive – and perhaps also the aesthetic – ones are metaphorically possessed. Furthermore, always in a metaphorical way does music instantiate those features of the extra-musical objects, processes, facts, etc. it may be recognized to express (like for example the movement of a spinner or of a locomotive).

Now, in both cases, music can also refer to the properties and features it literally or metaphorically instantiates. In fact, on the one hand, does music always – (quasi-)formalistically – refer to itself and to its structure, in which primarily its meaning lies. On the other, expression has reference (without) predication as a necessary condition [Section 2.1.2].

In being able to instantiate the features they refer to, musical symbols thus satisfy the necessary requirements for exemplification. At the formal level, the reference is intrinsic (intra-musical) and the instantiation literal. At the expressive one, the reference is extrinsic (extra-musical) and the instantiation metaphorical. From this, we can distinguish for convenience two modes of musical exemplification, literal or metaphorical, depending on the two modes in which musical symbols can refer to and instantiate properties.

Literal Musical Exemplification

As to the first mode of musical exemplification, music is taken to exemplify specifically musical properties, i.e. properties intrinsic to its formal structure. Take for instance a harpsichord sonata by Domenico Scarlatti. Among the different intrinsic musical properties it may serve as an exemplar for, there is that of being a bithematic binary sonata. On the one hand, the sonata refers to the property of being bithematic and binary in that, as all musical symbols, from the structural point of view, it is a self-referential symbol denoting nothing else but itself. On the other hand, it also instantiates that property, being namely a bithematic binary sonata. If the sonata both refers to and possesses the property of being bithematic and binary, it can be used as an exemplar for it.

In the same way, the sonata may exemplify all the other its intrinsic properties, i.e. all musical and formal properties it both (literally) instan-

tiates and denotes as a musical symbol¹¹. It may exemplify, for instance, the property of being a piece in a major (or minor) key, of having a quite short duration, of complying with the composing standards of baroque counterpoint, etc.

Of course, literal musical exemplification depends on the exemplificative context and on the intentions of the subject who sets up the example. Just in the context of a music composition class, for instance, the sonata by Scarlatti may potentially refer to, and thus exemplify, more than one of the formal properties it instantiates. Yet, as we know from [Section 2.2.1], examples must be selective in order to work properly, and exemplars cannot exemplify all their properties at once. What makes then the sonata refer to the property of being bithematic and binary and not to, say, that of being in a major key?

A possible way to answer this question is that of taking into account the specific *intentions* of the subject in using the musical symbol as an example for that property [Textor 2008]. Yet, the symbol's being intentionally used for exemplifying something is in some cases insufficient for explaining how the symbol refers exactly to the property it tries to exemplify [Vernazzani 2021, 14-15]. Something more is needed. The exemplificative context must be in fact implicitly or explicitly restricted, i.e. the range of properties the musical symbol may serve as an exemplar for. This means that, in the exemplar, the exemplified property must be *highlighted*. Correspondingly, other properties which are irrelevant and not functional to the exemplification must be necessarily *downplayed* [Elgin 2017b].

Highlighting the exemplified properties, however, should not be intended here as a mere *calling attention* to them. An exemplar is in fact cognitively valuable for the sake of understanding only if a subject can convey *by it* an *epistemic access* to the exemplified properties [Vernazzani 2021, 15]. A music teacher who uses the sonata by Scarlatti as exemplar of bithematic binary sonata form does not simply call attention to the sonata's referring to the musical form. She instead uses that exemplar in order to convey her students an epistemic access to the musical form

¹¹ On exemplification and self-referential symbols see also [Goodman 1968, 59].

in general, to make them *understand* what a bithematic binary sonata is, to make them able to recognize that musical property also in other musical contexts and objects (e.g. in music by other authors or from other historical periods).

It is important to note that, as a non-propositional and non-truth-apt symbol, the sonata performs its cognitive and epistemic function as an exemplar simply by showing, manifesting itself and namely exemplifying, not by verbally asserting anything. No verbal explanation, no *proposition-alization* is needed in order to connect the exemplar to the exemplified object, the actual sonata to the general musical form, and no explanation is required for understanding the latter by means of the former. All what we need is namely someone or something presenting the sonata to us ostensibly as an example of the musical form and just our careful, epistemically-aimed listening to the exemplar [Elgin 2017b, 188].

Finally, literal musical exemplification also depends on the background assumptions of the recipient subjects. Imagine the sonata by Scarlatti happens to be accidentally played on the radio in a waiting room. Its functioning as an example for, i.e. its referring to, the bithematic binary sonata form, would depend not only on possible intentional context restrictions by, say, the radio speaker giving technical and historical information about the sonata, but also on the background knowledge of the subjects sitting in the room. Undoubtedly, a subject who happens to be an expert of baroque sonata would implicitly recognize the sonata actually played on the radio as an exemplar for the bithematic binary sonata form – provided his (in this case auditive) approach to the work to be active, epistemically and aesthetically driven [Vernazzani 2021]. Conversely, for another subject who spent all her free time in listening to jazz music, it may be surely more difficult to directly take the sonata as exemplifying that musical property.

Metaphorical Musical Exemplification

Yet, music is not able to exemplify just intra-musical features, i.e. the formal properties intrinsic to its structure and literally instantiated by it. It can namely exemplify also extra-musical properties and features it

metaphorically possesses.

Consider for example the famous *Adagio for Strings* by Samuel Barber. At least for the average Western ear, this piece indubitably *expresses* sadness [Howard 2007]. This means that in this precise cultural context it refers to an expressive property – sadness – it can however only metaphorically instantiate. Sadness is in fact an extra-musical property in the sense that it does not belong to that realm of properties music can literally instantiate, i.e. the realm of the formal properties.

If, Barber's *Adagio* both instantiate and refers to sadness, and if in general, with [Goodman 1968, 85], all «what is expressed is metaphorically exemplified», then we can legitimately take the piece as a symbol exemplifying the property, the label of sadness – and indirectly also «the things (including the symbol itself) in the range of that label» [Goodman 1968, 92].

Metaphorical musical exemplification is contextual as well. Depending on the context in which it is intentionally used as an example or simply occurs as a musical symbol, Barber's piece may namely exemplify various properties among those it metaphorically instantiates. If played at a funeral, it may be taken to exemplify sadness. If used as the soundtrack of a funny video, it may be taken to exemplify irony, etc.

It is important to note that on the context depends also whether musical exemplification is literal or metaphorical. The same musical symbol may be used for exemplifying some of its formal properties in a context, and some of its expressive properties in another. For instance, besides metaphorically exemplifying extrinsic properties like sadness or irony in the way we saw, Barber's *Adagio* may be used in a music class for literally exemplifying 20th century musical neo-romanticism, the arch form, the key of *B♭* minor, the use of mixed meter time signature, the use of half cadence at the end of a piece, etc., i.e. things which *intrinsically* and literally belong to it as a musical symbol.

As pointed out earlier, which one of these properties will be exactly denoted by the exemplar (the *Adagio*) depends, besides context, also on the intentions of the subject in setting up the example. A property will be *selected* and *highlighted* on the basis of what the subject aims to provide

to others an *epistemic access* to.

Indeed, like in the case of literal musical exemplification, a musical exemplar providing epistemic access to the extra-musical property it exemplifies, is cognitively valuable in that it affords one an understanding of that property, the possibility to spot that property in different contexts, the possibility of rising questions and reflect further about it. And this, it is worth to notice again, not by saying, but simply by *showing* and namely exemplifying.

The Value of Musical Exemplification for (Musical) Understanding

Thus music can exemplify literally or metaphorically. In both cases, musical exemplification depends on the context. Furthermore, a musical exemplar generally provides epistemic access to the properties it exemplifies. This also means that music, when exemplifies, enhances one's understanding of what is exemplified. Indeed, this is the most interesting aspect of musical exemplification – and exemplification in general. What we have to do now is to examine it more closely and see what does it exactly mean to gain understanding through a musical exemplar and what kind of understanding is that deriving from musical exemplification.

In [Section 1.1], we said that the nature of the object of understanding generally defines the kind of understanding. Then, if music advances understanding by means of exemplification, what kind of understanding we are here speaking about eventually depends on what music exemplifies in a given context.

In literal exemplification music exemplifies properties which it literally possesses, namely musical and structural properties. Take for instance a given musical composition K to exemplify property φ of being a fugue. As an exemplar, K affords epistemic access to φ . So, through K a subject could understand property φ , i.e. what a fugue is. Then, she would understand K itself as individual composition, i.e. why and how K instantiates property φ . Furthermore, she would be able to recognize φ when instantiated by other different compositions (or other kinds of symbols somehow able to literally or metaphorically instantiate musical

properties among which φ). As pointed out by [Elgin 2017b], for a subject – think for example of a young composer – the understanding gained of φ might also take the form of *knowing-how* to apply and manipulate φ to further (musical) ends (see also [Section 2.3]). Or still, it may take the form of raising further questions, and fostering further research, about φ [Elgin 2017b, 216]. Since φ , the object of understanding, is a *musical* property, we would speak of the understanding of φ as *musical understanding*.

In the same way, music affords understanding of those extrinsic (extra-musical) properties it for example expresses, i.e. metaphorically exemplifies. At least in the context of classical harmony, it is common for instance to speak of the alternation between tension and rest as a characteristic aspect of the musical discourse. These are expressive properties which are generally metaphorically instantiated by dissonant and consonant chords. Thus, such musical symbols can not only express, but also exemplify them, which would give to a potential listener the opportunity to epistemically access and understand them in the sense described above.

Now, the understanding one gains of tension and rest from musical harmony is surely definable, on the one hand, as musical. Such expressive properties, in fact, even if not literally possessed by *tonal* music, play an undoubted structural role in it, for example by contributing to setting up the musical discourse. On the other hand, however, the understanding one gains of them via exemplification goes beyond mere musical understanding. Tension and rest are property which can be also metaphorically or even literally instantiated by several other non-musical symbols. A sculpture may instantiate rest, a person may manifest tension, a thought experiment could represent a physical system alternating states of tension and rest. Lot of the expressive, and in general extra-musical, properties metaphorically instantiated and exemplified by music can be found in several other different context, which are often very far from music, or even from the arts in general. This means that the understanding music can afford by metaphorical exemplification is very large in scope and is therefore truly definable as objectual in that it regards entire domains

and aspect of the human and natural world.

It is however not excluded that also literal (intrinsic) exemplification in music may lead to a broader understanding than just the musical one. There are in fact structural properties in music which are literally instantiated by it, albeit not specifically musical. One of such properties is for example that of symmetry. By recognizing, say, *Bach's Canon 1. à 2* from *Das Musikalische Opfer* as an exemplar for (musical) symmetry, a subject may have epistemic access to the general concept of symmetry and understand it to a given degree. This would give her the possibility to spot the same concept in other musical symbols, or in contexts where it happens to be (literally or metaphorically) instantiated even by non-musical symbols – think for example to architecture or mathematics.

In the same way, music may afford even political or historical understanding. With its quest for a musical system without tonal centre, instances of dodecaphonic music, for example, may be recognized as metaphorically symbolizing political values and features such as egalitarianism, democracy, autonomy, etc. So, in the right context, a composition by Schönberg or Webern – which has not to be necessarily thought as an explicitly political work – may provide epistemic access to those features, and give an understanding of them to the subject who approaches it epistemically. This would enable the subject to recognize the political dimension of aspects of the world, of the human life and of society which previously she has been considering apolitically. Equally, she would recognize those values also when instantiated by other musical works, artworks and symbolic structures, like, say, in a choreography by Yvonne Rainer [Elgin 2017b, 217].

In its refusal of the rigid schemes, rules and rationality of tonal harmony, according to which people in the West had only been making music in at least the last three centuries, dodecaphony (like other atonal compositional techniques) represented the musical dimension of what has passed into history as the “crisis of positivism”, which involved between 19th and 20th century also other disciplines from philosophy, painting and the arts, to physics, mathematics, biology, psychology and social sciences. Philosophical irrationalism, abstract art, non-euclidean

geometries, Einstein's relativity, quantum physics, Darwin's evolutionary theory, Freud's psychoanalysis, etc. partly developed as a critical reaction to the positivist ideal of rationality, an ideal of which the tonal system and the nineteenth century music aesthetics were also expression of.

Dodecaphonic music acts as a musical symbol exemplifying this reaction to, this distancing from, positivistic rationality. By epistemically approaching dodecaphonic music in the right context and with the right background knowledge, a subject may gain through it an epistemic access to this general development in the history of Western culture. Aesthetically appreciating how in a work by Schönberg, say, dissonant chords are introduced without preparation, how there is no leading tone resolving to a tonic, would correspond for her also to understanding the fact that the work by Schönberg, like all other works instantiating the same formal properties, distances itself from the rules, the intrinsic rationality of tonal music. This is namely something understandable not only intrinsically at the musical level, but also at the historical one, since music's distancing from tonality is namely a historical event. In other words, in this case the subject would gain from music a historical understanding, an understanding she may even enlarge by recognizing the same historical fact relative to the opposition to positivistic rationality as exemplified elsewhere in the sciences and the arts.

Such historical understanding may surely be incomplete: much is there to understand about the crisis of positivism between 19th and 20th century, and much cannot even be shown by music alone. In hindsight, however, one has to confess that even explicitly historical works are in this sense incomplete. With the words of [Elgin 2017b, 281]: «There is simply too much information about any historical episode to incorporate it all into a coherent, comprehensible account». In fact, an historian is always *selective* in writing history, exactly as a musical symbol must be when it (successfully) exemplifies something. A dodecaphonic work will never be able to show, to exemplify, everything about the crisis of tonal harmony. But it exemplifies, it selectively shows *enough* for providing an understanding of this historical event *at an even minimal degree*.

Similarly to what happens for other symbolic structures as for exam-

ple dance [Elgin 2017b, 216], another way in which music can advance understanding is by raising philosophical questions, both intrinsic to music (“Why do we find music beautiful?”, “What is the meaning of music?”, “Is performance necessary to music? Or does music remain music even if not performed?”), and extrinsic to it, when it metaphorical exemplifies something extra-musical (for example question about the nature of emotions, the relationship between emotion and cognition, the nature of time, etc.). A subject who epistemically approaches a musical composition – i.e. not only for simply enjoying it – generally takes it as a stimulus for further research about the things it intrinsically or extrinsically exemplifies, which is the first step towards a deeper understanding of them.

Finally, it has to be considered that musical exemplification (and thus understanding) depends on interpretation [Section 2.2.1]. As we pointed out, musical works, chords, themes, etc. are symbols that exemplify musical or extra-musical properties making them this way epistemically accessible. As such, we understand them similarly as we understand other symbols: «we know how to interpret the symbol system they belong to» [Elgin 2017b, 218] (see also [Vernazzani 2021]). This means, in order to grasp the cognitive content of music, in order to recognize what a musical work is an exemplar for, one has to possess the right background knowledge.

You would fail to understand a canon by Bach if you never heard of counterpoint. You would surely be able to appreciate it as an enjoyable work to listen to, but only hardy would you take it as exemplifying, say, symmetry. Likewise, you would fail to understand (and maybe also appreciate) a work by Berg if you are not aware of the fact that the work is composed according to a musical style which refuses the canons of tonal harmony. Being aware of that means not only to better understand the work, but possibly also to have the opportunity to better appreciate it aesthetically. From cases like this it is eventually clear that, at least in music, the *aesthetic understanding* and *appreciation* of a work closely depends on one’s *epistemically* approaching it. Aesthetic and epistemic understanding (the understanding one intrinsically has of a work) are, so to say, two sides of the same medal.

Still, you would fail to convey a historical understanding from dodecaphony in terms of renounce to nineteenth century positivist ideal of rationality, if you happen even to know everything about Western history, but just stuck to your traditional Japanese music all the life without hearing a single work from Western tonal tradition. In this respect, there is namely a quite extended evidence of the fact that being aware of the cultural background a musical composition belongs to is essential for understanding what features it metaphorically expresses (and exemplifies) [Patel 2010, 302] (see also [Huxley 1926] and [Goodman 1968, 89-90]).

To sum things up, what I tried to show here is that music can afford understanding by exemplification, both of music itself at more general level, and of various aspects of the world and of human experience. Music – like other arts (see [Elgin 2017b] for the case of dance) and ultimately also science (see [Section 3.1]) – exemplifies, makes salient and explicit features that, when instantiated elsewhere, are difficult or impossible to notice and to discern. In so doing, it calls our attention to them, it underlines their importance, it enables us to recognize them when we encounter them again in other contexts. It renders them epistemically accessible and, in a word, understandable.

When a musical composition is cognitively significant, it generally advances our understanding by revealing something to us. This does not mean that we could gain from music, say, the same historical understanding of event *abc* which a book about *abc* would provide to us. Yet, even the minimal understanding of *abc* music may afford would legitimately count as understanding: understanding does in fact come in degree. To this respect, music would moreover have also the cognitive virtue of stimulating the desire and the interest of a subject in deepening the understanding *abc* initially gained by music.

I do not however claim that every musical composition can do this job, can always epistemically work this way. Some music is banal and deprived of any interesting cognitive content, even at the intrinsically musical level. Interpretation is moreover a quite difficult task: some of our hypotheses about what a piece of music exemplifies, about what it is trying to convey us in terms of understanding, may be surely not sound. Yet, after all, are

maybe other symbols like verbal explanations, informative conversations, or even scientific experiments always interesting and never banal? Do they always lead to sound hypotheses? Still, some music, even when cognitively significant, may remain for us completely senseless. But is the awareness of the senselessness of something, Socratically, already not the beginning of our understanding of it?

Convinced of the fact that music can afford understanding by exemplification, we are ready for considering in the next chapter the main question of this work, which we can now reformulate in more precise terms: can music advance scientific understanding by exemplifying scientific concepts?

Before coming to this, however, let me digressively spend a few words about how exemplification may answer a third possible problem of [Strategy (ii)], namely that relative to the *transmission* of understanding in non-propositional and especially know-how-related domains.

2.3 Exemplification and Transmission of Understanding

Until now I showed how by the notion of exemplification it is possible to explain the functioning of understanding in non-propositional domains, which namely answers the two difficulties highlighted about [Strategy (ii)]. Yet, about this strategy another difficulty is often discussed in the literature, namely one concerning the transmission of understanding in non-propositional contexts and through non-propositional symbols.

Now, we do not need to address this issue explicitly in this work. In fact, we are already answering it: as we have seen in this chapter and shall see in the next one, exemplification is implicitly what makes this transmission possible. However, a further interesting aspect of this problem emerges if one were to consider *know-how* as a (non-propositional and non-propositionalizable) component of understanding [Section 1.1]. In this case the problem of transmission of understanding would come to concern also the possibility of transmitting *understanding-how*, i.e. that (non-propositional) form of understanding relative to tasks, epistemic and

even practical abilities.

As pointed out, the problem of *know-how* in understanding is a still controversial one, and discussing the issue of the transmission of *know-how*-related understanding would primarily require a thorough research about the extent to which it is namely possible to consider *know-how* as a (propositional or non-propositional) component of understanding. Although such research is something which cannot be done here (but perhaps in a next work), I want to expose some heuristic ideas about the issue, mostly derived from my own experience as a musician. So, in the following, I will try to show how the notion of exemplification may answer also the problem of the transmission of understanding in non-propositional, *know-how*-related contexts.

Above all, let us expose the problem more clearly. In [Chapter 1], I denied that understanding is necessarily propositional and therefore entirely reducible to knowledge. If so, it becomes difficult to explain how understanding is generally transmissible. In fact, the notion of *testimony*, used in epistemology for explaining the transmission of propositional knowledge (see e.g. [Lackey 1999], [Faulkner 2006], [Zagzebski 2009], [Carter and Nickel 2014], [Stump 2014], [Leonard 2021]), would no longer work for such a conception of understanding. Testimony itself namely presupposes propositionality: Jerry *knows* that p , after his grandma told him that p , whereby p has to be a (possibly true) proposition. Indeed, the notion would be at most adequate for explaining cases of transmission of propositional understanding (see e.g. [Malfatti 2019a], [Malfatti 2019b], [Malfatti 2020]), i.e. whereby p is namely a proposition. Yet, it seems insufficient for explaining understanding transmission in non-propositional, *know-how*-related domains, i.e. where p is a non-propositional symbol, or especially an epistemic or practical task. Solve a math exercise, lighting a fire, doing philosophy, carving wood, repairing a carburetor, articulating a *detaché* on an eighteen-century pipe organ, etc. In most of these cases, propositional and verbal testimony turns out to be insufficient or even superfluous for an effective transmission of understanding.

Yet, against this, some authors such as [Boyd 2017] and [Grimm 2020] have claimed that testimony may indeed have a role in such cases

for transmitting understanding, at least under particular circumstances. Consider for example a counterpoint teacher explaining four-part fugue to her class¹². In the previous semester students achieved a quite extensive understanding of three-part contrapuntal writing, as the good results obtained in the final exam showed. It is to notice that in the class there are some few individuals with excellent musical skills. Some grew up in a family of musicians, some possess a certain neuroanatomical brain configuration enhancing musical talent, some others are inspired directly by God: this has indeed no real epistemological relevance. According to those authors, in a so characterized context, verbal testimony would be sufficient to the teacher for transmitting to the students her understanding of four-part fugue. At least for the musical geni of the class this transmission of non-propositional understanding by verbal testimony should be immediately effective. By the time the teacher stops speaking, they should have already achieved a satisfying understanding of four-part fugue and could exhibit it directly by composing one without errors.

Thus, the effectivity of testimony as a means of transmission of understanding depends on contextual factors, as well as on the understander's background assumptions and degree of understanding relative to the given object¹³. As such, testimony should be therefore not excluded from the start, when it comes to explaining how understanding is transmissible.

Indeed, I do not want to refuse or underestimate such compelling considerations. In fact, I agree especially with [Boyd 2017] on the importance of context and background assumptions in relation to the efficacy of testimony as a means of transmission of understanding. Yet, as [Boyd 2017] himself points out, this efficacy is limited just to cases like that described above. Take for example the less skilled counterpoint student of the class, or one who does not have an even minimal understanding of counterpoint. Would the teacher's verbal testimony be here sufficient for a successful transfer of understanding of four-part counterpoint? It seems fair to answer this question negatively. Surely would verbal testimony still

¹² Equivalently, you can also consider the scientific example of a teacher explaining, say, compound pendulum to a class already possessing a good understanding of simple pendulum.

¹³ It is fair to believe that this conclusion also applies for propositional understanding.

play a role in the process, but not a leading one. Something else would be additionally required for a successful transmission, something in relation to which testimony would perform just a complementary function.

[Zagzebski 2009, 145-146], who instead categorically denies the possibility of a direct transmission of understanding by testimony, believes that this may be possible only indirectly in the form of «recreat[ing] the conditions that produce understanding». In non-propositional, *know-how*-related contexts of understanding, after all, no alternatives are available: for ensuring a successful transfer of her understanding of four-part counterpoint to *all* potential understanders, the teacher should rely not so much on direct testimony, treating the understanders as passive recipients. Rather, she should tend to recreate those conditions under which it is possible for the potential understanders to *actively* understand four-part counterpoint, because this is the only way in which the transmission of *know-how*-related understanding can work.

At this point the question however arises, how these conditions may be recreated. In this regard, many have referred again to the notion of explanation. The teacher could provide you an explanation of the task you are trying to understand. This would allegedly amount to providing you the conditions under which you can actively acquire an understanding of them.

However, in [Section 1.2] we already observed that explanation is in general not sufficient, nor necessary, for understanding. Finally, if one (arguably) takes explanations as being basically propositional, the appeal to them as possible indirect means of transmission of understanding would undergo the same criticisms made before about testimony. Verbal explanations may in fact work just in cases of propositional understanding, and in cases of non-propositional understanding only under some specific, context- and subjects-related, circumstances.

Indeed, I partly disagree with [Zagzebski 2009] on absolutely denying any importance to testimony in understanding transmission. As said with [Boyd 2017], in fact, I grant to testimony a *potential* function in transmitting understanding at least under those specific circumstances in which subjects, for example, already have appropriate background

knowledge of the object of the transmission. However, I must agree with [Zagzebski 2009] on the fact that, *in most cases*, understanding cannot be transmitted *in a successful and effective way* directly by testimony, but only indirectly by (re)creating the appropriate conditions under which it is possible for a potential understander to achieve the understanding one is trying to transmit.

Now, I believe that the most adequate tool for (re)creating such conditions is by exemplification. Let us take again the example of the counterpoint class. The only way in which the teacher could bring, *with certainty, each* member of the class, irrespectively of his or her background understanding of the subject matter, to the achievement of a satisfying understanding of four-part fugue, is namely that of using examples. So, for instance, in order to convey effectively to her students an understanding of, say, how to introduce the last countersubject entry at the end of a four-part fugue exposition, the teacher could simply *show*, as an example for that, how Bach does it in the fugues of *Die Kunst der Fuge*. Or still, the most effective way for a music performance teacher for transmitting an understanding of *how* to properly articulate *inégalité* in French baroque harpsichord music is that of giving examples for that by playing it himself and asking you to try to simulate what he is doing. At some point, you will be able to play *inégalité* on your own, which in other words means that you will have understood it.

In cases like these, examples are generally the only means for conveying understanding – there is no way you will properly understand four-part fugue or *inégalité* just by reading a handbook. Especially in non-propositional, *know-how*-related domains, examples (indirectly) provide those conditions under which the potential understander can actively acquire understanding of a given task. Examples are chosen or constructed by an agent who already possesses an understanding of the object the examples refer to. Their aim is that of (re)creating the conditions under which potential understanders can understand *through them* the object whose understanding the agent had initially intended to transmit. Therefore they act as an indirect means of transmission of understanding. Verbal testimony or verbal explanations, *per se* generally

insufficient for an effective transmission of understanding, may however still play a role. Yet, this would be just a complementary one with respect to exemplification. Associating examples with pieces of propositional information can help, for instance, in defining or clarifying the exemplificative and denotative context – i.e. the connection between exemplar and exemplified object – which in turn facilitates also the interpretation of the exemplars. This is roughly the role of titles, captions, descriptions, etc. Even without playing themselves an actual role in the transmission process, such complementary propositional elements can thus have in some cases still a crucial function in it, since, as we know, context and interpretation are extremely important for exemplification to serve its cognitive and epistemic functions in advancing understanding.

Chapter 3

Music and Transmission of Scientific Understanding

In the last chapter, I showed how music can advance understanding by means of exemplification, be this both musical understanding and understanding of other kinds. In this chapter, I shall try to give an answer to the question posed in the [Introduction], namely whether music can also convey scientific understanding.

I will answer this question positively and claim that the way in which music can do this is by exemplifying scientific concepts. To this aim, I will take a specific case study, namely the musical exemplification of the concept of entropy after Xenakis' formal theory of music.

In [Section 3.1] I carry out a preliminary theoretical inquiry about the possibility aesthetic symbols generally have of conveying scientific understanding. Then, in [Section 3.2.1] I give an overview of the concept of entropy in statistical mechanics, information theory and stochastic music theory, and pass in [Section 3.2.2] to show how music can advance scientific understanding of entropy by exemplifying it and its related properties.

3.1 Some Remarks on Scientific and Aesthetic Understanding

In [Chapter 1] we saw that understanding *in general* does not have necessarily to be explanatory and propositional. If so, it is reasonable

to think that also scientific understanding *in particular* does not have necessarily to be such – this is for example what the mentioned examples by [Lipton 2009a] show. Moreover, if scientific understanding does not have necessarily to be explanatory and propositional, this also means that even non-explanatory and non-propositional symbolic systems like music or in general the arts may, in a theoretically legitimate way, advance scientific understanding.

One of the possible ways they could do that is by exemplification. In [Chapter 2], we referred in particular to music and to its power of advancing musical understanding or understanding of other kinds depending on the intra-musical or extra-musical features it literally or metaphorically exemplifies. So, conceivably, music may advance scientific understanding when it exemplifies scientific concepts.

Exemplification is, after all, a possible and legitimate way through which scientific understanding as well may be conveyed. Namely, as Goodman and Elgin point out, as a mode of reference it largely occurs not only in the arts, but also in science. So, for instance, an orrery exemplifies the motion of the planets, Galileo’s thought experiment exemplifies the independence of gravitational acceleration from mass¹, and even laboratory experiments exemplify natural phenomena [Elgin 2017b, 222-229; 278-279]. In selectively exemplifying such properties and phenomena, these exemplars make them salient and therefore epistemically accessible. So, they advance scientific understanding.

If scientific understanding can be legitimately conveyed via exemplification, and music can exemplify, then music can theoretically be considered as legitimately capable of advancing scientific understanding by exemplifying scientific concepts. Yet, the question arises: why should we take a musical composition, which is properly speaking not a scientific

¹ As we know, exemplification requires both reference and instantiation. Thus, Galileo’s thought experiment, as an abstract entity, could strictly speaking not exemplify concrete features such as the falling of a body, since it cannot instantiate them: nothing in the experiment can *actually* fall, *actually* possesses the property of falling. Indeed, what is exemplified is not a concrete property, but an abstract mathematical one correspondent to the measure of the rate of fall, which can be namely both abstractly and concretely instantiated (see also [Elgin 2017b, 229-235] and [Section 2.2.1]).

symbol, as capable of advancing scientific understanding? Traditionally, in fact, only scientific symbols (theories, models, experiments, observations, findings, representations, etc.) are considered as capable of advancing scientific understanding.

From the methodological point of view, it is worth answering this question firstly by examining what a scientific symbol consists in, i.e. what scientists generally require from a given symbol to count as a vehicle of scientific understanding. Then, it may be considered whether musical and aesthetic symbols have some possibility, at least partly, to meet these requirements as well. To a certain extent, this boils down to a discussion about the differences between scientific and aesthetic symbols. As such, this discussion is part of the well-known art-science demarcation problem, which is however an issue impossible to address here exhaustively. I will therefore not give sufficient and necessary conditions for something to count as a scientific symbol, asking then whether aesthetic symbols may satisfy these conditions as well. Rather, I will focus just on a “symptomatic” – therefore not sufficient, nor perhaps always necessary – character of both, a character concerning the conditions of their interpretability as exemplars for given features. Whereas, in general, these conditions are for scientific symbols rigid, this is not so for aesthetic symbols. Yet, we will see that, despite such differences, the way interpretations are builded in science and aesthetics is similar, which ultimately gives the possibility to theoretically admit aesthetic and musical symbols as legitimate vehicles of scientific understanding.

Let us then begin with the “symptomatic” characterization of scientific symbols. Scientific symbols seem to be *relatively univocal*. What they symbolize or *exemplify* does not generally admit of many different interpretations within the same scientific domain. There is for instance quite much agreement about what an orrery or Galileo’s thought experiment exemplify, and no further interpretations of these symbols are generally allowed within physics. This does not however exclude that more symbols may have the same target. Both wave-based and particles-based models, for example, exemplify atomic phenomena.

Now, some of these symbols may be in part divergent: think for

example to GTR and NM in physics [Section 1.3.2]. If so, one may think, what symbol is used in science for exemplifying a given phenomenon is basically a subjective matter: there is no objective criterion for preferring a symbol to another, there is no objective criterion for discharging wrong interpretations of a symbol or adducing new ones, and there is no objective criterion for adducing a new symbol for a phenomenon. Yet, as [Elgin 2017b, Ch. 7] points out, science does not lack objectivity. Although a scientific symbol cannot be objective in the sense of embodying a *view from nowhere* on a given phenomenon [Nagel 1986] – which ultimately depends on the epistemic limits of human nature – it can still be objective in the sense of resulting from *objective procedures*. Let us see what this exactly means.

A finding, a model are admitted as scientific symbols if «they result from or are confirmed by objective procedures» which are intersubjectively endorsed, i.e. «devised, tested and certified by the epistemic communities, who understand their domains, their disciplines, and the available and appropriate means for investigating the phenomena» [Elgin 2017b, 159]. Whether a symbol counts as objective, whether a symbol is accepted as a vehicle of scientific understanding depends on whether it complies with the epistemic standards and «methodological and normative framing commitments that are reflectively endorsed» by those communities [Elgin 2017b, 273-274]. So, for example, scientists generally accept only «findings that are intersubjectively accessible, backed by empirical evidence, preferably testable, and defeasible». Similarly, which interpretation of a symbol counts as the standard one, is always something procedurally and intersubjectively defined. New interpretations of a scientific symbol – as in the case, say, of physics models applied in economics or different areas of physics – are admitted as long as they are builded by objective procedures.

This furthermore means that also many contrasting (or even mutually exclusive) symbols can be considered at the same time as objective and valuable for the sake of advancing scientific understanding. In fact, as [Elgin 2017b, 180] points out,

«different models make manifest different features of the target. Diverging

models afford different perspectives on the same reality. And it is no surprise that different perspectives reveal different aspects of that reality. There is no optimal model for the same reason there is no optimal perspective. Every perspective, in revealing some things, inevitably occludes others».

But since every perspective is procedurally defined, and procedures are objective, possibly more symbols of the same target can be accepted.

Science, in other words, «puts a premium on intersubjective agreement» and univocality, and for that it «limits precision and constraints repleteness» [Elgin 2017b, 288]. A symbol is more precise and *replete*, the more the dimensions are along which it symbolizes [Goodman 1968]. So, for example, in the report of an electrocardiogram (EKG) only the shape of the wave and the frequency with which it repeats matter, not, say, the size, the orientation, the color, the intensity of the line, the type of paper used, etc. The symbol namely possesses a low degree of repleteness, and this as an effect of an antecedent intersubjective decision, taken in the relative epistemic community, about the dimensions along which it can symbolize.

If science works this way and scientific symbols are so characterized, there is apparently no possibility for seriously taking works of art, including musical compositions, as capable of conveying scientific understanding. In fact, aesthetic (and musical) symbols in general have a different characterization than scientific ones. As [Goodman 1978, 67-68], bearing on [Goodman 1968, 252-255], highlights, an aesthetic symbol normally shows certain *symptomatic* characteristics: it is

- *syntactically dense*, since «the finest differences in certain respects constitute a difference between symbols» – e.g. two musical compositions similar in everything but one single note are two different symbols;
- *semantically dense*, since «symbols are provided for things distinguished by the finest differences in certain respects» – e.g. different symbols are provided for notes with same name but different accidental;
- capable of *multiple and complex reference*, since it «performs several integrated and interacting referential functions, some direct and some mediated through other symbols» – e.g. a musical composition refers to its form, and through its form to a music-historical period;

- able to *exemplify*, since «a symbol, whether or not it denotes, symbolizes by serving as a sample of properties it literally or metaphorically possesses – see [Section 2.2];
- *relatively replete*, «since comparatively many aspects of a symbol are significant» – e.g. a musical tone symbolizes along many of its dimensions (pitch, timbre, loudness, duration, etc.).

Whereas, up to exemplification, aesthetic and scientific symbols may share a common characterization, they often differ from each other as to repleteness. As we observed, namely, scientific symbols are generally not or just little replete. Not so for aesthetic symbols, of which repleteness is often one of the most distinctive characteristics. Imagine, for example, an artist uses the wave of an EKG report as basis for drawing the skyline of a hill range – or directly uses the EKG report in a piece of contemporary art. What would matter in his work would be not only the shape of the line and the frequency with which a same pattern repeats. The symbol, in fact, would also symbolize along many other dimensions: size, color, intensity of the line, type of paint, type of paper, etc. In other words, the symbol would come to possess in this case a high degree of repleteness.

As it might be intuited, if the arts do not renounce repleteness, they put, differently from science, no premium on intersubjective agreement and univocality of symbols [Elgin 2017b, Ch. 8, 13]. Not only there may be many different symbols for the same target, but also many different interpretations of the same symbol. None of these interpretations is the wrong one, none is the standard one (see [Goodman 1968], [Goodman 1978]).

In science, you can interpret the orrery as exemplifying something different than planetary motion, maybe say, the way grazing hippos move. You would also give *reasons* why other scientists should take the orrery as exemplifying that. You may highlight for instance some alternative features and symbolic dimensions of the orrery for making them recognize that it exemplifies indeed the motion of grazing hippos, with each puppy grazing circularly around its mom and each mom grazing circularly around the heaviest exemplar of the herd. You would also adduce that this is actually the best way for making sense, for understanding, an orrery and its function. You can surely do that. However, only by scarifying your

career as a scientist and as a member of the scientific community. As we know, in fact, given their inherent structure as symbols, alternative interpretations of scientific symbols are generally not allowed, *unless* resulting from objective procedures and complying with the epistemic standards and framing requirements intersubjectively set by the scientific community.

Not so in the arts. An interpretation of a given work may show us that the work exemplifies *a*, another one that it indeed exemplifies *b*². Each would bring its reasons why we should take it instead of the other as the right one, the standard one, as the interpretation that really makes sense of the work, as the interpretation that really makes the work *understandable*. Each would selectively highlight features of the work, symbolic dimensions of the aesthetic symbol, that the other perhaps completely disregards. Each would show that the features it highlights make evidence of the work's exemplifying respectively *a* or *b*. Each would therefore claim that those features are the most important ones to take into consideration for understanding the work, and each would do so simply by arguing that focusing on the features highlighted by the alternative is not necessary for making sense of it.

Nothing could settle the dispute. All what remains to do is actually acknowledging that in aesthetics, given the characterization, the structure itself of the aesthetic symbol and its repleteness, «multiple, divergent, but equally tenable interpretations» of it are admitted at the same time [Elgin 2017b, 176]. If so, aesthetics is apparently consigned to the realm of subjectivity, and as such inadequate for transmitting scientific understanding, which conversely depends, as we know, upon objectivity and intersubjective agreement. Why should we accept, in fact, an interpretation of a work of art which refers to it as exemplifying a scientific concept and thus affording scientific understanding, and not to an alternative, *but equally tenable* one which instead denies any scientific exemplification in the work? That the work aesthetically acts as a scientific symbol exemplifying and

² To this point, I rather follow [Vernazzani 2021] than [Elgin 2017b], who seems instead to suggest that what is exemplified by an artwork remains relatively stable independently of contexts, interpretations and subject's intentions.

affording understanding of a certain scientific concept is just one among the possible interpretations of the work. We have no objective reasons to accept it, to prefer it to another interpretation which instead perfectly makes sense of the work without treating it as a scientific symbol. If aesthetic symbols admit of different valid interpretations, they cannot apparently act as scientific symbols, whose condition of interpretation seem instead to be more rigid.

Yet, there are interpretations in aesthetics which are plainly wrong, and hence inadmissible [Elgin 2017b, 178]. Under no plausible interpretation does Géricault's *Le Radeau de la Méduse* refer to a masquerade ball. We indeed dismiss such interpretations and we do that because the admissibility of an interpretation does not actually depend on its rightness or "truth", but on whether it does or does not make the work somehow intelligible and understandable [Elgin 2017b, 179 and ff.]. But still, how should we understand a work, and what should we understand of a work, through interpretations which are both admissible but *divergent*? Each interpretation would in fact *selectively* highlight features of the work the other disregards. Each interpretation would claim that the work therefore exemplifies *a* and not *b*, and vice versa.

However, no interpretation can indeed pretend to be the only correct one in highlighting all the relevant features of a work necessary for its understanding. To this aim, in fact, many aspects of the work are cognitively relevant and therefore worth of being made manifest. As a result, many interpretations are hence needed. Plurality and divergence of interpretations should be regarded rather as a resource than a hindrance to the extension of our epistemic range, because only through many and different interpretation of a work we can understand it [Elgin 2017b, 181].

At a closer look, if we remember what said before, we notice that this is exactly what happens in science too at symbolic level. We cannot understand nature through one single model or theory. We need in fact many different symbols which highlight and exemplify of a phenomenon different features. This is indeed the only way at our disposal for making that phenomenon epistemically accessible and understandable. A model exemplifying all the features of a phenomenon *at once* – however buildable

would this concretely be – would make the phenomenon rather epistemically confused than intelligible. Hence, plurality and divergence of models is also in science an epistemic resource rather than an hindrance.

Furthermore, as we know, symbols (and interpretations of symbols) in science are admitted as long as they comply with precise epistemic standards and intersubjectively set methodological commitments. Indeed, this is not different in aesthetics. Like a scientific model, an *admissible* interpretation of a work of art, i.e. an interpretation that is epistemically valuable in that it affords understanding of the work, must be backed by «publicly available, publicly assessable reasons» [Elgin 2017b, 181]: «the issue is not whether a work has a standard interpretation, but whether it has an intersubjectively accessible and assessable interpretation» [Elgin 2017b, 285]. An interpretation of *Le Radeau de la Méduse* as exemplifying a masquerade ball is not backed by any such reasons, and therefore inadmissible, as much as an interpretation of an orrery as exemplifying the motion of grazing hippos is. For being admitted in their respective domains, such instances must in fact both comply with intersubjectively set epistemic and methodological standards, and result from procedures which are objective in that they are intersubjectively defined.

Again, that in aesthetics no standard interpretation of a work exists leaves no room for subjectivity. Each interpretation can be in fact procedurally objective, and, as [Elgin 2017b, 182] points out,

«if we are going to take the admissibility of incompatible alternative [interpretations] to discredit the claim to objectivity of [art] criticism, we seem forced to discredit the claim to objectivity of the sciences that admit alternative incompatible models. It seems preferable to recognize that any field that admits of reason giving and sets standards for the appropriateness, acceptability, and weight of reasons is one that admits of objectivity».

That being so, an answer could be provided to our initial question: an aesthetic (and thus musical) symbol which along some of its dimensions is recognized, in one of its possible *admissible* interpretations in the sense just defined, as exemplifying a scientific concept, could be taken as legitimately advancing scientific understanding, and would therefore work as a scientific symbol by functioning aesthetically. It would in fact comply

with the requirements on intersubjectivity and procedural objectivity of science in exactly the same way a scientific symbol (like a model) would do³.

Let us conclude this section by highlighting another point about the interpretation of aesthetic symbols which will be particularly important for us in the following. The interpretation of a symbol in music (and aesthetics in general) as exemplifying a scientific concept is possible not only on the ground of the interpretability conditions intersubjectively set within the discipline. In fact, this depends also on the subject's recognizing what the symbol exactly exemplifies on the basis of the mobilization of right background knowledge (see e.g. [Vernazzani 2021] and [Section 2.2]). In interpreting, in recognizing a musical symbol as exemplifying a scientific concept, it is reasonable to think that the subject mobilizes both her musical and her scientific background knowledge. She must have a sufficient knowledge of the references codes the symbol and what it exemplifies belong to in order to be able to recognize the scientific exemplification.

Furthermore, in the case of music, exemplification is borne by properties (formal, expressive, aesthetic) it literally or metaphorically instantiates [Section 2.2.2]. For this reason, recognizing a given musical composition as an exemplar for something scientific will always depend on treating that symbol primarily as an aesthetic one, and on experiencing (and understanding) it primarily as such. An *aesthetic experience*, in fact, has exactly these properties as its primary content. And only by attending such properties with understanding, which is namely necessary

³ As also [Elgin 2017b, 274] points out, scientific procedures, epistemic standards, methodological framing commitments, etc. are after all not utterly strict. They evolve as the discipline evolves and are better to be regarded as «strong presumptions but not rigid requirements». Theories, models and other symbols which were known in the past to reliably afford scientific understanding lie today outside science, and many of them through which we today convey scientific understanding will be probably deemed as unscientific in the future. Of course, this does not mean that *every* symbol, independently of the kind, has to be *potentially* regarded as scientifically admissible. Yet, the fact that those scientific procedures, standards and commitments are not something dogmatic and unchangeable leaves room for taking seriously the possibility that some symbols like the aesthetic ones, even if not expressly scientific, may perform, in some cases and under specific circumstances, a cognitive function in advancing scientific understanding, even to a minimal degree.

for experiencing the symbol aesthetically [Vernazzani 2021, 11], can a subject recognize the scientific concept it may exemplify – this being indeed true for every case of musical exemplification. In other words, the symbol advances scientific understanding by functioning aesthetically, and to convey scientific understanding from it depends for a subject primarily on understanding it aesthetically.

3.2 Musical Exemplification of Scientific Concepts

In the last section I showed why music can be *theoretically* taken as capable of transmitting scientific understanding: exemplification is both in science and in music a way through which understanding can be conveyed; by exemplifying scientific concepts also aesthetic symbols can be possibly interpreted as *legitimately* affording scientific understanding.

In this section, I explain how musical exemplification of scientific concepts *concretely* works and how music, as an aesthetic symbol, can advance scientific understanding. To this aim, I will focus in particular on the concept of entropy.

However, before coming to the core of the epistemological discussion about the musical exemplification of entropy, and about how and to what extent music can convey an understanding of it [Section 3.2.2], we need firstly to introduce the notion of entropy from the technical point of view⁴. In this respect, I will refer in [Section 3.2.1] to the concept of entropy in Boltzmann's statistical mechanics, Shannon's information theory and Xenakis stochastic theory of music⁵.

⁴ As we pointed out before, in fact, a certain knowledge of the reference code a symbol belongs to is necessary for recognizing what the symbol exemplifies. So, all musical composition, as we shall see, may theoretically exemplify entropy – we will just consider the eloquent case of a particular work by Xenakis, *Metastaseis*. Yet, they become interpretable as such, provided that one has an even minimal familiarity not only with music in general, but also with the way Xenakis and other stochastic composers conceive music and music composition.

⁵ Hopefully with just little sacrifice of accurateness, in the present exposition I will avoid, for place constraints and simplicity, any physical and mathematical formalism, for which I refer to [Appendix A] and [Appendix B] taken in turn from [Zanzarella I. 2021].

3.2.1 The Concept of Entropy in Science and Music

The Concept of Entropy in Science: From Boltzmann to Shannon

The concept of entropy firstly appeared during the 19th century in the field of thermodynamics, and more precisely in the context of an engineering problem, namely how, and how far, it is possible to improve the efficiency of a heat engine so as to have one capable of wholly transforming heat in useful mechanical work (see e.g. [Müller 2007]).

By the research of Sadi Carnot and Rudolf Clausius (see [Carnot 1824], [Clausius 1854], [Clausius 1865]) the conclusion could be reached that such an engine is physically impossible. Only in an ideal reversible thermodynamic system – i.e. a system operating on an ideal thermodynamic cycle with heat exchanges occurring also from cooler to warmer bodies – is the overall amount of heat exchanged equal to that of useful work done. Conversely, in any actual, irreversible engine, part of the heat exchanged get irremediably lost and cannot be converted into useful work. Whereas in the first case the sum of the ratios of the heat exchanged between the bodies and their respective temperatures is always equal to zero, in the second case it is (generally) less than zero (*Clausius theorem*). To this particular quantity Clausius gave the name of *entropy*, and also demonstrated that it can never decrease (*Second Law of Thermodynamics*, after one of its formulations) [Uffink 2007].

Clausius' thermodynamic entropy is a state function. This means, it just measures the ratio between exchanged heat and temperature in relation to individual states of a thermal system, states in which the system is in thermodynamic equilibrium. In other words, it does not regard its non-equilibrium states, the states in which it instead undergoes changes. Furthermore, this concept of entropy refers to macroscopic aspects of a thermodynamic system (like for example the temperature), without mentioning its microscopic ones (for example the behavior of the molecules). It is a non-probabilistic concept and offers «no intuitive interpretation as a measure of disorder, disorganization, or randomness»

[Frigg and Werndl 2011, 117].

A “microscopic” and probabilistic conception of entropy was instead the result of another fundamentally different approach, which had been developing not from an engineering problem, but from a truly scientific hypothesis: that the macroscopic properties of a thermodynamic system depend on, and are explainable by, the mechanical behavior of its microscopic components⁶. So, for example, changes in the pressure of a gas in a container are made dependent on changes of the momentum of the molecules composing it – a change occurring due the continuous collisions of them with the walls of the container; or still, changes in the temperature on the (mean) kinetic energy of the molecules, etc. Already Clausius – though anticipated by August Karl Krönig [Krönig 1856] – tried to interpret thermodynamic processes in such kinetic terms [Clausius 1857], actually laying the groundwork of modern kinetic theory of gases. However, a genuine turning point within this new approach to thermodynamics came with the research of James Clerk Maxwell and Ludwig Boltzmann.

With the development of kinetic (or mechanical) theory of heat, concepts and tools of Newtonian mechanics began to be employed with regard to the microscopic constituents of a thermodynamic system in order to explain its macroscopic behavior. Yet, if Newtonian mechanics was basically deterministic, the new “laws” of thermodynamics appeared to have rather a probabilistic and statistical character. So was Maxwell’s final formulation [Maxwell 1867] of his *distribution function* – later improved and extended by Boltzmann (see [Boltzmann 1868], [Müller 2007, 92-94] and [Uffink 2007, Sec. 3-4]), which gives the *probability* that a certain number of particles (atoms or molecules) of a gas in a container have a certain velocity (where the system is supposed to be in a state of thermodynamic equilibrium at a specific time). This number, in fact, cannot be deterministically set, for the particles, due to incessant collisions, undergo a continuous change of their velocities. The only way in which it is possible to interpret that number is then just as a *mean* or *expectation*

⁶ The first scientist to propose this hypothesis was Daniel Bernoulli in [Bernoulli 1738].

value, so that the distribution function rather amounts to a *probability distribution* [Frigg and Werndl 2011, 123-124].

Starting from the presuppositions of the new kinetic approach, a first probabilistic interpretation was given also to the Second Law of Thermodynamics and to the concept of entropy, which now began to be related not only, as in Clausius, to the macroscopic properties of a system, but also (in a mathematically equivalent way [Frigg and Werndl 2011, 127]) to the microscopic objects composing it. In particular, this was a task undertaken by Boltzmann (see [Boltzmann 1872] and [Boltzmann 1877]).

In its most mature formulation, Boltzmann's entropy is the measure of the possible *microstates* of a thermodynamic system like, say, a gas in a vessel (i.e. all its particles' having certain positions and momenta at certain instants of time) coherent, *with equal probability*, to a given *macrostate* of it (i.e. the system's having a certain temperature, pressure, etc. at a certain instant of time). Since states with *more* microstates coherent with a same macrostate are *more probable*, the system naturally tends to evolve towards states of higher entropy, until it finally reaches the state with maximum entropy (*thermodynamic equilibrium*), which is namely the *most probable* one as, in it, the *greatest* number of microstates is equiprobably coherent with the macrostate of the system. This all eventually amounted to the new kinetic and probabilistic justification given by Boltzmann of the Second Law and of why entropy is bound to constantly increase and never to decrease.

If, within the evolution of the system towards equilibrium, the number of microstates coherent, with the same probability, with a given macrostate increases, this also means that the randomness, the unpredictability, the disorganization and the disorder of the system grow, reaching the maximum namely at equilibrium. In this respect, entropy is also a measure of such disorder and randomness in the sense that the greater it becomes, the less it is possible to know about the *actual* microstate of the system given a macrostate of it.

This probabilistic notion of entropy was later further developed by Josiah Willard Gibbs [Gibbs 1902], who made of it one of the core

notions of the newborn discipline of statistical mechanics. Especially as a measure of the disorder and unpredictability of a system, it became a powerful tool also for analyzing and explaining different phenomena than thermodynamic ones, showing great theoretical efficacy within a large and heterogeneous number of disciplines, from cosmology, logic, biology, informatics and economics to hermeneutic, philosophy and the arts.

This was for example the case of Claude Shannon and his information theory (see [Shannon 1948] and [Shannon and Weaver 1949]). Shannon's aim was that of quantifying the information contained in a certain number of messages flowing from a source. Relying on some previous ideas by Hartley [Hartley 1928], he proposed to make this quantity (mathematically) dependent on the probability of each message. The more probable a message is, the less information we gain from its reception. From messages flowing from a source with exactly the same probability, we gain no information, since all messages have equal probability to contain information. As in the case of thermodynamic equilibrium, where all microstates are equiprobable, when all messages have the same probability to contain the same amount of information, the entropy (*Shannon information entropy*) is at maximum. On the contrary, the less probable a message is, the more uncertainty there is about its information content, the more information we gain from its reception removing the uncertainty and the lower entropy is. Information and uncertainty are for Shannon, in fact, two sides of the same coin.

The Concept of Entropy in Music: Xenakis' Stochastic Theory of Music

Outside science as well has the Boltzmann-Shannon probabilistic conception of entropy found great possibilities of application. A case is exactly that of Iannis Xenakis, who applied it in the theory of sound and of music composition for describing, or prescribing, the development of sounds and musical compositions [Xenakis 1992].

By applying mathematics, logic and especially probability theory in music, Xenakis' major aim has been that of developing a new theory of sound and of music composition by which it is possible, on the one hand,

to account for the evolution in time of complex and disordered sound events (indeed the most common in our experience, like for example the sound made by a car or the song of the cicadas) and, on the other, to produce musical compositions whose structure and evolution in time are truly ruled by randomness and chance. As he points out [Xenakis 1992, Ch. 1], in fact, the most common theories of sound and of music – tonal, serial or dodecaphonic music – rely on deterministic assumptions and models, and are therefore inadequate for fulfilling this purpose.

Xenakis considers the application of probabilistic and statistical methods to sound phenomena and music – i.e. nothing but sound phenomena which can be created by the composer – a natural consequence of their introduction into the sciences (in particular, statistical mechanics and quantum theory) as powerful tools for explaining the world, which apparently behaves in some respects not deterministically. As sound is part of this world, it is reasonable to think that a probabilistic, *stochastic*, theory of music can offer some useful insights into its nature and behavior, as well as some effective hints to composers for making sound evolve in their compositions in a truly stochastic and indeterministic way [Xenakis 1992, 4].

In his stochastic theory of music, Xenakis starts from the same idea at the basis of statistical mechanics and kinetic theory of gases, namely accounting for sonic events⁷, with their “macroscopically” audible properties (pitch, timbre, intensity, etc.), in “microscopic” terms. As already [Helmholtz 1862] showed, a sonic event is a system of complex sound waves which are in turn composed by a large number of simple sinusoidal waves behaving according to Fourier Analysis. The resulting sonic event ultimately depends on the quantity and amplitude (loudness) of the simple sinusoidal waves occurring it.

Xenakis takes these latter to be the microscopic components of a sonic event and, very similarly to what happens in statistical mechanics with particles [Frigg and Werndl 2011], represents them as points, called *sonic grains*, in an ideal mathematical space with dimensions frequency,

⁷ By “sonic event” Xenakis means every possible auditory phenomenon perceivable by the human ear, from sounds and noises to entire musical compositions.

intensity and duration or time, the *grains space*. The grain space is furthermore taken as divided along the time axis into a large, but finite, number of slices of time – *thick sonic screens* small at will and invariably equal in size. Then the *macrostate* of a sound event within a screen, i.e. its having certain pitch, timbre, intensity, etc. at a certain time, is describable (or prescribable) by referring to its *microstate* within that screen, i.e. to the quantity and distribution of the sonic grains over the grains space at that time.

Yet, since – like for particles in statistical mechanics – it is impossible to deterministically account for the exact quantity and distribution of the single gains in the grains space, the macroscopic evolution over time of a sonic event can be accounted for only in a probabilistic and statical way. To this aim, Xenakis devises some stochastic laws – derived from the theory of continuous probability – for calculating, given a sound event, i.e. a certain (mean) number of sonic grains in the whole grains space, the distribution of the grains along each dimension of the space.

So, given macrostates of a sonic event at different instants of time, the grains will be found – in a probabilistic way – within the correspondent sonic screens as having a certain distribution over that portion of the grains space, i.e. as having certain (mean) values of frequency, intensity and duration. This is indeed similar to what happens in statistical mechanics, where, given macrostates of a gas at different instants of time, its particles will be found – again, in a probabilistic way – as having certain (mean) values of momentum, position, etc.

Now, as we saw, in a thermodynamic system the entropy value can be calculated for each of its macrostates. As we noticed in the discussion of Boltzmann's entropy, in a thermodynamic system this value depends on the number of microstates (the particle's having certain values of momentum, position, etc. at a certain time) equiprobably coherent with a given macrostate. The greater this number is – i.e. the larger the range of values of momentum, position, etc. is which the particles can equiprobably take on coherently with a given macrostate – the higher the entropy.

Similarly, the entropy value can be calculated also for each of the

macrostates of a sonic event. As in thermodynamics, this value depends on the number of the grains microstates that is equiprobably coherent with a given sonic macrostate. The greater this number is – i.e. the larger the range of values of frequency, intensity and duration is which the grains can equiprobably take on coherently with a given sonic macrostate – the higher the entropy.

So, if the grains have the probability to take on just *one* single value of frequency, intensity and duration, we would indeed have just a single grain in the screen, which amounts to having a *pure sound* and an entropy value equal to 0. Conversely, if the grains can occupy, *with the same probability*, indifferently all regions of the screen, if they can in other words take on, with the same probability, *all possible values* of frequency, intensity and duration, we would have a *white sound* and maximum entropy. «Between these two limits the grains may be distributed in an infinite number of ways with mean entropies between 0 and the maximum and able to produce both the Marseillaise and a raw, dodecaphonic series» [Xenakis 1992, 64]. Between these two limits, with the means of statistics, every possible perceivable natural sonic event is describable or prescribable, every musical work of the past is explainable and every new musical work can be composed.

Of course, as the sonic event develops over time and the relative sonic screens flow, the overall entropy of the sonic event may undergo changes. In general, how entropy changes in a sonic event depends on the *modes* in which *transitions* between screens take place. Screens transitions are nothing but the final result of *transformations* of frequency, intensity and duration values from one screen to another. Transformations take place (or are prescribed by the composer) according to given probabilities. So two general modes of transformations and therefore transitions are distinguishable: the *deterministic* ones, occurring only with a probability of 1 or 0 and hence leaving the overall entropy value of the sonic event unaltered⁸; and the *stochastic* ones, occurring with probabilities strictly between 1 and 0. Do *all* transformations and screens transitions occur

⁸ Stochastic theory of music thus contains tonality, dodecaphony and other deterministic theories of music as limiting cases.

with *equal probability*, then would the evolution of the sonic event be completely indeterminate and totally ruled by chance. The transitions would be all equally unpredictable and thus, as happens in information theory when all messages can equiprobably contain information, the disorder and the entropy would be at maximum.

With respect to stochastic transformations, still two typologies are finally distinguishable: those towards disorder, bringing about increases in the overall value of entropy; and those towards order, bringing about decreases in the overall value of entropy. This means that, differently from what happens in the systems of statistical mechanics, in music entropy must not necessarily develop in a constrained direction, i.e. towards its maximum. Musical entropy necessarily increases as long as the sound to be analyzed or the musical composition to be composed present themselves as collections of screens developing in a direction from pure sound to white sound. In cases different from this, entropy may instead remain constant or even decrease.

Differently – in most cases – from natural sounds, in music compositions many kinds of transformations can be applied to screens in the course of the same composition. Thus, entropy may for example increase up to a certain point of the composition, and then decrease or remain constant. Or, it may increase with respect to intensity, but decrease with respect to frequency and duration – whereby the overall entropy value of each sonic screen is given every time by the arithmetic mean of the entropies calculated for each of its dimensions.

Hence, in this way, the variation of entropy over time becomes for the composer an additional aesthetic criterion for shaping music [Xenakis 1992, 75-78]. To this respect, in [Xenakis 1992, Ch. 3], Xenakis for example discusses some practical applications of stochastic music theory in music composition, by referring in particular to his own works *Analogique A* and *Analogique B*, both composed in 1958-59. There, he explains how, starting from the abstract probabilistic laws at the basis of stochastic music theory, it is possible to create new (musical) sonic events by defining screens, transformations, transitions, entropies, etc.

3.2.2 Musical Exemplification of Entropy and Scientific Understanding

In the previous section, we examined what the concept of entropy technically consists in, and how it is used in different contexts, from science – thermodynamics, statistical mechanics and information theory – to the arts – stochastic theory of music.

We are now ready for trying to provide an answer to the question posed at the beginning of [Section 3.2], namely how, by an aesthetic experience of music, a general understanding of the concept of entropy can be conveyed, that is, how music, by functioning properly as an aesthetic symbol, can transmit an understanding of entropy, which is a scientific understanding, as entropy is, strictly speaking, a scientific concept.

What Kind of Exemplification?

As observed in [Chapter 2], the way in which non-propositional symbols like music can in general transmit understanding is by exemplification. Thus, music can advance scientific understanding of entropy providing that it exemplifies it, and that it is part of one's aesthetic experience of music to recognize this latter as exemplifying it. Now, *that* music is capable of exemplifying and advancing understanding by exemplification is something for us no longer in need of justification and explanation. The question is rather that of *how* music can concretely exemplify a scientific concept like entropy, and advance in this way an understanding of it.

In [Section 2.2.2] we distinguished between two possible modes of musical exemplification: literal musical exemplification and metaphorical musical exemplification. Whereas in the latter music refers to extra-musical properties it can only metaphorically instantiate, in the former music refers to musical properties it literally instantiates. As a special case of literal musical exemplification, we moreover observed that music might refer to properties – such as symmetry – which, albeit not quite specifically musical, are yet literally instantiable by it as structural, formal properties.

Now, if we are trying to answer the question about how music

can exemplify entropy, we firstly have to see whether entropy can be instantiated by music literally or metaphorically. Let us briefly return to entropy in science. In physics, for example, entropy generally indicates a *physical*, therefore *intrinsic property* of a system – or more precisely, of a given state of it at a certain time. A thermodynamic system at equilibrium possesses a given entropy, a given degree of disorder, as a physical property, much like, say, Earth possesses a given mass as a physical property. Or similarly, in information theory, entropy is a property a message source, or more generally a random variable, possesses as a function of the probability of its outcomes’ containing information.

Indeed, things are not so different for sonic events. As observed in [Xenakis 1992], like thermodynamic systems, sonic events – or more precisely each of the sonic screens composing it – possess a given entropy or degree of disorder as a physical, intrinsic property (sonic events are namely physical phenomena). Order, disorder, entropy changes are things we *physically* perceive – plausibly as higher-order properties⁹ – not simply *in*, but *of* a sound. Then, if some of the sonic events are moreover musical compositions, this also means that entropy becomes in them also an intrinsic property of music, yet not just as a physical property, but now as a truly *musical* one, which possibly bears expressive or aesthetic functions as well. As pointed out before, in fact, variations of entropy over time are for the composer an additional aesthetic criterion for shaping music, are means for her to setting up the musical discourse and the structure of her musical compositions. Choosing whether to

⁹ This point is indeed controversial. If defenders of the so-called *rich content view of perceptual experience* would claim that order, disorder, entropy, etc. are higher-order physical properties of the sound a trained subject would perceive of it beyond other lower-order properties (pitch, intensity, etc.) defenders of the *sparse content view* would maintain that only the latter represent the content of our perceptions, whereas the former are the result of a cognitive intervention on those perceptions (see e.g. [Stokes 2018], [Siegel 2021]). Yet, as [Vernazzani 2021] points out, it is plausible to think that higher-order properties are mostly based on lower-order properties, with the result that the former are however partly perceived by a trained subject inasmuch she must necessarily pay attention to the latter, which namely ground them. So, in the case of musical compositions, entropy may be perceived as higher-order musical properties inasmuch one pays attention, within an aesthetic experience (see [Section 3.1]), to those structural, expressive and aesthetic properties of music grounding it.

make a (stochastic) composition develop towards order or disorder is for a composer as *musically* and *aesthetically* relevant as choosing whether to end a work, in a tonal environment, say, with a picardy cadence or not.

This is eloquently shown for example by Xenakis's work *Metastaseis B* for orchestra (1954). By the graphic scheme of the work [Figure 3.1], it is possible to observe how entropy changes along the entire duration of the piece, and that in a different way for every sonic dimension¹⁰.

At the beginning of the first section, for instance, entropy increases with respect to pitch (i.e. frequency, at the microscopic level). From one single tone played simultaneously by the whole orchestra, the piece begins to evolve, with a slow and gradual *glissando*, towards a complex dissonant chord, where almost every instrument plays a different tone: a state of higher entropy has been reached.

If in the first section of the piece entropy increases with respect to pitch, it remains practically constant with respect to time: instruments play all tones simultaneously and grains can take on just few different values of duration. As to time, entropy however changes from the first to the second and third section, in which it constantly increases. Instruments play with increasingly different

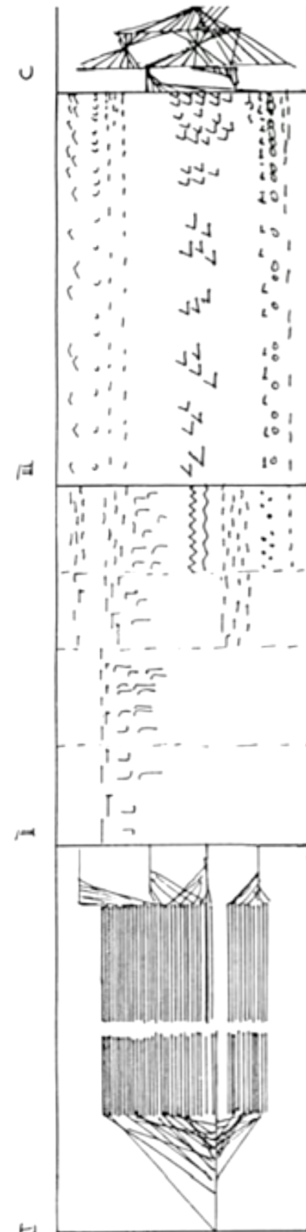


Figure 3.1
(read horizontally, left-to-right)

¹⁰ For a better understanding of my discourse, I recommend listening to the composition along with reading the scheme and its description (take e.g. [Xenakis 1988]).

rhythms and grains can correspondingly take on, with the same probability, different values of duration. Disorder grows, and hence so entropy. At the end, the piece finally evolves again towards order and lower entropy with respect to every sonic dimension [Figure 3.2].

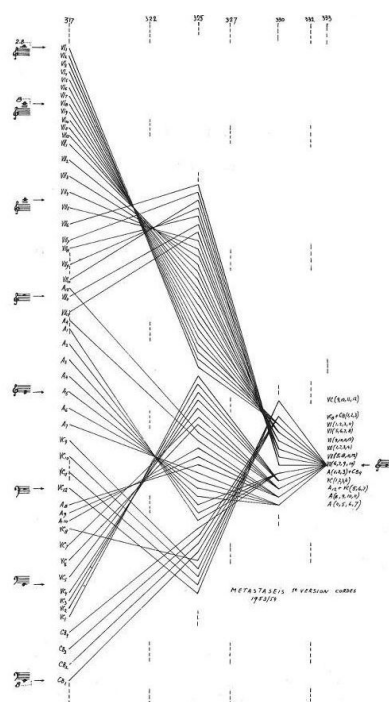


Figure 3.2
(read left-to-right)

Thus, in *Metastaseis*, we hear sound physically becoming more or less disordered. Yet, this is not just the simple auditory experience of a physical phenomenon. Being in a musical composition, the phenomenon namely takes on also an aesthetic and musical meaning. The particular entropy development of *Metastaseis* is in fact intrinsically part of the structure itself of the musical piece, and upon grasping it as such ultimately depends our aesthetic, namely musical, understanding of the work.

The same conclusion would be reached if we were to thematize musical entropy not as a measure of disorder, but rather, in informational terms, as a measure of unpredictability or uncertainty. As we know, as long as entropy increases, more possible evolutions of a sonic event become equally probable. It is for this reason that its development, by increasing entropy and disorder, becomes also for a listener more difficult to *predict*.

If a musical composition evolves according to the schemes, say, of tonal or dodecaphonic music – which are, as we know, highly deterministic – entropy will remain unchanged and the piece therefore predictable in its whole development: given a leading tone, it will be followed, with *certainty* – or at least high probability – by the tonic; given a dodecaphonic series, tones in it will follow each other, with certainty, as the series prescribes. Not so in a stochastic composition, where entropy does not remain constant. Since it would develop in rhythm, pitch, intensity, etc.

according to specific probabilities, its evolution would be for the listener not predictable at every point in time with the same degree of certainty.

There is already general consensus among musicologists, philosophers and psychologists of music about the fact that the «arousal and subsequent inhibition of expectant tendencies» have a central importance «in the shaping of musical experience» [Meyer 1967, 5]. Provided the subject's familiarity with the musical system and style of a given musical composition, her (musical) understanding of it also depends – at least at the structural level – on her conscious or unconscious capacity of making hypotheses, with a given degree of certainty, about the development of the musical discourse, as well as upon her hypotheses' being every time actually confirmed or disconfirmed by it (see also [Meyer 1956], [Loui and Wessel 2007], [Abdallah and Plumbley 2009], [Tillmann et al. 2014]). This is indeed something which has been even empirically proven in a number of experiments in psychology and cognitive science (see e.g. [Manzara et al. 1992]). Trying to anticipate, to predict the development of a musical composition on the basis of its probability is thus an aspect of *musical understanding*. If we take entropy, in its informational acceptance, to be the measure of the certainty of a subject about this development, or, equally, as the degree of predictability of this development, then entropy itself turns out to be in this sense implicitly characteristic of musical understanding too. To understand a musical composition is in part to grasp, its entropy in terms of degree of predictability of its development (consciously or not, depending on one's awareness of the possibility of accounting music in informational terms).

Thus, both as a measure or degree of (dis)order and of (un)predictability, entropy is a property music *literally* possesses – and, with this, one of the two necessary conditions on exemplification, namely instantiation, is fulfilled. Similarly to what happens with symmetry, however, the case of entropy is indeed a particular one, for, albeit *not* strictly speaking musical, entropy is a property music does literally (and not just metaphorically) instantiate.

Now, as we know from [Section 2.2], for a symbol to serve an exemplificative function, not only *instantiation* of, but also *reference* to,

the exemplified property is needed. And we know also that, in order to refer to a property, a symbol must be *intentionally* used as an exemplar, whereby, however, the exemplar works properly as such when it *selectively highlights* the exemplified property at expense of other irrelevant features it instantiates. Yet, a good exemplar is still not the one that just calls attention to that property. In fact, a good exemplar affords one *epistemic access* to it, as well as the possibility of recognizing it elsewhere. In a word, a good exemplar is therefore that enhancing one's *understanding* of something – an understanding whose kind ultimately depends upon the nature of the exemplified property. Reference of an exemplar to the exemplified property moreover depends on the *context*. A deliberate restriction or explicitation of the exemplificative context is sometimes required for making a symbol referring, as an exemplar, properly and exactly to the target property. In some cases, then, this is the function that, alternatively to *explicit* context restrictions, also the subject's background assumptions may *implicitly* perform.

Take again *Metastaseis*. The work may possibly refer to, and thus exemplify, a large number of properties – properties it can instantiate literally or metaphorically. If used as a soundtrack for a documentary film about the Second World War, it may metaphorically, thus extrinsically, exemplify the absurdity, the grief, the fear, the abjection of the war – whereby these are properties the work metaphorically possesses, i.e. expresses¹¹. In a music composition class, conversely, it might intrinsically exemplify stochastic music, but also atonality, polyrhythm and other properties it literally possesses – this essentially on the ground of its self-referentiality as a musical symbol [Section 2.1].

As we have seen, among these latter properties – that are namely formal, structural, intrinsically musical ones – there is also entropy, to which *Metastaseis* hence refers, and that it can therefore also exemplify. The disorder and unpredictability increase of the beginning, the correspondent decrease of the end are entropy-related properties the work

¹¹ Indeed, this could possibly make it an aesthetic symbol capable of advancing some form of historical understanding, as in particular also [Elgin 2017b, Ch. 13] would claim.

literally instantiates and to which it can refer, as said, because of its self-referentiality as a musical symbol.

Yet, this is not enough for *Metastaseis* to exemplify entropy. In fact, just in a single context – say again, the music composition class – the work may be taken as referring to, and thus exemplifying – beyond entropy, also other intrinsic (or extrinsic) properties it happens to possess. Actually, in order for *Metastaseis* to serve as an exemplar for entropy, what is needed is a selective highlighting of the work’s instantiating entropy and specific entropy developments, which correspondingly amounts to downplaying for example the facts that it is an atonal work, that it presents polyrhythm, and so on. As we know, an exemplar works namely as such only if it is selective¹².

Furthermore, as pointed out, an exemplar works properly as such when it provides *epistemic access* to the property it exemplifies. And this is for us namely an important point. What kind of entropy may in fact a musical exemplar such as *Metastaseis* provide epistemic access to? We said, in a music composition class, *Metastaseis* can be taken as an example for entropy or for a specific entropy development. In that particular context, however, exemplification would rather be *intrinsically* aimed at affording epistemic access to entropy as a genuine *musical property*. In so doing it would thus provide *musical understanding*. By that exemplar, a subject would understand what musical entropy is, and recognize the structural function entropy variations perform not only in *Metastaseis*, but also *generally* in music. By it, it would be possible to her to spot that property, maybe in the same way in which it is instantiated in *Metastaseis*, also in other musical works. So, her musical understanding would be enhanced¹³.

¹² As pointed out before, selectivity is indeed implicitly contextual: the fact that *Metastaseis* is discussed in a music composition class, implicitly excludes that it may be taken as metaphorically referring to, thus as extrinsically exemplify, for instance, the sorrow of the Second World War.

¹³ Indeed, grasping the structural function entropy plays in music is sufficient for musical understanding – this should eventually be clear from what we said before about music, information and predictability. As pointed out in [Section 2.1], in fact, musical understanding primarily depends on grasping the structure of music and the role its different formal elements play in it. This means that, should an entropy increase in a musical composition be also metaphorically *expressive* of something,

Yet, we noticed that, even if music can literally instantiate it, entropy is not quite a *specifically* musical property. There are in fact many other objects and phenomena, outside music, which can instantiate such property in the same terms of degree of disorder or unpredictability. We just examined for example two of them: the thermodynamic systems of statistical mechanics and the random variables of information theory – arguably, among many others from other different domains. If so, then music, just similarly to the case of symmetry, would be able to exemplify entropy and entropy-related properties even *extrinsically*, i.e. as extra-musical features.

Now, as pointed out in [Section 2.2.2], whether musical exemplification is literal or metaphorical, intrinsic or extrinsic, is something that depends on the context. Whether a composition such as *Metastaseis* works as an exemplar for musical entropy in particular, or as an exemplar for the concept of entropy in general, is something that depends on the context in which it occurs or is intentionally used as such. If in a music composition class the work may be used as an exemplar of musical entropy, within a course from a History, Philosophy and Culture of Science program, it could be used as an exemplar of entropy in the arts, or even of entropy in general.

Moreover, whether a musical composition works as an exemplar of either a musical or an extra-musical property – and, indeed, whether it works as an exemplar at all – depends on correctly *interpreting* it as such (see e.g. [Elgin 2017b, 218-220], [Section 2.2], [Section 3.1]). For that, a context explicitation may be sometimes required, in our case, for example in the form of a music teacher’s selectively and explicitly giving *Metastaseis* as an exemplar of musical entropy. Yet, this is not always strictly necessary. The subject’s background assumptions would *implicitly* play, in fact, a similar role. The students who already know Xenakis’ composition style, would be able, for instance, to take *Metastaseis* as an exemplar of musical entropy without any need of context explicitations.

then our musical understanding of the composition would be surely enhanced by grasping that expressive content as well, yet not exclusively and necessarily dependent on this.

Background assumption indeed allow of some interesting considerations. Firstly, an even minimal acquaintance with the symbolic system an exemplar belongs to is necessary for interpreting it, and interpreting it in the correct way (see e.g. [Elgin 2017b, 218], [Section 3.1]). You cannot pretend to understand *Metastaseis*, and how it instantiates specific entropy variations, without any knowledge of the basic structures of tonal and atonal music, and maybe also of the main concepts of Xenakis' music theory. Furthermore, within an unspecified exemplificative context – say, the waiting room from above – and provided one's attentive and epistemically (i.e. aesthetically) aimed listening, many interpretations of a work may be possible according to the different background assumptions possessed by each of the subjects involved in that context. So, Jerry, who possesses a relative extended knowledge of music theory, may be rather inclined to interpret *Metastaseis* primarily as an exemplar of musical entropy. Conversely, his grandma, who is rather interested in the natural sciences, would more or less immediately recognize the changes in the degree of disorder in the composition as referring to, and so exemplifying, the general concept of entropy instantiated for instance in thermodynamic systems.

At this point, however, an objection of circularity may arise. If we namely claim that a certain amount of background knowledge relative to musical or scientific entropy is partly required for recognizing *Metastaseis* or other musical works as exemplars for entropy, in which sense would those symbols provide one epistemic access to (and ultimately understanding of) the concept of entropy? What would be the epistemic gain of music exemplifying entropy, if in order to recognize this exemplification one must already know something about entropy? The answer to this objection lies indeed another time in the notion of *degree* of understanding [Section 1.1]. Jerry or his grandma may just have a very rough and confused idea about what entropy is – yet at least sufficient for recognizing it in a musical symbol. They may just *know* by hearsay the way the notions of order, disorder, unpredictability, etc. relate to the concept of entropy, but still lack an *understanding* of it, of what entropy increases or decreases actually are, etc. Now, exactly such understanding is what, at least to a minimal

degree, a musical exemplar may afford them. In not only knowing that the given work they are listening to exemplifies entropy, thus changes in the degree of disorder, unpredictability, ecc. but in auditively *experiencing* such things in the music, their general understanding of entropy would be enhanced, it would be possible for them perhaps to epistemically access aspects of this notion they were before not aware of, and also recognize them elsewhere.

The Value of Musical Exemplification for Scientific Understanding

As we said, music could thus be correctly interpreted by a subject – in the right context, and by the appropriate background assumptions – as exemplifying entropy and entropy-related properties also in an extrinsic way, i.e as extra-musical features. Now, by exemplifying them in that way, music would make them manifest and epistemically accessible, it would make them *recognizable* when instantiated even outside music. This is indeed important for us, for music may perform this cognitive function also when entropy and entropy-related properties happen to be instantiated by objects or phenomena which are commonly regarded as belonging to the scientific domain. And exactly by conveying us the possibility to spot these features in those objects and phenomena, to (macroscopically) recognize in them a certain (changing) degree of disorder or unpredictability – in a word, to look at them in “a new light” – music can enhance our scientific understanding of the world¹⁴.

By epistemically accessing, through musical exemplification, the general concept of entropy as a measure of disorder and unpredictability, we could better understand situations and phenomena which are part of our quotidian life but whose “entropic” character would be otherwise inaccessible, maybe because not apparent or hidden behind other properties. Music may work as a cognitive stimulus for approaching them with a scientific drive, and so help us to discern in them entropy-related features.

¹⁴ As a consequence of the dependence of musical exemplification on context and background assumptions of the subject, also which kind of understanding a musical symbol conveys by exemplification – whether musical or in this case scientific – is dependent on the same conditions.

So, it would enhance our scientific understanding by making us aware of the fact that they could be scientifically accountable namely in terms of entropy, that they could bear some sort of relation to the concept of entropy as this normally appears in science. When the same relation between information and predictability instantiated in music happens to be also instantiated in a card game, we would understand that this kind of situation is possibly accountable in terms of (informational) entropy. When we notice that the evolution from order to disorder of the beginning of *Metastaseis* is the same that makes itself visually appreciable in a number of physical phenomena from mixing fluids to engines and even organisms, we become aware of the possible connection this phenomena can have with the scientific concept of entropy.

Just to be aware of this possibility, just to be aware of the fact that a phenomenon may count as scientific, and may furthermore be accounted by science in terms of entropy, amounts to having scientific understanding. Just to stimulate someone towards *deepening* further that understanding, towards developing the simple understanding *that* those phenomena bear a connection to entropy, into an understanding also of *how* and *why* they show an entropic behavior, amounts for music to performing an authentic and valuable cognitive function as a vehicle of scientific understanding.

Of course, the scientific understanding music can transmit is not equal in degree to that you may convey for instance after a university course. Yet, as we pointed out in [Chapter 1], understanding can come in degrees, and an understanding of inferior or minimal degree does not count as no understanding at all. Similarly, it is reasonable to think that the scientific understanding of entropy of a Nobel prize in physics would be not even minimally enhanced by listening to Xenakis. Perhaps, by listening to his music she could conversely enlarge her musical or aesthetic understanding, for example by acknowledging that entropy and entropy-related features can be also instantiated by musical symbols¹⁵. She may appreciate the structural role they play in music. But only

¹⁵ To this respect it is interesting to notice that by exemplifying entropy music may also enhance our aesthetic understanding in general, as long as it render entropy and entropy-related features recognizable and discernible in other aesthetic objects like dances, paintings, etc. in which they namely happen to be instantiated as well.

hardly would she have from that musical experience an epistemic gain in terms of scientific understanding.

Yet, it becomes necessary at this point to recall an important character of understanding, that is, its *contextuality*. Not all people namely won a Nobel Prize, not all people attended physics courses at university. Some even dislike science or are bad at math. They just like listening to contemporary music, and never would open a more or less technical book on entropy in science. Some other can just rely on their sense of hearing, maybe because of some physical or mental disability. For them, reading a physics handbook or attending a physics course would be something very difficult, not quite perhaps as listening to music. So, clearly, *whether* music can transmit scientific understanding also depends on context and subjects involved, and, paraphrasing [Elgin 2017b, 281], it is clear that if music sometimes works as a vehicle of scientific understanding, it functions as such «not primarily to deepen the understanding of experts, but rather to deepen the understanding of ordinary people».

In this respect, music would moreover show still further cognitive advantages, especially in contrast to other, “more classical”, symbols and ways by which scientific understanding is commonly transmitted, from explanations and graphs to models and equations. One of this advantages is that concerning the epistemic role played by the component of *time* in understanding (see e.g. [Elgin 2017b, 185] on dynamic models and exemplification).

As pointed out in [Section 2.1.1], music is sound organized in a temporal structure (sounds have a given duration, follow each other in a given temporal order, and so on). Now, it is quite unproblematic to believe that actually listening to this sound constitutes a fundamental aspect of the *aesthetic* experience of music – otherwise, music would have surely not developed as an art form possessing in sound its most essential characteristic. Since time is part of the structure of music, and indeed what basically turns unorganized sound events in music, time itself represents a constitutive character of our aesthetic experience of music. We can gain an aesthetic understanding of a given musical composition because of its temporality, because sound in it is organized in a precise time

structure. By experiencing the composition in a way which is somehow incoherent with that structure (for example in revers, or, say, by every measure), we would fail to understand *that* music composition. This is allegedly not the case of other art forms like painting or architecture, which have apparently no particular temporal structure.

That being so, the understanding one gains of given entropy developments through a musical exemplar is intrinsically temporal. In the beginning of *Metastaseis*, we grasp the evolution of music towards disorder and unpredictability *in the time*, as taking place *in the time*. The intrinsic or extrinsic understanding of entropy one gains by *Metastaseis* is temporal in essence. Time plays a pivotal cognitive role in affording one epistemic access to the concept of entropy and entropy variation, which is something that, depending on the subject and on the context, other *static* and non-temporal symbols like an explanation or an equation may be not able to do, and to do successfully and effectively. Just by looking at an equation or a graph, or by reading an explanation, an ordinary subject may possibly fail to recognize entropy variations and entropy-related features in other situations and phenomena in the world, where they are obviously temporally instantiated – think for example to disorder increases taking place in mixing fluids, which is namely something happening *in the time*. And this is exactly what music, by exemplifying entropy also in a temporal sense, may help to do.

Another cognitive advantage of music in transmitting scientific understanding would moreover derive from music expressiveness and expression [Section 2.1]. Music literally instantiating given entropy developments also (metaphorically) possesses expressive properties or refers to extra-musical objects and states of affairs. So, for example, the most musically disordered passages of *Metastaseis* instantiate, at least for an average Western listener, expressive properties referring to emotions like upset, fear, anxiety, etc. It is not excluded that, depending on the subject, also properties like these may play a cognitive function in conveying an understanding of entropy. Changes in the degree of disorder or unpredictability could be namely experienced also in the form of “emotional” changes in the music. Similarly, what *Metastaseis* happens metaphorically to express

– which depends, as we know, on contexts and listeners – may also play a cognitive role in advancing understanding of entropy. For example, the title of the work (literally *meta*–=“beyond”, *–staseis*=“immobility”) makes it refer to the dialectic relationship between movement and stand-still [Harley 2004]. Being aware of this may help a subject in correctly contextualizing the work, in interpreting it as an exemplar for given entropy developments, and thus in conveying through it an understanding of the concept of entropy itself.

A Last Word on Interpretation and Aesthetic Functioning

Interpreting a musical work – and an aesthetic or musical symbol in general – mostly consists in putting forward hypotheses about what it exemplifies, i.e. what the *key* features of the work are by which it is possible to make sense of it, without which it would be ultimately ununderstandable [Elgin 2017b, 218-219]. Yet, interpreting a work in the correct way is not always an easy job. There are cases in which we may remain completely befuddled and incapable of advancing any hypothesis about it. Sometimes, any attempt to recognize key features in a work, to render it somewhat intelligible, may fail. Surely, as pointed out before, background assumptions and context may be helpful in interpreting a work. But still, depending on that, many, equally tenable, interpretations of it could be possible, especially when it allegedly bears an expressive content.

Yet, being musical works aesthetic symbols as well, it applies also for them what said in [Section 3.1]. Only those interpretations are admissible which are procedurally objective, intersubjectively accessible and which actually make sense of the work and render it in some way epistemically accessible and understandable. In this sense, would be inadmissible hypotheses and interpretations which were for example to regard the tonal character of *Metastaseis* the key feature for understanding the work, as much as would be inadmissible the interpretation of Bach’s organ *Passacaglia and Fugue* as expressive of the atmosphere of a gay field trip.

Now, what epistemic status has an interpretation, say, of *Metastaseis* as exemplifying entropy? More specifically, is this an admissible

interpretation? I suggest that the answer to this question is a positive one. As shown, in fact, entropy is considerable as an intrinsic, musical property of music, and an intrinsic, physical property of music as sound developing over time. An interpretation claiming that we can make sense of *Metastaseis* – and of any other musical work – not by simply disregarding, but by explicitly *denying* the fact given developments towards higher or lower degrees of order or predictability take place in it, would be wrong and *inadmissible*. It would be wrong, in the same sense in which it would obviously be such an interpretation denying that no modulation actually takes place in the adagio from Beethoven's *Moonlight Sonata*, and that this piece cannot serve as an exemplar for modulation in music. And it would be wrong in the same sense of an interpretation of Galileo's thought experiments denying its exemplification of the independence of acceleration from mass.

Of course, the interpretation of a musical symbol as exemplifying entropy is not the most accessible and immediate one. Nor it is strictly necessary to recognize the entropic character of a work in order to make it intelligible: one could namely understand a musical work – whereby musical understanding primarily depends on grasping the structure of music [Section 2.1.2] – also by *disregarding* entropy and *selectively focusing* instead on other alternative musical properties – after all, not all properties of a symbol can be exemplified at once [Section 2.2.1], and no single interpretation is sufficient for making sense of a work [Section 3.1]. Yet, in the right context, it becomes *possible* to interpret it actually as such. And such interpretation would even be utterly *admissible* because resting upon recognizing an intrinsic property of music which is in this sense objective, intersubjectively accessible and univocal. If so, for the reasons exposed in [Section 3.1], that symbol would also be legitimately admissible as a vehicle of scientific understanding, as a scientific symbol.

Would this symbol thus perform two distinct, separate functions, an aesthetic and a scientific one? Not really: a musical symbol namely conveys scientific understanding of entropy by functioning aesthetically. And this not only because it, as a species of aesthetic symbol, more or less shows the same symptoms of all aesthetic symbols (see [Goodman

1968, 179-192] and [Section 3.1]), but above all because entropy in itself, as a measure of disorder and unpredictability, is, as pointed out before, consciously or unconsciously part of our aesthetic experience of music. This is even clearer from what we said above about temporality of musical experience: in music, entropy variations happens, and are consequently grasped, in the time, and with time is necessarily connected our aesthetic experience and understanding of music in general, to the extent that we are able to interpret, recognize a musical composition as an exemplar of entropy, and so convey epistemically access to it, only when we treat and experience it aesthetically.

Conclusion

Let us briefly sum things up. I have claimed that understanding is non-explanatory, non-factual and objective. It does in general not depend on explanation, truth and propositionality. We also gain understanding of or through symbols which are totally or partly propositional and false, or non-propositional and thus non-truth-apt. If understanding does not depend on atomic propositions, this means that it should be regarded as holistic and objectual. One's understanding of something depends on her having an integrated and tenable account about it, in which also false or non-propositional elements can be present. The more extended this account is, the more one understands and is able to wield the object of understanding for further epistemic and practical ends. This means firstly that understanding involves in some cases a component of *know-how*, and secondly that differently from knowledge, it admits of degrees.

On the ground of this view on understanding, I defended that also non-propositional instances of understanding are therefore possible. In this regard, I specifically referred to the case of music. After demonstrating its non-propositionality, I showed how it can advance understanding by means of exemplification.

I claimed that it is part of our aesthetic experience of music to recognize it as exemplifying certain properties. Functioning aesthetically, music afford us epistemic access to those properties, which we are then able to spot also when they are instantiated elsewhere in the world. In so doing, music enhances our understanding of the world. Furthermore, some of the properties music can exemplify are instantiated by objects and phenomena we commonly regard as belonging to the scientific domain. If so, music is capable of enhancing our scientific understanding of the world.

An interpretation of a musical symbol as exemplifying scientific concepts or properties, and thus as a legitimate vehicle of scientific understanding, is admissible as long as it is a procedurally objective and intersubjectively accessible interpretation.

From the scientific point of view, the aim of the present work has been that of contributing to the present-day general debate about understanding, in particular by showing how the transmission of understanding is possible in non-propositional domains and through non-propositional symbols. Some commonalities between the way in which science and the arts – music in particular – epistemically work and advance understanding of the world have been moreover highlighted. This eventually represents a step forward in relation to the important epistemological problem of the unity of knowledge and culture.

Furthermore, by highlighting the cognitive value of music in transmitting scientific understanding, the work has tried to make an essential point in relation to another hotly debated topic in present epistemology, namely the reflection about new possible “unconventional” ways for communicating and transmitting science. Authors such as [Galison 2014, 204] have for example pointed out how visual symbols – from charts to photographs and films – represent a true epistemological contribution to scientific understanding. They are not just decorative elements, but sometimes even «constitutive parts of a scholarly argument», symbols by which genuine scientific understanding can be conveyed.

In the same terms of [Galison 2014], my work has tried to show that under the right circumstances music can legitimately function in the same way. It can advance scientific understanding and thus actually be accepted as a possible “unconventional” way for transmitting and communicating science. As we have seen, music would moreover possess also great cognitive advantages over other media, symbols and forms of art because of its expressiveness and its temporal structure.

Outside epistemology, all this may be also important especially in educational and popular contexts, where music can be proposed as an interactive means for spreading the interest in science and advancing

an understanding of various scientific concepts. For all those who find conventional scientific symbols technically inaccessible (and ultimately uninteresting), music may serve as a stimulus, and possibly as a more enjoyable way, for approaching science and scientific research.

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Notes on the Illustrations

After a long, thorough, but unsuccessful research, no reference for the graphic scheme of *Metastaseis* on [Figure 3.1] could be found. Even the *Centre Iannis Xenakis* (CIX)[†] – which curates the archive of Xenakis' works – has been asked after it. Yet, the only answer that it could give is that very probably the scheme is not by the composer himself. Nevertheless, the scheme has been used in this work, and that because of its clarity and immediacy in conveying the exposed ideas about musical entropy – which indeed constituted the major interest and the priority in the present work. Only hardly, in fact, would other written version of *Metastaseis* have conveyed those ideas with the same efficacy. Given this state of knowledge, the authorship of the scheme cannot be given but allegedly to the author of the internet video[‡] from which it has been taken.

[Figure 3.2] has been taken from Sharon Kanach's *Music and Architecture*^{††}. As mentioned there, the drawing is part of a private collection of Xenakis' family.

[†] Website: <http://www.centre-iannis-xenakis.org/>.

[‡] Website: <https://youtu.be/SZazYFchLRI>.

^{††} Kanach, S. (2008). *Music of Architecture (Iannis Xenakis)*. Pendragon Press. Hillsdale, New York, p. 47.

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Appendix A

Boltzmann's Concept of Entropy

Boltzmann gave essentially two formulations of the concept of entropy: in [Boltzmann 1872] and in [Boltzmann 1877]. In both papers, his aim was that of justifying the Second Law of Thermodynamics in the light of the kinetic theory of gases. This also meant to attempt to justify Clausius's thermodynamic entropy in kinetic and probabilistic terms. In the following we shall analyze these two formulations in all their technical details¹.

With regard to the 1872 formulation, let us examine an isolated thermodynamic system like a gas of n particles of mass m in a container of volume V . Given that each particle of the system has three degrees of freedom with respect to position and momentum, the system and its evolution over time can be mathematically described by considering a $6n$ -dimensional phase space², that we will call the γ -space (X_γ), in which each specific *microstate* of the system (i.e. all its particles' having a certain positions and momenta at a certain instant of time) will be denoted by a vector $x_\gamma \in X_\gamma$. X_γ is the Cartesian product of n copies of the 6-dimensions phase space X_μ (μ -space), relative to one single particle. The microstate of one single particle will be thus denoted by the vector $x_\mu \in X_\mu$, where $x_\mu = (x, y, z, p_x, p_y, p_z)$, with $\vec{r} = (x, y, z)$ and

¹ Given the intricate and old-fashioned form in which Boltzmann presented his results in the original papers (acknowledged among others even by [Müller 2007, 95]) we will follow in this analysis – rather systematically than historically aimed – [Frigg and Werndl 2011] and [Uffink 2007], which in turn refer back mostly to [Ehrenfest and Ehrenfest-Afanassjewa 1911] and [Tolman 1938].

² This “geometrical” language for describing thermodynamic systems was introduced by [Gibbs 1902] on the idea that changes in the system are “changes in phase” (see [Tolman 1938, 43 and ff.]).

$\vec{p} = (p_x, p_y, p_z)$, respectively, for position and momentum coordinates. From the kinetic theory of gases, we know that a given macrostate of the system at a certain time t (like the gas' having specific pressure and temperature) is describable by considering its microstate at that same time, that is, by individuating positions and momenta at time t of the particles. To this aim, Boltzmann refers to his improved and extended³ version of the theoretical and mathematical means for the analysis of the microstates of a thermodynamic system which was already developed in the decade before by Maxwell, namely the previously mentioned *Maxwell distribution*, in modern form:

$$f(x_\mu, t) = \frac{\chi_V(\vec{r}) (2\pi mkT)^{-\frac{3}{2}}}{\|V\|} \exp\left(-\frac{\vec{p}^2}{2mkT}\right), \quad (\text{A.1})$$

where $\chi_V(\vec{r})$ is the characteristic function of the set V , k the Boltzmann's constant, T the temperature of the gas, $\|V\|$ the volume of the container and $\vec{p}^2 := p_x^2 + p_y^2 + p_z^2$.

Today known as *Maxwell-Boltzmann distribution*, $f(x_\mu, t)$ gives the (mean) number of particles in the system with a position and momentum value lying within the infinitesimal interval $(x_\mu, x_\mu + dx_\mu)$ at time t . The study of the overall dynamic of the system can be carried out considering the evolution of $f(x_\mu, t)$ over time, namely

$$H_B(f) := \int_{X_\mu} f(x_\mu, t) \log f(x_\mu, t) dx_\mu, \quad (\text{A.2})$$

Relying on mechanical arguments, Boltzmann then moves on in his 1872 paper with the discussion of one of the most important results proposed in that work, namely the meaning of his famous *H*-theorem:

³ The *Maxwell distribution* [Maxwell 1867] describes only the speeds of the particles of a thermodynamic system, whereby the system is supposed to be at time t in thermodynamic equilibrium and the particles not interacting with each other except for very brief collisions without consequences on the overall microstate of the system. Later, Boltzmann extended the *Maxwell distribution* also to the description of the energies of the particles and to states of non-equilibrium of the system. The result was a nonlinear integro-differential equation, the so-called *Boltzmann equation*, which expresses – always in terms of probability distribution – how the number of particles having a certain energy (as well as a certain momentum and position) at time t *varies* within an infinitesimal time interval as consequence of collisions (*Stöße*) between particles (see [Boltzmann 1868], [Boltzmann 1872, 345 (Abh.)] and, for an overview, [Müller 2007, Ch. 4] and [Uffink 2007, Sec 4.2]).

due to incessant collisions of the particles with each other, the quantity $H_B(f)^4$ can only decrease towards a minimum value, after whose reaching it remains constant. This value is reached when $f(x_\mu, t)$ becomes the Maxwell-Boltzmann distribution [Equation A.1], i.e. exactly when the system reaches the state of equilibrium, where collisions have no longer effect on the positions and momenta distribution of the particles. He then passes to highlight the close connection existing between $H_B(f)$ and the thermodynamic entropy in equilibrium states, which takes the modern mathematical form of:

$$S_{B,f}(f) := -knH_B(f), \quad (\text{A.3})$$

where $S_{B,f}$ is the *fine-grained* (or *continuous*) *Boltzmann entropy*⁵ (therefore “B” and “f” in subscript). $S_{B,f}$ can only increase during the dynamic evolution over time of the system towards equilibrium, and reaches its maximum – remaining then constant – once the system has finally reached it and its microscopic distribution has thus become the Maxwell-Boltzmann distribution [Equation A.1].

In this way, Boltzmann provides a first kinetic proof of why entropy can only *increase* (or at least remain constant), delivering, in so doing, the coveted «mikroskopische Deutung des zweiten Hauptsatzes der Thermodynamik» [Boltzmann 1872, 116 (Brush 1976)] (see also [Boltzmann 1872, 313-314 (Abh.)] and [Uffink 2007, 46] on the comparison between Boltzmann’s and Clausius’ entropy). Moreover, as we have seen, the kinetic interpretation of the Second Law of Thermodynamics and of the concept of entropy expressed in [Equation A.2] and [Equation A.3] cru-

⁴ Boltzmann calls this quantity E .

⁵ More in particular, the connection between Boltzmann entropy $S_{B,f}$ and the thermodynamic entropy can be better recognized considering the Sackur–Tetrode Formula which describes the thermodynamic entropy of a monatomic ideal gas:

$$S_{TD} = nk \log \left(\left(\frac{T}{T_0} \right)^{3/2} \frac{V}{V_0} \right),$$

where S_{TD} is the thermodynamic entropy, T_0 the temperature of the gas and V_0 its volume. It can be shown that $S_{B,f} = S_{TD}$ up to an additive constant. It remains open to question if this result remains also valid for systems different from ideal gases (see [Frigg and Werndl 2011], [Emch and Liu 2013], [Reiss 1965], [Uffink 2007]).

cially rests in itself upon a probabilistic and statistical notion, namely the Maxwell-Boltzmann distribution. In his paper, in fact, [Boltzmann 1872, 316 (Abh.)] appears very aware of the fact that a kinetic explanation of the behavior of a thermodynamic system cannot but be a statistical and probabilistic one, for (my italics)

« [...] die Moleküle der Körper sind ja so zahlreich und ihre Bewegungen so rasch, daß uns nie etwas anderes als jene *Durchschnittswerte* wahrnehmbar wird. [...] Denn die Moleküle sind gleichsam ebenso viele Individuen, welche die verschiedensten Bewegungszustände haben, und nur dadurch, daß die Anzahl derjenigen, welche *durchschnittlich* einen gewissen Bewegungszustand haben, konstant ist, bleiben die Eigenschaften des Gases unverändert. *Die Bestimmung von Durchschnittswerten ist Aufgabe der Wahrscheinlichkeitsrechnung. Die Probleme der mechanischen Wärmetheorie sind daher Probleme der Wahrscheinlichkeitsrechnung.*»

The validity of Boltzmann's 1872 conclusion was however criticized already by his contemporaries (see e.g [Uffink 2007, 974-983] and [Ehrenfest and Ehrenfest-Afanassjewa 1911, 35-36]), which questioned many aspects and theoretical presuppositions of that research: from the specific role of probability in the argumentation and the exact mechanical characterization of the particles collisions to the generality of his H -theorem – pivotal for making his notion of entropy work – which is actually still today regarded as problematic (see [Emch and Liu 2013, 92-105] and [Uffink 2007, 962-974]).

For this reason, Boltzmann reformulated his kinetic theory of gases and, with it, the fundamental concept of entropy in his paper of 1877, where he intended to highlight even more the role of probability theory in thermodynamics, as well as the link between probability and entropy/Second Law⁶. Let us see briefly how in the following.

We take again the $6n$ -dimensional phase space X_γ for describing a thermodynamic system consisting of a gas in a vessel, as well as the phase space X_μ relative to one single particle of it. Since the system is completely isolated with constant energy and volume V , the state of each particle (its position and momentum at time t) can be represented only

⁶ This is also the reason why many commentators (see [Uffink 2007, 55], [Klein 1973, 83] and [ter Haar 1955]) consider this paper as clearly marking the transition from kinetic theory to statistical mechanics.

by points from a limited region of the phase space X_μ , which we will call the *accessible region of X_μ* , $X_{\mu,a}$.

We consider now a finite number of partitions⁷ $\omega = \{ \omega_i \mid i = 1, \dots, l \}$ of $X_{\mu,a}$ defined in the following way: $\forall i, j \in \{1, \dots, l\}$ with $i \neq j$, $\omega_i \neq \emptyset$, $\omega_i \cap \omega_j = \emptyset$, and $\bigcup_{i=1}^l \omega_i = X_{\mu,a}$. These partitions define cells on the 6-dimensional accessible region $X_{\mu,a}$, which are taken to be rectangular with respect to the position and momentum coordinates and to have fixed volume $\delta\omega$, defined through the Lebesgue measure μ on X_μ , $\mu(\omega_i) = \delta\omega$ for all $i = \{1, \dots, l\}$. Now, for each $x_{\mu_i} \in X_{\mu,a}$, i.e. for each *microstate* of the system, we define a *distribution of state* as follow: $Z := \{n_1, \dots, n_l\}$. This distribution indicates the number n_i of particles that, at a certain time t_i , are in a state (i.e. have certain position and momentum) which is represented by points of $X_{\mu,a}$ contained in the cell ω_i . It indicates, simply said, *how many* particles are in which cell ω of the accessible region $X_{\mu,a}$ at a certain instant of time. Obviously is $\sum_{i=1}^l n_i = n$ the total number of the particles of the system, and, for different time instants t_i , there are different Z_i distributions, where the ratio Z_i/n can be interpreted as being equivalent to the Maxwell distribution – $f(x_\mu, t_i)$ [Equation A.1]. Moreover, different distributions Z_i individuate univocally different *macrostates* of the system. Now, the association between the microstates $x_{\mu_i} \in X_{\mu,a}$ and a macrostate of the system depending on the particles distribution Z_i is defined in these terms: $Z_i := \{ x_{\mu_i} \in X_{\mu,a} \mid Z(x_{\mu_i}) = Z_i \}$. What it is important to notice here is that $Z(x_{\mu_i})$ is however not a bijective function. This means that possibly many microstates with the *same distribution* can correspond at time t_i to a same macrostate. In fact, the microstates can have different *internal arrangements*: even if the distribution (i.e. the number n_i of particles being in the cell ω_i) remains unchanged, the state of the particles within the cells can undergo rearrangements, for example by permutation. The number W of the possible arrangements compatible with a given distribution of state Z is determined in general by using

⁷ Boltzmann considered this kind of discretization as a mere mathematical tool and not as a physical hypothesis, as Max Plank will soon do (see [Müller 2007, 99]). This idea will be very important also for Xenakis (see [Appendix B]).

ordinary combinatorics:

$$W_Z = \frac{n!}{\prod_{i=1}^l n_i!}. \quad (\text{A.4})$$

Now, we can finally define a new concept of entropy, namely as:

$$S_{B,c} := k \log W_Z, \quad (\text{A.5})$$

which is called *combinatorial* (or *coarse-grained*) *Boltzmann entropy*⁸ (therefore “c” in subscript), which is thus directly proportional to the number W_Z of possible arrangements of a particles state distribution corresponding to a given macrostate of the system. This means that the more arrangements are compatible with a given particle distribution, the greater the entropy value will be.

Now, in order to understand the inherent probabilistic character of combinatorial entropy is moreover decisive to say that *all* arrangements of a given distribution are regarded as *equiprobable*. Thus, entropy is, more precisely, the measure of the number of possible arrangements coherent, *with equal probability*, to a given distribution. Now, Boltzmann states [Boltzmann 1877, 164-167 (Abh.)] that the system spontaneously tends to evolve *always* toward states with *more* arrangements coherent with a same distribution, until it reaches the state with *most* arrangements, namely the equilibrium, where entropy is therefore at maximum. If, within the evolution of the system toward equilibrium, the number of equally probable arrangements coherent with a distribution increase, this also means that the randomness, the unpredictability, the disorganization and the disorder of the system grow, reaching the maximum at equilibrium. Entropy is in fact a measure of such disorder and randomness in the sense that the greater it becomes, the less it is possible to know about the actual arrangement of the particles of the system on the basis of their distribution (where, we repeat, all arrangements compatible with a distribution are equiprobable). At the same time, however, entropy can be identified, Boltzmann says, with the probability of the total state of the system: in fact, equilibrium is also the *most probable* state – since

⁸ It can be shown that $S_{B,c}$ is formally (nearly) equivalent to $S_{B,f}$ (see [Uffink 2007, 56-57], [Frigg and Werndl 2011, 126-127] and [Müller 2007, 99-101]).

the *largest* number of arrangements correspond to its distribution – to the effect that the evolution of open thermodynamic systems always and spontaneously takes place from less probable to more probable states and never in the opposite way.

Recapitulating, a thermodynamic system generally evolves towards an equilibrium state, i.e. the most disordered and, at the same time, the the most probable one, whereby entropy is the measure of these disorder and probability. All this eventually amounted for Boltzmann to providing a new kinetic and probabilistic justification of the Second Law of Thermodynamics and of why entropy has constantly to increase⁹.

Boltzmann’s entropy formula [Equation A.5] can also take another form inasmuch as it is applied not to arrangements with respect to distributions, but, in a formally equivalent way¹⁰, to distributions (i.e. microstates) with respect to macrostates, where by distribution is meant how the particles-points of the phase space of the system are distributed in the cells. In this new form, it can represent the probability to find, given a macrostate of the system at a certain time, the corresponding distribution of the particles-points within the cells of the system phase space. Thus, assuming that in most systems $n_i \gg 1$ and using Stirling’s approximation formula for the factorials, $\log n! \approx n \log n - n$, we can derive from [Equation A.5]:

⁹ Yet, if [Müller 2007, in pt. 101] presents this conclusion as a more or less direct and explicit consequence of Boltzmann’s research, [Frigg and Werndl 2011, 125], drawing on [Ehrenfest and Ehrenfest-Afanassjewa 1911] and other commentators, regards it rather as a postulation in need of further assumptions and justifications (such as ergodicity or typicality).

¹⁰ See [Frigg and Werndl 2011, 125-127], [Uffink 2007, 56-57], [Müller 2007, 99-103], [Ehrenfest and Ehrenfest-Afanassjewa 1911, Ch. 2], [Tolman 1938, Ch. 4].

$$\begin{aligned}
S_{B,c} &= k \log W_Z \\
&= k \log \left(\frac{n!}{\prod_{i=1}^{\ell} n_i!} \right) \\
&= k \log(n!) - k \log(n_1!) - \dots - k \log(n_{\ell}!) \\
&\approx (nk \log n - n) - (n_1 k \log n_1 - n_1) - \dots - \\
&\quad - (n_{\ell} k \log n_{\ell} - n_{\ell}) \\
&= -k \sum_{i=1}^{\ell} n_i \log n_i,
\end{aligned}$$

whereby, letting $p_i = n_i/n$ be the probability of finding a randomly chosen particle in the cell ω_i , we therefore have:

$$S_{B,c} = -nk \sum_{i=1}^{\ell} p_i \log p_i, \quad (\text{A.6})$$

which links the macrostates of the system with the possible microstates of it (i.e. possible distributions of the particles in the cells), expressing, for a given macrostate, the probability to find at the microlevel its correspondent distribution¹¹. At equilibrium, the probability to find a randomly chosen particle in a given cell is equal for all cells, since the particles-points are distributed uniformly within the phase space. As in the case of arrangements and distributions, however, to a same macrostate several different microstates can correspond. In fact, due to the uniform distribution of the particles in the phase space and hence the equiprobability of all possible particles microstates (i.e. the equiprobability to find a particle in each of the cells of the phase space), at equilibrium a very large number of microstates are compatible with this macrostate of system, indeed the largest, and this is also the reason why the entropy of the system at equilibrium is at maximum. For non-equilibrium states, where the particles do not occupy quite homogeneously all regions of the phase space, but rather have all a specific and uniform value for position

¹¹ The two formulations of entropy are equal up to the additive constant $nk \log(\delta\omega)$ relative to the volume of the cells of the phase space, which we omitted for simplicity and irrelevance to the next argumentations. For the same reasons we avoided to derive this entropy formula in terms of proportionality between the number of microstates and the volume of the phase space. For a detailed derivation of the formula in current terms, see [Frigg and Werndl 2011, 125-127] or [Uffink 2007, 56-57].

and momentum (i.e. they occupy a specific region or cell of the phase space), the probability to find the microstate of the system corresponding to a given macrostate of it is ascertainable again through [Equation A.6], i.e. through finding out how many particles occupy which cells. And, the less *all* cells have the *same probability* to contain particles with position and momentum values compatible with that given macrostate, the lower the entropy and the more ordered the system will be.

[Equation A.6] never appears in the original scientific production by Boltzmann in the form in which we presented it. However, as it can be observed in some of his later writings [Boltzmann 1898], the idea behind it should have been already clear to him. The modern formula was proposed later by Gibbs [Gibbs 1902], who generalized it to different kinds of systems and system states, even more highlighting its statistical and probabilistic character. As a measure of the disorder and unpredictability of a system, the probabilistic notion of entropy expressed in [Equation A.6] became a powerful tool also in order to account for other kinds of phenomena.

This was for example the case of Shannon and his information theory (see [Shannon 1948] and [Shannon and Weaver 1949]). Shannon's aim was to provide a mathematical law for quantifying (and predicting) the quantity of information contained in a certain number of messages flowing from a source. Relying on some previous ideas by Hartley, [Hartley 1928] he proposed to make this quantity (mathematically) dependent on the probability of each message. The more probable a message is, the less information we gain from its reception. From messages flowing from a source with exactly the same probability, we gain no information, since all messages have equal probability to contain information. As in the case of thermal equilibrium, where all microstates are equiprobable, when all messages have the same probability to contain the same amount of information the entropy (*Shannon information entropy*) is at maximum. On the contrary, the less probable a message is, the more uncertainty there is about its information content, the more information we gain from its reception removing the uncertainty. Information and uncertainty are for Shannon, in fact, two sides of the same coin. Shannon noticed that

the only mathematical equation able to represent this relation between probability and information, as well as to satisfy other important formal requirements¹², was

$$H = -K \sum_{i=1}^n p_i \log_2 p_i, \quad (\text{A.7})$$

where H is the entropy, named by Shannon after Boltzmann's H -theorem¹³, K a positive constant, p_i the probability of the messages, and the binary logarithm referring to the fact that the entropy is expressed in bits, unit of measurement of information [Sedgewick and Wayne 2011, 185]. [Equation A.7], as we can see, is formally very similar to Boltzmann's entropy formula in the form of [Equation A.6]. And also the probabilistic concept of entropy it expresses, giving probabilities not to particles microstates but to messages, is by and large comparable to that expressed by the latter¹⁴.

¹² Continuity, additivity, monotonicity, branching and bit normalization (see [Shannon 1948, 10-11], [Frigg and Werndl 2011, 118]).

¹³ In Boltzmann thermodynamics, it can be in fact shown that if [Equation A.1] does not vary very much with respect to each cell of the phase space, [Equation A.6] and [Equation A.2] - i.e. the H -theorem - can be considered approximately equal up to the additive constant relative to the volume of the cells [Footnote A.11] (see [Frigg and Werndl 2011, 127]).

¹⁴ A lot of words have been spent until now about the relationship between Boltzmann's and Shannon's entropy. In particular, in epistemology, questions have been investigated such as if they both can be considered isomorphic concepts, if they interpret and apply probability in the same way, and still, if physical system as the thermodynamic ones are accountable in terms of information. Remarkable in this respect are for example the pioneering papers by Edwin Thompson Jaynes (in particular and [Jaynes 1957a] and [Jaynes 1957b]), who argued that Boltzmannian statistical mechanics can be seen just a particular application of Shannon's information theory.

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Appendix B

Xenakis' Concept of Entropy

It is worth starting the introduction of stochastic music theory from Xenakis' characterization of the primitive material of music, namely the sonic entity, the sound¹. Xenakis basically shares the modern physical account of sound as complex sound wave as described for example by [Helmholtz 1862]. Thus a complex sound, say a C₃ played on a piano, is composed by a large number of simpler sinusoidal waves (principal tone, overtones, summational tones, differential tones, etc.) which behave according to Fourier Analysis in producing the final sound. In particular, the quality of the sound is determined by the quantity and the amplitude of these simpler sound waves. Like Boltzmann and Gibbs in thermodynamics, Xenakis chooses to construct a geometrical representation of "macroscopic" sonic events. To this aim – almost recalling the particle-wave dualism of quantum physics – he considers the simple sound waves constituting these sonic events as *sound grains*, elementary sonic particles or sonic quanta, represented by points in a three-dimensional space with coordinates frequency, intensity and duration. Complex sounds are thus assemblages, *clouds* or *ensemble* of simple sound grains evolving over time. The macroscopic condition of every possible sound event becomes explainable, as in statistical mechanics, by referring to the microscopic condition of its granular constituents and its evolution over time. In other words, it suffices to consider the position of the sonic grains in the grains space relative to the complex sound in terms of their having specific frequency

¹ By "sound" or "sonic event" Xenakis means not only the "musical sound" distinguished from the noise, but every possible auditory phenomenon perceivable by the human ear. In the following we use the word "sound" in this acceptance.

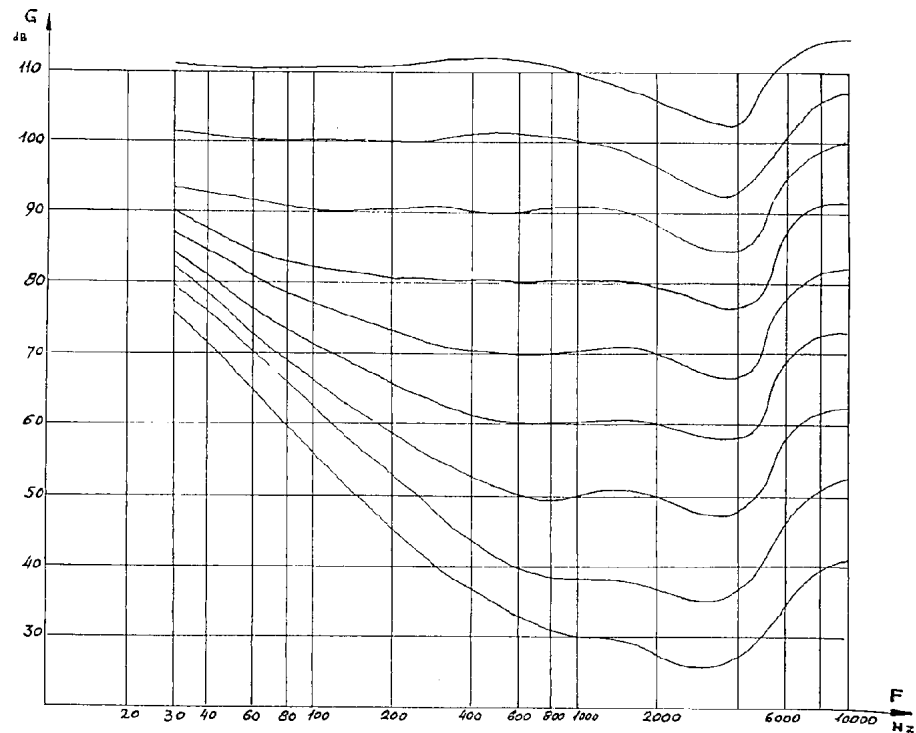


Figure B.1

and intensity coordinates as a function of time.

Like the phase space describing thermodynamic systems, however, the grains space is not infinite, for sounds are perceivable by human ears up to certain limits of duration, frequency and intensity. These limits have been firstly discovered in 1933 by Harvey Fletcher and Wilden A. Munson [Fletcher and Munson 1933], who proposed the famous equal-loudness contour diagram representing namely the *audible area* [Figure B.1], which gives for example the thresholds for the minimum perceptible duration of a sound as a function of its frequency F and its intensity G , or of its perceivable intensity in decibels compatible with its minimum frequency and duration. Thus we have to consider only this audible subregion of the grains space, i.e. only that containing the possible grains configurations microscopically corresponding to macroscopically audible sounds. For simplicity, we can operate a transformation on the curved space defined by the Fletcher-Munson diagram in order to gain, without alterations, a more regular space of rectangular form for representing the

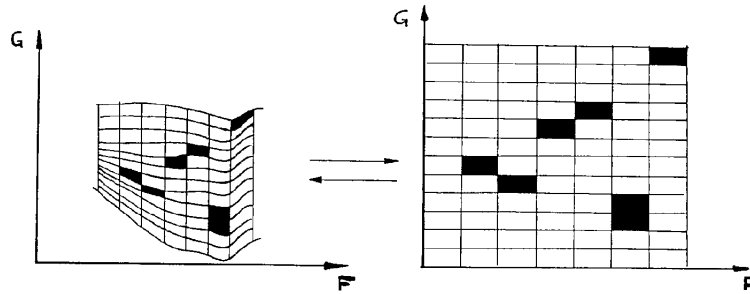


Figure B.2

audible subregion of the grains space [Figure B.2].

If we now assume that the (limited) duration of a given macroscopic sonic event can be divided into a large, but finite, number of slices of time ΔT_i , with $i \in [0, n]$, small at will and invariably equal in size, we can individuate for each T_i, T_{i+1} , where $T_{i+1} - T_i = \Delta T_i$, a certain microsound configuration, i.e. a certain distribution of the grains along the two dimensions of frequency F and intensity G coherent with the macrostate of the sound event in that interval of time. By flattening for simplicity the grains in the duration slice ΔT on the two-dimensional frequency-intensity space, we find that the entire macroscopic evolution of the sound event over time can be described by a finite series of n screens S_i [Figure B.3], with $i = \{0, \dots, n\}$ and $\bigcup_{i=0}^n S_i = FGT$, that succeed one another in time with different distributions and configurations of grains *in*² them.

As in statistical mechanics [Tolman 1938, 43-44], it is not important – if not completely impossible – considering directly the *single* microconstituents (single grains) of the sound and their respective behavior. In fact, we always refer to *clouds* of grains and to their evolution over time. Then, what is important for microscopically explaining the macroscopic condition of a sound at time T_i , is knowing the total number of the grains at time T_i – what Xenakis calls *density* – and their distribution over the i -th screen (i.e. their having specific frequency and intensity values

² We use the preposition “in” instead of “on” for speaking about grains with regard to screens even though, being screens two-dimensional objects, this may be grammatically incorrect. However, this will help us hereafter to keep well in mind that, as to the screens, we only artificially flattened their three-dimensionality, and that the grains *in* them exist in the their “time thickness” ΔT .

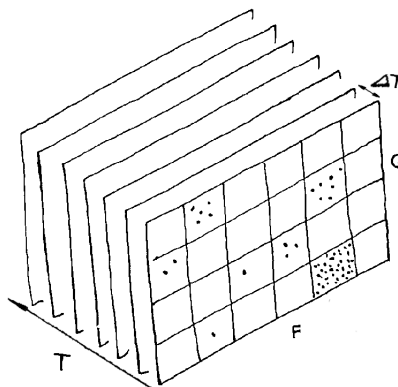


Figure B.3

at that instant of time). In other words, we need to know how many sound grains of the cloud occupy which region of the screen at time T_i , which means, how many sound grains have at this time instant specific frequency and intensity coordinates compatible with a given macrostate of the sound at that same time. The careful reader will already notice a certain resemblance between this problem and that answered by the Maxwell-Boltzmann formula (see [Equation A.1]). Xenakis actually considers the possibility to apply an adjusted version of this equation for calculating the distribution of the sonic grains within the screens. Yet, he notices that this would require firstly a reformulation of the geometrical environment of scalar type so far adopted – the grains space FGT should be indeed construct as a vector space – and secondly the introduction of the concept of speed [Xenakis 1992, 55-56]. Albeit mathematically possible, maybe due to practical reasons Xenakis does not further undertake this task and continues with his arguments based on the scalar grains space. Nevertheless, he shows that the mathematical³ and probabilistic ideas behind the Maxwell-Boltzmann distribution are very clear to him. In fact, the distribution of the grains in the screens at certain instants of time is anyway deterministically detectable [Xenakis 1992, 52], as the

³ It can be noticed from [Equation B.1], [Equation B.2] and [Equation B.3], in fact, that the mathematical idea of the Boltzmann-Maxwell distribution (specifying the *mean number* of microscopic objects lying in a certain region of a geometrical space individuated by an infinitesimal interval) is also applied by Xenakis for calculating the distribution of the grains within the screens with respect to coordinates D , T , F and G one at a time.

grains of natural sounds⁴ “fluctuate” around an equilibrium position of *mean* frequency and intensity. Also the density – i.e. the number of the grains in each screen – varies from one screen to another around a mean value. This eventually amount to say that the number of grains occupying at T_i a given region of S_i can be only *probabilistically* interpreted, just as a *mean* or *expectation* value – actually like in the case of particles distributions in thermodynamic systems.

How, then, does Xenakis account, from the mathematical point of view, for the distribution of the sound grains in the screens? First of all, he evidently embraces the theoretical presuppositions of the combinatorial approach of the “second” Boltzmann and of Gibbs: the distribution of the grains within the screens $FG\Delta T$ is only *statistically* determinable. Moreover, since we are not interested in the single grains, but in knowing which region of the screen a *cloud* of grains occupies, in order to find out that, we can take screens as divided into cells $\zeta = \{ \zeta_j \mid j = 1, \dots, l \}$ with $\bigcup_{j=1}^l \zeta_j = S_i$, defined as in [Appendix A]⁵ and of equal volume $\Delta F \Delta G \Delta T$ (remember: screens have a thickness ΔT) [Figure B.4], and consider the density ΔD of the grains for each cell, i.e. the (mean⁶) number of grains in it.

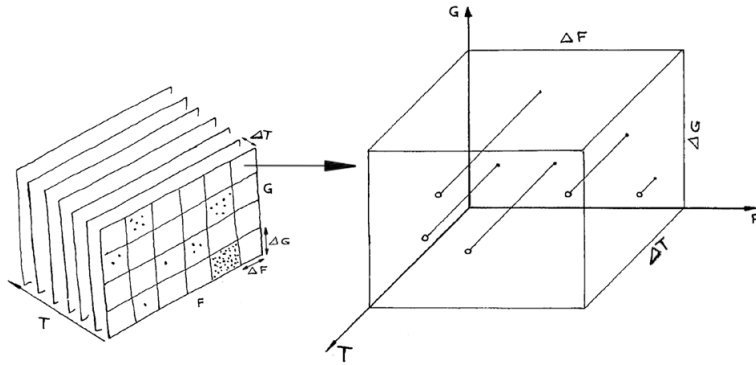


Figure B.4

⁴ An exception to this would be represented by electronic sound, whose grains are characterized by fixity.

⁵ The only difference being the possibility that $\zeta_j = \emptyset$ for some $j \in \{1, \dots, l\}$.

⁶ “Mean”, because, as we said, in natural sound the distribution of the grains in the screens and therefore also in their relative cells is not deterministically, but only probabilistically definable.

It is possible to calculate the density ΔD of the cells by the *mean density* D of the screen (ratio of the number of grains in it and its total volume) using Poisson's formula:

$$P_\mu = \frac{\mu_0^\mu}{\mu!} e^{-\mu_0} \quad (\text{B.1})$$

which expresses, given the fixed screen mean density μ_0 , the probability P_μ for a specific density μ (i.e. a given number of grains) to occur in a certain cell of the screen.

At this point, known the (mean) density of the grains per cell, we can calculate – always in a statistical way – the distribution of the grains along the other dimensions of the screen: the time T , the frequency F and the intensity G . Instead of giving a method for calculating this distribution directly with reference to *all* the three dimensions of the grains space *at one*, which would be mathematically very complex, Xenakis simplifies the calculation proposing *stochastic laws* – derived from the theory of continuous probability – for the distribution with reference to *one dimension at a time* [Xenakis 1992, 12-16; 52-54; 323-327].

As to the dimension of the time, the law that statistically gives the distribution of the grains along ΔT , i.e. the portion of the time axis T related to a screen, is

$$P_x = ce^{-cx} dx, \quad (\text{B.2})$$

where c is the linear density of the points on the axis, x the length of every possible segment taken on it and P the probability that the i -th segment will have a length x_i between x and $x + dx$. Hereby we consider the *points* on ΔT to be the *projections* t_j over this portion of the time axis T of the grains in the “thick” screen $FG\Delta T$. These points thus individuate segments *within* ΔT of length $t_{j+1} - t_j = x_j$, representing the duration of each grain of the screen. Then, [Equation B.2] expresses the *probability* to find on ΔT , taken on it *any* (even infinitesimal) duration δt , segments $x_j = t_j, t_{j+1}$ with $x_j = \delta t$ ⁷.

⁷ This same argument could be generalized from the duration of a single screen to that of the whole sound event: the statistical distribution of the grains along T can be also described through [Equation B.2].

With respect to frequency F and intensity G (as well as to each further variable one wishes to consider as characteristic of sound), the formula for calculating the distribution of the grains along ΔF and ΔG , the other two dimensions of the cells [Equation B.4], is

$$\theta(\gamma)d\gamma = \frac{2}{a} \left(1 - \frac{\gamma}{a}\right) d\gamma, \quad (\text{B.3})$$

where a is the length of the segments ΔF or ΔG on their respective axes. This equation thus gives the probability that a certain number of segments within ΔF and ΔG , defined, as in the case of the time, by points deriving from the projection of the grains in $FG\Delta T$ on the frequency F or intensity G axes, and representing intervals respectively of pitch or intensity, will be found to have a length between γ and $\gamma + d\gamma$, for *any* γ with $0 \leq \gamma \leq a$.

So, these are the different mathematical tools for statistically calculating the distribution of the sonic grains in a cell ζ_j of the screen S_i at time T_i with regard to its four dimensions⁸ $\Delta F\Delta G\Delta T\Delta D$, provided an overall screen mean density D_i in advance. They allow to individuate, for every chosen δt and γ , sets⁹ of durations, frequencies and intensities, $\delta t = \{\delta t_1, \dots, \delta t_n\}$ and $\gamma = \{\gamma_1, \dots, \gamma_n\}$, containing any possible value of these variables within the range of $\Delta F\Delta G\Delta T\Delta D$ compatible, with a certain probability, with δt or γ . In this sense, by them a probability distribution is assigned to the elements of these sets, formally in the following way¹⁰: $P = (p_1, \dots, p_n) := (p(\delta t_1), \dots, p(\delta t_n))$, with $p_i \geq 0$ and $p_1 + \dots + p_n = 1$. This means, every duration segment δt_j on ΔT has a certain probability p_i to correspond to a fixed duration segment δt . So, for instance, for $\delta t_1 = \delta t$, $p_1 = p(\delta t_1) = 1$, assuming that this probability is 0 for all the other elements.

There is obviously a correspondence between these sets, so that,

⁸ We regard density as fourth dimension of the cell.

⁹ These sets are finite for the following reasons: the number of possible perceivable duration intervals is limited (see equal-loudness contour diagram); the duration ΔT of the screens is limited; the overall duration T of complex sonic events is limited; the audible area FG is limited. They are furthermore totally ordered as subsets of totally ordered sets (respectively T , F and G).

¹⁰ We consider in the following only the case of δt , which is however completely similar to that of γ .

for example, grains with a given frequency and intensity have a certain duration, etc. If in a natural sound these correspondences are clearly independent of us, in music they can be set, even randomly, by the composer, who derives those values and correspondences by specifying the initial parameters, i.e. giving, for example, concrete values to D (mean screen density), j (number of cells the screens are divided into), δt , γ etc. and applying the above formulas to them [Xenakis 1992, 54; Ch. 1; Ch. 3]. In any case, by calculating with the same statistical procedures the grains distribution for *all cells* along their respective dimensions, and then considering the arithmetic means of the obtained values, we should be able to generalize our knowledge about the distribution of the grains at a certain “thick” instant of time to the whole screen. In this way, we would have a statistical, microscopic, *description* of the macroscopic state of a complex sonic event at that same instant of time, may this sound be natural or musical¹¹. But, as said, the composer could use these mathematical procedures also for actively building screens. In so doing, he would be able to *prescribe*, through a manipulation of the microscopic components of the sound, the macroscopically perceivable properties of a (musical) sound event at specific time instants of it. Examining in depth how stochastic music composition exactly works, how the values of the variables are concretely calculated and how the correspondences between them are set would however lead us too far away from our aim. Nevertheless, what has been generally said about that is already sufficient in order to introduce Xenakis’ concept of entropy.

Entropy is defined, with respect to each of the four dimensions $\Delta F \Delta G \Delta T \Delta D$ of a screen cell, as

$$H = -K \sum_{i=1}^n p_i \log p_i. \quad (\text{B.4})$$

As for the time, the minimum entropy is given in the case of simultaneity or isochrony (if they are emitted at regular intervals of time) of the grains. In such a case, in fact, we would have that $\delta t_1 = \dots = \delta t_n = \delta t$, that

¹¹ These mathematical methods are useful, in fact, not only for describing sonic events in general or composing new music, but, as Xenakis points out, also for analyzing and describing in a more accurate way *musical* works of the past [Xenakis 1992, 4].

is, only one element in the set δt , namely δt . Its *variety* (the number of *distinct* elements in a set or group) would be 1. Correspondingly, we would have $p(\delta t_1) = \dots = p(\delta t_n)$, thus $p_1 = \dots = p_n = 1$ (since the probability of the duration is $p_i = 1/n$ and the set has $n = 1$ elements), hence

$$H = -K \sum_{i=1}^n p_i \log p_i = -K \log 1 = 0.$$

On the contrary, if the variety of the set is different from 1 and, *theoretically*¹², $n \rightarrow \infty$, then also $H \rightarrow \infty$, inasmuch as $p(\delta t_1) = \dots = p(\delta t_n) = 1/n$, i.e. inasmuch as *all* durations of the grains, emitted according to [Equation B.2] become, in a fairly long succession of screens (see [Footnote B.8]), *equiprobable*.

As to the other dimensions, the argument unfolds quite similarly. The overall entropy of the cell is given by the arithmetic mean entropies calculated for each dimension, and that of the whole screen by the arithmetic mean of the entropies of the single cells composing it. For mean entropy $H = 0$, the cell or the screen will contain just one single sonic grain corresponding to a *pure sound*, with specific and constant frequency and intensity and emitted at regular intervals of time (the variety of the sets δt and γ will be 1) [Figure B.5 (A)]. In the case of maximum mean entropy (theoretically ∞ , see [Footnote B.12]), cells or screens will contain grains homogeneously distributed over all their regions, i.e. will have each one a different value for frequency, intensity and duration. The state of maximum entropy will thus correspond to a *white sound* [Figure

¹² This is however impossible in practice: in fact, firstly, there is a limit, as already pointed out, for the perceptibility of duration intervals, and secondly, a sonic event or a musical composition always has a *finite* number of grains and screens (see [Footnote B.9]). So, all in all, n will always depend, in a real sonic event, on the overall duration of this latter, and thus will also the probability distribution over the set δt of the duration intervals.

B.5 (C)] – that of malfunctioning (analog) radios and televisions.

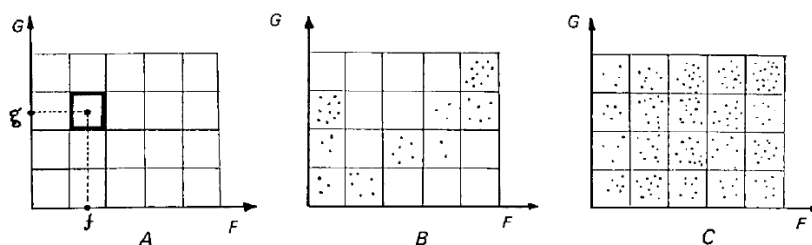


Figure B.5

And now something very fascinating: «Between these two limits the grains may be distributed in an infinite number of ways with mean entropies between 0 and the maximum and able to produce both the Marseillaise and a raw, dodecaphonic series» [Xenakis 1992, 64]. Between these two limits (e.g. [Figure B.5 (B)]), with the means of statistics, every possible perceivable natural sonic event is describable or prescribable, every musical work of the past is explainable and every new musical work can be composed.

So far we have discussed just about single screens, as descriptions at the microlevel of the state of a complex sonic event at a certain instant of time. Nothing has been said about the evolution in time of complex sonic events, that is, about the succession of the screens and how this happens and works. Xenakis distinguishes between *causes* and *modes* of this succession. The cause of a transition from a screen to another is to be traced back to the *physical* structure of the sound – which has been analyzed for example by [Helmholtz 1862] – or the *logical* structure of a musical composition – which Xenakis thoroughly analyzes in [Xenakis 1992, Ch. 6-8] using linear algebra, in particular interpreting these transitions as operations on vectors from a vector space with dimensions F , G and T . Again, an examination of these structures is something that can and need not be undertaken here. Rather than in the causes, what we are instead interested in modes in which the transitions between screens *generally* take place.

First of all, speaking about transitions between screens means speaking about transitions between different configurations of sonic grains along

frequency, intensity and duration. Thus, we can describe or prescribe a transition from these three different points of view. A *transition* is formally a succession of two *terms*, whereby the second one is called the *transform*. The terms can be denoted for instance by the letters of the alphabet (a, b, c, \dots). A *transformation* is finally a collection of transitions and can take the following form:

$$\begin{array}{c} \downarrow \\ \begin{array}{cccc} a & b & c & \dots \\ a & c & c & \dots \end{array} \end{array}$$

where (a, b, c, \dots) are for example pitches, intensities, durations, etc. A transformation can be also represented by a matrix. Taking the previous example:

$$\begin{array}{c|ccc} \downarrow & a & b & c \\ \hline a & 1 & 0 & 0 \\ b & 0 & 0 & 0 \\ c & 0 & 1 & 1 \end{array}$$

So, transformations of screens are generally the result of transformations of pitches, intensity, durations, etc, correspondingly represented by matrices. In music composition, in order to prescribe a certain overall evolution of the sound, the composer will construct a protocol for such transformations and engender the relative matrices, which may be independent from each other or possibly coupled.

Screen transformations represented by matrices wherein only values 1 and 0 appear are actually just a particular case of transformations, namely determined ones, with univocal and closed transitions. The general case is in fact that of *stochastic* transformations, whose transitions are many-valued and depend on probabilities. In the corresponding matrices, the values 1 and 0 are namely replaced by relative frequencies which specify the probability of a certain transition. Are these probabilities constant over a long period of time and dependent only on the first term of each transition, then we have a particular case of stochastic transformation, called *Markov chain*:

$$\begin{array}{c} a b a b b b a b a a b a b a b a b b b b a b a a b \\ a b b a a b a b b a b a a a b a b b a a b b a b b a \end{array}$$

The real frequencies of each transition occurring in the transformation are:

$$\begin{array}{rcl} a \rightarrow b & 17 & \text{times} \\ a \rightarrow a & 6 & \text{times} \\ \hline & 23 & \text{times} \end{array} \qquad \begin{array}{rcl} b \rightarrow a & 17 & \text{times} \\ b \rightarrow b & 10 & \text{times} \\ \hline & 27 & \text{times} \end{array}$$

from which we can construct a matrix of transition (i) as in the example above, and also a matrix of transition probabilities (ii) by converting the real into relative frequencies:

$$(i) \quad \begin{array}{c|cc} \downarrow & a & b \\ \hline a & 6 & 17 \\ b & 17 & 10 \end{array} \qquad (ii) \quad \begin{array}{c|cc} \downarrow & a & b \\ \hline a & 0.26 & 0.63 \\ b & 0.74 & 0.37 \\ \hline & 1.00 & 1.00 \end{array}$$

Now, the mode in which a screen transformation – and thus the evolution over time of a sonic event – take place, always implies certain changes in the overall value of the entropy of the screens of this event. If a transformation takes place in a determined mode (with regard to *all* the dimensions of the screens), that is, if all transitions in it occur according to a matrix of transition probabilities of the form

$$\begin{array}{c|cc} \downarrow & a & b \\ \hline a & 0 & 1 \\ b & 1 & 0 \end{array}$$

the overall entropy of the screens within the transformation remains unchanged. Since all transitions occur deterministically, the transformation brings about neither an increase nor a decrease of disorder along the temporal evolution of the sonic event¹³. In the opposite case, if all transitions occur with the same probability, thus according to a matrix of transition probabilities of the form

$$\begin{array}{c|cc} \downarrow & a & b \\ \hline a & 0.5 & 0.5 \\ b & 0.5 & 0.5 \end{array}$$

the transformation would be completely indeterminate and totally ruled

¹³ A case in which all transitions (must) always occur deterministically is that, for example, of tonal music.

by chance. The transitions would be all equally unpredictable and thus the disorder and the entropy of the transformation at maximum.

As said, transformations of screens are the result of transformations of pitches, intensities, durations, etc. Then, the overall entropy of the former (the extent to which they bring about changes in the entropy of the screens over time) corresponds to nothing but a mean entropy of the latter. Thus, a musical composition might develop, for example, deterministically as to time, towards disorder as to frequency, and so on, whereby the overall entropy of the composition, and of the general screen transformation involved in it, would be a mean value of the entropies of the transformations occurring along each of its three dimensions.

Hence, between the two limits set by determined and indeterminate transformations, we have stochastic transformation, in which transitions take place each with a different probability (see [Matrix (ii)] above). From the point of view of the entropy, these transformations are classifiable under three groups:

- (1) “Neutral” transformations, i.e. those in which the overall value of entropy does not change;
- (2) Transformation towards order, i.e. those bringing about decreases in the overall value of entropy;
- (3) Transformation towards disorder, i.e. those bringing about increases in the overall value of entropy.

Group (1) is that of determined and indeterminate transformations, which keep the value of entropy unchanged, respectively, at its minimum (zero) and its maximum (theoretically ∞). Conversely, to group (2) or (3) belong stochastic transformations inasmuch as the probabilities of their transitions are set as, or develop towards, those of the transitions in determined or indeterminate transformations. Differently – in most cases – from natural sound, in music compositions many kinds of transformations can be applied to screens in the course of the same composition.

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Notes on the Illustrations

All images in [Appendix B] have been taken from Iannis Xenakis' *Formalized Music*^{‡‡}. In [Figure B.3] and [Figure B.4] some minor and non-altering elements (arrows, symbols for the name of the axes, etc.) have been added for facilitating the understanding of Xenakis' arguments as we exposed them.

^{‡‡} Xenakis, I. (1992). *Formalized Music: Thought and Mathematics in Composition*. Pendragon Press, Hillsdale, New York, pp. 48-51, 53.

Eidesstattliche Erklärung

Hiermit versichere ich, dass ich diese Masterarbeit selbstständig verfasst und keine anderen als die angegebenen Quellen und Hilfsmittel benutzt habe, alle Ausführungen, die anderen Schriften wörtlich oder sinngemäß entnommen wurden, kenntlich gemacht sind und die Arbeit in gleicher oder ähnlicher Fassung noch nicht Bestandteil einer Studien- oder Prüfungsleistung war.

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