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Research Article

An Approach to Interval-Valued Hesitant Fuzzy Multiattribute Group Decision Making Based on the Generalized Shapley-Choquet Integral

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The purpose of this paper is to develop an approach to multiattribute group decision making under interval-valued hesitant fuzzy environment. To do this, this paper defines some new operations on interval-valued hesitant fuzzy elements, which eliminate the disadvantages of the existing operations. Considering the fact that elements in a set may be interdependent, two generalized interval-valued hesitant fuzzy operators based on the generalized Shapley function and the Choquet integral are defined. Then, some models for calculating the optimal fuzzy measures on the expert set and the ordered position set are established. Because fuzzy measures are defined on the power set, it makes the problem exponentially complex. To simplify the complexity of solving a fuzzy measure, models for the optimal 2-additive measures are constructed. Finally, an investment problem is offered to show the practicality and efficiency of the new method.

1. Introduction

The socioeconomic environment becomes more and more complex; it is impractical to require an expert to give his/her exact attribute values of every alternative. Based on fuzzy set theory [1], decision making under fuzzy environment is rapidly developed [2–6]. Since Zadeh [1] first introduced fuzzy sets, many extending forms are developed such as interval-valued fuzzy sets [7], type-2 fuzzy sets [8], interval type-2 fuzzy sets [9], and fuzzy multiset [10]. With the development of fuzzy set theory, the corresponding fuzzy decision-making theory is developed such as interval-valued fuzzy decision making [11, 12], type-2 fuzzy decision making [13, 14], interval type-2 fuzzy decision making [15, 16], and fuzzy multiset decision making [17].

Although there are several families of fuzzy sets, all of the above-mentioned fuzzy sets only consider the membership information. As Atanassov [18] noted, in some situations, it is insufficient to only know the membership degree for a certain fuzzy concept. Thus, Atanassov [18] introduced the concept of intuitionistic fuzzy sets (IFSs), which are characterized by a membership degree, a nonmembership degree, and a hesitancy degree. Since then, many intuitionistic fuzzy decision-making methods are proposed [19–21]. To further extend the application of IFSs, Atanassov and Gargov [22] introduced the concept of interval-valued intuitionistic fuzzy sets (IVIFSs), which are characterized by an interval membership function and an interval nonmembership function rather than real numbers. Such a generalization is further facilitated effectively to represent inherent imprecision and uncertainty in the human decision-making analysis. Many theories and methods on IVIFSs have been put forward and used to solve decision-making problems [23–27].

Recently, Torra and Narukawa [28] noted when an expert makes a decision, there may be several possible values for one thing. To deal with this situation, Torra [29] introduced the concept of hesitant fuzzy sets (HFSs) that permit the membership to have a set of possible values. Later, Xia and Xu [30] defined some operational laws on HFSs and presented some aggregation operators for hesitant fuzzy elements. Furthermore, Xia et al. [31] defined a series of hesitant fuzzy aggregation operators with the aid of quasi-arithmetic means and developed an approach to hesitant fuzzy multiple attribute decision making. Motivated by the ideal of prioritized aggregation operators, Wei [32] developed the hesitant fuzzy prioritized weighted average (HFPWA) operator and the hesitant fuzzy prioritized weighted geometric (HFPWG) operator, whilst Zhu et al. [33] introduced the weighted hesitant fuzzy geometric Bonferroni mean (WHFGBM) operator. More researches can be seen in the literature [34–37]. Just as interval type-2 fuzzy sets and IVIFSs, in some situations, it is still difficult to require an expert to give the exact possible values for one thing. Very recently, Chen et al. [38] introduced the concept of interval-valued hesitant fuzzy sets (IVHFSs) and defined some aggregation operators. Farhadinia [39] investigated the relationship between the entropy, the similarity measure, and the distance measure for HFSs and IVHFSs. Wei and Zhao [40] presented several induced hesitant interval-valued fuzzy Einstein aggregation operators and applied them to multiattribute decision making. Meanwhile, Wei et al. [41] defined two hesitant interval-valued fuzzy Choquet operators and studied their application in interval-valued hesitant multiattribute decision making. Meng and Chen [42] introduced two induced generalized interval-valued hesitant fuzzy hybrid Shapley operators that globally consider the interactions between the weights of elements in a set. It is noteworthy that all these aggregation operators are based on the operational laws presented by Chen et al. [38]. These operations cannot preserve the order relationship under multiplication by a scalar. It means that monotonicity is not always true. Thus, when these operators are used in decision making, it cannot guarantee to obtain the best choice. Furthermore, Meng et al. [43] researched the correlation coefficients of IVHFSs that need not consider the lengths of interval-valued hesitant fuzzy elements (IVHFEs). However, the correlation coefficients only consider the weights of attributes and disregard that of orders.

To address the above-mentioned issues for decision making with IVHFSs, this paper continues to study group decision making under interval-valued hesitant fuzzy environment. First, some new operations that eliminate the existing issues are defined. To deal with the situation where the elements in a set are correlative, two generalized interval-valued hesitant fuzzy dependent operators are defined, which can be seen as an extension of some hesitant fuzzy operators. Then, a distance measure on IVHFSs is defined, which does not consider the length of IVHFEs and the arrangement of their possible interval membership degrees. Based on the Shapley function and the defined distance measure, models for the optimal fuzzy measures and the optimal 2-additive measures are constructed, respectively. Finally, approach to interval-valued hesitant fuzzy multiattribute group decision making is developed. Comparing the existing methods, the new approach includes the following four features: (i) it uses the new defined operations that avoid the nonmonotonic problem; (ii) it applies the aggregation operator based on fuzzy measures that can address the interactive situations; (iii) when the weighting vector is partly known, models for the optimal fuzzy measure and the optimal 2-additive measure are built; (iv) because the experts' knowledge, skills, and experiences are different, the new method gives the experts' weights with respect to each attribute.

The paper is organized as follows: In Section 2, some basic concepts related to IVHFSs are reviewed, and some new operations on IVHFSs are defined. In Section 3, some generalized interval-valued hesitant fuzzy Choquet operators are defined, and some special cases are examined. Meanwhile, to simplify the complexity of solving a fuzzy measure, a generalized interval-valued hesitant fuzzy operator based on 2-additive measures is introduced. In Section 4, a new distance measure is defined, and then models for the optimal fuzzy measure and the optimal 2-additive measure on the associated set are built, respectively. After that, an approach to multiattribute group decision making under interval-valued hesitant fuzzy environment is developed. In Section 5, an illustrative example is provided to show the concrete application of the proposed procedure. Conclusions are made in the last section.

2. Some Basic Concepts

To address the situation where the membership degree of an element has several possible interval values, Chen et al. [38] presented the concept of interval-valued hesitant fuzzy sets (IVHFSs), which is an extension of hesitant fuzzy sets (HFSs) [29].

Definition 1 (see [38]). Let $X = \{x_1, x_2, \ldots, x_n\}$ be a finite set, and IVHFS in X is in terms of a function that when applied to X returns a subset of $D[0,1]$, denoted by

$$\overline{A} = \left\{ \left\langle x_i, \overline{h}_{\overline{A}}(x_i) \right\rangle \mid x_i \in X \right\}, \tag{1}$$

where $\overline{h}_{\overline{A}}(x_i)$ is a finite set of all possible interval-valued membership degrees of the element $x_i \in X$ to the set $\overline{A}$ with $D[0,1]$ being the set of all closed subintervals in $[0,1]$. For convenience, Chen et al. [38] called $\overline{h} = \overline{h}_{\overline{A}}(x_i)$ an interval-valued hesitant fuzzy element (IVHFE) and $\overline{H}$ is the set of all IVHFEs.

If all possible interval-valued membership degrees of each element $x_i \in X$ degenerate to real numbers, it derives an HFS [29].

Similar to the operational laws on HFEs [30], Chen et al. [38] defined the following operations on IVHFEs. Let $\overline{h}$, $\overline{h}_1$, and $\overline{h}_2$ be any three IVHFEs in $\overline{H}$, then

(i) $\overline{h}^{\lambda} = \bigcup_{\overline{r}=[r^l,r^u]\in\overline{h}}\{[r^{l^{\lambda}}, r^{u^{\lambda}}]\}$ $\lambda > 0$;

(ii) $\lambda\overline{h} = \bigcup_{\overline{r}=[r^l,r^u]\in\overline{h}}\{[1-(1-r^l)^{\lambda}, 1-(1-r^u)^{\lambda}]\}$ $\lambda > 0$;

(iii) $\overline{h}_1 \oplus \overline{h}_2 = \bigcup_{\overline{r}_1=[r_1^l,r_1^u]\in\overline{h}_1,\overline{r}_2=[r_2^l,r_2^u]\in\overline{h}_2}\{[r_1^l + r_2^l - r_1^l r_2^l, r_1^u + r_2^u - r_1^u r_2^u]\}$.;

(iv) $\overline{h}_1 \otimes \overline{h}_2 = \bigcup_{\overline{r}_1=[r_1^l,r_1^u]\in\overline{h}_1,\overline{r}_2=[r_2^l,r_2^u]\in\overline{h}_2}\{[r_1^l r_2^l, r_1^u r_2^u]\}$.

Let $\bar{a} = [a^l, a^u]$ and $\bar{b} = [b^l, b^u]$ be any two intervals; their order relationship is given using the possible degree formula as follows [42]:

$$p\left(\bar{a} \geq \bar{b}\right) = \max\left\{1 - \max\left(\frac{b^u - a^l}{d(\bar{a}) + d\left(\bar{b}\right)}, 0\right), 0\right\}, \quad (2)$$

where $d(\bar{a}) = a^u - a^l$ and $d(\bar{b}) = b^u - b^l$.

If $0 \leq p(\bar{a} \geq \bar{b}) < 0.5$, then $\bar{a} < \bar{b}$; if $p(\bar{a} \geq \bar{b}) = 0.5$, then $\bar{a} = \bar{b}$; if $0.5 < p(\bar{a} \geq \bar{b}) \leq 1$, then $\bar{a} > \bar{b}$.

Based on this possible degree formula on intervals, Chen et al. [38] introduced the following order relationship on IVHFEs.

Definition 2 (see [38]). For an IVHFE $\bar{h}$, $S(\bar{h}) = \sum_{\bar{r}=[r^l,r^u]\in\bar{h}}[r^l/\#\bar{h}, r^u/\#\bar{h}]$ is called the score function of $\bar{h}$ with $\#\bar{h}$ being the number of interval-valued membership degrees in $\bar{h}$, and $S(\bar{h})$ is an interval value in $[0, 1]$. For any two IVHFEs $\bar{h}_1$ and $\bar{h}_2$, if $S(\bar{h}_1) > S(\bar{h}_2)$, then $\bar{h}_1 > \bar{h}_2$; if $S(\bar{h}_1) = S(\bar{h}_2)$, then $\bar{h}_1 = \bar{h}_2$.

However, the operations given by Chen et al. [38] have some undesirable properties. For example, $(\lambda\bar{h})^\beta = \lambda^\beta\bar{h}^\beta$ and $(\bar{h}_1 \oplus \bar{h}_2)^\lambda = \bar{h}_1^{\ \lambda} \oplus \bar{h}_2^{\ \lambda}$ are not always true. See Example 3.

Example 3. Let $\bar{h} = ([0.2, 0.3], [0.5, 0.7])$, $\lambda = 0.2$, and $\beta = 0.3$; it derives

$$\begin{aligned}\left(\lambda\bar{h}\right)^\beta &= ([0.39, 0.45], [0.54, 0.63]),\\ \lambda^\beta\bar{h}^\beta &= ([0.45, 0.52], [0.64, 0.76]).\end{aligned} \quad (3)$$

It means $(\lambda\bar{h})^\beta \neq \lambda^\beta\bar{h}^\beta$.

Furthermore, take $\bar{h}_1 = \bar{h}$ and $\bar{h}_2 = ([0.3, 0.5])$; it gets

$$\begin{aligned}\left(\bar{h}_1 \oplus \bar{h}_2\right)^\lambda &= ([0.85, 0.92], [0.92, 0.97]),\\ \bar{h}_1^{\ \lambda} \oplus \bar{h}_2^{\ \lambda} &= ([0.11, 0.19], [0.19, 0.32]).\end{aligned} \quad (4)$$

It means $(\bar{h}_1 \oplus \bar{h}_2)^\lambda \neq \bar{h}_1^{\ \lambda} \oplus \bar{h}_2^{\ \lambda}$.

In addition, as Beliakov et al. [19] noted for intuitionistic fuzzy sets, the operations given by Chen et al. [38] cannot preserve the order relationship under multiplication by a scalar: $\bar{h}_1 < \bar{h}_2$ does not necessarily imply $\lambda\bar{h}_1 < \lambda\bar{h}_2$, where λ is a scalar. See Example 4.

Example 4. Take $\bar{h}_1 = ([0.21, 0.48])$, $\bar{h}_2 = ([0.31, 0.39])$, and $\lambda = 0.3$. Because $p(S(\bar{h}_1) \geq S(\bar{h}_2)) = 0.4857$, $\bar{h}_1 < \bar{h}_2$. However, $\lambda\bar{h}_1 = ([0.0683, 0.1781])$, $\lambda\bar{h}_2 = ([0.1053, 0.1378])$, and $p(S(\lambda\bar{h}_1) \geq S(\lambda\bar{h}_2)) = 0.5114$, so $\lambda\bar{h}_1 > \lambda\bar{h}_2$. Thus, $\bar{h}_1 < \bar{h}_2$, does not imply $\lambda\bar{h}_1 < \lambda\bar{h}_2$.

To avoid these disadvantages, we adopt the following operations on IVHFEs. Let $\bar{h}, \bar{h}_1$, and $\bar{h}_2$ be any three IVHFEs in $\bar{H}$,

(I) $\bar{h}^\lambda = \bigcup_{\bar{r}=[r^l,r^u]\in\bar{h}}\{[(r^l)^\lambda, (r^u)^\lambda]\}$ $\lambda > 0$;

(II) $\lambda\bar{h} = \bigcup_{\bar{r}=[r^l,r^u]\in\bar{h}}\{[\lambda r^l, \lambda r^u]\}$ $0 \leq \lambda \leq 1$;

(III) $\bar{h}_1 \times \bar{h}_2 = \bigcup_{\bar{r}_1=[r_1^l,r_1^u]\in\bar{h}_1, \bar{r}_2=[r_2^l,r_2^u]\in\bar{h}_2}\{[r_1^l r_2^l, r_1^u r_2^u]\}$;

(IV) $\bar{h}_1 + \bar{h}_2 = \bigcup_{\bar{r}_1=[r_1^l,r_1^u]\in\bar{h}_1, \bar{r}_2=[r_2^l,r_2^u]\in\bar{h}_2}\{[r_1^l + r_2^l, r_1^u + r_2^u]\}$ with $\bar{h}_1 + \bar{h}_2$ being an IVHFE, namely, $[r_1^l + r_2^l, r_1^u + r_2^u] \subseteq [0, 1]$ for all $\bar{r}_1 = [r_1^l, r_1^u] \in \bar{h}_1$ and $\bar{r}_2 = [r_2^l, r_2^u] \in \bar{h}_2$.

It is easy to verify that the new defined operations can eliminate the issues listed above. Without special explanation, this paper adopts the operations on IVHFEs defined by (I)–(IV).

In some cases, the possible degree formula (2) fails to distinguish two distinct IVHFEs. For example, let $\bar{h}_1 = \{[0.1, 0.8], [0.3, 0.6]\}$ and $\bar{h}_2 = \{[0.2, 0.3], [0.6, 0.7]\}$, then their scores are respective of $S(\bar{h}_1) = [0.2, 0.7]$ and $S(\bar{h}_2) = [0.4, 0.5]$. From (2), it gets $p(S(\bar{h}_1) \geq S(\bar{h}_2)) = p(S(\bar{h}_2) \geq S(\bar{h}_1)) = 0.5$ and $\bar{h}_1 = \bar{h}_2$. However, they are obviously different. To increase the identification of IVHFEs, we here adopt the following ranking method.

Let $\bar{a} = [a^l, a^u]$ and $\bar{b} = [b^l, b^u]$ be any two intervals; if $(a^l + a^u)/2 \leq (b^l + b^u)/2$ or $(a^l + a^u)/2 = (b^l + b^u)/2$ and $(b^u - b^l)/2 \leq (a^u - a^l)/2$, then $\bar{a} \leq \bar{b}$; otherwise, $\bar{a} \geq \bar{b}$.

3. Several Generalized Interval-Valued Hesitant Fuzzy Dependent Aggregation Operators

Let us consider the following example: "We are to evaluate three companies according to three attributes: {economic benefits, environment benefits, social benefits}, we want to give more importance to environment benefits than to economic benefits or social benefits, but on the other hand we want to give some advantage to companies that are good in environment benefits and in any of economic benefits and social benefits". In this situation, the aggregation operator based on additive measures seems to be insufficient.

To address the situation where the elements in a set are correlative, many aggregation operators based on the Choquet integral [44] are defined [45–52]. Using the Shapley function [53], Zhang et al. [54] defined the intuitionistic fuzzy Shapley weighted operator, Meng et al. [55] introduced some uncertain generalized Shapley aggregation operators, and Meng et al. [56] defined two linguistic hesitant fuzzy hybrid Shapley aggregation operators. More researches about decision making based on the Shapley function can be seen in the literature [57–60].

To obtain the comprehensive attribute values and reflect the interactions between attributes as well as the ordered positions, this section introduces several interval-valued hesitant fuzzy operators based on the Choquet integral and the generalized Shapley function. First, let us review the following definitions.

Definition 5 (see [61]). A fuzzy measure on finite set $N = \{1, 2, \ldots, n\}$ is a set function μ: $P(N) \to [0, 1]$ satisfying

(i) $\mu(\varnothing) = 0$, $\mu(N) = 1$,

(ii) If $A, B \in P(N)$ and $A \subseteq B$, then $\mu(A) \le \mu(B)$,

where $P(N)$ is the power set of N.

From the definition of fuzzy measures, we know that the fuzzy measure does not only give the importance of every element but also consider the importance of all their combinations. Corresponding to fuzzy measures, fuzzy integrals are important tools to aggregate information with interactive characteristics. The Choquet integral is one of the most important fuzzy integrals, which can be seen as an extension the ordered weighted averaging (OWA) operator. Grabisch [62] gave the following expression of the Choquet integral on discrete sets.

Definition 6 (see [62]). Let f be a positive real-valued function on $X = \{x_1, x_2, \dots, x_n\}$ and μ be a fuzzy measure on $N = \{1, 2, \dots, n\}$. The discrete Choquet integral of f for μ is defined as

$$C_\mu\left(f\left(x_{(1)}\right), f\left(x_{(2)}\right), \dots, f\left(x_{(n)}\right)\right) = \sum_{i=1}^{n} f\left(x_{(i)}\right)\left(\mu\left(A_i\right) - \mu\left(A_{i+1}\right)\right), \quad (5)$$

where $(\cdot)$ indicates a permutation on N such that $f(x_{(1)}) \le f(x_{(2)}) \le \cdots \le f(x_{(n)})$, and $A_i = \{i, \dots, n\}$ with $A_{(n+1)} = \varnothing$.

Remark 7. From Definition 6, one can see that the fuzzy measure μ only relates to the positions. It does not consider which element in the position.

From Definition 6, we know that the Choquet integral only considers the correlations between the ordered subsets A_i and A_{i+1} $(i = 1, 2, \dots, n)$. If there are interdependences, it seems to be insufficient. To globally reflect the interactions between the ordered subsets, the generalized Shapley function [63] seems to be a good choice, denoted as

$$\varphi_S\left(\mu, N\right) = \sum_{T \subseteq N \setminus S} \frac{(n-s-t)!t!}{(n-s+1)!}\left(\mu\left(S \cup T\right) - \mu\left(T\right)\right) \quad \forall S \subseteq N, \quad (6)$$

where μ is a fuzzy measure on $N = \{1, 2, \dots, n\}$, and s, t, and n denote the cardinalities of the coalitions S, T, and N, respectively.

Form (6), we know that it is an expect value of the overall marginal contributions between the coalition $S \subseteq N$ and any coalition in $N \setminus S$. When $S = \{i\}$, it degenerates to the famous Shapley function [53]:

$$\varphi_i\left(\mu, N\right) = \sum_{T \subseteq N \setminus i} \frac{(n-1-t)!t!}{n!}\left(\mu\left(i \cup T\right) - \mu\left(T\right)\right) \quad \forall i \subseteq N. \quad (7)$$

From (7), we know that when the elements in N are uncorrelated, then their Shapley values equal to their own importance, namely, $\varphi_i(\mu, N) = \mu(i)$ for all $i = 1, 2, \dots, n$.

Definition 8. Let f be a positive real-valued function on $X = \{x_1, x_2, \dots, x_n\}$, and μ be a fuzzy measure on $N = \{1, 2, \dots, n\}$. The discrete generalized Shapley-Choquet integral of f for μ is defined as

$$C_\mu\left(f\left(x_{(1)}\right), f\left(x_{(2)}\right), \dots, f\left(x_{(n)}\right)\right) = \sum_{i=1}^{n} f\left(x_{(i)}\right)\left(\varphi_{A_i}\left(\mu, N\right) - \varphi_{A_{i+1}}\left(\mu, N\right)\right), \quad (8)$$

where $(\cdot)$ indicates a permutation on N such that $f(x_{(1)}) \le f(x_{(2)}) \le \cdots \le f(x_{(n)})$, φ is the generalized Shapley on N, and $A_i = \{i, \dots, n\}$ with $A_{(n+1)} = \varnothing$.

From Definition 8, one can see that the generalized Shapley-Choquet integral overall considers the interactions between any two adjacent coalitions. Now, let us introduce the generalized interval-valued hesitant fuzzy Shapley-Choquet weighted averaging (G-IVHFSCWA) operator as follows.

Definition 9. Let $\overline{h}_i$ $(i = 1, 2, \dots, n)$ be a collection of IVHFEs in $\overline{H}$ and μ be a fuzzy measure on the ordered set $N = \{1, 2, \dots, n\}$. The generalized interval-valued hesitant fuzzy Shapley-Choquet weighted averaging (G-IVHFSCWA) operator is defined as

$$\begin{aligned} \text{G-IVHFSCWA}\left(\overline{h}_1, \overline{h}_2, \dots, \overline{h}_n\right) &= \left(\sum_{i=1}^{n}\left(\varphi_{A_i}\left(\mu, N\right) - \varphi_{A_{i+1}}\left(\mu, N\right)\right)\overline{h}_{(i)}^{\lambda}\right)^{1/\lambda} \\ &= \bigcup_{\overline{r}_{(1)} \in \overline{h}_{(1)}, \overline{r}_{(2)} \in \overline{h}_{(2)}, \dots, \overline{r}_{(n)} \in \overline{h}_{(n)}} \left[\left(\sum_{i=1}^{n}\left(\varphi_{A_i}\left(\mu, N\right) - \varphi_{A_{i+1}}\left(\mu, N\right)\right)\left(r_{(i)}^{l}\right)^{\lambda}\right)^{1/\lambda}, \left(\sum_{i=1}^{n}\left(\varphi_{A_i}\left(\mu, N\right) - \varphi_{A_{i+1}}\left(\mu, N\right)\right)\left(r_{(i)}^{u}\right)^{\lambda}\right)^{1/\lambda}\right], \end{aligned} \quad (9)$$

where $\lambda > 0$, $(\cdot)$ indicates a permutation on $\overline{A}$ such that $\overline{h}_{(1)} \le \overline{h}_{(2)} \le \cdots \le \overline{h}_{(n)}$ and $\varphi_{A_i}(\mu, N)$ is the generalized Shapley value of $A_i = \{i, \dots, n\}$ with $A_{n+1} = \varnothing$.

Remark 10. If $\lambda = 1$, then the G-IVHFSCWA operator degenerates to the interval-valued hesitant fuzzy Shapley-Choquet weighted averaging (IVHFSCWA) operator

$$\text{IVHFSCWA}\left(\overline{h}_1,\overline{h}_2,\ldots,\overline{h}_n\right) = \sum_{i=1}^{n}\left(\varphi_{A_i}(\mu,N)-\varphi_{A_{i+1}}(\mu,N)\right)\overline{h}_{(i)}. \quad (10)$$

Remark 11. If $\lambda = 2$, then the G-IVHFSCWA operator degenerates to the interval-valued hesitant fuzzy Shapley-Choquet quadratic weighted averaging (IVHFSCQWA) operator

$$\text{IVHFSCQWA}\left(\overline{h}_1,\overline{h}_2,\ldots,\overline{h}_n\right) = \left(\sum_{i=1}^{n}\left(\varphi_{A_i}(\mu,N)-\varphi_{A_{i+1}}(\mu,N)\right)\overline{h}_{(i)}^{\,2}\right)^{1/2}. \quad (11)$$

From Definition 9, we know that the G-IVHFSCWA operator only gives the importance of the ordered positions. To further consider the importance of elements and reflect their correlations, we introduce the interval-valued hesitant fuzzy Shapley-Choquet hybrid operator that considers the importance of the attributes (or experts) and their ordered positions as well as reflects their interactions.

Definition 12. Let $\overline{h}_i$ $(i = 1,2,\ldots,n)$ be a collection of IVHFEs in $\overline{H}$, ν be a fuzzy measure on $\overline{A} = \{\overline{h}_1,\overline{h}_2,\ldots,\overline{h}_n\}$, and μ be a fuzzy measure on the ordered set $N = \{1,2,\ldots,n\}$. The generalized interval-valued hesitant fuzzy Shapley-Choquet hybrid weighted averaging (G-IVHFSCHWA) operator is defined as

$$\begin{aligned}\text{G-IVHFSCHWA}\left(\overline{h}_1,\overline{h}_2,\ldots,\overline{h}_n\right) &= \left(\frac{\sum_{i=1}^{n}\left(\varphi_{A_i}(\mu,N)-\varphi_{A_{i+1}}(\mu,N)\right)\left(\varphi_{\overline{h}_{(i)}}(\nu,\overline{A})\overline{h}_{(i)}\right)^{\lambda}}{\sum_{i=1}^{n}\left(\varphi_{A_i}(\mu,N)-\varphi_{A_{i+1}}(\mu,N)\right)\left(\varphi_{\overline{h}_{(i)}}(\nu,\overline{A})\right)^{\lambda}}\right)^{1/\lambda} \\ &= \bigcup_{\overline{r}_{(1)}\in\overline{h}_{(1)},\overline{r}_{(2)}\in\overline{h}_{(2)},\ldots,\overline{r}_{(n)}\in\overline{h}_{(n)}}\left[\left(\frac{\sum_{i=1}^{n}\left(\varphi_{A_i}(\mu,N)-\varphi_{A_{i+1}}(\mu,N)\right)\left(\varphi_{\overline{h}_{(i)}}(\nu,\overline{A})r_{(i)}^{l}\right)^{\lambda}}{\sum_{i=1}^{n}\left(\varphi_{A_i}(\mu,N)-\varphi_{A_{i+1}}(\mu,N)\right)\left(\varphi_{\overline{h}_{(i)}}(\nu,\overline{A})\right)^{\lambda}}\right)^{1/\lambda},\left(\frac{\sum_{i=1}^{n}\left(\varphi_{A_i}(\mu,N)-\varphi_{A_{i+1}}(\mu,N)\right)\left(\varphi_{\overline{h}_{(i)}}(\nu,\overline{A})r_{(i)}^{u}\right)^{\lambda}}{\sum_{i=1}^{n}\left(\varphi_{A_i}(\mu,N)-\varphi_{A_{i+1}}(\mu,N)\right)\left(\varphi_{\overline{h}_{(i)}}(\nu,\overline{A})\right)^{\lambda}}\right)^{1/\lambda}\right],\end{aligned} \quad (12)$$

where $\lambda > 0$, $(\cdot)$ indicates a permutation on $\overline{A}$ such that $\varphi_{\overline{h}_{(1)}}(\nu,\overline{A})\overline{h}_{(1)} \le \varphi_{\overline{h}_{(2)}}(\nu,\overline{A})\overline{h}_{(2)} \le \cdots \le \varphi_{\overline{h}_{(n)}}(\nu,\overline{A})\overline{h}_{(n)}$, $\varphi_{\overline{h}_i}(\nu,\overline{A})$ is the Shapley value of $\overline{h}_i$, and $\varphi_{A_i}(\mu,N)$ is the generalized Shapley value of $A_i = \{i,\ldots,n\}$ with $A_{n+1} = \varnothing$.

Remark 13. If $\varphi_{\overline{h}_i}(\nu,\overline{A}) = 1/n$ for each $i \in N$, then the G-IVHFSCHWA operator degenerates to the G-IVHFSCWA operator.

Remark 14. If $\lambda = 1$, then the G-IVHFSCHWA operator degenerates to the interval-valued hesitant fuzzy Shapley-Choquet hybrid weighted averaging (IVHFSCHWA) operator

$$\text{IVHFSCHWA}\left(\overline{h}_1,\overline{h}_2,\ldots,\overline{h}_n\right) = \frac{\sum_{i=1}^{n}\left(\varphi_{A_i}(\mu,N)-\varphi_{A_{i+1}}(\mu,N)\right)\varphi_{\overline{h}_{(i)}}(\nu,\overline{A})\overline{h}_{(i)}}{\sum_{i=1}^{n}\left(\varphi_{A_i}(\mu,N)-\varphi_{A_{i+1}}(\mu,N)\right)\varphi_{\overline{h}_{(i)}}(\nu,\overline{A})}. \quad (13)$$

Remark 15. If $\lambda = 2$, then the G-IVHFSCHWA operator degenerates to the interval-valued hesitant fuzzy Shapley-Choquet quadratic hybrid weighted averaging (IVHFSCQHWA) operator

$$\text{IVHFSCQHWA}\left(\overline{h}_1,\overline{h}_2,\ldots,\overline{h}_n\right) = \left(\frac{\sum_{i=1}^{n}\left(\varphi_{A_i}(\mu,N)-\varphi_{A_{i+1}}(\mu,N)\right)\left(\varphi_{\overline{h}_{(i)}}(\nu,\overline{A})\overline{h}_{(i)}\right)^{2}}{\sum_{i=1}^{n}\left(\varphi_{A_i}(\mu,N)-\varphi_{A_{i+1}}(\mu,N)\right)\left(\varphi_{\overline{h}_{(i)}}(\nu,\overline{A})\right)^{2}}\right)^{1/2}. \quad (14)$$

Although the fuzzy measure can address the situation where the elements in a set are correlative, they define the power set. It makes the problem exponentially complex. Thus, it is not easy to solve a fuzzy measure when the set is large. To reflect the interactions between elements and simplify the complexity of solving a fuzzy measure, we introduce a special case of the G-IVHFSCHWA operator using 2-additive measures.

Let $f : \{0,1\} \to \mathbb{R}$ be a pseudo-Boolean function. Grabisch [64] noted that any fuzzy measure μ can be seen as a particular case of pseudo-Boolean function and put under a multilinear polynomial in n variables:

$$\mu(A) = \sum_{T\subseteq N}\left[a_T\prod_{i\in T}y_i\right] \quad \forall A \subseteq N, \quad (15)$$

where $a_T \in \mathbb{R}$, $y = (y_1, y_2, \ldots, y_n) \in \{0,1\}^n$, and $y_i = 1$ if and only if $i \in A$.

The set of coefficients a_T $(T \subseteq N)$ corresponds to the Möbius transform, denoted by $a_T = \sum_{S\subseteq T}(-1)^{t-s}\mu(S)$. Because the transform is inversible, μ can be recovered from a_T by $\mu(A) = \sum_{B\subseteq A} a_B$.

Definition 16 (see [64]). A fuzzy measure μ on $N = \{1,2,\ldots,n\}$ is said to be *k-additive* if its corresponding pseudo-Boolean function is a multilinear polynomial of degree k, i.e., $a_T = 0$ for all T such that $t > k$, and there exists at least one subset T with k elements such that $a_T \neq 0$.

Particularly, when $k = 2$, it gets a 2-additive measure. For a 2-additive measure μ, one can easily get [64], for any $S \subseteq N$, with $s \ge 2$,

$$\mu(S) = \sum_{i=1}^{n} a_i x_i + \sum_{\{i,j\}\subseteq N} a_{ij}x_i x_j = \sum_{i\in S} a_i + \sum_{\{i,j\}\subseteq S} a_{ij}$$

$$= \sum_{\{i,j\}\subseteq S} \mu(i,j) - (s-2)\sum_{i\in S}\mu(i), \quad (16)$$

where $\mu(i) = a_i$ and $\mu(i,j) = a_i + a_j + a_{ij}$.

For a 2-additive measure, we only need $n(n+1)/2$ coefficients to determine it for a set with n elements.

Theorem 17 (see [64]). *Let μ be a fuzzy measure on $N = \{1, 2, \ldots, n\}$, then μ is a 2-additive measure if and only if there exist coefficients $\mu(i)$ and $\mu(i, j)$ for all $i, j \in N$ that satisfy the following conditions:*

(i) $\mu(i) \geq 0\ \forall i \in N$,

(ii) $\sum_{\{i,j\}\subseteq N} \mu(i,j) - (n-2)\sum_{i\in N}\mu(i) = 1$,

(iii) $\sum_{i\subseteq S\backslash k}(\mu(i,k) - \mu(i)) \geq (s-2)\mu(k)\ \forall S \in N$ *s.t.* $k \in S$ *and* $s \geq 2$.

Theorem 18 (see [46]). *Let μ be a 2-additive measure on $N = \{1, 2, \ldots, n\}$, then the generalized Shapley function φ with respect to μ can be expressed as*

$$\varphi_S(\mu, N) = \sum_{\{i,j\}\subseteq S} \mu(i,j) + \frac{1}{2}\sum_{i\in S, j\in N\backslash S}(\mu(i,j) - s\mu(j)) - \frac{n+s-4}{2}\sum_{i\in S}\mu(i) \quad (17)$$

for any $S \subseteq N$ such that $s \geq 2$ and for any $\{i\} = S \subseteq N$,

$$\varphi_i(\mu, N) = \frac{3-n}{2}\mu(i) + \frac{1}{2}\sum_{j\in N\backslash i}(\mu(i,j) - \mu(j)). \quad (18)$$

In Definition 12, if v and μ are both a 2-additive measure, then it derives the generalized interval-valued hesitant fuzzy 2-additive Shapley-Choquet hybrid weighted averaging (G-IVHF2SCHWA) operator.

4. An Approach to Multiattribute Group Decision Making

Because of various reasons, the weighting information may be incompletely known. To solve this situation, this section first establishes models for the optimal fuzzy measure and the optimal 2-additive measure on the associated sets. Then, an approach to multiattribute group decision making under interval-valued hesitant fuzzy environment with incomplete weighted information and interactive characteristics is developed.

Let $A = \{a_1, a_2, \ldots, a_m\}$ be the set of alternatives, let $C = \{c_1, c_2, \ldots, c_n\}$ be the set of attributes, and let $E = \{e_1, e_2, \ldots, e_q\}$ be the set of experts. Assume that $\overline{h}_{ij}^k$ is the IVHFE of the alternative a_i for the attribute c_j given by the expert e_k ($i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$; $k = 1, 2, \ldots, q$). By $\overline{H}^k = (\overline{h}_{ij}^k)_{m\times n}$, we denote the interval-valued hesitant fuzzy decision matrix given by the expert e_k ($k = 1, 2, \ldots, q$). Let $N = \{1, 2, \ldots, n\}$ and $Q = \{1, 2, \ldots, q\}$ be respective of the ordered sets for the attribute set C and the expert set E.

4.1. Models for the Optimal Fuzzy Measure. Before building models for the optimal fuzzy measure, let us first introduce a new distance measure. Let $\overline{h}_1$ and $\overline{h}_2$ be any two IVHFEs, Chen et al. [38] defined the following distance measures for IVHFEs, denoted as

$$d_C^1(\overline{h}_1, \overline{h}_2) = \frac{1}{2l}\sum_{j=1}^{l}\left(\left|r_{\overline{h}_1(j)}^l - r_{\overline{h}_2(j)}^l\right| + \left|r_{\overline{h}_1(j)}^u - r_{\overline{h}_2(j)}^u\right|\right), \quad (19)$$

$$d_C^2(\overline{h}_1, \overline{h}_2) = \sqrt{\frac{1}{2l}\sum_{j=1}^{l}\left(\left|r_{\overline{h}_1(j)}^l - r_{\overline{h}_2(j)}^l\right|^2 + \left|r_{\overline{h}_1(j)}^u - r_{\overline{h}_2(j)}^u\right|^2\right)}, \quad (20)$$

where $(\cdot)$ is a permutation on the possible interval value in $\overline{h}_1$ and $\overline{h}_2$ with $\overline{r}_{\overline{h}_1(j)} = [r_{\overline{h}_1(j)}^l, r_{\overline{h}_1(j)}^u]$ and $\overline{r}_{\overline{h}_2(j)} = [r_{\overline{h}_2(j)}^l, r_{\overline{h}_2(j)}^u]$ being the jth largest values in $\overline{h}_1$ and $\overline{h}_2$, respectively; let $l = \max\{l(\overline{h}_1), l(\overline{h}_2)\}$ with $l(\overline{h}_1)$ and $l(\overline{h}_2)$ being the numbers of possible interval-valued membership degrees in $\overline{h}_1$ and $\overline{h}_2$. For $l(\overline{h}_1) \neq l(\overline{h}_2)$, the authors adopted the method that extends the shorter one until both of them have the same length by adding the biggest interval several times.

Different from this distance measure, we define another one that need not consider the length of IVHFEs.

Definition 19. Let $\overline{h}_1$ and $\overline{h}_2$ be any two IVHFEs, then the generalized distance measure between $\overline{h}_1$ and $\overline{h}_2$ is defined as

$$d^p(\overline{h}_1, \overline{h}_2) = \left[\frac{1}{2}\left\{\frac{\sum_{\overline{r}_1\in\overline{h}_1}\min_{\overline{r}_2\in\overline{h}_2}\left(\left|r_1^l - r_2^l\right|^p + \left|r_1^u - r_2^u\right|^p\right)}{2\#\overline{h}_1} + \frac{\sum_{\overline{r}_2\in\overline{h}_2}\min_{\overline{r}_1\in\overline{h}_1}\left(\left|r_2^l - r_1^l\right|^p + \left|r_2^u - r_1^u\right|^p\right)}{2\#\overline{h}_2}\right\}\right]^{1/p}, \quad (21)$$

where $p > 0$ and $\#\overline{h}_1$ and $\#\overline{h}_2$ denote the number of the possible interval value in $\overline{h}_1$ and $\overline{h}_2$, respectively.

For example, let $\overline{h}_1 = \{[0.2, 0.3], [0.4, 0.6], [0.7, 0.8]\}$ and $\overline{h}_2 = \{[0.1, 0.4], [0.5, 0.6]\}$. From (19), it derives $d_C^1(\overline{h}_1, \overline{h}_2) = 0.1167$. By (20), it gets $d_C^2(\overline{h}_1, \overline{h}_2) = 0.1353$. Furthermore, by (21) it gives $d^1(\overline{h}_1, \overline{h}_2) = 0.0958$ for $p = 1$ and $d^2(\overline{h}_1, \overline{h}_2) = 0.1136$ for $p = 2$.

4.1.1. Models for the Optimal Fuzzy Measure on the Expert Set E. For each interval-valued hesitant fuzzy decision matrix $\overline{H}^k = (\overline{h}_{ij}^k)_{m\times n}$ ($k = 1, 2, \ldots, q$), we calculate the score matrix $S(\overline{H}^k) = (S(\overline{h}_{ij}^k))_{m\times n}$ with $S(\overline{h}_{ij}^k) = \sum_{\overline{r}_{ij}^k = [(r_{ij}^k)^l, (r_{ij}^k)^u]\in\overline{h}_{ij}^k}[(r_{ij}^k)^l/\#\overline{h}_{ij}^k,$

$(r_{ij}^k)^u/\#\overline{h}_{ij}^k] = [S(r_{ij}^k)^l, S(r_{ij}^k)^u]$. Because the experts' knowledge, skills, and experiences are different, it is unreasonable to give the same weight of an expert for different attributes.

Let $d_{ij}^k = |S(r_{ij}^k)^l - (\sum_{k=1}^q S(r_{ij}^k)^l)/q| + |S(r_{ij}^k)^u - (\sum_{k=1}^q S(r_{ij}^k)^u)/q|$. With respect to the attribute c_j, $j = 1, 2, \ldots, n$, if the weighting information on the expert set is partly known, the following model is established:

$$
\begin{aligned}
\min \quad & \sum_{k=1}^{q}\sum_{i=1}^{m}\varphi_{e_k}\left(v_j^E, E\right) d_{ij}^k \\
\text{s.t.} \quad & B^j\left(v_j^E\left(S_1\right), \ldots, v_j^E\left(S_{k_1}\right)\right) \le \alpha^j, \\
& \qquad S_l \subseteq E,\ l = 1, 2, \ldots, k_1 \\
& G^j\left(v_j^E\left(T_1\right), \ldots, v_j^E\left(T_{k_2}\right)\right) = \beta^j, \\
& \qquad T_l \subseteq E,\ l = 1, 2, \ldots, k_2 \\
& v_j^E(E) = 1 \\
& v_j^E(S) \le v_j^E(T) \quad \forall S, T \subseteq E \text{ s.t. } S \subseteq T \\
& v_j^E\left(e_k\right) \in W_{e_k}^j, \quad v_j^E\left(e_k\right) \ge 0,\ k = 1, 2, \ldots, q,
\end{aligned}
\tag{22}
$$

where B^j and G^j are the coefficient matrices, α^j and β^j are the constant vectors, $B^j(v_j^E(S_1), v_j^E(S_2), \ldots, v_j^E(S_{k_1})) \le \alpha^j$ and $G^j(v_j^E(T_1), v_j^E(T_2), \ldots, v_j^E(T_{k_2})) = \beta^j$ are the known constraints, v_j^E is the fuzzy measure on the expert set E with respect to the attribute c_j, $\varphi_{e_k}(v_j^E, E)$ is the Shapley value of the expert e_k, and $W_{e_k}^j$ is the known weighting information.

If v_j^E is a 2-additive measure, by (18) it gets the following model:

$$
\begin{aligned}
\min \quad & \sum_{i=1}^{m}\sum_{k=1}^{q}\frac{d_{ij}^k}{2}\left((3-n)\, v_j^E\left(e_k\right) + \sum_{e_l \in E \setminus e_k}\left(v_j^E\left(e_k, e_l\right) - v_j^E\left(e_l\right)\right)\right) \\
\text{s.t.} \quad & \widetilde{B}^j\left(v_j^E\left(E_j\right), v_j^E\left(E_i, E_j\right),\ i, j = 1, \ldots, q,\ i \ne j\right) \le \widetilde{\alpha}^j \\
& \widetilde{G}^j\left(v_j^E\left(E_j\right), v_j^E\left(E_i, E_j\right),\ i, j = 1, \ldots, q,\ i \ne j\right) = \widetilde{\beta}^j \\
& \sum_{e_l \in S \setminus e_k}\left(v_j^E\left(e_k, e_l\right) - v_j^E\left(e_l\right)\right) \ge (s-2)\, v_j^E\left(e_k\right), \quad \forall S \subseteq E,\ \forall e_k \in S,\ s \ge 2 \\
& \sum_{\{e_k, e_l\} \subseteq E} v_j^E\left(e_k, e_l\right) - (q-2)\sum_{e_l \in E} v_j^E\left(e_l\right) = 1 \\
& v_j^E\left(e_k\right) \in W_{e_k}^j, \quad v_j^E\left(e_k\right) \ge 0,\ k = 1, 2, \ldots, q,
\end{aligned}
\tag{23}
$$

where $\widetilde{B}^j$ and $\widetilde{G}^j$ are the coefficient matrices, $\widetilde{\alpha}^j$ and $\widetilde{\beta}^j$ are the constant vectors, $\widetilde{B}^j(v_j^E(E_j), v_j^E(E_i, E_j),\ i, j = 1, \ldots, q,\ i \ne j) \le \widetilde{\alpha}^j$, and $\widetilde{G}^j(v_j^E(E_j), v_j^E(E_i, E_j),\ i, j = 1, \ldots, q,\ i \ne j) = \widetilde{\beta}^j$ are the equivalent expressions of the known constraints given in model (22) with respect to the 2-additive measure v_j^E.

The optimal fuzzy measure obtained from this model has the following desirable characteristics: the closer an expert's evaluation values are to the other experts', the larger the fuzzy measure will be. This can decrease the influence of the unduly high or low evaluation value induced by the experts' limited knowledge or expertise.

4.1.2. Models for the Optimal Fuzzy Measure on the Ordered Set Q. To construct the model for the optimal fuzzy measure on the ordered set Q, the following procedure is needed.

Step 1. Calculate the interval-valued hesitant fuzzy Shapley weighted decision matrices $\overline{H}_{\varphi_{e_k}(\mu^E, E)}^k = (\overline{h}_{ij}'^k)_{m\times n}$, $k \in Q$, where

$$
\overline{h}_{ij}'^k = \bigcup_{\overline{r}_{ij}^k = [(r_{ij}^k)^l, (r_{ij}^k)^u] \in \overline{h}_{ij}^k}\left[\varphi_{e_k}\left(\mu_j^E, E\right)\left(r_{ij}^k\right)^l, \varphi_{e_k}\left(\mu_j^E, E\right) \cdot \left(r_{ij}^k\right)^u\right]. \tag{24}
$$

Step 2. Calculate the score matrices $S(\overline{H}_{\varphi_{e_k}(\mu^E, E)}^k) = (S(\overline{h}_{ij}'^k))_{m\times n}$, $k \in Q$, where

$$
\begin{aligned}
S\left(\overline{h}_{ij}'^k\right) &= \sum_{\overline{r}_{ij}'^k = [(r_{ij}'^k)^l, (r_{ij}'^k)^u] \in \overline{h}_{ij}'^k}\left[\frac{\left(r_{ij}'^k\right)^l}{\#\overline{h}_{ij}'^k}, \frac{\left(r_{ij}'^k\right)^u}{\#\overline{h}_{ij}'^k}\right] \\
&= \left[S\left(r_{ij}'^k\right)^l, S\left(r_{ij}'^k\right)^u\right].
\end{aligned}
\tag{25}
$$

Step 3. Calculate the mid-width matrices $P^k = (p_{ij}^k)_{m\times n}$, $k \in Q$, where

$$p_{ij}^{k}=\frac{S\left(r_{ij}^{\prime k}\right)^{l}+S\left(r_{ij}^{\prime k}\right)^{u}}{S\left(r_{ij}^{\prime k}\right)^{l}+S\left(r_{ij}^{\prime k}\right)^{u}+S\left(r_{ij}^{\prime k}\right)^{u}-S\left(r_{ij}^{\prime k}\right)^{l}}=\frac{S\left(r_{ij}^{\prime k}\right)^{l}+S\left(r_{ij}^{\prime k}\right)^{u}}{2S\left(r_{ij}^{\prime k}\right)^{u}}. \quad (26)$$

Step 4. For each pair (i, j), we rearrange each p_{ij}^{k}, $k \in Q$, such that $p_{ij}^{(1)} \le p_{ij}^{(2)} \le \cdots \le p_{ij}^{(q)}$.

Because there is no inferior for the ordered positions with respect to the different attributes, if the weighting information on the ordered set Q is not exactly known, the following model for the optimal fuzzy measure is built:

$$\begin{aligned}
\max\quad & \sum_{k=1}^{q}\sum_{j=1}^{n}\sum_{i=1}^{m}\varphi_k\left(\mu^Q, Q\right)p_{ij}^{(k)}\\
\text{s.t.}\quad & B\left(\mu^Q\left(S_1\right),\ldots,\mu^Q\left(S_{p_1}\right)\right)\le\alpha,\\
& \qquad S_l\subseteq Q,\ l=1,2,\ldots,p_1\\
& G\left(\mu^Q\left(T_1\right),\ldots,\mu^Q\left(T_{p_2}\right)\right)=\beta,\\
& \qquad T_l\subseteq Q,\ l=1,2,\ldots,p_2\\
& \mu^Q(Q)=1\\
& \mu^Q(S)\le\mu^Q(T)\quad \forall S,T\subseteq Q \text{ s.t. } S\subseteq T\\
& \mu^Q(k)\in W_k,\quad \mu^Q(k)\ge 0,\ k=1,2,\ldots,q,
\end{aligned} \quad (27)$$

where B and G are the coefficient matrices, α and β are the constant vectors, $B(\mu^Q(S_1),\ldots,\mu^Q(S_{p_1})) \le \alpha$ and $G(\mu^Q(T_1),\ldots,\mu^Q(T_{p_2})) = \beta$ are the known constraints, μ^Q is the fuzzy measure on the ordered set Q, $\varphi_k(\mu^Q, Q)$ is the Shapley value of the kth position, and W_k is the known weighting information.

If μ^Q is a 2-additive measure, by (18) it gets the following model:

$$\begin{aligned}
\max\quad & \sum_{k=1}^{q}\sum_{j=1}^{n}\sum_{i=1}^{m}\frac{p_{ij}^{(k)}}{2}\left((3-n)\mu^Q(k)+\sum_{l\in Q\backslash k}\left(\mu^Q(k,l)-\mu^Q(l)\right)\right)\\
\text{s.t.}\quad & \widetilde{B}\left(\mu^Q(j),\mu^Q(i,j),\ i,j=1,\ldots,q,\ i\ne j\right)\le\widetilde{\alpha}\\
& \widetilde{G}\left(\mu^Q(j),\mu^Q(i,j),\ i,j=1,\ldots,q,\ i\ne j\right)=\widetilde{\beta}\\
& \sum_{l\in S\backslash k}\left(\mu^Q(k,l)-\mu^Q(l)\right)\ge(s-2)\mu^Q(k),\quad \forall S\subseteq Q,\ \forall k\in S,\ s\ge 2\\
& \sum_{\{k,l\}\subseteq Q}\mu^Q(k,l)-(q-2)\sum_{l\in Q}\mu^Q(l)=1\\
& \mu^Q(k)\in W_k,\quad \mu^Q(k)\ge 0,\ k=1,2,\ldots,q,
\end{aligned} \quad (28)$$

where $\widetilde{B}$ and $\widetilde{G}$ are the coefficient matrices, $\widetilde{\alpha}$ and $\widetilde{\beta}$ are the constant vectors, $\widetilde{B}(\mu^Q(j),\mu^Q(i,j),\ i,j=1,\ldots,q,\ i\ne j)\le\widetilde{\alpha}$, and $\widetilde{G}(\mu^Q(j),\mu^Q(i,j),\ i,j=1,\ldots,q,\ i\ne j)=\widetilde{\beta}$ are the equivalent expressions of the known constraints given in model (27) with respect to 2-additive measure μ^Q.

4.1.3. Models for the Optimal Fuzzy Measure on the Attribute Set C. Next, let us consider the optimal fuzzy measure on the attribute set C. Assume that $\overline{H}=(\overline{h}_{ij})_{m\times n}$ is the comprehensive interval-valued hesitant fuzzy decision matrix. Let $\overline{h}_j^{+}=\max_{i=1}^{m}\overline{h}_{ij}$ and $\overline{h}_j^{-}=\min_{i=1}^{m}\overline{h}_{ij}$ for each $j=1,2,\ldots,n$.

By (21), we calculate the distance $d^p(\overline{h}_{ij},\overline{h}_j^{+})$ between $\overline{h}_{ij}$ and $\overline{h}_j^{+}$ as well as the distance $d^p(\overline{h}_{ij},\overline{h}_j^{-})$ between $\overline{h}_{ij}$ and $\overline{h}_j^{-}$ for each pair (i, j). Because all alternatives are noninferior, if the weighting information on the attribute set C is not exactly known, the following models for the optimal fuzzy measure are constructed:

$$\begin{aligned}
\min\quad & \sum_{j=1}^{n}\sum_{i=1}^{m}\varphi_{c_j}\left(v^C, C\right)d^p\left(\overline{h}_{ij},\overline{h}_j^{+}\right)\\
\text{s.t.}\quad & R\left(v^C\left(S_1\right),\ldots,v^C\left(S_{q_1}\right)\right)\le\varepsilon,\\
& \qquad S_l\subseteq C,\ l=1,\ldots,q_1\\
& H\left(v^C\left(T_1\right),\ldots,v^C\left(T_{q_2}\right)\right)=\eta,\\
& \qquad T_{t_2}\subseteq C,\ t_2=1,\ldots,q_2\\
& v^C(C)=1\\
& v^C(S)\le v^C(T)\quad \forall S,T\subseteq C \text{ s.t. } S\subseteq T\\
& v^C\left(c_j\right)\in W_{c_j},\quad v^C\left(c_j\right)\ge 0,\ j=1,2,\ldots,n,
\end{aligned} \quad (29)$$

$$\begin{aligned} \max \quad & \sum_{j=1}^{n}\sum_{i=1}^{m}\varphi_{c_j}\left(v^C, C\right) d^p\left(\overline{h}_{ij}, \overline{h}_j^-\right) \\ \text{s.t.} \quad & R\left(v^C\left(S_1\right), \ldots, v^C\left(S_{q_1}\right)\right) \le \varepsilon, \\ & \qquad S_l \subseteq C,\ l = 1, \ldots, q_1 \\ & H\left(v^C\left(T_1\right), \ldots, v^C\left(T_{q_2}\right)\right) = \eta, \\ & \qquad T_{t_2} \subseteq C,\ t_2 = 1, \ldots, q_2 \\ & v^C(C) = 1 \\ & v^C(S) \le v^C(T) \quad \forall S, T \subseteq C \text{ s.t. } S \subseteq T \\ & v^C\left(c_j\right) \in W_{c_j}, \quad v^C\left(c_j\right) \ge 0,\ j = 1, 2, \ldots, n, \end{aligned} \tag{30}$$

where $d^p(\overline{h}_{ij}, \overline{h}_j^+)$ and $d^p(\overline{h}_{ij}, \overline{h}_j^-)$ are defined in Definition 19, R and H are the coefficient matrices, ε and η are the constant vectors, $R(v^C(S_1), \ldots, v^C(S_{q_1})) \le \varepsilon$ and $H(v^C(T_1), \ldots, v^C(T_{q_2})) = \eta$ are the known constraints, v^C is the fuzzy measure on the attribute set C, $\varphi_{c_j}(v^C, C)$ is the Shapley value of the attribute c_j, and W_{c_j} is the known weighting information.

Because models (29) and (30) have the same constraints and all alternatives are noninferior, they can be combined to formulate the following linear programming:

$$\begin{aligned} \min \quad & \sum_{i=1}^{m}\sum_{j=1}^{n}\varphi_{c_j}\left(v^C, C\right) \frac{d^p\left(\overline{h}_{ij}, \overline{h}_j^+\right)}{d^p\left(\overline{h}_{ij}, \overline{h}_j^-\right) + d^p\left(\overline{h}_{ij}, \overline{h}_j^+\right)} \\ \text{s.t.} \quad & R\left(v^C\left(S_1\right), \ldots, v^C\left(S_{q_1}\right)\right) \le \varepsilon, \\ & \qquad S_l \subseteq C,\ l = 1, \ldots, q_1 \\ & H\left(v^C\left(T_1\right), \ldots, v^C\left(T_{q_2}\right)\right) = \eta, \\ & \qquad T_{t_2} \subseteq C,\ t_2 = 1, \ldots, q_2 \\ & v^C(C) = 1 \\ & v^C(S) \le v^C(T) \quad \forall S, T \subseteq C \text{ s.t. } S \subseteq T \\ & v^C\left(c_j\right) \in W_{c_j}, \quad v^C\left(c_j\right) \ge 0,\ j = 1, 2, \ldots, n. \end{aligned} \tag{31}$$

If v^C is a 2-additive measure, then it derives the following model:

$$\begin{aligned} \min \quad & \sum_{i=1}^{m}\sum_{j=1}^{n}\frac{d^p\left(\overline{h}_{ij}, \overline{h}_j^+\right)}{2\left(d^p\left(\overline{h}_{ij}, \overline{h}_j^+\right) + d^p\left(\overline{h}_{ij}, \overline{h}_j^-\right)\right)}\left((3-n)\, v^C\left(c_j\right) + \sum_{c_i \in C \setminus c_j}\left(v^C\left(c_j, c_i\right) - v^C\left(c_i\right)\right)\right) \\ \text{s.t.} \quad & \widetilde{R}\left(v^C\left(c_j\right), v^C\left(c_i, c_j\right),\ i, j = 1, \ldots, n,\ i \ne j\right) \le \widetilde{\varepsilon} \\ & \widetilde{H}\left(v^C\left(c_j\right), v^C\left(c_i, c_j\right),\ i, j = 1, \ldots, n,\ i \ne j\right) = \widetilde{\eta} \\ & \sum_{c_i \in S \setminus c_j}\left(v^C\left(c_i, c_j\right) - v^C\left(c_i\right)\right) \ge (s-2)\, v^C\left(c_j\right), \quad \forall S \subseteq C,\ \forall c_j \in S,\ s \ge 2 \\ & \sum_{\{c_i, c_j\} \subseteq C} v^C\left(c_i, c_j\right) - (n-2)\sum_{c_i \in C} v^C\left(c_i\right) = 1 \\ & v^C\left(c_j\right) \in W_{c_j}, \quad v^C\left(c_j\right) \ge 0,\ j = 1, 2, \ldots, n, \end{aligned} \tag{32}$$

where $\widetilde{R}$ and $\widetilde{H}$ are the coefficient matrices, $\widetilde{\varepsilon}$ and $\widetilde{\eta}$ are the constant vectors, and $\widetilde{R}(v^C(c_j), v^C(c_i, c_j),\ i, j = 1, \ldots, n,\ i \ne j) \le \widetilde{\varepsilon}$ and $\widetilde{H}(v^C(c_j), v^C(c_i, c_j),\ i, j = 1, \ldots, n,\ i \ne j) = \widetilde{\eta}$ are the equivalent expressions of the known constraints given in model (30) with respect to 2-additive measure v^C.

4.1.4. Models for the Optimal Fuzzy Measure on the Ordered Set N. Let

$$z_{ij} = \frac{d^p\left(\overline{h}_{ij}, \overline{h}_j^+\right)}{d^p\left(\overline{h}_{ij}, \overline{h}_j^+\right) + d^p\left(\overline{h}_{ij}, \overline{h}_j^-\right)} \tag{33}$$

for each pair (i, j).

For each $i = 1, 2, \ldots, m$, we rearrange $z_{i1}, z_{i2}, \ldots, z_{in}$ such that $z_{i(1)} \le z_{i(2)} \le \cdots \le z_{i(n)}$. Similar to model for the optimal fuzzy measure on the attribute set C, if the weighting vector on the ordered set N is incompletely known, the following model is established:

$$\begin{aligned} \min \quad & \sum_{j=1}^{n}\sum_{i=1}^{m}\varphi_j\left(\mu^N, N\right) z_{i(j)} \\ \text{s.t.} \quad & W\left(\mu^N\left(S_1\right), \ldots, \mu^N\left(S_{h_1}\right)\right) \le \pi, \\ & \qquad S_l \subseteq N,\ l_1 = 1, \ldots, h_1 \\ & P\left(\mu^N\left(T_1\right), \ldots, \mu^N\left(T_{h_2}\right)\right) = \tau, \end{aligned}$$

$$T_l \subseteq N, \; l = 1, \ldots, h_2$$

$$\mu^N(N) = 1$$

$$\mu^N(S) \le \mu^N(T) \quad \forall S, T \subseteq N \text{ s.t. } S \subseteq T$$

$$\mu^N(j) \in W_j, \quad \mu^N(j) \ge 0, \; j = 1, 2, \ldots, n, \tag{34}$$

where W and P are the coefficient matrices, π and τ are the constant vectors, $W(\mu^N(S_1), \ldots, \mu^N(S_{h_1})) \le \pi$ and $P(\mu^N(T_1), \ldots, \mu^N(T_{h_2})) = \tau$ are the known constraints, μ^N is the fuzzy measure on the ordered set N, $\varphi_j(\mu^N, N)$ is the Shapley value of the jth position, and and W_j is the known weighting information.

If μ^N is a 2-additive measure, then it derives the following model:

$$\begin{aligned}
\min \quad & \sum_{i=1}^{m} \sum_{j=1}^{n} \frac{z_{i(j)}}{2} \left((3-n)\mu^N(j) + \sum_{i \subseteq N \setminus j} \left(\mu^N(i,j) - \mu^N(i) \right) \right) \\
\text{s.t.} \quad & \widetilde{W}\left(\mu^N(j), \mu^N(i,j), \; i, j = 1, \ldots, n, \; i \ne j\right) \le \widetilde{\pi} \\
& \widetilde{P}\left(\mu^N(j), \mu^N(i,j), \; i, j = 1, \ldots, n, \; i \ne j\right) = \widetilde{\tau} \\
& \sum_{i \in S \setminus j} \left(\mu^N(i,j) - \mu^N(i) \right) \ge (s-2)\mu^N(j), \quad \forall S \subseteq N, \; \forall j \in S, \; s \ge 2 \\
& \sum_{\{i,j\} \subseteq N} \mu^N(i,j) - (n-2) \sum_{i \in N} \mu^N(i) = 1 \\
& \mu^N(j) \in W_j, \quad \mu^N(j) \ge 0, \; j = 1, 2, \ldots, n,
\end{aligned} \tag{35}$$

where $\widetilde{W}$ and $\widetilde{P}$ are the coefficient matrices, $\widetilde{\pi}$ and $\widetilde{\tau}$ are the constant vectors, and $\widetilde{W}(\mu^N(j), \mu^N(i,j), \; i, j = 1, \ldots, n, \; i \ne j) \le \widetilde{\pi}$, and $\widetilde{P}(\mu^N(j), \mu^N(i,j), \; i, j = 1, \ldots, n, \; i \ne j) = \widetilde{\tau}$, are the equivalent expressions of the known constraints given in model (34) with respect to 2-additive measure μ^N.

Remark 20. In built models, we apply the elements' Shapley values as their weights that overall consider their interactions. Furthermore, if the elements in a set are independent, the built models degenerate to models for the optimal additive measure vector on the associated sets.

4.2. An Approach to Multiattribute Group Decision Making. Based on the analysis above, this section introduces an approach to interval-valued hesitant fuzzy multiattribute group decision making with incomplete weighting information and interactive characteristics. The main decision procedure to obtain the most desirable alternative(s) can be described as follows.

Step 1. If all attributes are benefits (i.e., the bigger the better), then the attribute values need not transformation. Otherwise, we need to transform the interval-valued hesitant fuzzy decision matrix $\overline{A}^k = (\overline{a}_{ij}^k)_{m \times n}$ into $\overline{H}^k = (\overline{h}_{ij}^k)_{m \times n}$, $k \in Q$, where

$$\overline{h}_{ij}^k = \begin{cases} \overline{a}_{ij}^k & \text{for benefit attribute } c_j \\ \left(\overline{a}_{ij}^k\right)^c & \text{for cost attribute } c_j \end{cases} \tag{36}$$

$$(i = 1, 2, \ldots, m; \; j = 1, 2, \ldots, n)$$

with $(\overline{a}_{ij}^k)^c = \bigcup_{[(a_{ij}^k)^l, (a_{ij}^k)^u] \in \overline{a}_{ij}^k} [1 - (a_{ij}^k)^u, 1 - (a_{ij}^k)^l]$.

Step 2. Use model (22) to calculate the optimal fuzzy measure on the expert set E with respect to each attribute.

Step 3. Use model (27) to calculate the optimal fuzzy measure on the ordered set Q.

Step 4. Utilize the G-IVHFSCHWA operator to calculate the interval-valued hesitant fuzzy element $\overline{h}_{ij}$; it derives the comprehensive interval-valued hesitant fuzzy matrix $\overline{H} = (\overline{h}_{ij})_{m \times n}$.

Step 5. Use model (31) to calculate the optimal fuzzy measure on the attribute set C.

Step 6. Use model (34) to calculate the optimal fuzzy measure on the ordered set N.

Step 7. Again utilize the G-IVHFSCHWA operator to calculate the comprehensive interval-valued hesitant fuzzy element $\overline{h}_i$ of the alternative a_i, $i = 1, 2, \ldots, m$.

Step 8. According to the comprehensive value $\overline{h}_i$ of the alternative a_i, we calculate the score

$$S(\overline{h}_i) = \sum_{\overline{r}_i = [r_i^l, r_i^u] \in \overline{h}_i} \left[\frac{r_i^l}{\# \overline{h}_i}, \frac{r_i^u}{\# \overline{h}_i} \right], \quad i = 1, 2, \ldots, m. \tag{37}$$

Then, we rank the comprehensive IVHFEs $\overline{h}_i$, $i = 1, 2, \ldots, m$, and select the best alternative(s).

Step 9. End.

Table 1: The interval-valued hesitant fuzzy matrix $\overline{A}_1$.

	c_1	c_2	c_3	c_4
a_1	([0.2, 0.3], [0.5, 0.7])	([0.4, 0.5])	([0.4, 0.6])	([0.6, 0.7])
a_2	([0.2, 0.4])	([0.4, 0.5])	([0.6, 0.8])	([0.4, 0.6])
a_3	([0.3, 0.4], [0.6, 0.7])	([0.5, 0.6])	([0.5, 0.7])	([0.2, 0.4], [0.6, 0.7])
a_4	([0.4, 0.6])	([0.5, 0.7])	([0.3, 0.4], [0.6, 0.7])	([0.2, 0.3], [0.5, 0.6])

Table 2: The interval-valued hesitant fuzzy matrix $\overline{A}_2$.

	c_1	c_2	c_3	c_4
a_1	([0.2, 0.3])	([0.2, 0.4])	([0.3, 0.5])	([0.4, 0.5])
a_2	([0.4, 0.5])	([0.3, 0.6])	([0.1, 0.3], [0.5, 0.6])	([0.3, 0.4], [0.6, 0.7])
a_3	([0.2, 0.6])	([0.5, 0.7])	([0.5, 0.6])	([0.4, 0.5])
a_4	([0.3, 0.5])	([0.4, 0.6])	([0.3, 0.5])	([0.2, 0.4])

Table 3: The interval-valued hesitant fuzzy matrix $\overline{A}_3$.

	c_1	c_2	c_3	c_4
a_1	([0.4, 0.5])	([0.4, 0.6])	([0.3, 0.5])	([0.1, 0.3], [0.6, 0.8])
a_2	([0.3, 0.5])	([0.2, 0.4])	([0.1, 0.2])	([0.7, 0.9])
a_3	([0.5, 0.7])	([0.3, 0.6])	([0.2, 0.3], [0.5, 0.7])	([0.6, 0.7])
a_4	([0.3, 0.6])	([0.1, 0.3], [0.6, 0.8])	([0.3, 0.5])	([0.4, 0.5])

5. A Practical Example

Let us consider an investment company that wants to invest a sum of money in the best option [65]. There is a panel with four possible alternatives in which to invest the money: a_1 is a car company, a_2 is a computer company, a_3 is a TV company, and a_4 is a food company. The investment company must make a decision according to the following four attributes: c_1 is the risk index, c_2 is the growth index, c_3 is the social-political impact index, and c_4 is the environmental impact index. The four possible alternatives $A = \{a_1, a_2, a_3, a_4\}$ are evaluated by three experts $E = \{e_1, e_2, e_3\}$ using the IVHFEs under the above four attributes $C = \{c_1, c_2, c_3, c_4\}$. The interval-valued hesitant fuzzy matrices are listed as shown in Tables 1–3.

Based on the expert's reputation, experience, and expertise, the weighting information on the expert set E with respect to each attribute is, respectively, given as follows:

$$\begin{gathered}
0.1 \le v_1^E(e_2), \\
0.1 \le v_1^E(e_1) - v_1^E(e_2) \le 0.2, \\
0 \le v_1^E(e_1) - v_1^E(e_3) \le 0.1, \\
0.6 \le v_1^E(e_1, e_2) \le 0.8, \\
v_1^E(e_2, e_3) \le v_1^E(e_1, e_3), \\
v_1^E(e_1, e_2) \le v_1^E(e_1, e_3); \\
0.2 \le v_2^E(e_k), \\
0.1 \le v_2^E(e_1) - v_2^E(e_k) \le 0.3, \\
k = 2, 3, \\
v_2^E(e_2, e_3) \le v_2^E(e_1, e_3) = v_2^E(e_1, e_2), \\
0.4 \le v_2^E(e_2, e_3) \le 0.6; \\
0.1 \le v_3^E(e_1), \\
0.1 \le v_3^E(e_2) - v_3^E(e_1) \le 0.3, \\
0 \le v_3^E(e_3) - v_3^E(e_1) \le 0.2, \\
0.3 \le v_3^E(e_1, e_2) \le 0.5, \\
v_3^E(e_1, e_2) \le v_3^E(e_1, e_3) \le v_3^E(e_2, e_3); \\
0.15 \le v_4^E(e_1), \\
v_4^E(e_2) \le 0.6, \\
v_4^E(e_1) \le v_4^E(e_3) \le v_4^E(e_2), \\
v_4^E(e_1, e_k) + 0.2 \le v_4^E(e_2, e_3), \\
k = 2, 3, \\
v_4^E(e_1, e_2) = v_4^E(e_1, e_3), \\
0.7 \le v_4^E(e_2, e_3) \le 0.9.
\end{gathered} \tag{38}$$

In addition to the usual weighting information on experts taken separately, the weighting information on any combination of experts is also defined. Take the fuzzy measure v_1^E, for example, with respect to the other two experts; the importance of the expert e_2 is no less than 0.1. Furthermore,

Table 4: The interval-valued hesitant fuzzy matrix $\overline{H}_1$.

	c_1	c_2	c_3	c_4
a_1	([0.3, 0.5], [0.7, 0.8])	([0.4, 0.5])	([0.4, 0.6])	([0.3, 0.4])
a_2	([0.6, 0.8])	([0.4, 0.5])	([0.2, 0.4])	([0.4, 0.6])
a_3	([0.3, 0.4], [0.6, 0.7])	([0.5, 0.6])	([0.3, 0.5])	([0.3, 0.4], [0.6, 0.8])
a_4	([0.4, 0.6])	([0.5, 0.7])	([0.3, 0.4], [0.6, 0.7])	([0.4, 0.5], [0.7, 0.8])

the importance of the expert e_1 is no smaller than that of the expert e_2 or e_3; their differences belong to the intervals $[0.1, 0.2]$ and $[0, 0.1]$, respectively. Moreover, the importance of the combination of the experts e_1 and e_3 is no less than that of the combination of the experts e_1 and e_2 as well as the combination of the experts e_2 and e_3.

Furthermore, the weighting information on the ordered set Q is defined as follows:

$$\begin{aligned} & 0.2 \le \mu^Q(1), \\ & \mu^Q(3) \le 0.5, \\ & \mu^Q(1) \le \mu^Q(2) \le \mu^Q(3) \\ & \mu^Q(1,2) \le \mu^Q(1,3) \le \mu^Q(2,3), \\ & 0.5 \le \mu^Q(1,2), \\ & \mu^Q(2,3) \le 0.9. \end{aligned} \tag{39}$$

From the weighting information above, it indicates that the importance is increasing with respect to the ordered positions. The range of their individual weights is $[0.2, 0.5]$, and the range of the combinations of any two ordered positions' weights is $[0.5, 0.9]$.

Considering the following facts: "These four companies belong to one state that has a stable social-political environment. Its government always attaches great importance to environmental protection. In addition, with the help of the government, they have a certain antirisk ability". The weighting information on the attribute set C is given as follows:

$$\begin{aligned} & \nu^C(c_3) \ge 0.1, \\ & \nu^C(c_1) - \nu^C(c_3) \ge 0.1, \\ & \nu^C(c_2) - \nu^C(c_1) \ge 0.1, \\ & \nu^C(c_4) - \nu^C(c_2) \ge 0.2, \\ & \nu^C(c_1,c_3) \le \nu^C(c_2,c_3) \le \nu^C(c_3,c_4) \le \nu^C(c_1,c_2) \\ & \quad \le \nu^C(c_1,c_4) \le \nu^C(c_2,c_4), \\ & \nu^C(c_2,c_4) - \nu^C(c_1,c_3) \ge 0.3, \\ & \nu^C(c_1,c_2,c_3) \le \nu^C(c_1,c_3,c_4) \le \nu^C(c_2,c_3,c_4) \\ & \quad \le \nu^C(c_1,c_2,c_4), \\ & \nu^C(c_1,c_2,c_4) \ge 0.8. \end{aligned} \tag{40}$$

Similar to the weights on Q, the weighting information on the ordered set N is defined as follows:

$$\begin{aligned} & \mu^N(1) \ge 0.1, \\ & \mu^N(4) \ge 0.3, \\ & \mu^N(3,4) \ge 0.6, \\ & \mu^N(2,3,4) \le 0.9, \\ & \mu^N(j) - \mu^N(j+1) \le -0.1, \quad j = 1,2,3 \\ & \mu^N(1,2) - \mu^N(1,3) \le -0.1, \\ & \mu^N(1,3) - \mu^N(1,4) \le -0.1, \\ & \mu^N(2,3) - \mu^N(2,4) \le -0.1, \\ & \mu^N(2,4) - \mu^N(3,4) \le -0.1, \\ & \mu^N(1,2,3) - \mu^N(1,2,4) \le -0.1, \\ & \mu^N(1,2,4) - \mu^N(1,3,4) \le -0.1, \\ & \mu^N(1,3,4) - \mu^N(2,3,4) \le -0.1. \end{aligned} \tag{41}$$

In the following, we can utilize the proposed procedure to obtain the most desirable alternative(s).

Step 1. Because the attributes c_1, c_3, and c_4 are cost and the attribute c_2 is benefit, it needs to transform the decision matrix $\overline{A}^k$ into $\overline{H}^k$, $k = 1,2,3$. Take $\overline{A}^1$, for example; the decision matrix $\overline{H}^1$ is given as shown in Table 4.

Step 2. According to model (22), the following linear programming is constructed:

$$\begin{aligned} \min \quad & -0.022\left(\nu_1^E(e_1) - \nu_1^E(e_2,e_3)\right) \\ & + 0.069\left(\nu_1^E(e_2) - \nu_1^E(e_1,e_3)\right) \\ & - 0.047\left(\nu_1^E(e_3) - \nu_1^E(e_1,e_2)\right) + 0.544 \\ \text{s.t.} \quad & 0.1 \le \nu_1^E(e_2) \\ & 0.1 \le \nu_1^E(e_1) - \nu_1^E(e_2) \le 0.2 \end{aligned}$$

Table 5: The optimal fuzzy measures.

	$\{e_1\}$	$\{e_2\}$	$\{e_3\}$	$\{e_1,e_2\}$	$\{e_1,e_3\}$	$\{e_2,e_3\}$	E
v_1^E	0.3	0.1	0.3	0.6	1	0.3	1
v_2^E	0.5	0.4	0.2	1	1	0.4	1
v_3^E	0.1	0.4	0.1	0.4	0.4	1	1
v_4^E	0.15	0.6	0.15	0.676	0.676	0.876	1

Table 6: The experts' Shapley values.

	c_1	c_2	c_3	c_4
e_1	0.533	0.6	0.083	0.192
e_2	0.083	0.25	0.533	0.517
e_3	0.383	0.15	0.383	0.292

$$\begin{aligned}&0\le v_1^E(e_1)-v_1^E(e_3)\le 0.1\\&0.6\le v_1^E(e_1,e_2)\le 0.8\\&v_1^E(e_2,e_3)-v_1^E(e_1,e_3)\le 0\\&v_1^E(e_1,e_2)-v_1^E(e_1,e_3)\le 0\\&v_1^E(S)\le v_1^E(T)\\&\qquad\forall S,T\subseteq\{e_1,e_2,e_3\}\ \text{s.t. } S\subseteq T.\end{aligned}\tag{42}$$

Solving the above model, it derives

$$\begin{aligned}&v_1^E(e_1)=v_1^E(e_3)=v_1^E(e_2,e_3)=0.3,\\&v_1^E(e_2)=0.1,\\&v_1^E(e_1,e_2)=0.6,\\&v_1^E(e_1,e_3)=v_1^E(e_1,e_2,e_3)=1.\end{aligned}\tag{43}$$

Similar to the calculation of the optimal fuzzy measure v_1^E, the optimal fuzzy measures with respect to each attribute are obtained as shown in Table 5.

From Table 5, the experts' Shapley values with respect to each attribute are obtained as shown in Table 6.

Step 3. Calculating the Shapley weighted matrices $\overline{H}^k_{\varphi_{e_k}(\mu^E,E)}=(\overline{h}'^k_{ij})_{m\times n}$, $k\in Q$, take $\overline{H}^1$, for example; the Shapley weighted matrix $\overline{H}^1_{\varphi_{e_1}(\mu^E,E)}$ is obtained as shown in Table 7.

According to model (27), the following linear programming is constructed:

$$\begin{aligned}\max\quad&-0.423\left(\mu^Q(1)-\mu^Q(2,3)\right)\\&+0.03\left(\mu^Q(2)-\mu^Q(1,3)\right)\\&+0.393\left(\mu^Q(3)-\mu^Q(1,1)\right)+13.7\\\text{s.t.}\quad&0.2\le\mu^Q(1),\mu^Q(3)\le 0.5\\&0.5\le\mu^Q(1,2),\mu^Q(2,3)\le 0.9\\&\mu^Q(1)-\mu^Q(2)\le 0\\&\mu^Q(2)-\mu^Q(3)\le 0\\&\mu^Q(1,2)-\mu^Q(1,3)\le 0\\&\mu^Q(1,3)-\mu^Q(2,3)\le 0\\&\mu^Q(S)\le\mu^Q(T)\quad\forall S,T\subseteq\{1,2,3\}\ \text{s.t. } S\subseteq T.\end{aligned}\tag{44}$$

Solving the above model, it derives

$$\begin{aligned}&\mu^Q(1)=0.2,\\&\mu^Q(2)=\mu^Q(3)=\mu^Q(1,2)=\mu^Q(1,3)=0.5,\\&\mu^Q(2,3)=0.9,\\&\mu^Q(1,2,3)=1.\end{aligned}\tag{45}$$

Step 4. Let $\lambda=2$, by the G-IVHFSCHWA operator the comprehensive interval-valued hesitant fuzzy matrix is obtained as shown in Table 8.

Take $\overline{h}_{11}$, for example,

$$\begin{aligned}&\text{G-IVHFSCHWA}\left(\overline{h}^1_{11},\overline{h}^2_{11},\overline{h}^3_{11}\right)\\&=\bigcup_{\overline{r}^1_{11}\in\overline{h}^1_{11},\overline{r}^2_{11}\in\overline{h}^2_{11},\overline{r}^3_{11}\in\overline{h}^3_{11}}\left(\left[\left(\frac{0.15\times\left(0.533\times\left(r^1_{11}\right)^l\right)^2+0.4\times\left(0.383\times\left(r^3_{11}\right)^l\right)^2+0.45\times\left(0.083\times\left(r^2_{11}\right)^l\right)^2}{0.15\times0.533^2+0.4\times0.383^2+0.45\times0.083^2}\right)^{1/2},\right.\right.\\&\left.\left.\left(\frac{0.15\times\left(0.533\times\left(r^1_{11}\right)^l\right)^2+0.4\times\left(0.383\times\left(r^3_{11}\right)^l\right)^2+0.45\times\left(0.083\times\left(r^2_{11}\right)^l\right)^2}{0.15\times0.533^2+0.4\times0.383^2+0.45\times0.083^2}\right)^{1/2}\right]\right)\end{aligned}$$

Table 7: The Shapley weighted matrix $\overline{H}^1_{\varphi_{e_1}(\mu^E,E)}$.

	c_1	c_2	c_3	c_4
a_1	([0.16, 0.27], [0.37, 0.43])	([0.24, 0.3])	([0.03, 0.05])	([0.06, 0.08])
a_2	([0.76, 0.89])	([0.23, 0.3])	([0.02, 0.03])	([0.08, 0.12])
a_3	([0.16, 0.21], [0.32, 0.37])	([0.3, 0.36])	([0.02, 0.04])	([0.06, 0.08], [0.12, 0.15])
a_4	([0.21, 0.32])	([0.3, 0.42])	([0.02, 0.03], [0.05, 0.06])	([0.08, 0.1], [0.13, 0.15])

Table 8: The comprehensive interval-valued hesitant fuzzy matrix $\overline{H}$.

	c_1	c_2	c_3	c_4
a_1	([0.44, 0.57], [0.6, 0.69])	([0.39, 0.5])	([0.5, 0.7])	([0.46, 0.56], [0.5, 0.62])
a_2	([0.58, 0.78])	([0.39, 0.5])	([0.58, 0.67], [0.73, 0.9])	([0.3, 0.42], [0.56, 0.67])
a_3	([0.3, 0.44], [0.52, 0.64])	([0.5, 0.61])	([0.37, 0.5], [0.51, 0.61])	([0.46, 0.56], [0.47, 0.57])
a_4	([0.4, 0.62])	([0.48, 0.68], [0.49, 0.69])	([0.5, 0.69], [0.5, 0.7])	([0.53, 0.68], [0.59, 0.73])

$$\begin{aligned}
&= \Bigg(\Bigg[\left(\frac{0.15\times(0.533\times0.3)^2+0.4\times(0.383\times0.5)^2+0.45\times(0.083\times0.7)^2}{0.15\times0.533^2+0.4\times0.383^2+0.45\times0.083^2}\right)^{1/2},\\
&\left(\frac{0.15\times(0.533\times0.5)^2+0.4\times(0.383\times0.6)^2+0.45\times(0.083\times0.8)^2}{0.15\times0.533^2+0.4\times0.383^2+0.45\times0.083^2}\right)^{1/2}\Bigg],\\
&\Bigg[\left(\frac{0.15\times(0.533\times0.7)^2+0.4\times(0.383\times0.5)^2+0.45\times(0.083\times0.7)^2}{0.15\times0.533^2+0.4\times0.383^2+0.45\times0.083^2}\right)^{1/2},\\
&\left(\frac{0.15\times(0.533\times0.8)^2+0.4\times(0.383\times0.6)^2+0.45\times(0.083\times0.8)^2}{0.15\times0.533^2+0.4\times0.383^2+0.45\times0.083^2}\right)^{1/2}\Bigg]\Bigg)\\
&= ([0.44, 0.57], [0.6, 0.69]).
\end{aligned} \tag{46}$$

Step 5. Let $p = 1$; according to model (31), the following linear programming is constructed:

$$\begin{aligned}
\min \quad & -0.095\left(v^C(c_1)-v^C(c_2,c_3,c_4)\right)\\
&+0.056\left(v^C(c_2)-v^C(c_1,c_3,c_4)\right)\\
&+0.072\left(v^C(c_3)-v^C(c_1,c_2,c_4)\right)\\
&-0.034\left(v^C(c_4)-v^C(c_1,c_2,c_3)\right)\\
&-0.019\left(v^C(c_1,c_2)-v^C(c_3,c_4)\right)\\
&-0.011\left(v^C(c_1,c_3)-v^C(c_2,c_4)\right)\\
&-0.064\left(v^C(c_1,c_4)-v^C(c_2,c_3)\right)+2.062\\
\text{s.t.} \quad & 0.1\le v^C(c_3),\\
&0.8\le v^C(c_1,c_2,c_4)\\
&v^C(c_3)-v^C(c_1)\le -0.1\\
&v^C(c_1)-v^C(c_2)\le -0.1\\
&v^C(c_2)-v^C(c_4)\le -0.2\\
&v^C(c_1,c_3)-v^C(c_2,c_4)\le -0.3\\
&v^C(c_1,c_3)-v^C(c_2,c_3)\le 0\\
&v^C(c_2,c_3)-v^C(c_3,c_4)\le 0\\
&v^C(c_3,c_4)-v^C(c_1,c_2)\le 0\\
&v^C(c_1,c_2)-v^C(c_1,c_4)\le 0\\
&v^C(c_1,c_4)-v^C(c_2,c_4)\le 0\\
&v^C(c_1,c_2,c_3)-v^C(c_1,c_3,c_4)\le 0\\
&v^C(c_1,c_3,c_4)-v^C(c_2,c_3,c_4)\le 0\\
&v^C(c_2,c_3,c_4)-v^C(c_1,c_2,c_4)\le 0\\
&v^C(S)\le v^C(T)\\
&\qquad \forall S,T\subseteq\{c_1,c_2,c_3,c_4\}\ \text{s.t. } S\subseteq T.
\end{aligned} \tag{47}$$

Solving the above model, it derives

$$\begin{aligned}
&\nu^C(c_1) = 0.2,\\
&\nu^C(c_2) = \nu^C(c_1, c_3) = \nu^C(c_2, c_3) = 0.3,\\
&\nu^C(c_3) = 0.1,\\
&\nu^C(c_4) = \nu^C(c_1, c_2) = \nu^C(c_3, c_4) = \nu^C(c_1, c_2, c_3)\\
&\quad = 0.5765,\\
&\nu^C(c_1, c_4) = \nu^C(c_2, c_4) = \nu^C(c_1, c_2, c_4) = \nu^C(c_1, c_3, c_4)\\
&\quad = \nu^C(c_2, c_3, c_4) = \nu^C(c_1, c_2, c_3, c_4) = 1.
\end{aligned} \tag{48}$$

Using the Shapley function, it derives

$$\begin{aligned}
\varphi_1(\nu^C, C) &= 0.1833,\\
\varphi_2(\nu^C, C) &= 0.21667,\\
\varphi_3(\nu^C, C) &= 0.03333,\\
\varphi_4(\nu^C, C) &= 0.5667.
\end{aligned} \tag{49}$$

Step 6. According to model (34), the following linear programming is constructed:

$$\begin{aligned}
\min \quad & -0.71\left(\mu^N(1) - \mu^N(2,3,4)\right)\\
& -0.183\left(\mu^N(2) - \mu^N(1,3,4)\right)\\
& +0.381\left(\mu^N(3) - \mu^N(1,2,4)\right)\\
& +0.513\left(\mu^N(4) - \mu^N(1,2,3)\right)\\
& -0.447\left(\mu^N(1,2) - \mu^N(3,4)\right)\\
& -0.165\left(\mu^N(1,3) - \mu^N(2,4)\right)\\
& -0.099\left(\mu^N(1,4) - \mu^N(2,3)\right) + 2.13\\
\text{s.t.} \quad & \mu^N(j) - \mu^N(j+1) \le -0.1, \quad j = 1,2,3\\
& \mu^N(1,l) - \mu^N(1,l+1) \le -0.1, \quad l = 2,3\\
& \mu^N(2,3) - \mu^N(2,4) \le -0.1\\
& \mu^N(2,4) - \mu^N(3,4) \le -0.1\\
& \mu^N(1,2,3) - \mu^N(1,2,4) \le -0.1\\
& \mu^N(1,2,4) - \mu^N(1,3,4) \le -0.1\\
& \mu^N(1,3,4) - \mu^N(2,3,4) \le -0.1\\
& 0.1 \le \mu^N(1),\\
& 0.3 \le \mu^N(4),\\
& 0.6 \le \mu^N(3,4)\\
& \mu^N(2,3,4) \le 0.9\\
& \mu^N(S) \le \mu^N(T)\\
& \qquad \forall S, T \subseteq \{1,2,3,4\} \text{ s.t. } S \subseteq T.
\end{aligned} \tag{50}$$

Solving the above model, it derives

$$\begin{aligned}
&\mu^N(1) = 0.1,\\
&\mu^N(2) = 0.2,\\
&\mu^N(3) = \mu^N(2,3) = 0.3,\\
&\mu^N(4) = \mu^N(2,4) = 0.4,\\
&\mu^N(1,2) = 0.5,\\
&\mu^N(1,3) = \mu^N(3,4) = \mu^N(1,2,3) = 0.6,\\
&\mu^N(1,4) = \mu^N(1,2,4) = 0.7,\\
&\mu^N(1,3,4) = 0.8,\\
&\mu^N(2,3,4) = 0.9,\\
&\mu^N(1,2,3,4) = 1.
\end{aligned} \tag{51}$$

Step 7. Let $\lambda = 2$, by the G-IVHFSCHWA operator the comprehensive IVHFEs are obtained as follows:

$$\begin{aligned}
\bar{h}_1 &= ([0.46, 0.56], [0.47, 0.58], [0.49, 0.6], [0.51, 0.62]);\\
\bar{h}_2 &= ([0.35, 0.47], [0.35, 0.48], [0.54, 0.66],\\
&\quad [0.55, 0.67]);\\
\bar{h}_3 &= ([0.46, 0.56], [0.47, 0.57], [0.46, 0.56], [0.47, 0.57],\\
&\quad [0.48, 0.57], [0.47, 0.58], [0.47, 0.57], [0.48, 0.58]);\\
\bar{h}_4 &= ([0.52, 0.68], [0.57, 0.72], [0.52, 0.68], [0.57, 0.72],\\
&\quad [0.52, 0.68], [0.57, 0.72], [0.52, 0.67], [0.57, 0.72]).
\end{aligned} \tag{52}$$

Step 8. According to the comprehensive IVHFEs, the scores are obtained as follows:

$$\begin{aligned}
S(\bar{h}_1) &= [0.4813, 0.5895],\\
S(\bar{h}_2) &= [0.45, 0.5735],\\
S(\bar{h}_3) &= [0.467, 0.5715],\\
S(\bar{h}_4) &= [0.5471, 0.701].
\end{aligned} \tag{53}$$

Because $S(\bar{h}_4) > S(\bar{h}_1) > S(\bar{h}_3) > S(\bar{h}_2)$, the best choice is the food company a_4.

Table 9: Ranking orders based on the G-IVHFSCHWA operator.

	$S(\overline{h}_1)$	$S(\overline{h}_2)$	$S(\overline{h}_3)$	$S(\overline{h}_4)$	Ranking orders
$\lambda = 0.1$	[0.487, 0.626]	[0.532, 0.675]	[0.464, 0.577]	[0.509, 0.69]	$S(\overline{h}_2) > S(\overline{h}_4) > S(\overline{h}_1) > S(\overline{h}_3)$
$\lambda = 0.2$	[0.486, 0.621]	[0.523, 0.666]	[0.465, 0.578]	[0.512, 0.69]	$S(\overline{h}_4) > S(\overline{h}_2) > S(\overline{h}_1) > S(\overline{h}_3)$
$\lambda = 0.5$	[0.483, 0.608]	[0.500, 0.641]	[0.467, 0.578]	[0.519, 0.692]	$S(\overline{h}_4) > S(\overline{h}_2) > S(\overline{h}_1) > S(\overline{h}_3)$
$\lambda = 1.0$	[0.481, 0.596]	[0.474, 0.608]	[0.468, 0.577]	[0.532, 0.695]	$S(\overline{h}_4) > S(\overline{h}_2) > S(\overline{h}_1) > S(\overline{h}_3)$
$\lambda = 2.0$	[0.481, 0.59]	[0.45, 0.574]	[0.467, 0.572]	[0.547, 0.701]	$S(\overline{h}_4) > S(\overline{h}_1) > S(\overline{h}_3) > S(\overline{h}_2)$
$\lambda = 5.0$	[0.484, 0.591]	[0.432, 0.546]	[0.464, 0.565]	[0.558, 0.706]	$S(\overline{h}_4) > S(\overline{h}_1) > S(\overline{h}_3) > S(\overline{h}_2)$
$\lambda = 10$	[0.485, 0.592]	[0.429, 0.541]	[0.464, 0.565]	[0.558, 0.706]	$S(\overline{h}_4) > S(\overline{h}_1) > S(\overline{h}_3) > S(\overline{h}_2)$
$\lambda = 20$	[0.485, 0.592]	[0.429, 0.541]	[0.464, 0.565]	[0.558, 0.706]	$S(\overline{h}_4) > S(\overline{h}_1) > S(\overline{h}_3) > S(\overline{h}_2)$

Table 10: Ranking orders based on the G-IVHF2SCHWA operator.

	$S(\overline{h}_1)$	$S(\overline{h}_2)$	$S(\overline{h}_3)$	$S(\overline{h}_4)$	Ranking orders
$\lambda = 0.1$	[0.489, 0.629]	[0.544, 0.684]	[0.470, 0.583]	[0.512, 0.691]	$S(\overline{h}_2) > S(\overline{h}_4) > S(\overline{h}_1) > S(\overline{h}_3)$
$\lambda = 0.2$	[0.487, 0.622]	[0.531, 0.670]	[0.472, 0.585]	[0.514, 0.691]	$S(\overline{h}_4) > S(\overline{h}_2) > S(\overline{h}_1) > S(\overline{h}_3)$
$\lambda = 0.5$	[0.480, 0.604]	[0.496, 0.631]	[0.477, 0.588]	[0.521, 0.693]	$S(\overline{h}_4) > S(\overline{h}_2) > S(\overline{h}_1) > S(\overline{h}_3)$
$\lambda = 1.0$	[0.470, 0.583]	[0.456, 0.583]	[0.481, 0.592]	[0.530, 0.695]	$S(\overline{h}_4) > S(\overline{h}_3) > S(\overline{h}_1) > S(\overline{h}_2)$
$\lambda = 2.0$	[0.461, 0.568]	[0.426, 0.542]	[0.484, 0.593]	[0.540, 0.699]	$S(\overline{h}_4) > S(\overline{h}_3) > S(\overline{h}_1) > S(\overline{h}_2)$
$\lambda = 5.0$	[0.459, 0.565]	[0.423, 0.529]	[0.485, 0.592]	[0.551, 0.702]	$S(\overline{h}_4) > S(\overline{h}_3) > S(\overline{h}_1) > S(\overline{h}_2)$
$\lambda = 10$	[0.462, 0.567]	[0.435, 0.538]	[0.484, 0.589]	[0.557, 0.704]	$S(\overline{h}_4) > S(\overline{h}_3) > S(\overline{h}_1) > S(\overline{h}_2)$
$\lambda = 20$	[0.466, 0.57]	[0.447, 0.55]	[0.483, 0.588]	[0.559, 0.704]	$S(\overline{h}_4) > S(\overline{h}_3) > S(\overline{h}_1) > S(\overline{h}_2)$

Table 11: Ranking results based on the GIVHFHA operator.

	$S(\overline{h}_1)$	$S(\overline{h}_2)$	$S(\overline{h}_3)$	$S(\overline{h}_4)$	Ranking orders
$\lambda = 0.1$	[0.524, 0.661]	[0.471, 0.628]	[0.544, 0.677]	[0.515, 0.707]	$S(\overline{h}_4) > S(\overline{h}_3) > S(\overline{h}_1) > S(\overline{h}_2)$
$\lambda = 0.2$	[0.612, 0.736]	[0.550, 0.705]	[0.621, 0.746]	[0.592, 0.771]	$S(\overline{h}_3) > S(\overline{h}_4) > S(\overline{h}_1) > S(\overline{h}_2)$
$\lambda = 0.5$	[0.617, 0.739]	[0.555, 0.707]	[0.626, 0.750]	[0.598, 0.774]	$S(\overline{h}_3) > S(\overline{h}_4) > S(\overline{h}_1) > S(\overline{h}_2)$
$\lambda = 1.0$	[0.626, 0.745]	[0.564, 0.712]	[0.636, 0.755]	[0.606, 0.778]	$S(\overline{h}_3) > S(\overline{h}_4) > S(\overline{h}_1) > S(\overline{h}_2)$
$\lambda = 2.0$	[0.643, 0.755]	[0.582, 0.720]	[0.654, 0.766]	[0.623, 0.787]	$S(\overline{h}_3) > S(\overline{h}_4) > S(\overline{h}_1) > S(\overline{h}_2)$
$\lambda = 5.0$	[0.688, 0.785]	[0.626, 0.743]	[0.693, 0.791]	[0.661, 0.808]	$S(\overline{h}_3) > S(\overline{h}_1) > S(\overline{h}_4) > S(\overline{h}_2)$
$\lambda = 10$	[0.731, 0.818]	[0.663, 0.769]	[0.729, 0.815]	[0.695, 0.828]	$S(\overline{h}_1) > S(\overline{h}_3) > S(\overline{h}_4) > S(\overline{h}_2)$
$\lambda = 20$	[0.762, 0.847]	[0.691, 0.794]	[0.758, 0.840]	[0.723, 0.849]	$S(\overline{h}_1) > S(\overline{h}_3) > S(\overline{h}_4) > S(\overline{h}_2)$

With respect to the comprehensive interval-valued hesitant fuzzy matrix $\overline{H}$, if the different values of λ are used to calculate the comprehensive IVHFEs of the alternatives, the ranking orders are obtained as shown in Table 9.

From Table 9, one can that different ranking orders are obtained. However, all ranking orders show that the food company a_4 is the best choice except for $\lambda = 0.1$.

If the G-IVHF2SCHWA operator is applied to calculate the comprehensive IVHFEs of the alternatives, ranking orders are obtained as shown in Table 10.

Table 10 shows that the different ranking orders are obtained. However, the best choices are the same as that obtained from the G-IVHFSCHWA operator.

In this example, if we assume that there are no interactions. Furthermore, if we adopt the operational laws given by Chen et al. [38], using the generalized interval-valued hesitant fuzzy hybrid averaging (GIVHFHA) operator [38], ranking orders are obtained as shown in Table 11.

From Table 11, it can be observed that the best choices obtained by the GIVHFHA operator are completely different from that derived by the G-IVHFSCHWA or G-IVHF2SCHWA operator. It may be caused by the following two aspects: the GIVHFHA operator does consider the interactions between elements, and the adopted operations cannot preserve the order relationship.

Furthermore, if the aggregation operators presented by Wei and Zhao [40] and Wei et al. [41] are applied in this example, the ranking results with respect to the comprehensive interval-valued hesitant fuzzy matrix $\overline{H}$ are obtained as shown in Table 12.

Table 12 indicates that the different ranking results and optimal choices are obtained too. The main reason is that they are based on the different point of views. The ranking order obtained from the HIVFWA operator and the HIVFCOG operator is the same as that derived from the G-IVHFSCHWA and G-IVHF2SCHWA operators for $\lambda = 0.2, 0.5$. Furthermore, the ranking order obtained from the HIVFOWA operator, the HIVFOWG operator, the HIVFCOA operator, the I-HIVFEOWA operator, and the I-HIVFEOWG operator is the same as that derived from the

Table 12: Ranking results with respect to different aggregation operators.

	$S(\bar{h}_1)$	$S(\bar{h}_2)$	$S(\bar{h}_3)$	$S(\bar{h}_4)$	Ranking orders
The HIVFWA operator	[0.482, 0.604]	[0.495, 0.643]	[0.478, 0.588]	[0.522, 0.699]	$S(\bar{h}_4) > S(\bar{h}_2) > S(\bar{h}_1) > S(\bar{h}_3)$
The HIVFWG operator	[0.447, 0.567]	[0.441, 0.572]	[0.444, 0.557]	[0.487, 0.672]	$S(\bar{h}_4) > S(\bar{h}_1) > S(\bar{h}_2) > S(\bar{h}_3)$
The HIVFOWA operator	[0.494, 0.643]	[0.576, 0.739]	[0.455, 0.571]	[0.514, 0.691]	$S(\bar{h}_2) > S(\bar{h}_4) > S(\bar{h}_1) > S(\bar{h}_3)$
The HIVFOWG operator	[0.488, 0.633]	[0.546, 0.691]	[0.445, 0.564]	[0.508, 0.689]	$S(\bar{h}_2) > S(\bar{h}_4) > S(\bar{h}_1) > S(\bar{h}_3)$
The HIVFCOA operator	[0.479, 0.620]	[0.537, 0.693]	[0.449, 0.566]	[0.512, 0.689]	$S(\bar{h}_2) > S(\bar{h}_4) > S(\bar{h}_1) > S(\bar{h}_3)$
The HIVFCOG operator	[0.473, 0.608]	[0.502, 0.637]	[0.439, 0.559]	[0.507, 0.688]	$S(\bar{h}_4) > S(\bar{h}_2) > S(\bar{h}_1) > S(\bar{h}_3)$
The I-HIVFEOWA operator	[0.493, 0.642]	[0.572, 0.734]	[0.453, 0.570]	[0.513, 0.690]	$S(\bar{h}_2) > S(\bar{h}_4) > S(\bar{h}_1) > S(\bar{h}_3)$
The I-HIVFEOWG operator	[0.489, 0.635]	[0.551, 0.699]	[0.446, 0.565]	[0.509, 0.689]	$S(\bar{h}_2) > S(\bar{h}_4) > S(\bar{h}_1) > S(\bar{h}_3)$

G-IVHFSCHWA and G-IVHF2SCHWA operators for $\lambda = 0.1$.

If there is no special explanation that the elements in a set are independent, we recommend that the experts adopt the aggregation operators based on fuzzy measures. Furthermore, to eliminate the disadvantages of the existing operational laws [38], we suggest the experts to use the operations defined in this paper.

6. Conclusions

With respect to interval-valued hesitant fuzzy multiattribute group decision making, we first research the issues of the existing operational laws on IVHFEs. Then, we define some new operations that can avoid these issues. To consider the fact that there may be some degree of interactions between the weights of elements in a set; this paper defines the generalized interval-valued hesitant fuzzy Shapley-Choquet weighted averaging (G-IVHFSCWA) operator. Because this operator only reflects the importance of the ordered positions, we further introduce the generalized interval-valued hesitant fuzzy Shapley-Choquet hybrid weighted averaging (G-IVHFSCHWA) operator, which does not only consider the importance of elements and the ordered positions but also reflect their interactions. To reflect the interactions between elements and reduce the complexity of solving a fuzzy measure, an aggregation operator using 2-additive measures is introduced. To cope with the case that the weighting information is not exactly known, using the defined distance measure, models for the optimal fuzzy measure and the optimal 2-additive measure are built. Then, an approach to interval-valued hesitant fuzzy multiattribute group decision making is developed. It is noteworthy that the defined operators and the built models can be directly used in the setting of hesitant fuzzy sets.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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