The Growth of Modern Acoustics:
From Modern Science of Music to Helmholtz's Theory of Sound

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Abstract

This essay aims to inquiry into the main factors responsible for the growth of modern acoustics, which basically have to be traced back to the empirical turn occurred in science of music around 1600. Helmholtz's theory of sound will be regarded as most scientifically significant archetype of modern acoustics. In Section 1 a general historical overview of the science of music will be given and its importance for the development of modern science and mathematics considered. In Section 2 the internal historical roots of modern acoustics, thus of Helmholtz's theory of sound will be analyzed. Finally, in Section 3 positive and negative elements of Helmholtz's acoustic will be discussed and its external historical roots as well as its actual epistemological relevance examined.

Summary

1 Introduction: What is Science of Music?

Our present common opinion about the constitution of the western human culture is largely based on the concept of 'split'. There exists a split between scientific and humanistic culture, but also, within sciences and humanities themselves, between various disciplines and specializations[1]. Thus, it sounds a little weird from the present point of view the fact that words like 'science', 'mathematics' and 'music' can be somehow matched. Indeed, the former refer for us to the world of the 'pure reason', of the exact laws and rigorous calculations and the letter rather to that of the irrational passions, the free creativity and whimsical imagination. Of course, this impression is not completely wrong, nevertheless considering it acritically does not allow to gain a genuine historical and theoretical understanding neither of what music today really is, nor of why science and mathematics for some aspects today are as they are.

More than other fine arts, music has always had a privileged relation with science, and it was even a science itself at least until the first half of the 18th century. The Babylonians and the ancient Egyptians already began to speculate on the musico-scientific problems that later became some of the most important in the whole culture of the ancient Greece, whose different solutions and approaches were still discussed during the Middle Ages, Renaissance and Modern Ages.

In ancient Greek and Middle Ages culture music had a quite specific position in the 'hierarchy' of the human activities. It was part of both the ἐπιστήμαι and the τέχναι (Lat. *artes liberales* and *artes mechanicae*), i.e. intellectual activities and manual activities[2]. Hence, music certainly had a mere esthetical function - which by the way played an important social[3] and later in the Christianity religious[4] role, but it also had an epistemic one. Music was in fact a mathematical science subordinated to arithmetic and sometimes to geometry which dealt with musical intervals expressed as proportions between (mainly discrete) quantities. So, there existed the *cantor* who could sing or play a musical instrument, but with no understanding of the mathematical musical theory and the *musicus*, often a philosopher or a mathematician who speculated on the mathematical theory of music but could not play any instrument[5].

From the late Middle Ages onward, the distinction between *cantor* and *musicus* gradually blurred, so that important personalities like Francisco de Salinas, Gioseffo

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[1] Cf. C.P. Snow, *The two cultures and a Second Look*, Cambridge University Press, Cambridge 1959.

[2] Cf. L. Borzacchini, *La crisi dell'armonia. Musica e matematica nel Settecento e nell'antropologia di Kant*, in *Atti del convegno Musica e Filosofia/3*, Edizioni dal Sud, Bari 2014.

[3] Cf. Plato, *Republic*; E. Fubini, *L'estetica musicale dall'antichità al Settecento*, Einaudi, Torino 1964.

[4] Cf. Augustine, *Sermones*; *Enarrationes in Psalmos*; *Confessiones*; *Retractationes.*

[5] Cf. J. Ciconia, *Nova Musica and De Proportionibus*, tr. by O.B. Ellsworth, University of Nebraska Press, Lincoln 1993.

Zarlino or Vincenzo Galilei (the father Galileo's), at the same time musicians and musical theorists, could later arise. Like in the Middle Ages, music continued to be a mathematical science and to constitute an essential part of the lower university education[6]. Due to the rediscovery of the Pythagorean and Platonic mathematical philosophy the mathematical music theory also knew a period of prosperous renewal, although a gradual but crucial ontological and epistemological change in the conception of music phenomena was about to happen roughly at the beginning of the Early Modern Ages.

Namely, at the end of the Renaissance, the inquiry into musico-scientific issues starts to be no longer performed in mathematical terms, but through a new kind of 'conceptual vocabulary', that of physics. The epistemic categories of the traditional theory of music (ratio, harmony, consonance, string lengths, etc.) assume new physical meanings and new ones like pulse, vibration, frequency, pitch, wave, etc. appear in order to give a scientific account of phenomena which are now ontologically conceived in a totally different way, no longer as something mathematical in itself and somehow metaphysical, but mechanistic and just therefore *mathematizable*[7].

This important transformation in the modern science of music finally leads not only to a new separation between *cantor* and *musicus*, but also to a radical dichotomy within the music itself between esthetical and scientific dimension. On the one hand, the analysis of musical tones becomes a matter of acoustic physics, which treats it in the same methodological and epistemological way of the other natural phenomena; Further traditional musico-scientific problems like the perception of the tones and the experience of the sweetness of the consonances become area of study of physiology and much later neurobiology. On the other hand, the analysis of harmonic rules and the study of the compositional techniques become exclusive appanage of practicing musicians and composers. There no longer exists something similar to a 'musico-scientist' and the two dimensions of music do not interact and influence each other anymore as in the past.

At the beginning of the 19th century this transformation process seems to be already completed. Both historians of science and of music are still discussing about the causes of this sudden change, nevertheless it is undoubted that the Romantic

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[6] In medieval and early modern university system, music was a part of the *Quadrivium* with astronomy, arithmetic and geometry. The seven liberal arts of *quadrivium* and *Trivium* - rhetoric, dialectic and grammatic - constituted the core of the lower university education as obligatory step for attending later the higher faculties - medicine, theology and law. For more about this cf. L. Moulin, *La vie des étudiants au Moyen Âge*, Éditions Albin Michel, Paris 1991.

[7] Cf. H.F. Cohen, *Quantifying Music. The science of music at the First Stage of the Scientific Revolution, 1580-1650*, Dordrecht, Reidel 1984;

movement, «with its one-sided emphasis on the autonomy of the artistic inspiration»[8] plays therein a quite important role: music is conceived as the mirror of the Absolute and shies away from any rationalistic attempt to be scientifically explained[9]. Musical beauty no longer depends on mathematical proportions and objective, universal facts, but only on subjective esthetical factors. Of this *cognitive and anthropological shift*[10] even Kant seems to be aware: «Mathematics, certainly, plays not the slightest part in the charm and movement of the mind produced by music»[11].

Thus, from a historical point of view the relation between music and scientific disciplines is an undeniable fact, even if it is today no longer so evident[12]. Moreover, as pointed out by Cohen[13] and Borzacchini[14] music carries out in history of science a crucial heuristic function for the development of the modern science and the modern mathematics, especially at the time of the Scientific Revolution, and the transformation itself in the science of music - occurred basically at that time - shares with the epistemological transformations in the other sciences many revolutionary elements, both as cause and as effect of this transformation. Hence, acknowledging the «importance of treating science of music as one of the sciences, on a par with more obvious domains of science like mechanics and optics»[15] is necessary within a historiographical inquiry into music and science in general.

From the ancient Greeks to the 17th century music was for example «the laboratory in which the new ideas about the role of quantity were forged»[16], and this was

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[8] H.F. Cohen, op. cit. (1984), p. xiii.

[9] Cf. E. Fubini, *L'estetica musicale dal Settecento a oggi*, Einaudi, Torino 1964, ch. III.

[10] Cf. L. Borzacchini, op. cit.

[11] «An dem Reize und Gemüthsbewegung, welche die Musik hervorbringt, hat die Mathematik sicherlich nicht den mindesten Antheil», I. Kant, *Kritik der Urtheilskraft*, rev. B. Erdmann, Leopold Voss Verlag, Hamburg-Leipzig 1884, En. tr. by J.C. Meredith, Oxford University Press, 1978.

[12] However, it must be noted that mathematics, for example, still has an important structural function in the modern theory of music (for practicing musicians, i.e. not the acoustic theory of sound!) and represents an interesting and fruitful conceptual toolbox for music composition already since the Baroque era. For more about this cf. I. Zanzarella, *Where Opposites Meet: Mathematics Between Science And Humanities*, in *Scienza e Filosofia*, 22 (2019), pp. 302-321; D. Tymoczko, *A Geometry of Music. Harmony and Counterpoint in the Extended Common Practice*, Oxford University Press, 2011; D.R. Hofstadter, *Gödel, Escher, Bach: An Eternal Golden Braid*, Basic Books, New York 1979.

[13] H.F. Cohen, op. cit.; H.F. Cohen, *Music as a test-case*, in *Studies in History and Philosophy of Science*, A 16(4) (1985), pp. 351-378.

[14] L. Borzacchini, op. cit.; L. Borzacchini, *Incommensurability, Music and Continuum: A Cognitive Approach*, in *Archive for History of Exact Sciences*, 61 (2007), pp. 273–302; L. Borzacchini, *Einstein's violin and the cognitive roots of the Scientific Revolution*, in *Music Education*, Nova Science Publisher, New York 2011, pp. 39-64; L. Borzacchini, *La nascita dell'armonia. Estetica e matematica nella paideia di Platone*, in *Scienza e Valori. Il bello, il buono, il vero*, Armando Editore, Roma 2014.

[15] H.F. Cohen, op. cit. (1984), p. xiii.

[16] «[...] il laboratorio in cui si forgiavano le nuove idee relativamente al ruolo della quantità», L. Borzacchini, op. cit. (2014a), p. 5, my tr..

extremely important for the development of mathematics. The beginners of the science of music were surely the Pythagoreans. Their mathematical science of music, the so-called *harmonics*, studied the pure consonances[17], by means of a particular musico-scientific instrument, the κανών or *monochord*, consisting of a string and of a moveable bridge used for dividing it. By division of the string at $\frac{1}{2}$, $\frac{2}{3}$ and $\frac{3}{4}$ the three principal consonances were obtained: the octave, the fifth and the fourth. These consonances were with a fourth one, the unison (ratio $\frac{1}{1}$), the basis of the broader concept of 'harmony', which pervaded all dimensions of Pythagorean philosophy and mathematics, but also of the practical life of their adherents (Pythagoreanism had a sect-organization[18]) as well as the entire cosmos. Music was very important in the Pythagorean philosophy and it is traditionally accepted that Pythagoras himself could formulate his entire philosophy just due to his musical investigations[19]. However, it is important to note that for the Greeks and in particular for Pythagoreans tones have no to do with modern acoustical concepts like wave, pitch or vibration. On the contrary, they are conceived as nothing but pure numbers and arithmetical proportions (by the way, as the rest of the nature, according to the motto 'everything is numbers'[20]), albeit not as abstract entities. In fact, the Pythagorean notion of mathematics and especially of arithmetic is not the modern 'abstract' one, but a quite concrete one, denoted by the expression 'arithmo-geometry'[21]. Thus, each number could be geometrically represented by concrete and spatial elements or figures, for example by the lengths of a string - as for tones and consonances - or by the ψῆφοι (Lat. *calculi*, En. pebbles) in a τετρακτύς: They were treated as discrete 'monadic' geometrical quantities. But now the *Glundlagekrise* of the Pythagorean philosophy: According to the continuous material nature of the string it is possible to set the bridge in any of its infinite material points and so to obtain a potentially infinite number of intervals whose ratios cannot be however always expressed by positive integer, the only possible numbers in

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[17] A melodic or harmonic juxtaposition of two tones which sounds somehow 'pleasant' to our ears is called *consonance*, otherwise *dissonance*. 'Why there is only a small, finite number of consonances'; 'Why some of them are 'more consonant' than the others'; 'What the sensation of pleasure connected with the consonance depends on' are the main questions that from Pythagoras until today the science of music has tried to answer, first in a mathematical way, then in a mechanistical and physical one.

[18] Cf. C. Riedweg, *Pythagoras: His Life, Teachings, and Influence*: Cornell University Press, Ithaca 2005, pp. 27-28.

[19] Cf. C. Riedweg, op. cit., p 31 et seq.

[20] Aristoteles, *Metaphysics*, A5.985b23-986a3; 58.B.4 *D-K*1:451ff.

[21] G. Reale, *A History of Ancient Philosophy I: From the Origins to Socrates.* State University of New York Press, 1987, p. 64.

Pythagorean and, more in general, ancient mathematics[22]. Moreover, those *irrational*[23] intervals are not consonances, because Pythagorean consonances derive exclusively from ratios expressed by the first four integers. So, from the disagreement between the discreetness of arithmo-geometrical quantities and the continuousness of geometrical space and real physical magnitudes the mathematical problem of the incommensurability[24] finally arises, remarkably «not in geometry or simply in arithmetic, but in harmonics»[25], in music. The problems of incommensurability and of the opposition discrete/continuous, arithmetic/geometry were destined to be discussed for ages by a lot of philosophers and mathematicians and they could partially be solved only by the introduction of real numbers and analytic geometry between 16th and 17th century[26].

Music and mathematics influence each other in Middle Ages as well, also because of new novelties occurred in music itself. On the one hand, for example, the increasingly common use in musical composition of major thirds and major sixths[27] as consonances in addition to the four ancient ones compels musical theorists to new mathematical speculations: The new system of consonances namely needs to be founded again as logically consistent. In this regard, really new consonance theories are put forward only much later, for example the *senario* of Gioseffo Zarlino[28] and the geometric theory of Johannes Kepler[29]. Cohen[30] quite broadly discusses these theories, classifying them in the early modern 'mathematical approach' to the problem of consonance. On the other hand, polyphony arises whit the need of «representing the *measure*, the length of musical times and pauses, i.e. 'lacks' of sound»[31]. For this, the

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[22] L. Borzacchini, op. cit. (2014a), p. 2.

[23] For the Pythagoreans, only ratios expressed by natural numbers (as those of the consonances) were admissible and so 'rational'. Otherwise, ratios were described as 'irrational'. Thus, as pointed out by Borzacchini in op. cit. (2011), p. 14, our modern «equality between ratios and real number», and therefore between rational (or irrational) ratios and rational (or irrational) numbers is virtually anticipated by Pythagorean science of music. This equality became explicit not before Newton, nevertheless it was already developed in ancient Islamic mathematics.

[24] L. Borzacchini, op. cit. (2014a), p. 6; Cf. also L. Borzacchini, op. cit. (2007).

[25] «[...] non in geometria o nella semplice aritmetica» L. Borzacchini, op. cit. (2014a), p. 4, my tr.; Cf. also L. Borzacchini, op. cit. (2007).

[26] Cf. C.B. Boyer, U.C. Merzbach, *A History of Mathematics*, John Wiley & Sons, Hoboken 2011, chs. XV-XVII.

[27] In just intonation with ratios respectively of $\frac{3}{5}$ and $\frac{4}{5}$.

[28] Cf. G. Zarlino, *Le Istitutioni harmoniche*, Venezia 1558.

[29] Cf. J. Kepler, *Harmonices Mundi*, Linz, 1619.

[30] Cf. H.F. Cohen, op. cit., pp. 3-32.

[31] G. Reese, *Music in the Middle Ages*, W.W. Norton and Co., New York 1940, p. 234, in L. Borzacchini, op. cit. (2007), p. 10, (emphasis in the original).

mensural notation[32] was developed already at the end of the 13th century as a substitute of the classical alphabetical one. It permitted to express the values of notes and pauses exactly and laid the groundwork of the modern musical notation. Instead, for indicating the pitch of the notes, the *diastematic-neumatic notation* was developed between 9th and 10th century and later improved with the introduction - attributed to Guido d'Arezzo - of the *tetragram* and of *square notation*[33]. According to Borzacchini[34], this new system composed of «a symbolic representation with [...] mensural and rhythmic signs and a geometric representation by neumes on the tetragram» is «the first 'syntactic' symbolism we can recognize after the Euclidean use of letters in geometry, the Aristotelean terminology in syllogistic logic, the alphabetical notation of Greek», a system whose «doubleness [... precedes] by many centuries the analogous one (symbolic algebra and analytical geometry) characteristic of Descartes' mathematics»[35]. It connects two (physical) dimensions existing in music, time (continuous horizontal lines of the tetragram) and pitch (discrete squares placed vertically on or between the lines), and therefore can have somehow philosophically influenced the contemporary attempts of medieval mathematicians like the Mertionian *Calculatores* and Nicolas Oresme to quantify, measure and geometrize the traditional Aristotelian qualities (speed, weight, temperature, in some contexts time, etc.). The early geometrical representation of uniform acceleration by Oresme - the famous Merton Rule[36] - can be a good example for this, by the way very probable, since the fervid musico-scientific interests of Oresme[37] himself.

Hence, on grounds of this few (but certainly not unique) examples, the heuristic function carried out by music towards mathematics cannot be questioned: Music contributed to develop and to define some of the most important problems and concepts of modern mathematics and consequently to reformulate the role of mathematics within sciences and towards reality itself, which represented a nonnegligible *cognitive root*[38] of the upcoming Scientific Revolution. But also for other aspects music played an important role in this important historic-scientific event: «The science of music [...] was one of the sciences that made up the revolution»[39], and namely it is not very

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[32] Cf. W. Apel, *The Notation of Polyphonic Music, 900–1600*, The Medieval Academy of America, Cambridge (MA) 1961.

[33] Cf. W. Apel, *Gregorian Chant*, Indiana University Press, Bloomington 1958.

[34] L. Borzacchini, op. cit. (2011), pp. 44-45; Cf. also L. Borzacchini, op. cit. (2007).

[35] Ibidem.

[36] Cf. E. Grant, *A Source Book in Medieval Science*, Harvard University Press, 1974, p. 252.

[37] Cf. e.g. N. Oresme, *De configurationibus qualitatum et motuum*, tr. by M. Clagett, University of Wisconsin Press, Madison 1968.

[38] L. Borzacchini, op. cit. (2011).

[39] H.F. Cohen, op. cit. (1984), p. 247; So also L. Borzacchini, op. cit. (2011), p. 19.

surprising to see a lot of its main characters involved in musico-scientific researches, from Kepler, Galileo or Stevin to Beeckman, Mersenne, Descartes, Leibniz, Huygens and even Newton. These aspects mainly concern the empirical turn of science characteristic of the Scientific Revolution. Drake[40], for instance, traces it back to the music-theoretical work of Giovanni Battista Benedetti and Vincenzo Galilei, which began the transformation of the traditional *harmonic science of music* in the modern *acoustic science of music* just on the basis of physical experiments. Although, agreeing in this with Cohen[41], one cannot accept Drake's thesis of seeing music as *unique* root of the modern experimentalism, it seems quite tenable to consider it as a contributing factor. The same can be said of Cohen's own thesis[42] - by the way, a very interesting and perhaps more acceptable one. The blur of the classical distinction between sciences and techniques is accepted as external root of the scientific revolution and, as we have seen above, it already occurred in music during the late Middle Ages and the early Renaissance (between *cantor* and *musicus*), so explicitly maybe before than in other fields. So, also the artisan or the practicing musician 'meets' the theorist: Practical and building problems or knowledges in music - e.g. according to which temperament to tune a harpsichord, how to divide the octave in order to allow larger possibilities of modulation and transposition - call the attention of scientists and mathematicians, in the same way as practical problems and knowledges in agriculture, architecture or civil engineering[43]. Musical and acoustical «phenomena [become] the subject of scientific analysis that were already quite familiar to instrument makers either as disturbing factor or as rules of thumb»[44]. For example, Isaac Beeckman was the first to speak about the physical phenomenon of beats in a scientific theory[45], having learnt about it just from an organ player. Moreover, it is quite certain that this phenomenon was already known by musicians and instrument makers since the Middle Ages or even before, due to his use for tuning purposes (which applies still today). However, regardless of who is right, whether Drake or Cohen, one can indubitably say that music contributed to the development of the modern empirical and practical dimension of science, essentially for two reasons: In fact, music is maybe one of the first traditional sciences in history to make use of *experimental instruments* (Pythagoras' monochord) and probably the first human activity to make use of

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[40] Cf. S. Drake, *Renaissance Music and Experimental Science*, in *Journal of the History of Ideas*, 31/4 (1970), pp. 483-500.

[41] H.F. Cohen, op. cit. (1984), p. 247.

[42] H.F. Cohen, op. cit. (1984), p. 248.

[43] Cf. e.g. J.D. Bernal, *Science in History*, MIT Press, Cambridge (MA) 1971.

[44] Ibidem.

[45] I. Beeckman, *Journal tenu par Isaac Beeckman de 1604 à 1634*, C. de Waard (ed.), M. Nijhoff, Den Haag 1939-1953.

complex *artificial instruments*[46] at all for intellectual and creative aims, namely - it is quite unexpected as obvious to affirm - the *musical instruments*, which, when not playing for pompous courtesans or praying congregations, often served as conscious or unconscious observative starting point and empirical test bench for scientific theories about music[47], the only possible one and even before the Scientific Revolution. However 'scientific' or not we may today define some of these theories, the fact remains that the way of scientific working they are results of contributed to the development of the empirical attitude in science, explicitly in the Scientific Revolution.

Finally, at the end of this brief introduction in the history of the musico-scientific problem, we are able not only to acknowledge that musical phenomena have been (and sill are) scientific relevant and that the three words mentioned above are 'compatible' with each other, but also to state whit some certainty that, because of the evident role played by music in the development of modern science, without its influences our science and mathematics themselves would probably not be today exactly as they are.

2 From Traditional Science of Music to Modern Acoustics: The Roots of Helmholtz's Theory of Sound

Let us concentrate now on what we have mentioned before: the 'revolution' occurred in science of music around 1600, i.e. the change from a mathematical ontology and epistemology of science of music to a physical one, started in particular with the empirical researches of Vincenzo Galilei and Giovanni Battista Benedetti (independently of each other) and carried on by those of other musico-scientists of the *experimental approach* and the *mechanistic approach*[48] to the musico-scientific problem in Modern Ages. These researches led to a new theory of sound and of consonance - a physical one, the so-called *coincidence theory of consonance* - which, over a period of few decades, was surprisingly already able to replace the other two main rival theories at the time, those of Zarlino and Kepler - still based on traditional mathematical and metaphysical grounds. Nevertheless, its explanatory power was in

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[46] That is, as manipulation of natural elements for intellectual purposes.

[47] In fact, one can easily speculate on the regularity of celestial or terrestrial motions and forces 'passively' or without particular instruments - as has really been done for a long time, but this would be more difficult, perhaps impossible with for example just intervals, inexistent as such in nature and investigable only by manipulating the acoustic phenomenon itself within an artificial situation, like that represented by a musical instrument. In order to get *proper* experimental devices for acoustic phenomena we have to wait even until the 19th century, but even then, musical instrument will continue to have an important role in scientific research.

[48] Definitions introduced by H.F. Cohen, op. cit. (1984).

comparison much weaker: In fact, the coincidence theory in the end «failed to solve [the original problem of consonance] it had sed out to solve»[49], contained a lot of *ad hoc* hypotheses, and «entailed [...] predictions that were all incompatible with the available data»[50]. Owing to many and different internal and external factors it however became the only theory capable of explaining musical and acoustic phenomena. Its epistemic and methodologic core represented «the beginning of the science of acoustics»[51] and a scientific legacy for the next generations of musico-scientists.

The new science of acoustics developed throughout the whole 17th, 18th and 19th century, reaching many important theoretical achievements both about physical topics like the production and the propagation of sound and physiological ones like the anatomic constitution of the human ear, the perception of sound, etc. However, «to the contemporary onlooker they [the achievements] were not pieces of one overall [acoustic theory], but rather disparate elements of several distinct sciences»[52]. There existed no consistent and homogeneous 'research program' of acoustics and scientists and musical theorists worked about acoustic topics quite independently of each other.

However, in 1863 a brilliant scientist was about to establish the modern science of acoustics by welding together in a consistent, but also very 'creative' way all the previous isolate achievements about acoustic phenomena. He was none other than the 'Reichskanzler der Physik' Hermann Helmholtz, with his masterpiece *Die Lehre von den Tonempfindungen als physiologische Grundlage für die Theorie der Musik*[53], certainly «the greatest masterpiece in the science of music ever written»[54].

Now, we can finally come to the principal aim of our historical inquiry into the science of music. Hereafter we will investigate the principal factors which allowed the modern science of music, as developed after the great seventeenth-century empirical transformation, to evolve into modern acoustics, represented by Helmholtz's theory of sound as its best instance. Therefore, the investigation on these factors will take the concrete form of the investigation on the internal and external roots of Helmholtz's acoustic theory, namely from the point of view of mathematics, physics and physiology on the one hand and aesthetics of music and philosophy on the other. Particular

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[49] H.F. Cohen, op. cit. (1984), p. 255.

[50] Ibidem; In Cohen, op. cit. (1985) the author takes this curious case of theory replacement in the history of science of music as a 'test case' in order to handle an interesting philosophic-scientific inquiry into how and why scientific theories are replaced.

[51] Ibidem, p. 234.

[52] Ibidem, p. 237.

[53] H. Helmholtz *Die Lehre von den Tonempfindungen als physiologische Grundlage für die Theorie der Musik*, F. Vieweg & So., Braunschweig 1863, En. tr. by A.J. Ellis (1885), Dover Publications, Mineola (NY) 1954.

[54] H.F. Cohen, op. cit. (1984), p. 238.

attention will be however paid, more than only to the mere description of the historical facts, to the historiographical problematization of the *elements of continuity* within this evolution, a point that, as far as I know, has not been thoroughly enough investigated until today. Finally, the relevance of this theory for the present acoustics and aesthetics of music will be briefly discussed in the last part of our inquiry and in particular its merits and demerits, accomplishments and critical points.

2.1 Some General and Biographical Remarks about Helmholtz's Theory

From a present point of view no definition of 'science of music' would be more appropriate than that of an interdisciplinary science. This is obvious because of the manifoldness and complexity themselves of the phenomenon concerned. Scientists involved in musico-scientific researches (or musicians involved in scientific investigations) often had more than only a superficial or partial knowledge in physics, mathematics, physiology and naturally music - and we can certainly say that today this still applies in some cases as well. Therefore, it is not by chance that just a personality like Helmholtz was able to formulate such a brilliant theory of acoustics, whose 'synthetizing' aim was that of connecting «the boundaries of two sciences, which, although drawn towards each other by many natural affinities, have [...] remained practically distinct - [i.e.] the boundaries of *physical and physiological acoustics* on the one side, and of [*musicology*] *and aesthetics* on the other»[55]. In fact, Helmholtz was a professional physician with very strong mathematical, physical and philosophical interests and, in addition to this, he was also an excellent pianist with great musical culture[56]: another example, before Einstein, of the fruitful cognitive effects of musical education and activity on human intelligence[57]!

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[55] «[...] die Grenzgebiete von Wissenschaften zu vereinigen, welche, obgleich durch viele natürliche Beziehungen aufeinander hingewiesen, [...] doch ziemlich getrennt nebeneinander gestanden haben, die Grenzgebiete nämlich einerseits der *physikalischen und physiologischen Akustik*, andererseits der *Musikwissenschaft und Ästhetik*» (emphasis in the original); H. Helmholtz, op. cit. (1954), p. 1. The expression 'science of music' in the original translation could be ambiguous today, since Helmholtz does not therewith mean 'acoustics' i.e. scientific theory of music, like us here; Therefore, it seemed us more useful to replace it with the modern equivalent word 'musicology' (inexistent at that time). Moreover, the original German text provides a further justification of this decision (Gr. Musikwissenschaft, equivalent of today's En. musicology).

[56] Cf. M. Meulders, *Helmholtz: From Enlightenment to Neuroscience*, MIT Press, Cambridge (MA) 2010, pp. 155-158. For more about the biography of Helmholtz cf. D. Cahan, *Helmholtz: A Life in Science*, University of Chicago Press, 2018.

[57] For more about this cf. L. Borzacchini, op. cit. (2011), p. 62, in which the author draws a parallel between the fruitfulness of science of music in the cognitive development in human history (especially in history of science) and of music education in individual cognitive development; E. Jensen, *Music with the Brain in Mind*, The Brain Store, San Diego 2000.

Today, every university student of music cannot not have met in a course on music theory or acoustics the theory of Helmholtz, a theory that still today gives a quite valid scientific account (albeit not entirely confirmed) of nearly all the most important historical musico-scientific problems: How tones are produced; How they propagate; How they are perceived and elaborated by the human ear; Whether, and if, where the boundary between consonance and dissonance has to be drawn and whether a scale of degrees of consonances exists; What pitch, intensity and timbre of tones depend on; What resonance is; How sound is produced in different types of musical instruments; What the best division of octave and the best temperament are, etc. Helmholtz himself was moreover sure to have solved them definitively: «The enigma which, about 2500 years ago, Pythagoras proposed to science, [...] has been solved»[58]. As we will see, such an affirmation was completely in accordance with the general scientific-philosophical credo which Helmholtz too believed in, namely the optimistic mechanicism of the second half of the 19th century. Yet we will see also that the theory of Helmholtz, however positive and extraordinary it may undeniably be, is not totally without problems and incompleteness of different nature, above all on the physiological side - how tones are perceived and elaborated, where our modern research on the brain and on the nervous system were obviously not possible yet.

Helmholtz researches in acoustics began in 1855, as we know from a letter of him to his friend, the physician Emil Du Bois-Reymond[59]. The scientific interest in explaining acoustic and auditory phenomena probably derived, besides from his closeness to the art of music, also from the desire for scientific completeness after having researched on the eye and on the vision in the previous years[60]: In 1856 the first part of his greatest, long-term work in physiology the *Handbuch der physiologischen Optik*[61] was published (the full work in 1867). So, with a research on acoustic phenomena, an exhaustive, scientifically and logically consistent explanation of everything concerning 'sensation' and 'perception' would have been provided, from physics to physiology and, in the case of acoustics[62], even beyond to aesthetics.

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[58] «Die Lösung des Räthsels, welches vor 2500 Jahren Pythagoras, der nach den Gründen der Dinge forschenden Wissenschaft, aufgegeben hat [...], hat sich nun [...] ergeben». H. Helmholtz, op. cit. (1954), p. 229.

[59] Cf. C. Kirsten, *Dokumente einer Freundschaft. Briefwechsel zwischen Helmholtz und Emil du Bois-Reymond (1846 – 1894)*, Akademie-Verlag, Berlin 1986, p. 267.

[60] Cf. M. Meulders, op. cit., p. 153. Cf. S. Vogel, *Sensation of Tone, Perception of Sound, and Empiricism Helmholtz's Physiological Acoustics*, in *Hermann Von Helmholtz and the Foundations of Nineteenth-century Science*, D. Cahan. (ed.), *California Studies in the History of Science*, 12, University of California Press, Berkeley 1993, pp. 259-287.

[61] H. Helmholtz, *Handbuch der physiologischen Optik*, L. Voss, Leipzig 1867.

[62] Esthetical consequences from the inquiry into optical phenomena are however not drawn by Helmholtz, maybe because of lack of interest or of sufficient skills in history and aesthetics of visual arts.

But what was exactly available to Helmholtz in term of concrete mathematical tools, physical theories, experimental instruments or practices and methodologies in the field of acoustics? How far did the science of music exactly come since the empirical turn at the beginning of the 17th century? What was moreover the state of affairs in the research on the physiology of hearing? And finally, what were Helmholtz's reference philosophical background and aesthetics of music? These questions will be answered in the following three sections and in the conclusion.

2.2 The Basic Starting Point: The Coincidence Theory of Consonance

The basic root of Helmholtz's acoustic theory is indubitably the mentioned empirical transformation of science of music occurred around 1600. As we know, the coincidence theory was the first example of this turn. In fact, it was formulated by means of a new epistemological vocabulary, namely that of physics and no longer of mathematics alone. G.B. Benedetti, who was the first to propose it[63], considered the sound as a wave, i.e. as a regular percussion of the air caused by a vibrating body. On this new account of the sound, he could finally explain the *consonance* of two tones by making the *coincidence* of air percussions the *physical cause* of it, whereas, solely some decades before, it was a mathematical fact to be responsible for it, i.e. the fact that the two tones (conceived in term of string lengths) were in a ratio expressed by only the first few integers. Benedetti also observed that the shorter the vibrating portion of a string was, the faster its vibration was and that the faster the vibration, i.e. the percussions of the air was, the higher the tone produced. Hence, he discovered nothing but what later will be known as *frequency*, i.e. number of vibration (percussions of air) per unit of time, directly proportional to the pitch of a tone and inversely proportional to the length of the string producing it[64]. This correlation between frequency and pitch was one of the most important novelties introduced in science of music.

Now, after this account consonance derives no longer from a proportion between strings whose length is in a ratio of 1:1 for the unison, 1:2 for the octave, 2:3 for the fifth and so on, but from a proportion between different pitches, curiously in a ratio of always the same integers, but obviously mathematically 'inversed' (2:1 for the octave, 3:2 for the fifth, etc.): For example, octave is consonant because every air

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[63] Cf. G.B. Benedetti, *Diversarum speculationum mathematicarum et physicarum liber*, Venice 1585.

[64] Indeed, a more accurate investigation on the direct proportionality between pitch and frequency was made by Galileo Galilei in his *Discorsi e dimostrazioni matematiche intorno a due nuove scienze attenenti alla meccanica & i movimenti locali* (1638), on the basis of studies about the isochrony of the pendulum and the sympathetic resonance. No quantitative explanation was however produced.

percussion of the longer string coincides with every second percussion of the shorter one producing faster percussions (exactly twice as much in the same unit of time).

Thus, from the mathematical point of view the theory of Benedetti was not particularly innovative. This can be however affirmed also from an ‘ontological’ one: Benedetti’s account of propagation of sound as wave or regular percussion of the air was already discussed by the ancient Greeks[65] beside an atomistic one (in modern times supported only by I. Beeckman)[66]. In addition to this, no attention was paid to important physical and physiological aspects like for example the influence of tension and material of the vibrating string on the pitch, the calculation of the pitch of the single tones or the sensation of the tones itself. These aspects would have been investigated - also on the basis of experiments - particularly by Vincenzo Galilei (tension and material of vibrating string), Marin Mersenne (pitch of tones), by Beeckman and Descartes (sensation of tone). Nevertheless, Benedetti managed to mix with brilliant intuition the elements of this legacy from ancient mathematics and physics and to set with his theory a first important milestone of modern acoustic, namely the ontological shift of the problem of sound and of consonance from mathematics to physics.

2.3 New Mathematical Tools for the Science of Acoustics

Between the 17th and the first decades of the 19th century mathematics knew some of its most revolutionary developments of all time. Most of them soon found an application in other sciences, especially in physics, which could so increase its explanatory and predictive power, extend it gradually to still unexplored fields and explain natural phenomena better than previous theories. The logarithms by Napier (1614), the analytic geometry by Descartes (1637), the theory of probability by Fermat, Pascal (1654) and Huygens (1657), the mathematical analysis by Leibniz and Newton (1684), the rigorous foundation of imaginary and complex number by Euler (1748) and Gauß (1797), and finally the introduction of the first Non-Euclidean Geometries for example that by Gauß (1813) and of the Fourier Analysis (1822): This is only a part of the most significant developments in mathematics during this period of time.

Some of these mathematical theories proved to be very useful also to deal with musico-scientific phenomena and contributed a lot to give account of long-standing problems like the division of the octave, the propagation of sound or the problem of consonance. Obviously, mathematics was here no longer the means of a metaphysical speculation about the tones à la Pythagoras or à la Kepler, but rather an *ancilla philosophiae naturalis* as it was also becoming for other sciences like for example

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[65] In particular Chrysippus of Soli.

[66] H.F. Cohen, op. cit. (1984), p. 77.

mechanics. Nevertheless, it could perhaps be interesting to note that, while an ontological shift from a mathematical ontology to a mere physical one was needed in music, the natural philosophy developing from the Galilean epistemology[67] seems to have followed in part a revers path, namely back to the Pythagorean (and Platonic) ontological idea of an intrinsically mathematical Universe: another odd historical characteristic of the science of music!

The first new mathematical idea to be applied in music was just the logarithm. In the classical theory of music intervals could be compared, i.e. added and subtracted by means of (repeated) multiplications and divisions[68]. Borzacchini points out that «the correspondence between the sum of intervals and the product of ratios» was even «an essential ingredients (if not the source) of a long lasting reflection that ultimately caused the emergence of [...] the idea of logarithm [...]»[69]. Logarithms was largely used in 17th century in order to compare music intervals, but also in order to measure their relative sizes. This was a problem in Greek theory of music, since intervals were conceived there as ratios and not as magnitudes. Then, the purpose was to find a sort of 'unit of measure' for the intervals on the basis of which to measure their size, but this was impossible for the Greeks, because for them the tone (8:9) was not divisible in two equal sized intervals expressible with a ratio of integers, but one had from its division a greater *chromatic semitone* and a smaller *diatonic semitone*, with a difference of 1 comma. So, two of the most difficult (and still controversial) mathematical (and philosophical) problems of western culture arose: the problem of the division of the octave and of temperament, i.e. whether to preserve the purity of the intervals - expressed by ratios of integers - but with huge limitations in the practical and esthetical dimension of music[70], or to give up the purity of intervals (and therewith rational ratios) dividing the octave in equal semitones, 12 to be exact. In this respect many authors, since the Greeks proposed different solutions and in 17th century logarithms were the mathematical means to reflect about the division of the octave. In 1661, for example, Huygens discovered the possibility to calculate mathematically 31 equal divisions of the octave by finding 30 mean proportional in between 1 and 2[71], an idea already considered by Mersenne and Salinas but practically unrealizable without

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[67] Cf. G. Galilei, *Il Saggiatore*, G. Mascardi (ed.), Roma 1623, ch. VI.

[68] For example (in term of string lengths), the sum of the fifth (2:3) and the fourth (3:4) is the octave: $\frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$, or the subtraction of the fifth (2:3) and the fourth (3:4) is the tone: $\frac{2}{3} \div \frac{3}{4} = \frac{8}{9}$.

[69] L. Borzacchini, op. cit. (2011), pp. 51.

[70] Pure intervals are too large to be *all*, without excess, in the space of an octave, i.e. the starting point of the circle of *pure* fifths cannot coincide with its conclusion.

[71] This discovery was published only 30 years later: C. Huygens, *Brief betreffende de harmonische cyclus*, in *Histoire des Ouvrages des Sçavans*, H.B. de Beauval, Rotterdam, October 1691, pp. 78-88.

logarithms, only later discovered[72]. Descartes[73] (1618), Mercator (1650-1680) and Newton (around 1665) also used logarithms in order calculate the size of the intervals and to represent the division of the octave[74] and, although at the end of 18th century the equal-tempered system was established as standard temperament among practicing musicians, its historical and structural problems will be still discussed even by Helmholtz himself[75], who also proposed his own temperament (later developed by Ellis), the *schismatic temperament*, definable as a sort of 'middle way' between tempered and Pythagorean system[76]. However, about the problem of temperament, as about other topics in his musico-scientific work, Helmholtz exhibits only a partial historical knowledge of the state of affairs (generally limited to the Greeks and to the 17th-18th century). Therefore, it is not surprisingly to find summary historical judgments of him about previous theories, theoreticians and discoveries. Yet this seems in some cases justifiable given the fact that many sources about the early science of music became known only recently[77].

The second mathematical discovery which became a useful tool for the science of music was the analysis, especially the differential equations, particularly able to describe important acoustic phenomena as the wave propagation and the string vibration. In this regard the figure of the English mathematician Brook Taylor (1685-1731) is very important. In 1708 he discovered a new solution of the problem of the 'center of oscillation', published only in 1713 on the *Philosophical Transactions*[78] and in 1715 he introduced what today is known as 'finite difference method' for solving differential equations. This method is his most important mathematical contribution to the science of acoustic from our point of view: In fact, by it Taylor could *describe the form of the movement of a vibrating string*[79]. It was the first step to the complete mechanization and modern 'mathematization' of sound phenomena.

The crucial point in this process was however represented by Fourier analysis, which constituted the principal root of Helmholtz's acoustics from the mathematical

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[72] Cf. H.F. Cohen, op. cit. (1984), pp. 219-220.

[73] Cf. R. Descartes, *Compendium musicae*, in *Oeuvres*, vol. X, Editions du Cerf, Paris 1897-1913.

[74] Cf. B. Wardhaug, *Musical logarithms in the seventeenth century: Descartes, Mercator, Newton*, in *Historia Mathematica,* 35 (2008), pp. 19-36.

[75] Cf. H. Helmholtz, op. cit. (1954), pp. 321-327.

[76] Cf. ibidem, p. 435.

[77] For example, the work of Simon Stevin on equal temperament, discovered only in 20th century. For more about this cf. H.F. Cohen, op. cit. (1984), p. 45-67.

[78] Cf. B. Taylor, *De inventione centri* oscillationis, in *Philosophical transactions of the Royal Society*, 28/337 (1713), pp. 11-21. Studies on this subject were being made in the same period also by Johann Bernoulli, which after Taylor's publication led to a dispute between the two mathematicians about the authorship of the idea.

[79] Cf. B. Taylor, *Methodus Incrementorum Directa et Inversa*, W. Innys, London 1715.

point of view, without which the largest part of nineteenth-century acoustics and therefore the work itself of the German physician would certainly not have been possible. In 1822 the French mathematician and physicist Jean Baptiste Joseph Fourier (1768-1830) published a revolutionary work in mathematics, the *Théorie analytique de la chaleur*[80]. The main mathematical intuition of Fourier - although partially present already in Euler, D. Bernoulli and d'Alembert - is that of 'decomposition' of a function in many simpler ones: Every mathematical function is representable as linear combination of harmonically related sinusoidal functions - in this regard one will speak of *Fourier series*-development of the function. This idea naturally constitutes the basis of what we know as *Fourier analysis.* So, in his paper Fourier used trigonometric series to study the partial differential equation of heat, which describes the evolution over time of the distribution of a quantity (as heat, for example) in a solid medium. However, Fourier's work will be important not only in thermodynamics, but also for the study of waves in general (the differential wave equation can be derived from heat equation) and in particular of acoustic waves, i.e. of sound. And it will be exactly here that Helmholtz, as already Ohm, will use the mathematical idea of Fourier to give his scientific account of the constitution of sound, the nature of consonance and to build up a physiological model of sound perception.

2.4 The Modern Science of Music

After the great transformation occurred at the beginnings of the 17th century, the research in the science of music continued *almost* exclusively in the empirical and mechanistical way. The traditional mathematical one, in fact, was not immediately abandoned. Descartes, albeit one of the first supporters of the modern mechanicism, demonstrate in his work about music theory[81] to be still close to the Renaissance, zarlinian, mathematical way of musical theorizing, relegating the discussion of the physical properties of sound to the physicists[82]. In the next two generations of scientists even masterminds as Leibniz and Euler[83] proposed mathematical theories of music which, abstracting from the physical and physiological dimension of acoustic phenomena, were really dissonant with the majority empirical approach supporting the

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[80] Cf. J.B.J. Fourier, *Théorie analytique de la chaleur*, F. Didot, Paris 1822.

[81] The already mentioned *Compendium* Musicae, the first work ever written by Descartes (1618, published posthumously in 1650) and dedicated to I. Beeckman.

[82] Cf. H.F. Cohen, op. cit. (1984), p. 161-179.

[83] Cf. H.F. Cohen, op. cit. (1984), p. 237.

physical theory of coincidence, and therefore historically ineffective on following scientific studies about music[84].

On the contrary, this latter theory became the most popular one among physicists and music theoreticians. After the introduction by Benedetti, it was systematized firstly by Mersenne[85], whose general aim in the science of music was that of quantifying all sound phenomena. He formalized properties of sound discovered before and remained only nothing but mere intuitions. For example, he defined precisely the relations between vibration, frequency and pitch[86] and between amount of vibrating air and loudness, calculated the absolute pitch of single tones and tried to extend the new theory of sound from vibrating strings to vibrating air columns, all by massive use of experiments, mostly carried out by means of musical instruments and with the assistance of instrument makers[87]. Two observations of Mersenne are very important as root of the modern acoustics. His experimental approach allowed him to investigate important acoustic phenomena like the *overtones* and the *beats*, both very important in Helmholtz's theory too.

Any tone, independently of the source, is a compound of many tones of different higher frequency, and this was already observed even by Aristoteles. Thus, a tone, as such a *complex tone*, is composed of many *partial* or *simple tones*, including the *fundamental tone* (the first 'partial'), producing the other. The *overtones* or *upper partial tones* are all the partials excluding the fundamental. If they have a frequency integer multiple of that of the fundamental, they are called *harmonic overtones*, otherwise *inharmonic overtones*. One can hear them also without particular instruments, only paying great attention and concentration after an adequate musical training, as Mersenne himself recommended; Although Helmholtz was essentially of the same mind, he developed a series of instruments to hear them with more accuracy, the today's *Helmholtz resonators* - each of them of different size for a specific overtone - based on

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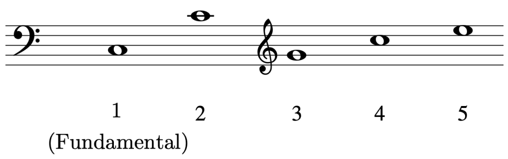
[84] These theories, however, influenced the philosophy of many music composers (see Footnote 12): An example is J.S. Bach, who, having also an education background in mathematics, often used mathematical concepts as framework for musical works. Moreover, it can also be very probably that Euler's mathematics itself influenced Bach's music, during a period, by the way, the German composer devoted entirely to his *musica mathematica*, i.e. to his more theoretical and formal counterpoint production (cf. *Die Kunst der Fuge, Musikalisches Opfer*): In 1747 he visited the court of King Frederick II of Prussia in Potsdam, and met the Swiss mathematician who had been working there since 1741. About the biography of Bach, his philosophy and mathematical methods of music composing cf. I. Zanzarella, op. cit.; E. Fubini, op. cit. (1964b), ch. II; A. Basso, *Frau Musika. La vita e le opere di J. S. Bach*, EDT, vols. I e II, Torino 1983.

[85] Cf. M. Mersenne, *Harmonie Universelle, contenant la théorie et la pratique de la musique*, S. Cramoisy (ed.), Paris 1636.

[86] Indeed, the quantitative explanation was taken by Messene from a geometrical proof by Beeckman (1614-1615), who later (1629) communicated it to him.

[87] Cf. H.F. Cohen, op. cit. (1984), p. 102.

the principle of air resonance in a cavity or *Helmholtz sympathetic resonance*[88]. Mersenne managed to distinguish the first four overtones, with a frequency defined by the first four integers (Fig. 1)[89]:



(Fig. 1)

As we will see below, this phenomenon will be mathematically explained by Helmholtz, just by means of Fourier analysis. He will make it the ground of his explanation of consonance as well as of other important properties of sound, for example the timbre. The second important phenomenon observed by Mersenne was that of the beats. The beats originate when pure consonances are slightly mistuned, which causes periodical alterations in the loudness of the resulting sound. The larger the deviation from pure consonance is, the higher the period of the beats is. However, Mersenne gave no quantitative account of the phenomenon, but he just noticed it. Indeed, as already mentioned, a semi-quantitative account of the phenomenon in terms of air strokes[90] was given around 1628 by Beeckman[91]. He supported, as unique case in history of modern science of music, a *corpuscular theory of sound* which would have however been practically ineffective for the following developments of acoustics, since he never published it. Yet Beekman's ideas, communicated by him to his best correspondents Mersenne and Descartes, did not fail to serve sometime them as important intellectual stimulus, as seen for example in Footnote 85.

Contrary to what one may think, the empirical turn in science of music and the coincidence theory of consonance did not directly introduce a *wave theory of sound*, i.e. a theory in which sound is regarded as mechanical longitudinal wave propagating by continuous compressions and rarefactions of a medium (air or water). Yet the properties of sound, quantitatively defined throughout the 17th century in terms of succession of pulses (frequency) or amount of air struk (loudness), could be unproblematically suited to the forthcoming wave theory of sound. There was only a property, which could not be quantitatively well explained without a wave account of sound,

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[88] Cf. H. Helmholtz, op. cit. (1954), pp. 36-49.

[89] [Frequency of the *n-th* overtone] = [Frequency of the fundamental] × n, with $2 \leqslant n \leqslant 5$, $n \in \mathbb{N}$.

[90] Cf. H.F. Cohen, op. cit. (1984), p. 144.

[91] Cf. I. Beeckman, *Journal tenu par Isaac Beeckman de 1604 à 1634*, C. de Waard (ed.), M. Nijhoff, Den Haag 1939-1953, v. 3 p. 51.

i.e. the timbre. In fact, although Mersenne had already rightly guessed that it depends on the behavior of the overtones, he did not success to formulate any quantitative hypothesis about this, for which we must wait until Helmholtz.

The transition - thoroughly described by Dostrovsky[92] and Wardhaug[93] - from an empirical but ontologically not yet well 'determined' theory of sound to the wave theory of sound took place gradually from the second half of 17th century and the first of the 18th. Many factors contributed to it, both experimental and theoretical. As already mentioned, experiments on physical variables of a vibrating string were carried out already by V. Galilei and Mersenne. Further experimental researches on frequency, resonance, overtones and beat and attempts to describe them quantitatively were made by Huygens, Hook, Wallis and Sauveur. The latter two also discovered the cause of the harmonic series of the overtones: the existence of *nodal points* on a string vibrating in sympathetic resonance with another one tuned at the pitch of one of its partials.

A proper turning point in this regard was represented however by Newton's *Principia*[94], in which a quantitative account of pressure waves in a compressible medium was given. Moreover, observations about optical phenomena led Newton already before, in 1685[95], to the concept of wavelength - guessed previously by Huygens - which he then applied to sound. Thus, with Newton the concept of wave becomes full-fledged part of physics' ontology. Yet the proper quantification and the mathematization of sound waves and acoustic phenomena can finally begin, also by means of new mathematical tools (see Section 2.3): The work of Taylor and J. Bernoulli on differential equations allows the first mathematical models for the shape of a vibrating string and for the propagation of the sound and later that of d'Alambert, Euler, Daniel Bernoulli, Laplace and Lagrange will lead to a quite comprehensive mathematical understanding of mechanical wave phenomena, in particular with the discovery of the one- and three-dimensional wave equations. Another important discovery in contemporary acoustics was made by the Italian violinist and music theoretician Tartini around 1750[96], the discovery of *differential tones*, whose pitch is given by the difference of the frequencies of two tones in a given interval. Discussing this '*terzo suono*' by Tartini, Helmholtz affirmed the discovery by himself even of a '*quarto suono*' -

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[92] Cf. S. Dostrovsky, *Early Vibration Theory: Physics and Music in the Seventeenth Century*, in *Archive for History of Exact Sciences,* 14/3 (1975), pp. 169–218.

[93] Cf. B. Wardhaug, *Music, Experiment and Mathematics in England (1653-1705)*, Routledge, London 2017.

[94] Cf. I. Newton, *Philosophiae Naturalis Principia Mathematica*, B. Motte, London 1687, II-XLVII/L.

[95] Cf. I.B. Cohen (ed.), *Isaac Newton's Papers and Letters on Natural Philosophy*, Harvard University Press, Cambridge 1958, p. 178.

[96] Indeed, the phenomenon was already known to the German organist and composer Georg A. Sorge.

indeed, predicted by his theory itself - i.e. the *summational tone*, originated by the sum of the frequencies of two tones of a given intervals[97]. The two phenomena - called by him *combinational tones* - will be explained in the second part of his work and will be especially important to him to define the property of sound quality, i.e. the timbre.

The new generalized laws of wave mechanics finally made acoustics become only a branch of rational mechanics. So, the great science of music, with roots in the ancient Pythagorean philosophy, definitively stopped to be an autonomous science. Sound phenomena were completely mechanized and quantified, so that their so complicate quantitative explanation became unimportant and useless for practicing musician - above all because things like the temperament, the techniques of instruments making, the absolute pitch of the tones, once important musico-scientific issues, were already standardized in this period. On the other side, the work of practicing musicians began to have practically nothing to do with scientific activity. In conclusion, the mutual influence between science of music and art of music, as it always existed in the past, became no longer possible, if not undesirable at all both by the growing generation of positivist scientists and by that of romanticist musicians (see Section 1). In fact, as far as I know, in the whole 19th century we have not a single case of a musico-scientist in traditional sense - except in part Helmholtz, who, in addition to his regular musical activity as composer or virtuoso performer, describes, maybe in a scientific laboratory, the behavior of sound waves by means of differential equations[98]. Of course, the evolution of the educational system and of the society in general, the specialization and professionalization of culture play a decisive role as causal factor in this process.

The change occurred in the status of science of music introduced an important transformation in the concrete experimental practices of acousticians. If until the first decades of the 18th century musical instruments was the first experimental basis of acoustic researches, in particular string instruments, the generalization of the laws on the vibration, the resonance and the propagation of sound made it possible to extend the empirical basis of acoustic researches also to other sound sources, for example air columns (organ pipes)[99] or bells[100]. At the beginning of the 19th, however, new experimental instruments for acoustic researches were introduced, which neither came from practiced music, nor had a particular ‘esthetical application’ outside the laboratories

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[97] Cf. H. Helmholtz, op. cit. (1954), p. 153.

[98] In this respect, the French composer and music theoretician Jean-Philippe Rameau can be probably considered as the last musico-scientist in traditional sense.

[99] Already Huygens and Newton gave a quantitative account of vibration of air columns by means of the concept of wavelength

[100] The first experiments on the propagation of sound waves in vacuum was attempted with bells at the end of the 18th century.

and beyond the scientific utilization: a separation between *episteme* und *techne* occurred quite late compared to the rest of history of science! With the only exception of the turning force, invented already before by the English trumpeter, lutenist and acoustician John Shore in 1711, these were essentially the plates, used for example by the German acoustician Ernst Chladni, the sonometer, invented by Felix Savart, and the siren, invented by the French engineer and physicist Charles Caignard de Latour in 1819 and later improved by many scientists, including Heinrich Wilhelm Dove (1851) and Helmholtz himself.

From the point of view of the experimental practices these inventions will be very important for Helmholtz's researches. In fact, to carry them out, especially to study overtones and combinational tones, he will make assiduous use of tuning forks and sirens. But not only that: He will demonstrate a great creativity and cleverness in inventing by himself new versions of these instruments permitting more precise observations and more accurate measures. Example of these inventions are the already mentioned resonators, the double siren and the electromagnetic pure tone generator[101].

However, until these inventions spread in acoustic research around 1830, after the great 'exploit' of the second half of the 18th century «acoustic [became] a virtually moribund research field»[102] and it was de facto only «a very minor branch of the emerging discipline of physics»[103]. In Germany, for example, during the first quarter of the 19th century only few articles on acoustics appeared on physics journals and no university chairs of acoustics existed. Instead, the previous studies on vibrating strings in rational mechanics were of a great heuristic value in mathematics, especially in the development of theories on differential and partial differential equations. Remarkable, however, were the researches of Chladni on the vibration of plates (1787) and the publication of the first monography of experimental acoustic (1802), in which he described all the experiments carried out by himself and other previous scientists in the field and gave a systematic account of all known acoustical phenomena. Moreover, he discovered longitudinal vibrations and measured the velocity of sound in different physical media, including gases. But it is only with the improvement of the siren around 1830 that the field of acoustics got new importance. The siren now introduced a new experimental way of investigating sound phenomena and even inspired new conceptions of tone, regarded in some cases no longer as sinusoidal wave produced by a vibrating body, but rather only as a series of periodic pulse transmitted to the auditory nerve troughout the air with a certain periodical regularity - an idea that

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[101] For a description of the functioning of these instruments cf. M. Meulders, op. cit. pp. 159-170 and the dedicated sections in Helmholtz's texts.

[102] S. Vogel, op. cit., p. 261.

[103] S. Vogel, op. cit., p. 262.

now could be surely expressed with stronger mathematics and supported by better experimental observations, but that was not at all so innovative, as we know. Giving a new account of the tone became, during the years immediately before Helmholtz's researches, the topic of a famous controversy between Thomas J. Seebeck and Georg Ohm. The former supported a pulse-interpretation, the latter a sinusoidal wave-interpretation of the tone. Both made use of Fourier analysis as mathematical tools for sound phenomena and of new experimental instruments as the siren., Even if Seebeck had to give up his inconsistent sound interpretation, the controversy finished indeed without a concrete agreement. Nevertheless, it did not fail to bring a renewed attention in acoustics to still problematic properties of sound and sound phenomena, which will be the starting point for Helmholtz's researches: the combinational tones, the beats and the quality of sound (timbre).

Now, Helmholtz will however focus not so much on the *physical acoustic*, «the *physical* part of the *theory of sound* that has been almost exclusively treated at length, [... and is] nothing but a section of the theory of the motions of elastic bodies»[104]. In this respect, he will only expose in an organic and systematical way all what was previously known in rational mechanics about sound waves, vibration, etc., including new observations by himself and following Ohm's same mathematical Fourier-approach for the quantitative description of sound. Instead, what Helmholtz will consider really new of his theory is the accurate investigation into «the processes that take place within the ear itself»[105], i.e. the *physiological* part of the theory of sound, something about which many data were collected, he says, but that never was the principal object of acoustic researches.

From this idea the structure of his entire acoustic research develops: The first *physical part of theory of sound* will be followed by the analysis of the mechanisms of *sound perceptions*, i.e. of the anatomy of human ear (*physiological part of theory of sound*). Then, the ways in which the auditory nerves 'decode' the mechanical sound perception originating the *sensation of tone* - to which the sensations of consonance, dissonance, beats, compositional tones and timbre belong - will be investigated (*psychological part of theory of sound*). Finally, the *musical* and *aesthetical consequences* of this scientific analysis - construction of scales, discussion of different temperaments,

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[104] «Bisher ist von der Lehre vom Schall fast nur der physikalische Teil ausführlich behandelt worden, [...]. [Diese physikalische Akustik ist ...] nichts als ein Teil der Lehre von den Bewegungen der elastischen Körper». H. Helmholtz, op. cit. (1954), p. 3.

[105] «[...] die Vorgange im Ohr selbst [...]». H. Helmholtz, op. cit. (1954), p. 4.

etc. - will be derived. Their correctness will even constitute for Helmholtz the «verification of the correctness of the physical and physiological views advanced»[106].

Yet already Helmholtz was aware of the fact that «not many results have as yet been established with certainty»[107] about the physiological and psychological part of the theory of sound. After the ancient attempts to explain sense perception and sensations, a renewed interest on this topic developed especially with the modern empiricism, which, however, did not move at the end too far from the traditional, more or less metaphysical, conceptual vocabulary used to describe them. Moreover, less attention was paid to the sense of hearing in particular. Apart from Kepler's explanation (1619), still based by on metaphysical and scholastic assumption[108], historically more interesting theories[109] were proposed by mechanists as Beeckman[110] (1631) and Descartes[111] (1633). They are probably the initiators of modern physiology, based, on the one hand, on a mechanistical view of human body and, on the other, on the empirical study of human anatomy, that is, by means of anatomic dissections. In fact, both authors - extremely acquainted with medicine - «tried to account for sense perception in terms of motions of certain particles of matter»[112] and, hereto, they also performed empirical observations on human bodies and dissections by themselves. The explanation of how the outer ear perceive sound as motion of particles (Beeckman) or infinitely divisible matter (Descartes) and that of how this *perception* is turned in the *sensation* of consonance within the inner ear by the auditory ossicles, the auditory nerve etc. introduce in physiology and in particular in physiology of hearing for the first time a new conceptual vocabulary, very different from the traditional, 'Galenian' one. Of course, it is still too poor to provide an accurate and exhaustive solution of the problem of the perception and sensation of tone, nevertheless his introduction has the merit to have started a genuine revolution in the field at stake.

Hence, the empirical turn in science of music coincides with an empirical and mechanistic turn in the physiology of hearing. We can notice, in fact, that musico-scientists in early modern times are already interested in connecting their physical investigation on the nature of tone and of consonance with the physiological one on

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[106] «[dem Physiologen muß die Richtigkeit dieser Folgerungen] eine Unterstützung für die Richtigkeit der vorgetragenen physikalischen und physiologischen Ansichten gelten». H. Helmholtz, op. cit. (1954), p. 5.

[107] «[...] noch nicht viel sichergestellte Ergebnisse [...] gewonnen sind». Cf. ibidem.

[108] Cf. H.F. Cohen, op. cit. (1984), pp. 29-32.

[109] Cf. H.F. Cohen, op. cit. (1984), pp. 139-143 and 172-175.

[110] Cf. I. Beeckman, op. cit., v. 3, p. 125, p. 187

[111] Cf. R. Descartes, *Traité de l'homme*, in *Oeuvres de Descartes,* C. Adam, P. Tannery (eds.), vol. XI, Editions du Cerf, Paris 1897-1913.

[112] H.F. Cohen, op. cit. (1984), p. 172.

sound perception and sound sensation, as well as in providing on the base of the mechanical explanation of acoustic phenomena *aesthetical criteria* concerning the temperament, the division of the octave or the scale of degree of the consonances. In this respect, Helmholtz certainly appears as the founder of modern acoustics, but, like every great revolutionary in history of science, also as the *last representant of the tradition*, that is, in *continuity* with it. A proof for this is also the fact that, after Helmholtz, only few scientists will research on acoustics accounting for all the 'three dimensions' of its problems, the physical, the physiological-psychological and the aesthetical one. On the contrary, the acoustic research will develop separately in each of this three fields on his own, independently of the others.

So, the early modern physiology of hearing and the empirical turn in physiology itself occurred at that time represent undeniable roots of Helmholtz's acoustic. Nevertheless, the development in this field troughout 17th, 18th and 19th century can be not compared with that in physics during the same period. In fact, the possibility to investigate empirically the nature of the ear, nerves, brain, etc. was then very limited, before the introduction of modern methodologies and instruments like the microscopical anatomy, the magnetic resonance imaging or the computed tomography, occurred only in 20th century. Nevertheless, concrete important progresses in physiology and psychology of hearing and, in general, of perception were made during the first half of the 19th century: In 1851, exploring the inner ear, the Italian anatomist Alfonso Corti discovered the organ that will become known with his name, the Corti's organ[113]; The first attempts to quantify sensations themselves analyzing the correlation between physical stimuli and perceptions and sensations produced were made by Gustav T. Fechner and Ernst H. Weber, founders of psychophysics[114]. These latter researches on the physiology and psychology of sensation and hearing will be very important for Helmholtz too.

In fact, the German polymath will discuss in his work on acoustics the functioning of Corti's organ and will even hypothesize that it is responsible for the sensation of tone itself. This will be indeed the core of his entire theory, whose physical and psychophysiological parts are hereafter briefly summarized: According with Ohm's acoustics, every tone is conceived by Helmholtz as a complex mixture of a certain quantity of partial and compound tones; The tone, from the physical point of view, is a complex sinusoidal wave deriving from the undisturbed superposition of simpler ones, those of the partial and compound tones; The mathematical representation of the tone

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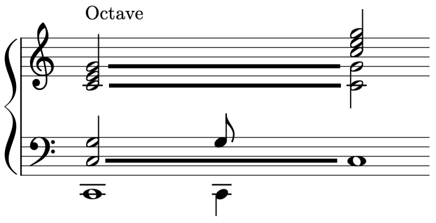
[113] Corti's organ is an organ of the inner ear in which mechanical impulses coming from the outer ear - through the action of the eardrum and of the auditory ossicles - are transformed by hair cells in nerve impulses headed to the brain.

[114] See Weber-Fechner Law.

as such a compound is given by Fourier's theorem; Certain quantity, frequency and loudness of partial and combinational tones determine a certain form of the resulting complex sound wave, i.e. a certain timbre of the tone itself; What the ear perceives as tone of a certain quality (timbre) is not a specific form of the complex wave corresponding to the tone, but the undisturbed combination of the simpler waves that compose it - i.e. the variety of tone qualities depends on the different possible (harmonic or inharmonic) combinations; Therefore, for what concerns perception and sensation of tone - the ear, and in particular Corti's organ, works as 'Fourier-analyzer' according to the laws of sympathetic resonance: it 'decodes' complex sounds resolving them into simple pendular (i.e. regular, periodical) oscillations. Therein quantification processes of perceptions take place which resemble in part the psychophysical idea of Fechner.

At this point the problem of consonance can be explained: «*Consonance is a continuous, dissonance an intermittent sensation of tones*»[115]. A *perfect* or *pure consonance* between two sounds gives rise to a completely *undisturbed combination* of *all of* their partial and combinational tones: This is for example the case of the octave (Fig. 2, only the first partial and combinational tones are represented), but also that of the three other Pythagorean consonances (expressed as ratio between integers), whose «enigma»[116], as mentioned before, has gotten now finally his (mechanical) explanation.



(𝅝 = fundamentals; 𝅗𝅥 = upper partials; ♩ = summational tones; ♪ = differential tones)

(Fig. 2)

For Helmholtz, the condition of consonance is however not the coincidence of all the partial and combinational tones of two sound in a given interval - impossible, by the way, for all intervals in equal temperament[117], the temperament in use as standard

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[115] «*Konsonanz ist eine continuierliche, Dissonanz eine intermittierende Tonempfindung*» (emphasis in the original). H. Helmholtz, op. cit. (1954), p. 226.

[116] See Footnote 58.

[117] The size of the intervals in equal temperament are not represented by ratios between frequencies expressible by integer numbers, i.e. by a rational number, but only by irrational numbers (with exception of unison and octave); In particular, the size of each interval in equal temperament is a

among musicians today as already at Helmholtz's time[118], but the coincidence of a *sufficient number* of them[119]. This 'threshold of consonance' defines the extent to which the deviation from the perfection of consonance can be still perceived as tolerable by the ear. Yet deviating considerably from it, the sensation of consonance is gradually lost, because increasingly less partial and combinational tones coincide, and the interval, now dissonant, begins to *beat* to a quite intolerable extent[120]. Hence, whereas the sensation of consonance is that of a *continuous, undisturbed tone*, that of dissonance is instead that of an *intermittent, disturbed, periodically beating tone.*

It is impossible to fail to notice, as Cohen also does[121], the resemblance of the core idea of Helmholtz's theory of consonance with that of the coincidence theory, namely consisting in the concept of 'coincidence'. The difference between the two theories concerns the application of this concept, namely related for the latter only to the fundamentals, for the former (also) to the partial and combinational tones, even if, already Mersenne affirmed overtones were certainly supposed to play any role in the phenomena of consonance and dissonance. «Thus the coincidence theory, too, live[s] on in Helmholtz's account as a kind of limiting case»[122]: This is another proof of the fact that, on the one hand, the consonance theory originating from the modern empirical turn in science of music constitutes an undoubted root of Helmholtz's acoustics and that, on the other, there exists a historical continuity in the development of science of music, from early modern times until Helmholtz.

In the third part of his work, Helmholtz finally proceeds to discuss the aesthetical consequences of these physical and psychophysiological conclusions, that is, topics regarding the division of the octave, the temperament, the construction of musical scales and chords, etc. The detailed analysis of these consequences exceeds the aim of our present inquiry. Some remarks will be however made in the next section for what concerns the epistemological examination of Helmholtz's acoustics.

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power of $\sqrt[12]{2}$, the size of an equal-tempered semitone (i.e. ca. 4,5 comma), the smallest, no further divisible interval of the equal-tempered octave.

[118] This is a clear example of how, one more time, the art of music influenced a scientific theory. This issue in particular, regarding equal temperament, will be more thoroughly discussed in Section 3.

[119] Helmholtz says in this respect *two*: The *condition of consonance* is that at least two of the lowest partial tones should have exactly the same frequency.

[120] Only in pure consonances there exists complete absence of beats.

[121] Cf. H.F. Cohen, op. cit. (1984), p. 242.

[122] Ibidem.

3 Conclusion: Last Remarks and Criticisms between Aesthetics and Philosophy

In Section 2 we have analyzed all the main historical developments in science of music, mathematics, physics and physiology that led to the emergence of modern acoustic, represented by the work of Helmholtz as its more concrete, synthetic and striking example. We noticed that the problems Helmholtz faces up to are very old and date back to the time of Pythagoras and probably even before. Moreover, we saw that the roots of Helmholtz's acoustics are to be traced back at least to the scientific and philosophical transformation in the science of music occurred at the beginning of the 17th century and that his theory of sound and of consonance presents an unquestionable continuity with this past, albeit not always consciously admitted by the author. Nonetheless, this appears justifiable given the absence of too deep historical aims in the work and the already mentioned lack of sufficient historical sources available to the time of the author about musico-scientific topics.

From the present point of view, the more interesting and still valid part of Helmholtz's theory is that regarding the physical analysis of tones. In this respect, it appears scientifically very accurate and it has a great explanatory power as well as a considerable predictive one. Many of the predictions[123] concern aesthetical problems of great importance in the practiced art of music, which in this case constitutes the verification bench of the physical and physiological theory of sound, as admitted by Helmholtz himself in the preface to his work. These predictions have been largely confirmed, even by the author, and this provides, on the *logical level*, the desired verification of the theory. Nevertheless, the theory presents many objectionable points on the *epistemological level*.

First of all, it provides only a partial explanation of the physiological and especially psychological mechanisms responsible for the perception and sensation of tone. As far as I know, the Fourier-hypothesis about the functioning of the ear and in particular of Corti's organ cannot still today take advantage of an exhaustive empirical proof, although very advanced techniques - more than those available to Helmholtz - exist today in the research on the ear, the nerves and the brain. In fact, one can state that the problem of the sensation of tone still persists in modern neuroanatomy, neurobiology and psychoacoustics[124]. 'Why we perceive sometimes two sound as consonant', 'Why we perceive a certain harmony as pleasant' are still fascinating

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[123] For example, a (confirmed) prediction regards the fact that the major third D - F♯ will sound better if a clarinet plays D and an oboe F♯ than the contrary, that on a matter of a better combination of partial and combinational tones produced by the two instruments.

[124] Cf. e.g. F.C. Rose, *Neurology Of Music*, World Scientific, Singapore 2010.

questions in modern science. And again, 'Why a certain interval is considered as consonant or pleasant in the context of only a certain tonal system and tuning system of a certain historical period', 'Where the boundary between consonance and dissonance has exactly to be traced', 'What the musical beauty depends on' are also still discussed questions in modern aesthetics of music that, in my opinion, require a 'Helmholtzian' method of investigation, i.e. based on an integrated and interdisciplinary approach with contribution of sciences, art and theory of art.

Secondly, the contributions given by Helmholtz with his science of music (i.e. physical and physiological part) to aesthetics of music and especially to the practiced art of music appear to be not so effective as expected by the author. In fact, his acoustics leads to blur the boundary between consonance and dissonance and makes it a matter of degree. Therefore, the problem of the difference between consonance and dissonance, important from the aesthetical and artistical point of view, is not really solved. Moreover, he clearly states that the definition of the boundary depends basically on external factors, as the historical evolution of the tonal systems[125]. This could be seen as an inconsistent rejection of the aim itself of the entire work - i.e. defining scientifically (and mechanistically) acoustical and psychoacoustical phenomena as namely the consonance, and maybe it is really so. But the main critical point consists in the fact that, about the problem of consonance, it is rather the practiced art of music to have influenced certain scientific conclusions than the opposite case , indeed desired by the author. However empirically proved these conclusions may be, one has to see, in fact, that a factor plays a key role in Helmholtz's definition of consonance, which is however rather subjective and intentional, namely the definition of a *minimum number* of coincident partial and differential tone as condition of consonance (see Section 2.4). The individuation of this number, i.e. the 'pragmatic' and not-absolute boundary between consonance and dissonance, is influenced by the practiced art of music *of the time* again, where already the equal system with its impossible pure consonances is accepted and used as standard by musicians. Therefore, Helmholtz's theory of consonance seems only to justify this state of affairs. So, criticisms about equal temperament or conceptions of new tuning systems characteristic of the third part of the work remain a mere speculative activity whose practical consequences are excluded circularly *a priori* by the tacit (aesthetical) presuppositions of the theory itself.

Thus, the utilization of equal temperament as standard tuning system characteristic already of the nineteenth-century practiced art of music becomes the main external (and probably unconscious) root of Helmholtz's acoustics. There are, however,

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[125] Cf. H. Helmholtz, op. cit. (1954), p. 229.

other important aesthetical elements that take the form of external roots towards Helmholtz's acoustic theory. They are especially connected with the general idea of 'music' the German physician reveals to support in the preface to his work. During the first half of 19th century music was also affected by the growing positivism which characterized other fields of the science and of the culture in general[126]. In this period namely arose several new disciplines which considered music not as an art to enjoy, but as a mere object to study from different point of views: musicology, aesthetics of music, music criticism, history of music, music paleography, philology of music and, as we know, acoustics and psychophysiology of music. This practically declared the sunset of the romantic era, with its own conception of music as means of concrete representation and expression of sentiments. This epilogue had been however theoretically already anticipated by the influent ideas of the Austrian music critic Eduard Hanslick in the essay *Vom Musikalisch-Schönen*[127] - even quoted by Helmholtz himself in the mentioned preface - which introduced a new formalistic and intellectualistic conception of music: Music is pure form without meaning; Tones express or represent nothing, they have any purpose but themselves. This idea of self-referentiality of music - albeit not in absolute absence of romantic elements[128] - is clearly shared by Helmholtz: «Music stands in a much closer connection with pure sensation than any of the other arts»[129], in fact, unlike these, «in *music*, the sensations of tone are the material of the art»[130] itself, i.e. form and matter in music are exactly the same, tones immediately (literally, without any mediation) represent a meaning, a meaning that however consists in the tones themselves and not in a representation or expression of an external object. Then, his aim will be to provide a scientific explanation of the artistical enjoying of music, depending not on external factors, but only on the pure sensation of tone, all this by means, as we know, of a physical and psychophysiological account of the sensations of tone themselves:

> «In this sense it is clear that music has a more immediate connection with pure sensation than any other of the fine arts, and, consequent, that the theory of the sensations of hearing is destined to play a much more important part in musical esthetics, than, for example, the theory of *chiaroscuro* or of perspective in painting. Those theories are certainly useful to the artist, as means for attaining the most perfect representation of nature, but they have no part in the artistic effect of his work. In music, on the other hand, no such

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[126] For more about music aesthetics in 19th century cf. E. Fubini, op. cit. (1964a), ch. V.

[127] Cf. E. Hanslick, *Vom Musikalisch-Schönen*, R. Weigel (ed.), Leipzig 1854.

[128] Cf. E. Fubini, op. cit. (1964b), pp. 214-215.

[129] «Die Musik steht in einem viel näheren Verhältnis zu den reinen Sinnesempfindungen, als sämtliche übrigen Künste»; Cf. H. Helmholtz, op. cit. (1954), p. 2.

[130] «In der *Musik* dagegen sind es wirklich geradezu die Tonempfindungen, welche das Material der Kunst bilden» (emphasis in the original); Cf. H. Helmholtz, op. cit. (1954), p. 3.

> perfect representation of nature is aimed at; tones and the sensations of tone exist for themselves alone, and produce their effects independently of anything behind them»[131].

Before concluding, we have to take into account a last brief observation about the philosophical roots of Helmholtz's acoustics, putting together all the single remarks sporadically made during our inquiry. The entire scientific work of Helmholtz develops during a very important period for the history of science and of physics in particular. The successes of Newtonian rational mechanics even since the end of the 18th century with great scientists like Laplace and Lagrange, important discoveries in physics like those of the principle of energy conservation by Helmholtz himself in 1847[132] together (but independently) with Mayer and Joule placed physics, and in particular theoretical physics, in a lead position among sciences. A scientific optimism was now widespread; Almost every scientist believed that the most important laws of the Universe were known and that everything would have been soon explained, it was only a matter of time and several calculations.

Now, we can quite certainly state that Helmholtz was basically one of these scientists. He largely shared the positivistico-physicalistic optimism and was even one of its principal founders; His scientific work was also influenced by a mechanistic conception of the nature, that he extended, as we have seen, to the human body too, showing a certain continuity with that idea previously characteristic of Descartes and Beeckman that later became so successful in the development of modern physiology.

However, the mechanicism and positivism do not translate into a strictly demarcationist or even reductionist scientific methodology in Helmholtz, which is confirmed just by his work on acoustics. The extent to which such a brilliant scientific mind has taken into account a great art as music, acknowledging its particular status among the arts as well as the epistemological relevance of its problems both for the science and for practiced music itself clearly shows, in fact, his scientific open-mindedness, whose most relevant concretization is represented by the interdisciplinarity of his scientific approach. Helmholtz's interdisciplinarity can be for us historically

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[131] «In diesem Sinne ist es klar, daß die Musik eine unmittelbarere Verbindung mit der sinnlichen Empfindung hat, als irgendeine der anderen Künste; und daraus folgt denn, daß die Lehre von den Gehörempfindungen berufen sein wird, in der musikalischen Ästhetik eine viel wesentlichere Rolle zu spielen, als etwa die Lehre von der Beleuchtung oder der Perspektive in der Malerei. Diese letzteren sind allerdings dem Künstler nützlich, um eine möglichst vollendete Naturwahrheit zu erreichen, haben aber mit der künstlerischen Wirkung des Werkes nichts zu tun. In der Musik dagegen wird gar keine Naturwahrheit erstrebt, die Tone und Tonempfindungen sind ganz allein ihrer selbst wegen da und wirken ganz unabhängig von ihrer Beziehung zu irgendeinem äußeren Gegenstande» (emphasis only in the English version). Cf. H. Helmholtz, op. cit. (1954), p. 3.

[132] Cf. H. Helmholtz, *Über die Erhaltung der Kraft*, G. Reimer (ed.), Berlin 1847.

interesting, because it still reflects a methodology of scientific working of an already distant past, in which there did not exist sharp boundaries between disciplines or entire ‘cultures’ yet, but also epistemologically very important, still in our present, and this especially for the case of all that disciplines, scientific or not, today involved in explaining music in its different aspects. Besides the unquestionable scientific values, in fact, thinking science of music in an interdisciplinary way is probably the greatest epistemological legacy of Helmholtz’s theory of sound.

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