

Research Article

Design of Nonfragile State Estimator for Discrete-Time Genetic Regulatory Networks Subject to Randomly Occurring Uncertainties and Time-Varying Delays

Yanfeng Zhao,^{1,2} Jihong Shen,³ and Dongyan Chen⁴

¹College of Automation, Harbin Engineering University, Harbin 150001, China

²Graduate Department, Harbin University of Science and Technology, Harbin 150080, China

³College of Science, Harbin Engineering University, Harbin 150001, China

⁴College of Science, Harbin University of Science and Technology, Harbin 150080, China

Correspondence should be addressed to Yanfeng Zhao; zhaoyanfeng@hrbust.edu.cn

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We deal with the design problem of nonfragile state estimator for discrete-time genetic regulatory networks (GRNs) with time-varying delays and randomly occurring uncertainties. In particular, the norm-bounded uncertainties enter into the GRNs in random ways in order to reflect the characteristic of the modelling errors, and the so-called randomly occurring uncertainties are characterized by certain mutually independent random variables obeying the Bernoulli distribution. The focus of the paper is on developing a new nonfragile state estimation method to estimate the concentrations of the mRNA and the protein for considered uncertain delayed GRNs, where the randomly occurring estimator gain perturbations are allowed. By constructing a Lyapunov-Krasovskii functional, a delay-dependent criterion is obtained in terms of linear matrix inequalities (LMIs) by properly using the discrete-time Wirtinger-based inequality and reciprocally convex combination approach as well as the free-weighting matrix method. It is shown that the proposed method ensures that the estimation error dynamics is globally asymptotically stable and the desired estimator parameter is designed via the solutions to certain LMIs. Finally, we provide two numerical examples to illustrate the feasibility and validity of the proposed estimation results.

1. Introduction

Genetic regulatory networks (GRNs), which are biochemically dynamical systems describing highly complicated interactions among DNA, RNA, protein, and metabolic product in the interactive transcriptional and translational processes, have become an attractive research field in the systems biology and biomedical sciences during the past few decades. One of the main objectives in systems biology is to understand how mRNAs and proteins work collectively and interact with each other to perform the complicated biological functions (e.g., the process of transcriptions and translations). In the past few years, a great deal of attention has been devoted to the theoretical analysis and experimental investigation on GRNs. Accordingly, a large amount of important results has been reported in literature making significant contributions

for understanding both static and dynamic behaviors of biological systems in detail (see [1–3] and the references therein). Among them, two main classes of models used to characterize the GRNs include discrete models such as Boolean model [4, 5] and continuous models such as differential equation models [6, 7].

When modelling GRNs, we should bear in mind the fact that GRNs models are unavoidably affected by modelling uncertainties, including parametric errors, time delays, and stochastic noises. More specifically, the time delays are unavoidable due primarily to the slow process of transcription/translation, translocation, or the finite switching speed of amplifiers. The existence of the time delays could affect the whole system performance and may lead to oscillation, divergence, and instability of the genetic networks. According to the occurrence way of time delays, they can generally be

classified into three types: interval time-varying delays [8, 9], distributed delays [10], and random time delays [11, 12]. In addition, gene regulation is an intrinsically noisy process due to intracellular and extracellular noise perturbations, which are derived from random births and deaths of individual molecules and environmental fluctuations. Accordingly, a great deal of effort has been made to deal with the analysis problem of GRNs with time-varying delays as well as stochastic noises and a growing number of approaches have been provided in the literature to examine the dynamical behaviors of the addressed GRNs [2, 3, 13]. On the other hand, the discrete-time version of GRNs receives increasing research interests due to the reason that the continuous-time GRNs are commonly discretized for computer simulation and experimental purposes [14, 15]. In recent years, much effort has been made to tackle the synthesis problems of discrete GRNs with time delays. For example, the robust H_∞ control problem has been investigated in [16] for discrete delayed stochastic GRNs. Moreover, the asymptotic stability problems have been investigated in [17–19] for discrete-time uncertain GRNs with time-varying delays and stochastic fluctuations, where some sufficient conditions have been presented to ensure the stability of the addressed GRNs via the linear matrix inequality approach.

In practice, for the purpose of both the drug design and disease diagnosis, it is worth mentioning that biologists hope to gain actual concentrations of gene products in GRNs. However, due to the existence of the model errors, time delays, and external disturbances, the actual values of GRNs can hardly be obtained and only partial information about the gene states is commonly available in the measurement outputs. As such, the filtering and state estimation problems for complex dynamics systems have been widely investigated; see, for example, [20–26]. To mention a few, the set-values filtering problem has been investigated in [20] for a class of GRNs with time-varying uncertainties and bounded external noise, where new filtering scheme has been given for the addressed GRNs. In [22], the state estimation problem has been studied for a class of discrete-time GRNs with random delays. Moreover, the H_∞ state estimation problem has been discussed in [24] for discrete stochastic GRNs with Markovian jumping parameters and time-varying delays. More recently, the sampling-data state estimation algorithm has been provided in [25] for a class of GRNs subject to time-varying delays. On the other hand, the parameter uncertainties may be changed in a random way with certain types and intensity due to various effects, for instance, the changing subsystem interconnections, network-induced random failures, repairs of components and sudden environmental perturbations, and so on. Hence, it is more significant to deal with the effects from the randomly occurring parameter uncertainties onto the networked systems [27–32]. In addition, as discussed in [33], a small or even tiny drift/fluctuation/error of the presented controller/estimator during the parameter implementation may result in unexpected fragility/degradation for whole system performance. In other words, there exist the deviations between the parameters of actually implemented controller/estimator and their expected values, and hence there is a need to design the robust

controller/estimator with certain degree of tolerance against the possible deviations. In the past decade, the nonfragile estimation problems have gained much attention with respect to the implementation errors of proposed estimators [34–36]. However, the nonfragile state estimation problem has not been fully studied yet for discrete delayed GRNs, not to mention the case where the randomly occurring parameter uncertainties are also considered. As such, the main purpose of the paper is to provide a robust state estimation method against the mentioned phenomena.

Motivated by the aforementioned analysis, in this paper, we aim to investigate the robust nonfragile state estimation problem for a class of discrete GRNs with time-varying delays and randomly occurring uncertainties. The Lyapunov-Krasovskii functional is chosen which utilizes more information about the time delays and a new set of conditions is added to calculate the difference of the designed Lyapunov-Krasovskii functional. Furthermore, as mentioned in [37], it is clearly seen that the Wirtinger-based inequality can reduce the conservatism and provide a more tighter lower delay bound of the summation terms. Therefore, we revisit the nonfragile state estimation problem for discrete GRNs with time-varying delays and randomly occurring uncertainties by using the discrete-time Wirtinger-based inequality and the reciprocally convex combination inequality. A delay-dependent estimation criterion is established in terms of feasibility of a set of LMIs. The main contributions of this paper can be summarized as follows: (1) the discrete Wirtinger-based inequality is utilized for the first time to investigate the nonfragile state estimation problem for GRNs with time-varying delays and randomly occurring uncertainties, where the estimator gain perturbations occur in a random way; (2) by utilizing the discrete-time Wirtinger-based inequality and the reciprocally convex combination approach, the summation terms are addressed and more information of time delays is reflected in the proposed algorithm; and (3) more free-weighting matrices are introduced by adding to new conditions with hope to further reduce the conservativeness induced by time delays. Finally, examples and simulation are given to illustrate the effectiveness and advantages of the proposed main results.

Notations. For a matrix A , $A > 0$ denotes that A is a symmetric positive-definite matrix. $E[\cdot]$ stands for the mathematical expectation operator. The superscript T and (-1) represent the transpose and the inverse of the matrix, respectively. I represents an identity matrix with appropriate dimension. For simplicity, $*$ represents the term that is induced by symmetry in symmetric block matrices.

2. Model Description and Preliminaries

In this paper, we consider the following discrete GRNs with time-varying delays, n mRNA, and n proteins:

$$\begin{aligned} M(k+1) &= AM(k) + Bf(P(k-\tau(k))) + L, \\ P(k+1) &= CP(k) + DM(k-\delta(k)), \end{aligned} \quad (1)$$

where $M(k) = [M_1(k), M_2(k), \dots, M_n(k)]^T$, $P(k) = [P_1(k), P_2(k), \dots, P_n(k)]^T \in R^n$ are the concentrations of mRNA and protein at instant k , respectively. $A = \text{diag}\{a_1, a_2, \dots, a_n\}$ and $C = \text{diag}\{c_1, c_2, \dots, c_n\}$ are real constant diagonal matrices representing the decay rates of mRNA and protein with entries $|a_i| < 1$ and $|c_i| < 1$, $D = \text{diag}\{d_1, d_2, \dots, d_n\}$, d_i is the translation rate, and $B = (b_{ij})_{n \times n}$ is the coupling matrix of the genetic networks. $L = [l_1, l_2, \dots, l_n]^T$ stands for the basal rates of degradation. $\tau(k) > 0$ and $\delta(k) > 0$ are the time-varying delays denoting the feedback regulation delay and the translation delay satisfying $0 < \tau_m \leq \tau(k) \leq \tau_M$ and $0 < \delta_m \leq \delta(k) \leq \delta_M$. In addition, the nonlinear function $f(k) = [f_1(k), f_2(k), \dots, f_n(k)]^T$ represents the feedback regulation of the protein on the transcription. It is a monotonic function in the Hill form; that is, $f_j(x) = (x/\beta_j)^{H_j}/(1 + (x/\beta_j)^{H_j})$, $x \in R$, where H_j is the Hill coefficient and β_j is a positive constant.

Let $[M^{*T}, P^{*T}] = [M_1^*, M_2^*, \dots, M_n^*, P_1^*, P_2^*, \dots, P_n^*]$ be an equilibrium point of system (1). Then, it is easy to obtain that

$$\begin{aligned} M^* &= AM^* + Bf(P^*) + L, \\ P^* &= CP^* + DM^*. \end{aligned} \quad (2)$$

Subsequently, we can shift the equilibrium point $[M^{*T}, P^{*T}]$ of system (1) to the origin point via the transformations $x(k) = M(k) - M^*$ and $y(k) = P(k) - P^*$. Then, system (1) can be converted into the following form:

$$\begin{aligned} x(k+1) &= Ax(k) + Bg(y(k - \tau(k))), \\ y(k+1) &= Cy(k) + Dx(k - \delta(k)). \end{aligned} \quad (3)$$

with $g(y(k)) = f(y(k) + P^*) - f(P^*)$.

As mentioned above, the GRNs are often the large-scale networks and it should be noted that system parameter uncertainties may be subject to random changes in real circumstances due to some factors such as repairs of components and sudden environmental disturbances. Thus, they may occur in a probabilistic way with various types and intensities. By considering the randomly occurring uncertainties, system (3) becomes

$$\begin{aligned} x(k+1) &= (A + \alpha_1(k) \Delta A(k)) x(k) \\ &\quad + (B + \beta_2(k) \Delta B(k)) g(y(k - \tau(k))), \\ y(k+1) &= (C + \beta_1(k) \Delta C(k)) y(k) \\ &\quad + (D + \alpha_2(k) \Delta D(k)) x(k - \delta(k)), \end{aligned} \quad (4)$$

where $\Delta A(k)$, $\Delta B(k)$, $\Delta C(k)$, and $\Delta D(k)$ denote the parameter uncertainties satisfying the following condition:

$$\begin{aligned} [\Delta A(k) \quad \Delta B(k) \quad \Delta C(k) \quad \Delta D(k)] \\ = E_1 F(k) [M_1 \quad M_2 \quad M_3 \quad M_4], \end{aligned} \quad (5)$$

where E_1 , M_1 , M_2 , M_3 , and M_4 are known constant matrices and $F(k)$ is an unknown time-varying matrix satisfying $F^T(k)F(k) \leq I$. The stochastic variables $\alpha_i(k)$ and $\beta_i(k)$ ($i = 1, 2$) are used to characterize the randomly occurring uncertainties and are mutually independent Bernoulli-distributed white noise sequences satisfying

$$\begin{aligned} \text{Prob}\{\alpha_i(k) = 1\} &= \alpha_i, \\ \text{Prob}\{\alpha_i(k) = 0\} &= 1 - \alpha_i, \\ \text{Prob}\{\beta_i(k) = 1\} &= \beta_i, \\ \text{Prob}\{\beta_i(k) = 0\} &= 1 - \beta_i, \end{aligned} \quad (6)$$

where α_i and $\beta_i \in [0, 1]$ ($i = 1, 2$) are known constants.

The state components of the GRNs are usually not completely accessible. Consequently, it is necessary to make use of the available output information and design a state estimator to estimate the state vector of the addressed GRNs. For this case, assume that the measurement outputs are given as follows:

$$\begin{aligned} z_x(k) &= W_1 x(k), \\ z_y(k) &= W_2 y(k), \end{aligned} \quad (7)$$

where $z_x(k)$ and $z_y(k)$ are the measurements of model (4); W_1 and W_2 are the known matrices.

The main objective of this paper is to estimate the concentrations of mRNA and protein in (4) from the available network outputs in (7). In the sequel, we design the following nonfragile state estimator:

$$\begin{aligned} \hat{x}(k+1) &= A\hat{x}(k) + Bg(\hat{y}(k - \tau(k))) \\ &\quad + (K_1 + \gamma_1(k) \Delta K_1(k)) [z_x(k) - W_1 \hat{x}(k)], \\ \hat{y}(k+1) &= C\hat{y}(k) + D\hat{x}(k - \delta(k)) \\ &\quad + (K_2 + \gamma_2(k) \Delta K_2(k)) [z_y(k) - W_2 \hat{y}(k)], \end{aligned} \quad (8)$$

where $\hat{x}(k)$, $\hat{y}(k)$ are the estimations of $x(k)$ and $y(k)$, and the nonfragile estimator gain matrices K_1 , K_2 are to be determined. The real-valued matrices $\Delta K_1(k)$ and $\Delta K_2(k)$ represent possible estimator gain fluctuations. It is assumed that $\Delta K_1(k)$ and $\Delta K_2(k)$ have the following structure:

$$[\Delta K_1(k) \quad \Delta K_2(k)] = E_2 F(k) [M_5 \quad M_6], \quad (9)$$

where E_2 , M_5 , and M_6 are the known constant matrices and $F(k)$ is the unknown time-varying matrix satisfying $F^T(k)F(k) \leq I$. The stochastic variables $\gamma_i(k)$ ($i = 1, 2$)

represent the perturbations in the estimator gain matrices and are mutually independent Bernoulli-distributed white noise sequences taking on values of 0 or 1 with

$$\begin{aligned} \text{Prob} \{ \gamma_i(k) = 1 \} &= \gamma_i, \\ \text{Prob} \{ \gamma_i(k) = 0 \} &= 1 - \gamma_i, \end{aligned} \quad (10)$$

where $\gamma_i \in [0, 1]$ ($i = 1, 2$) are known constants.

Letting the estimation error be $\tilde{x}(k) = x(k) - \hat{x}(k)$ and $\tilde{y}(k) = y(k) - \hat{y}(k)$, the estimation error dynamics can be described as follows:

$$\begin{aligned} \tilde{x}(k+1) &= (A - K_1 W_1) \tilde{x}(k) + Bg(\tilde{y}(k - \tau(k))) \\ &\quad + \alpha_1(k) \Delta A(k) x(k) \\ &\quad - \gamma_1(k) \Delta K_1(k) W_1 \tilde{x}(k) \\ &\quad + \beta_2(k) \Delta B(k) g(y(k - \tau(k))), \\ \tilde{y}(k+1) &= (C - K_2 W_2) \tilde{y}(k) + D\tilde{x}(k - \delta(k)) \\ &\quad + \beta_1(k) \Delta C(k) y(k) \\ &\quad - \gamma_2(k) \Delta K_2(k) W_2 \tilde{y}(k) \\ &\quad + \alpha_2(k) \Delta D(k) x(k - \delta(k)), \end{aligned} \quad (11)$$

where $g(\tilde{y}(k - \tau(k))) = g(y(k - \tau(k))) - g(\hat{y}(k - \tau(k)))$. Initial conditions for the uncertain GRNs (4) and state estimator system (8) are assumed to be $(x(k), y(k)) = (\phi(k), \psi(k))$ and $(\hat{x}(k), \hat{y}(k)) = (\hat{\phi}(k), \hat{\psi}(k))$ on $[-\tau, 0]$, where $\tau = \max\{\tau_M, \delta_M\}$.

To facilitate further derivations, set

$$\begin{aligned} e_x(k) &= \begin{bmatrix} x(k) \\ \tilde{x}(k) \end{bmatrix}, \\ e_y(k) &= \begin{bmatrix} y(k) \\ \tilde{y}(k) \end{bmatrix}. \end{aligned} \quad (12)$$

Combining (4) with (11), we obtain the augmented estimation error dynamics:

$$\begin{aligned} e_x(k+1) &= \bar{A}_1 e_x(k) + \bar{B}_1 g(e_y(k - \tau(k))) \\ &\quad + \bar{E}_1 \varrho_1(k) + \bar{E}_2 \varrho_2(k), \\ e_y(k+1) &= \bar{C}_1 e_y(k) + \bar{D}_1 e_x(k - \delta(k)) + \bar{E}_1 \varrho_3(k) \\ &\quad + \bar{E}_2 \varrho_4(k), \\ \varrho_i(k) &= \bar{F}(k) \varsigma_i(k), \quad i = 1, 2, 3, 4, \end{aligned}$$

$$\begin{aligned} \varsigma_1(k) &= \alpha_1(k) \bar{M}_1 e_x(k) \\ &\quad + \beta_2(k) \bar{M}_2 g(e_y(k - \tau(k))), \end{aligned}$$

$$\varsigma_2(k) = \gamma_1(k) \bar{M}_5 e_x(k),$$

$$\begin{aligned} \varsigma_3(k) &= \beta_1(k) \bar{M}_3 e_y(k) \\ &\quad + \alpha_2(k) \bar{M}_4 e_x(k - \delta(k)), \end{aligned}$$

$$\varsigma_4(k) = \gamma_2(k) \bar{M}_6 e_y(k), \quad (13)$$

where

$$\begin{aligned} \bar{A}_1 &= \begin{bmatrix} A & 0 \\ 0 & A - K_1 W_1 \end{bmatrix}, \\ \bar{C}_1 &= \begin{bmatrix} C & 0 \\ 0 & C - K_2 W_2 \end{bmatrix}, \\ \bar{B}_1 &= \begin{bmatrix} B & 0 \\ 0 & B \end{bmatrix}, \\ \bar{D}_1 &= \begin{bmatrix} D & 0 \\ 0 & D \end{bmatrix}, \\ \bar{E}_1 &= \begin{bmatrix} E_1 & 0 \\ 0 & E_1 \end{bmatrix}, \\ \bar{E}_2 &= \begin{bmatrix} E_2 & 0 \\ 0 & E_2 \end{bmatrix}, \\ \bar{F}(k) &= \begin{bmatrix} F(k) & 0 \\ 0 & F(k) \end{bmatrix}, \\ \bar{M}_1 &= \begin{bmatrix} M_1 & 0 \\ M_1 & 0 \end{bmatrix}, \\ \bar{M}_2 &= \begin{bmatrix} M_2 & 0 \\ M_2 & 0 \end{bmatrix}, \\ \bar{M}_3 &= \begin{bmatrix} M_3 & 0 \\ M_3 & 0 \end{bmatrix}, \\ \bar{M}_4 &= \begin{bmatrix} M_4 & 0 \\ M_4 & 0 \end{bmatrix}, \\ \bar{M}_5 &= \begin{bmatrix} 0 & 0 \\ 0 & -M_5 W_1 \end{bmatrix}, \\ \bar{M}_6 &= \begin{bmatrix} 0 & 0 \\ 0 & -M_6 W_2 \end{bmatrix}. \end{aligned} \quad (14)$$

Assumption 1. The nonlinear function $g_i(\cdot)$ ($i = 1, 2, \dots, n$) is a monotonically increasing function and satisfies the following condition:

$$0 \leq \frac{g_i(x_i)}{x_i} \leq l_i, \quad x_i \neq 0, \quad g_i(0) = 0, \quad (15)$$

which is equivalent to

$$g^T(x)(g(x) - Lx) \leq 0, \quad (16)$$

where $L = \text{diag}\{l_1, l_2, \dots, l_n\} > 0$ and l_i ($i = 1, 2, \dots, n$) are known constants.

Definition 2. System (8) is said to be a robust asymptotic state estimator of the GRNs (4) if estimation error system (13) is globally robustly asymptotically stable in the mean square; that is,

$$\lim_{k \rightarrow +\infty} E \{ \|\tilde{x}(k)\|^2 + \|\tilde{y}(k)\|^2 \} = 0. \quad (17)$$

To end of this section, we introduce the following lemmas which will be frequently used in the subsequent developments.

Lemma 3 ([38] (Schur complement)). *Given constant matrices S_{11} , S_{12} , and S_{22} with appropriate dimensions, where $S_{11} = S_{11}^T$ and $S_{22} = S_{22}^T < 0$, then $S_{11} - S_{12}S_{22}^{-1}S_{12}^T < 0$ if and only if*

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix} < 0. \quad (18)$$

Lemma 4 ([37] (discrete-time Wirtinger-based inequality)). *For a given positive-definite matrix R and three nonnegative integers a, b, k satisfying $a \leq b \leq k$, denote*

$$\chi(k, a, b) = \begin{cases} \frac{1}{b-a} \left[\left(2 \sum_{j=k-b}^{k-a-1} x(j) \right) + x(k-a) - x(k-b) \right], & (a < b) \\ 2x(k-a), & (a = b). \end{cases} \quad (19)$$

Then, one has

$$\begin{aligned} & -(b-a) \sum_{j=k-b}^{k-a-1} \eta^T(i) R \eta(i) \\ & \leq - \begin{bmatrix} \Omega_0 \\ \Omega_1 \end{bmatrix}^T \begin{bmatrix} R & 0 \\ 0 & 3R \end{bmatrix} \begin{bmatrix} \Omega_0 \\ \Omega_1 \end{bmatrix}, \end{aligned} \quad (20)$$

where

$$\begin{aligned} \eta(k) &= x(k+1) - x(k), \\ \Omega_0 &= x(k-a) - x(k-b), \\ \Omega_1 &= x(k-a) + x(k-b) - \chi(k, a, b). \end{aligned} \quad (21)$$

Lemma 5 (see [39]). *For given positive integers n, m , a scalar $\alpha \in (0, 1)$, an $n \times n$ -matrix $R > 0$, and two $n \times m$ -matrices S_1, S_2 , for all vector $\xi \in \mathbb{R}^m$, the function $\Theta(\alpha, R)$ is given by*

$$\Theta(\alpha, R) = \frac{1}{\alpha} \xi^T S_1^T R S_1 \xi + \frac{1}{1-\alpha} \xi^T S_2^T R S_2 \xi. \quad (22)$$

*If there is a matrix $X \in \mathbb{R}^{n \times n}$ such that $\begin{bmatrix} R & X \\ * & R \end{bmatrix} > 0$, then the following inequality holds:*

$$\min \Theta(\alpha, R) \geq \begin{bmatrix} S_1 \xi \\ S_2 \xi \end{bmatrix}^T \begin{bmatrix} R & X \\ * & R \end{bmatrix} \begin{bmatrix} S_1 \xi \\ S_2 \xi \end{bmatrix}. \quad (23)$$

3. Main Results

In this section, we first consider that there are no parameter uncertainties in GRNs (4). Also, the estimator can be selected as (8); then augmented error system (13) can be rewritten as

$$\begin{aligned} e_x(k+1) &= \bar{A}_1 e_x(k) + \bar{B}_1 g(e_y(k - \tau(k))) \\ &\quad + \bar{E}_2 \varrho_2(k), \\ e_y(k+1) &= \bar{C}_1 e_y(k) + \bar{D}_1 e_x(k - \delta(k)) + \bar{E}_2 \varrho_4(k), \\ \varrho_i(k) &= \bar{F}(k) \varsigma_i(k), \quad i = 2, 4, \\ \varsigma_2(k) &= \gamma_1(k) \bar{M}_5 e_x(k), \\ \varsigma_4(k) &= \gamma_2(k) \bar{M}_6 e_y(k). \end{aligned} \quad (24)$$

Our main aim is to design the nonfragile state estimator to guarantee that estimation error system (24) is globally asymptotically stable. In other words, by designing a proper Lyapunov-Krasovskii functional, together with the discrete-time Wirtinger-based inequality, reciprocally convex combination approach, and the free-weighting matrices method, we are interested in looking for the estimator gain matrices K_1 and K_2 such that the error dynamics governed by (24) is globally asymptotically stable. Further, the result is extended to handle the nonfragile state estimation problem for GRNs (4) with randomly occurring uncertainties.

Theorem 6. *For given positive scalars $\delta_m, \delta_M, \tau_m$, and τ_M , estimation error system (24) is globally asymptotically stable, if there exist scalars $\varepsilon_i > 0$ ($i = 2, 4$), matrices $P_1 > 0, P_2 > 0, R > 0, Z_j > 0$ ($j = 1, 2, 3, 4$), and $Q_l > 0$ ($l = 1, 2, \dots, 6$), positive diagonal matrices Λ_1, Λ_2 , and Λ_3 , matrices $Y_i = \begin{bmatrix} Y_{i1} & Y_{i2} \\ * & Y_{i3} \end{bmatrix}$ ($i = 1, 2$), and block diagonal matrices N_1, N_2, N_3, N_4, X_1 , and X_3 , such that the following LMIs hold:*

$$\Sigma = \begin{bmatrix} \Xi^0 & \varepsilon_2 \Psi_2^T & \varepsilon_4 \Psi_4^T \\ * & -\varepsilon_2 I_{2n} & 0 \\ * & * & -\varepsilon_4 I_{2n} \end{bmatrix} < 0, \quad (25)$$

where

$$\Psi_2 = [\gamma_1 \bar{M}_5 \quad 0_{n \times 19n}],$$

$$\Psi_4 = [0_{n \times 4n} \quad \gamma_2 \bar{M}_6 \quad 0_{n \times 15n}],$$

$$\Xi^0 = \sum_{i=1}^9 \Xi_i,$$

$$\Xi_1 = 2e_1 P_1 e_9^T + e_9 P_1 e_9^T + 2e_5 P_2 e_{10}^T + e_{10} P_2 e_{10}^T,$$

$$\begin{aligned} \Xi_2 = e_1 (Q_1 + (1 + \delta_1) Q_2 + Q_3) e_1^T - e_3 Q_1 e_3^T \\ - e_2 Q_2 e_2^T - e_4 Q_3 e_4^T, \end{aligned}$$

$$\begin{aligned} \Xi_3 = e_5 (Q_4 + (1 + \tau_1) Q_5 + Q_6) e_5^T - e_7 Q_4 e_7^T \\ - e_6 Q_5 e_6^T - e_8 Q_6 e_8^T, \end{aligned}$$

$$\Xi_4 = e_{11} (1 + \tau_1) R e_{11}^T - e_{12} R e_{12}^T,$$

$$\begin{aligned} \Xi_5 = e_9 (\delta_1 Z_1 + \delta_m Z_2) e_9^T + e_{10} (\tau_1 Z_3 + \tau_m Z_4) e_{10}^T \\ - \frac{1}{\delta_m} \rho_1 \tilde{Z}_2 \rho_1^T - \frac{1}{\delta_1} \rho_7 \Theta_1 \rho_7^T - \frac{1}{\tau_m} \rho_4 \tilde{Z}_4 \rho_4^T \\ - \frac{1}{\tau_1} \rho_8 \Theta_2 \rho_8^T, \end{aligned}$$

$$\begin{aligned} \Xi_6 = 2e_5 L \Lambda_1 e_{11}^T - 2e_{11} \Lambda_1 e_{11}^T + 2e_6 L \Lambda_2 e_{12}^T \\ - 2e_{12} \Lambda_2 e_{12}^T + 2(e_5 - e_6) L \Lambda_3 (e_{11} - e_{12})^T \\ - 2(e_{11} - e_{12}) \Lambda_3 (e_{11} - e_{12})^T, \end{aligned}$$

$$\Xi_7 = -e_{19} \varepsilon_2 I_{2n} e_{19}^T - e_{20} \varepsilon_4 I_{2n} e_{20}^T,$$

$$\begin{aligned} \Xi_8 = 2e_9 N_1 \bar{B}_1 e_{12}^T + 2e_9 N_1 \bar{E}_2 e_{19}^T - 2e_9 N_1 e_9^T \\ + 2e_1 N_2 \bar{B}_1 e_{12}^T + 2e_1 N_2 \bar{E}_2 e_{19}^T - 2e_1 N_2 e_9^T \\ + 2e_{10} N_3 \bar{D}_1 e_2^T + 2e_{10} N_3 \bar{E}_2 e_{20}^T - 2e_{10} N_3 e_{10}^T \\ + 2e_5 N_4 \bar{D}_1 e_2^T + 2e_5 N_4 \bar{E}_2 e_{20}^T - 2e_5 N_4 e_{10}^T, \end{aligned}$$

$$\begin{aligned} \Xi_9 = 2e_9 (N_1 \bar{A}_1 - X_1 \bar{W}_1) e_1^T \\ + 2e_1 (N_2 \bar{A}_1 - X_1 \bar{W}_1) e_1^T \\ + 2e_{10} (N_3 \bar{C}_1 - X_3 \bar{W}_2) e_5^T \\ + 2e_5 (N_4 \bar{C}_1 - X_3 \bar{W}_2) e_5^T, \end{aligned}$$

$$e_i = [0_{2n \times (i-1)2n} \quad I_{2n \times 2n} \quad 0_{2n \times (20-i)2n}]^T,$$

$$i = 1, 2, \dots, 20,$$

$$\rho_1 = [e_1 - e_3 \quad \sqrt{3}(e_1 + e_3 - e_{13})],$$

$$\rho_2 = [e_2 - e_4 \quad \sqrt{3}(e_2 + e_4 - e_{14})],$$

$$\rho_3 = [e_3 - e_2 \quad \sqrt{3}(e_3 + e_2 - e_{15})],$$

$$\rho_4 = [e_5 - e_7 \quad \sqrt{3}(e_5 + e_7 - e_{16})],$$

$$\rho_5 = [e_6 - e_8 \quad \sqrt{3}(e_6 + e_8 - e_{17})],$$

$$\rho_6 = [e_7 - e_6 \quad \sqrt{3}(e_7 + e_6 - e_{18})],$$

$$\rho_7 = [\rho_2 \quad \rho_3],$$

$$\rho_8 = [\rho_5 \quad \rho_6],$$

$$\tilde{Z}_i = \begin{bmatrix} Z_i & 0 \\ 0 & Z_i \end{bmatrix}, \quad (i = 1, 2, 3, 4),$$

$$\Theta_1 = \begin{bmatrix} \tilde{Z}_1 & Y_1 \\ * & \tilde{Z}_1 \end{bmatrix},$$

$$\Theta_2 = \begin{bmatrix} \tilde{Z}_3 & Y_2 \\ * & \tilde{Z}_3 \end{bmatrix},$$

$$\bar{A}_1 = \begin{bmatrix} A - I & 0 \\ 0 & A - I \end{bmatrix},$$

$$\bar{C}_1 = \begin{bmatrix} C - I & 0 \\ 0 & C - I \end{bmatrix},$$

$$\bar{W}_1 = \begin{bmatrix} 0 & 0 \\ 0 & -W_1 \end{bmatrix},$$

$$\bar{W}_2 = \begin{bmatrix} 0 & 0 \\ 0 & -W_2 \end{bmatrix}.$$

(26)

And the remaining terms are zero. Then, the estimator gain matrices are determined by

$$K_1 = N_{12}^{-1} X_1,$$

$$K_2 = N_{32}^{-1} X_3.$$

(27)

Proof. Set

$$\eta_1(k) = e_x(k+1) - e_x(k),$$

$$\eta_2(k) = e_y(k+1) - e_y(k),$$

$$\delta_1 = \delta_M - \delta_m,$$

$$\tau_1 = \tau_M - \tau_m.$$

(28)

We construct the following Lyapunov-Krasovskii functional for discrete-time GRNs (24):

$$V(k) = \sum_{i=1}^5 V_i(k), \quad (29)$$

where

$$\begin{aligned}
V_1(k) &= e_x^T(k) P_1 e_x(k) + e_y^T(k) P_2 e_y(k), \\
V_2(k) &= \sum_{i=k-\delta_m}^{k-1} e_x^T(i) Q_1 e_x(i) + \sum_{i=k-\delta_M}^{k-1} e_x^T(i) Q_3 e_x(i) \\
&\quad + \sum_{i=k-\delta(k)}^{k-1} e_x^T(i) Q_2 e_x(i) \\
&\quad + \sum_{j=-\delta_M+1}^{-\delta_m} \sum_{i=k+j}^{k-1} e_x^T(i) Q_2 e_x(i), \\
V_3(k) &= \sum_{i=k-\tau_m}^{k-1} e_y^T(i) Q_4 e_y(i) + \sum_{i=k-\tau_M}^{k-1} e_y^T(i) Q_6 e_y(i) \\
&\quad + \sum_{i=k-\tau(k)}^{k-1} e_y^T(i) Q_5 e_y(i) \\
&\quad + \sum_{j=-\tau_M+1}^{-\tau_m} \sum_{i=k+j}^{k-1} e_y^T(i) Q_5 e_y(i), \\
V_4(k) &= \sum_{i=k-\tau(k)}^{k-1} g^T(e_y(i)) R g(e_y(i)) \\
&\quad + \sum_{j=-\tau_M+1}^{-\tau_m} \sum_{i=k+j}^{k-1} g^T(e_y(i)) R g(e_y(i)), \\
V_5(k) &= \sum_{j=-\delta_M}^{-\delta_m-1} \sum_{i=k+j}^{k-1} \eta_1^T(i) Z_1 \eta_1(i) \\
&\quad + \sum_{j=-\delta_m}^{-1} \sum_{i=k+j}^{k-1} \eta_1^T(i) Z_2 \eta_1(i) \\
&\quad + \sum_{j=-\tau_M}^{-\tau_m-1} \sum_{i=k+j}^{k-1} \eta_2^T(i) Z_3 \eta_2(i) \\
&\quad + \sum_{j=-\tau_m}^{-1} \sum_{i=k+j}^{k-1} \eta_2^T(i) Z_4 \eta_2(i).
\end{aligned} \tag{30}$$

For convenience, define the following notations:

$$\begin{aligned}
\chi_1(k) &= \chi(k, 0, \delta_m), \\
\chi_2(k) &= \chi(k, \delta(k), \delta_M), \\
\chi_3(k) &= \chi(k, \delta_m, \delta(k)), \\
\chi_4(k) &= \chi(k, 0, \tau_m), \\
\chi_5(k) &= \chi(k, \tau(k), \tau_M),
\end{aligned}$$

$$\begin{aligned}
\chi_6(k) &= \chi(k, \tau_m, \tau(k)), \\
\xi^T(k) &= [e_x^T(k), e_x^T(k - \delta(k)), e_x^T(k - \delta_m), \\
&\quad e_x^T(k - \delta_M), e_y^T(k), e_y^T(k - \tau(k)), e_y^T(k - \tau_m), \\
&\quad e_y^T(k - \tau_M), \eta_1^T(k), \eta_2^T(k), g^T(e_y(k)), \\
&\quad g^T(e_y(k - \tau(k))), \chi_1^T(k), \chi_2^T(k), \chi_3^T(k), \chi_4^T(k), \\
&\quad \chi_5^T(k), \chi_6^T(k), \varrho_2^T(k), \varrho_4^T(k)].
\end{aligned} \tag{31}$$

Calculate the difference of $V(k)$ by defining $\Delta V(k) = V(k+1) - V(k)$ along the solutions of (24):

$$\begin{aligned}
\Delta V_1(k) &= V_1(k+1) - V_1(k) \\
&= e_x^T(k+1) P_1 e_x(k+1) - e_x^T(k) P_1 e_x(k) \\
&\quad + e_y^T(k+1) P_2 e_y(k+1) - e_y^T(k) P_2 e_y(k) \\
&= (\eta_1(k) + e_x(k))^T P_1 (\eta_1(k) + e_x(k)) \\
&\quad - e_x^T(k) P_1 e_x(k) \\
&\quad + (\eta_2(k) + e_y(k))^T P_2 (\eta_2(k) + e_y(k)) \\
&\quad - e_y^T(k) P_2 e_y(k) = \xi^T(k) \Xi_1 \xi(k),
\end{aligned}$$

$$\begin{aligned}
\Delta V_2(k) &= e_x^T(k) Q_1 e_x(k) \\
&\quad - e_x^T(k - \delta_m) Q_1 e_x(k - \delta_m) \\
&\quad + e_x^T(k) Q_3 e_x(k) \\
&\quad - e_x^T(k - \delta_M) Q_3 e_x(k - \delta_M) \\
&\quad + \delta_1 e_x^T(k) Q_2 e_x(k) \\
&\quad + \sum_{i=k+1-\delta(k+1)}^k e_x^T(i) Q_2 e_x(i) \\
&\quad - \sum_{i=k-\delta(k)}^{k-1} e_x^T(i) Q_2 e_x(i) \\
&\quad - \sum_{j=-\delta_M+1}^{-\delta_m} e_x^T(k+j) Q_2 e_x(k+j) \\
&\leq e_x^T(k) Q_1 e_x(k) + (1 + \delta_1) e_x^T(k) Q_2 e_x(k) \\
&\quad + e_x^T(k) Q_3 e_x(k) \\
&\quad - e_x^T(k - \delta_m) Q_1 e_x(k - \delta_m)
\end{aligned}$$

$$\begin{aligned}
& -e_x^T(k-\delta(k))Q_2e_x(k-\delta(k)) \\
& -e_x^T(k-\delta_M)Q_3e_x(k-\delta_M) \\
& = \xi^T(k)\Xi_2\xi(k).
\end{aligned} \tag{32}$$

In the same way, we obtain

$$\begin{aligned}
\Delta V_3(k) & \leq e_y^T(k)Q_4e_y(k) + (1+\tau_1)e_y^T(k)Q_5e_y(k) \\
& + e_y^T(k)Q_6e_y(k) \\
& - e_y^T(k-\tau_m)Q_4e_y(k-\tau_m) \\
& - e_y^T(k-\tau(k))Q_5e_y(k-\tau(k)) \\
& - e_y^T(k-\tau_M)Q_6e_y(k-\tau_M) \\
& = \xi^T(k)\Xi_3\xi(k),
\end{aligned} \tag{33}$$

$$\begin{aligned}
\Delta V_4(k) & = \tau_1 g^T(e_y(k))Rg(e_y(k)) \\
& + \sum_{i=k+1-\tau(k+1)}^k g^T(e_y(i))Rg(e_y(i)) \\
& - \sum_{i=k-\tau(k)}^{k-1} g^T(e_y(i))Rg(e_y(i)) \\
& - \sum_{j=-\tau_M+1}^{-\tau_M} g^T(e_y(k+j))Rg(e_y(k+j)) \\
& \leq (1+\tau_1)g^T(e_y(k))Rg(e_y(k)) \\
& - g^T(e_y(k-\tau(k)))Rg(e_y(k-\tau(k))) \\
& = \xi^T(k)\Xi_4\xi(k),
\end{aligned} \tag{34}$$

$$\begin{aligned}
\Delta V_5(k) & = \delta_1 \eta_1^T(k)Z_1\eta_1(k) - \sum_{i=k-\delta_M}^{k-\delta_m-1} \eta_1^T(i)Z_1\eta_1(i) \\
& + \delta_m \eta_1^T(k)Z_2\eta_1(k) - \sum_{i=k-\delta_m}^{k-1} \eta_1^T(i)Z_2\eta_1(i) \\
& + \tau_1 \eta_2^T(k)Z_3\eta_2(k) - \sum_{i=k-\tau_M}^{k-\tau_m-1} \eta_2^T(i)Z_3\eta_2(i) \\
& + \tau_m \eta_2^T(k)Z_4\eta_2(k) \\
& - \sum_{i=k-\tau_m}^{k-1} \eta_2^T(i)Z_4\eta_2(i).
\end{aligned} \tag{35}$$

By Lemma 4, it follows that

$$\begin{aligned}
& - \sum_{i=k-\delta_m}^{k-1} \eta_1^T(i)Z_2\eta_1(i) \\
& \leq -\frac{1}{\delta_m} \left[\begin{pmatrix} e_x(k) - e_x(k-\delta_m) \\ e_x(k) + e_x(k-\delta_m) - \chi_1(k) \end{pmatrix}^T \right. \\
& \cdot \begin{pmatrix} Z_2 & 0 \\ 0 & 3Z_2 \end{pmatrix} \begin{pmatrix} e_x(k) - e_x(k-\delta_m) \\ e_x(k) + e_x(k-\delta_m) - \chi_1(k) \end{pmatrix} \left. \right] \\
& = -\frac{1}{\delta_m} \xi^T(k) \rho_1 \bar{Z}_2 \rho_1^T \xi(k).
\end{aligned} \tag{36}$$

Next, it follows from Lemmas 4 and 5 that

$$\begin{aligned}
& - \sum_{i=k-\delta_M}^{k-\delta_m-1} \eta_1^T(i)Z_1\eta_1(i) = - \sum_{i=k-\delta_M}^{k-\delta(k)-1} \eta_1^T(i)Z_1\eta_1(i) \\
& - \sum_{i=k-\delta(k)}^{k-\delta_m-1} \eta_1^T(i)Z_1\eta_1(i) \leq -\frac{1}{\delta_M - \delta_m} \xi^T(k) \\
& \cdot \rho_2 \bar{Z}_1 \rho_2^T \xi(k) - \frac{\delta(k) - \delta_m}{\delta_M - \delta_m} \frac{1}{\delta_M - \delta(k)} \xi^T(k) \\
& \cdot \rho_2 \bar{Z}_1 \rho_2^T \xi(k) - \frac{1}{\delta_M - \delta_m} \xi^T(k) \rho_3 \bar{Z}_1 \rho_3^T \xi(k) \\
& - \frac{\delta_M - \delta(k)}{\delta_M - \delta_m} \frac{1}{\delta(k) - \delta_m} \xi^T(k) \rho_3 \bar{Z}_1 \rho_3^T \xi(k) \\
& = -\frac{1}{\delta_M - \delta_m} \left[\frac{\delta_M - \delta_m}{\delta_M - \delta(k)} \xi^T(k) \rho_2 \bar{Z}_1 \rho_2^T \xi(k) \right. \\
& \left. + \frac{\delta_M - \delta_m}{\delta(k) - \delta_m} \xi^T(k) \rho_3 \bar{Z}_1 \rho_3^T \xi(k) \right] \leq -\frac{1}{\delta_M - \delta_m} \\
& \cdot \xi^T(k) \rho_7 \Theta_1 \rho_7^T \xi(k).
\end{aligned} \tag{37}$$

Based on (35)–(37), we have

$$\begin{aligned}
\Delta V_5(k) & \leq \eta_1^T(k) (\delta_1 Z_1 + \delta_m Z_2) \eta_1(k) + \eta_2^T(k) (\tau_1 Z_3 \\
& + \tau_m Z_4) \eta_2(k) - \xi^T(k) \left[\frac{1}{\delta_m} \rho_1 \bar{Z}_2 \rho_1^T + \frac{1}{\delta_1} \rho_7 \Theta_1 \rho_7^T \right. \\
& \left. + \frac{1}{\tau_m} \rho_4 \bar{Z}_4 \rho_4^T + \frac{1}{\tau_1} \rho_8 \Theta_2 \rho_8^T \right] \xi(k) = \xi^T(k) \Xi_5 \xi(k).
\end{aligned} \tag{38}$$

From Assumption 1, for diagonal matrices Λ_1 , Λ_2 , and $\Lambda_3 > 0$ and a known matrix $L = \text{diag}\{l_1, l_2, \dots, l_n\} > 0$, it is not difficult to see that the inequalities

$$2g^T(e_y(k))\Lambda_1g(e_y(k)) - 2e_y^T(k)L\Lambda_1g(e_y(k)) \leq 0, \quad (39)$$

$$2g^T(e_y(k-\tau(k)))\Lambda_2g(e_y(k-\tau(k))) - 2e_y^T(k-\tau(k))L\Lambda_2g(e_y(k-\tau(k))) \leq 0, \quad (40)$$

$$\begin{aligned} & 2[g(e_y(k)) - g(e_y(k-\tau(k)))]^T \\ & \cdot \Lambda_3[g(e_y(k)) - g(e_y(k-\tau(k)))] \\ & - 2[e_y(k) - e_y(k-\tau(k))]^T \\ & \cdot L\Lambda_3g(e_y(k) - e_y(k-\tau(k))) \leq 0 \end{aligned} \quad (41)$$

hold. In addition, since $\eta_1(k) = e_x(k+1) - e_x(k)$, $\eta_2(k) = e_y(k+1) - e_y(k)$, by introducing relaxation matrices $N_1 = \text{diag}\{N_{11}, N_{12}\}$, $N_2 = \text{diag}\{N_{21}, N_{12}\}$, $N_3 = \text{diag}\{N_{31}, N_{32}\}$, and $N_4 = \text{diag}\{N_{41}, N_{32}\}$ with appropriate dimensions, we obtain

$$2(\eta_1^T(k)N_1 + e_x^T(k)N_2)[(\bar{A}_1 - I_{2n})e_x(k) + \bar{B}_1g(e_y(k-\tau(k))) + \bar{E}_2\varrho_2(k) - \eta_1(k)] = 0, \quad (42)$$

$$2(\eta_2^T(k)N_3 + e_y^T(k)N_4)[(\bar{C}_1 - I_{2n})e_y(k) + \bar{D}_1e_x(k-\delta(k)) + \bar{E}_2\varrho_4(k) - \eta_2(k)] = 0. \quad (43)$$

From $F^T(k)F(k) \leq I$, we have

$$\varrho_i^T(k)\varrho_i(k) \leq \zeta_i^T(k)\zeta_i(k), \quad (i = 2, 4). \quad (44)$$

Then, there exist positive scalars ε_2 and ε_4 satisfying

$$\varepsilon_i[\xi^T(k)\Psi_i^T(k)\Psi_i(k)\xi(k) - \varrho_i^T(k)\varrho_i(k)] \geq 0. \quad (45)$$

Now, combining (32)–(45) and taking mathematical expectation, one can obtain the following inequality:

$$\begin{aligned} & \mathbb{E}[\Delta V(k)] \\ & \leq \mathbb{E}\left[\xi^T(k)\left(\sum_{i=1}^9 \Xi_i + \varepsilon_2\Psi_2^T\Psi_2 + \varepsilon_4\Psi_4^T\Psi_4\right)\xi(k)\right] \\ & < 0. \end{aligned} \quad (46)$$

Using the Schur complement Lemma, it can be shown that the considered uncertain system (24) is globally asymptotically

stable in mean square sense, which completes the proof of the theorem. \square

Remark 7. It is worthwhile to mention that two constraint conditions (i.e., (39)–(40)) on feedback regulatory function were used on dealing with the state estimation problem for discrete GRNs with time-varying delays. In this paper, we add inequality (41) with hope to reduce the conservativeness introduced by time delays.

Now, we are in a position to deal with the nonfragile estimator design for GRNs (4) with time-varying delays and randomly occurring uncertainties.

Theorem 8. For given positive scalars δ_m , δ_M , τ_m , and τ_M , system (8) is said to be a globally robustly asymptotic state estimator of GRNs (4), if there exist scalars $\varepsilon_i > 0$ ($i = 1, 2, 3, 4$), matrices $P_1 > 0$, $P_2 > 0$, $R > 0$, $Z_j > 0$ ($j = 1, 2, 3, 4$), and $Q_l > 0$ ($l = 1, 2, \dots, 6$), positive diagonal matrices Λ_1 , Λ_2 , and Λ_3 , matrices $Y_i = \begin{bmatrix} Y_{i1} & Y_{i2} \\ * & Y_{i3} \end{bmatrix}$ ($i = 1, 2$), and block diagonal matrices N_1 , N_2 , N_3 , N_4 , X_1 , and X_3 with appropriate dimensions, such that the following LMIs hold:

$$\tilde{\Sigma} = \begin{bmatrix} \sum_{i=1}^{11} \Xi_i & \varepsilon_1\Psi_1^T & \varepsilon_2\Psi_2^T & \varepsilon_3\Psi_3^T & \varepsilon_4\Psi_4^T \\ * & -\varepsilon_1I & 0 & 0 & 0 \\ * & * & -\varepsilon_2I & 0 & 0 \\ * & * & * & -\varepsilon_3I & 0 \\ * & * & * & * & -\varepsilon_4I \end{bmatrix} < 0, \quad (47)$$

with

$$\Psi_1 = [\alpha_1\bar{M}_1 \quad 0_{n \times 10n} \quad \beta_2\bar{M}_2 \quad 0_{n \times 10n}],$$

$$\Psi_2 = [\gamma_1\bar{M}_5 \quad 0_{n \times 21n}],$$

$$\Psi_3 = [0_{n \times n} \quad \alpha_2\bar{M}_4 \quad 0_{n \times 2n} \quad \beta_1\bar{M}_3 \quad 0_{n \times 17n}],$$

$$\Psi_4 = [0_{n \times 4n} \quad \gamma_2\bar{M}_6 \quad 0_{n \times 17n}], \quad (48)$$

$$\begin{aligned} \Xi_{10} &= 2[e_9N_1 + e_1N_2]\bar{E}_1e_{21}^T \\ &+ 2[e_{10}N_3 + e_5N_4]\bar{E}_1e_{22}^T, \end{aligned}$$

$$\Xi_{11} = -e_{21}\varepsilon_1I_{2n}e_{21}^T - e_{22}\varepsilon_3I_{2n}e_{22}^T,$$

and the other parameters are defined as in Theorem 6. Moreover, the estimation gain matrices are determined by

$$K_1 = N_{12}^{-1}X_1, \quad (49)$$

$$K_2 = N_{32}^{-1}X_3.$$

Proof. The proof of this theorem follows from Theorem 6 directly. According to (13), for any matrices N_1, N_2, N_3 , and N_4 , we have

$$\begin{aligned} & 2\left(\eta_1^T(k)N_1 + e_x^T(k)N_2\right)\left[\left(\bar{A}_1 - I_{2n}\right)e_x(k) \right. \\ & \quad \left. + \bar{B}_1g\left(e_y(k - \tau(k))\right) + \bar{E}_1\varrho_1(k) + \bar{E}_2\varrho_2(k) \right. \\ & \quad \left. - \eta_1(k)\right] = 0, \end{aligned} \quad (50)$$

$$\begin{aligned} & 2\left(\eta_2^T(k)N_3 + e_y^T(k)N_4\right)\left[\left(\bar{C}_1 - I_{2n}\right)e_y(k) \right. \\ & \quad \left. + \bar{D}_1e_x(k - \delta(k)) + \bar{E}_1\varrho_3(k) + \bar{E}_2\varrho_4(k) - \eta_2(k)\right] \\ & = 0. \end{aligned} \quad (51)$$

Similarly, we get

$$\varrho_i^T(k)\varrho_i(k) \leq \zeta_i^T(k)\zeta_i(k), \quad (i = 1, 2, 3, 4). \quad (52)$$

Then, for positive scalars ε_i , we have

$$\begin{aligned} \varepsilon_i \left[\tilde{\xi}^T(k)\Psi_i^T(k)\Psi_i(k)\tilde{\xi}(k) - \varrho_i^T(k)\varrho_i(k) \right] \geq 0, \\ (i = 1, 2, 3, 4), \end{aligned} \quad (53)$$

with

$$\begin{aligned} \tilde{\xi}^T(k) &= \left[e_x^T(k), e_x^T(k - \delta(k)), e_x^T(k - \delta_m), \right. \\ & \quad e_x^T(k - \delta_M), e_y^T(k), e_y^T(k - \tau(k)), e_y^T(k - \tau_m), \\ & \quad e_y^T(k - \tau_M), \eta_1^T(k), \eta_2^T(k), g^T(e_y(k)), \\ & \quad g^T(e_y(k - \tau(k))), \chi_1^T(k), \chi_2^T(k), \chi_3^T(k), \chi_4^T(k), \\ & \quad \left. \chi_5^T(k), \chi_6^T(k), \varrho_2^T(k), \varrho_4^T(k), \varrho_1^T(k), \varrho_3^T(k) \right]. \end{aligned} \quad (54)$$

Combining the above and taking the mathematical expectation, one has

$$\mathbb{E}[\Delta V(k)] \leq \mathbb{E}\left[\tilde{\xi}^T(k)\tilde{\Sigma}\tilde{\xi}(k)\right]. \quad (55)$$

By using the Schur complement Lemma, it is easy to conclude that the proof of this theorem is complete. \square

Remark 9. In this paper, a nonfragile state estimation algorithm has been presented for discrete-time GRNs with time-varying delays and randomly occurring uncertainties. It is worth mentioning that it is the first time to use the discrete Wirtinger-based inequality and the reciprocally

convex inequality to handle the state estimation problem for discrete-time delayed GRNs subject to randomly occurring uncertainties and estimator gain perturbations. In particular, the reciprocally convex approach, the discrete Wirtinger-based inequality, and the free-weighting matrix method are fully utilized in order to reduce the conservativeness of the developed estimation criterion. Besides, by introducing two new zero equations (42) and (43), the free-weighting matrices N_1, N_2, N_3 , and N_4 have been introduced in the main results with hope to further reduce the possible conservativeness induced by time-varying delays, which could be utilized to deal with related state estimation problems for complex systems with time delays.

Remark 10. It should be noted that the presented method in this paper is based on LMIs. The standard LMI system has a polynomial-time complexity bounded by $O(MN)$, where M is the total row size of the LMI system and N is the total number of scalar decision variables. For example, let us look at the LMIs condition for the addressed GRNs (as described in Theorem 6); we have $M = 44n$ and $N = 2 + 13n(n + 1)/2 + 2(n(n + 1) + n^2) + 3n + 8n^2 + 2n^2 = (1/2)(41n^2 + 23n + 4)$. Therefore, the computational complexity of the LMI-based state estimation criterion can be represented by $O(902n^3 + 506n^2 + 88n)$.

Remark 11. It is worthwhile to point out that our major focus is on the nonfragile state estimation problem for discrete-time GRNs subject to time-varying delays and randomly occurring uncertainties. Due to such a complicated system, we have decided to carry out research on the feasibility problem with satisfactory performance requirements for addressed GRNs; that is, we made great efforts to propose a new state estimation method against the above-mentioned phenomena. Nevertheless, in case that the convergence precision of the proposed state estimation method becomes a concern, some additional conditions can be provided, which constitutes one of our future research topics.

4. Numerical Simulations

In this section, we present two numerical examples to illustrate the effectiveness and correctness of the proposed state estimation scheme.

Example 1. Consider three-node GRNs with the following parameters:

$$A = D = 0.3I,$$

$$C = 0.1I,$$

$$F(k) = \text{diag}\{\sin(k), \cos(k), 0\},$$

$$B = 0.5 \times \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix},$$

$$\begin{aligned}
W_1 &= \begin{bmatrix} 0.3 & 0.5 & 0 \\ 0.2 & 0.1 & 0.15 \end{bmatrix}, \\
W_2 &= \begin{bmatrix} 0.4 & 0.1 & -0.5 \\ -0.2 & 0.1 & 0.3 \end{bmatrix}, \\
E_2 &= \begin{bmatrix} 0.2 & 0.3 & 0 \\ 0.1 & 0 & 0.2 \\ 0 & 0.3 & 0 \end{bmatrix}, \\
M_5 &= \begin{bmatrix} -0.3 & -0.05 \\ -0.4 & 0.06 \\ -0.1 & 0.02 \end{bmatrix}, \\
M_6 &= \begin{bmatrix} 0.1 & 0.03 \\ 0.02 & 0.1 \\ 0.03 & 0.1 \end{bmatrix}.
\end{aligned} \tag{56}$$

The nonlinear genetic regulatory function $g(s)$ is taken to be as usual Hill function. That is, $g(s) = s^2/(1 + s^2)$. Then, it is easy to see that

$$L = \text{diag}\{0.65, 0.65, 0.65\}. \tag{57}$$

This means that Assumption 1 holds. Moreover, assume $\delta_m = 1$, $\delta_M = 5$, $\tau_m = 2$, $\tau_M = 4$, $\gamma_1 = 0.3$, and $\gamma_2 = 0.7$. By using the LMIs control Toolbox in Matlab to solve the LMIs in Theorem 6, we can obtain feasible solutions. Due to space consideration, only the following few parameters are listed: $\varepsilon_2 = 1.1713$; $\varepsilon_4 = 1.0898$; the nonfragile state estimator can be designed with the estimator gain matrices described by

$$\begin{aligned}
K_1 &= \begin{bmatrix} 0.0968 & -0.1305 \\ -0.1321 & 0.2728 \\ 0.0909 & -0.2096 \end{bmatrix}, \\
K_2 &= \begin{bmatrix} -0.2167 & 0.1296 \\ 0.5065 & 0.8443 \\ -0.0759 & 0.1388 \end{bmatrix}.
\end{aligned} \tag{58}$$

The numerical simulation affirms our theoretical results. Figure 1 plots the trajectories of mRNA concentrations, its estimates, and the estimation errors with initial conditions $\phi(k) = [0.2 \ 0.2 \ 0.4]^T$ and $\hat{\phi}(k) = [0.8 \ 0.5 \ 0.9]^T$. Figure 2 depicts the trajectories of protein concentrations, its estimates, and the estimation errors with initial conditions $\psi(k) = [0.1 \ 0.5 \ 0.3]^T$ and $\hat{\psi}(k) = [0.5 \ 0.8 \ 0.3]^T$. It follows from Figures 1 and 2 that the estimation error between the original GRNs and its estimation approaches

zero asymptotically. In addition, in order to illustrate the advantage of the main results, we can set $E_2 = 0$. Then, error system (24) is equivalent to system (6) in [27]. Subsequently, it is not difficult to check that when the feedback regulation delay $\tau(k)$ and the translation delay $\delta(k)$ satisfy $2 \leq \tau(k) \leq 5$ and $2 \leq \delta(k) \leq 9$, respectively, LMI (25) in Theorem 6 of our paper is feasible while LMI condition (7) in [27] is infeasible. Therefore, it is concluded that our results are less conservative than those in [27]. The major reason is that we have made additional efforts to propose new results with more information of time delays by taking the advantage of the discrete-time Wirtinger-based inequality, the reciprocally convex combination approach, and the free-weight matrix technique. On the other hand, it should be mentioned that, compared to [27] without consideration of estimator gain perturbations, the developed state estimation approach is more applicable.

Example 2. To demonstrate the effectiveness of Theorem 8, we consider the following parameters as in Example 1, with

$$\begin{aligned}
E_1 &= \begin{bmatrix} 0.2 & 0 & 0.1 \\ 0.2 & -0.1 & 0 \\ 0 & 0.2 & 0.3 \end{bmatrix}, \\
E_2 &= \begin{bmatrix} 0.1 & 0 & -0.2 \\ 0.2 & 0.1 & 0 \\ 0.1 & 0 & 0.3 \end{bmatrix}, \\
M_1 &= \begin{bmatrix} 0.2 & 0.05 & -0.3 \\ 0.2 & -0.1 & 0 \\ 0.2 & 0.3 & 0.6 \end{bmatrix}, \\
M_2 &= \begin{bmatrix} 0.1 & 0.3 & 0 \\ 0.2 & 0.1 & 0.2 \\ 0.1 & 0 & 0.3 \end{bmatrix}, \\
M_3 &= \begin{bmatrix} -0.1 & 0 & -0.06 \\ 0 & 0.1 & 0.2 \\ 0.3 & 0.2 & -0.2 \end{bmatrix}, \\
M_4 &= \begin{bmatrix} 0.2 & 0.1 & 0.2 \\ -0.5 & 0.2 & 0.1 \\ -0.3 & 0.2 & 0 \end{bmatrix}, \\
M_5 &= \begin{bmatrix} -0.1 & 0.02 \\ 0.05 & 0.1 \\ 0.1 & 0.02 \end{bmatrix}, \\
M_6 &= \begin{bmatrix} 0.2 & 0.03 \\ 0.3 & 0.1 \\ -0.1 & 0 \end{bmatrix}.
\end{aligned} \tag{59}$$

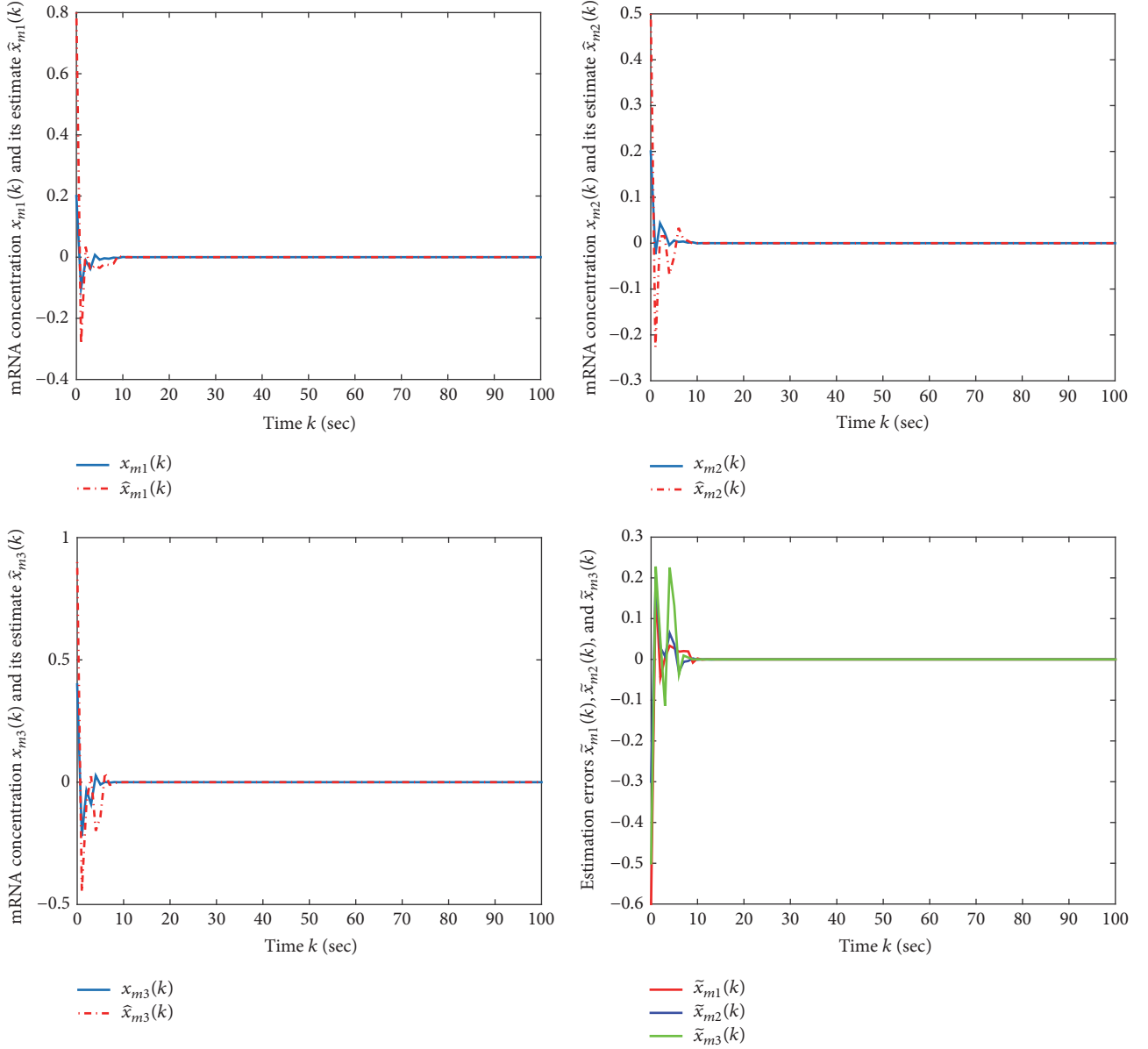


FIGURE 1: mRNA concentrations, its estimates, and the estimation error of Example 1.

Let the delays $\delta(k)$ and $\tau(k)$ vary randomly within the intervals $[2, 5]$. The mathematical expectations of Bernoulli-distributed white sequences can be taken as $\alpha_1 = 0.5$, $\alpha_2 = 0.4$, $\beta_1 = 0.3$, $\beta_2 = 0.2$, $\gamma_1 = 0.4$, and $\gamma_2 = 0.3$. By solving the LMIs in Theorem 8 with the help of Matlab LMI toolbox, we can obtain feasible solutions and scalars $\varepsilon_1 = 0.1385$, $\varepsilon_2 = 0.6089$, $\varepsilon_3 = 0.0910$, and $\varepsilon_4 = 0.5866$. According to Theorem 8, the nonfragile state estimator with randomly occurring uncertainties can be designed with the estimator gain matrices defined as follows:

$$K_1 = \begin{bmatrix} 0.1198 & -0.2160 \\ -0.2708 & 0.4029 \\ 0.1412 & -0.3462 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} 0.0842 & 0.0778 \\ 0.3287 & 0.5749 \\ 0.0143 & 0.1117 \end{bmatrix}.$$

(60)

The simulation results are shown in Figures 3 and 4. The actual state responses $x(k)$ and $y(k)$, their estimates $\hat{x}(k)$ and $\hat{y}(k)$, and the estimation error are depicted in Figures 3 and 4. The initial conditions are taken as $\phi(k) = [0.5 \ 0.3 \ 0.6]^T$, $\hat{\phi}(k) = [0.3 \ 0.8 \ 0.2]^T$, $\psi(k) = [0.7 \ 0.3 \ 0.4]^T$, and $\hat{\psi}(k) = [0.2 \ 0.9 \ 0.8]^T$. It is also observed from Figures 3 and 4 that the estimation error between the original GRNs

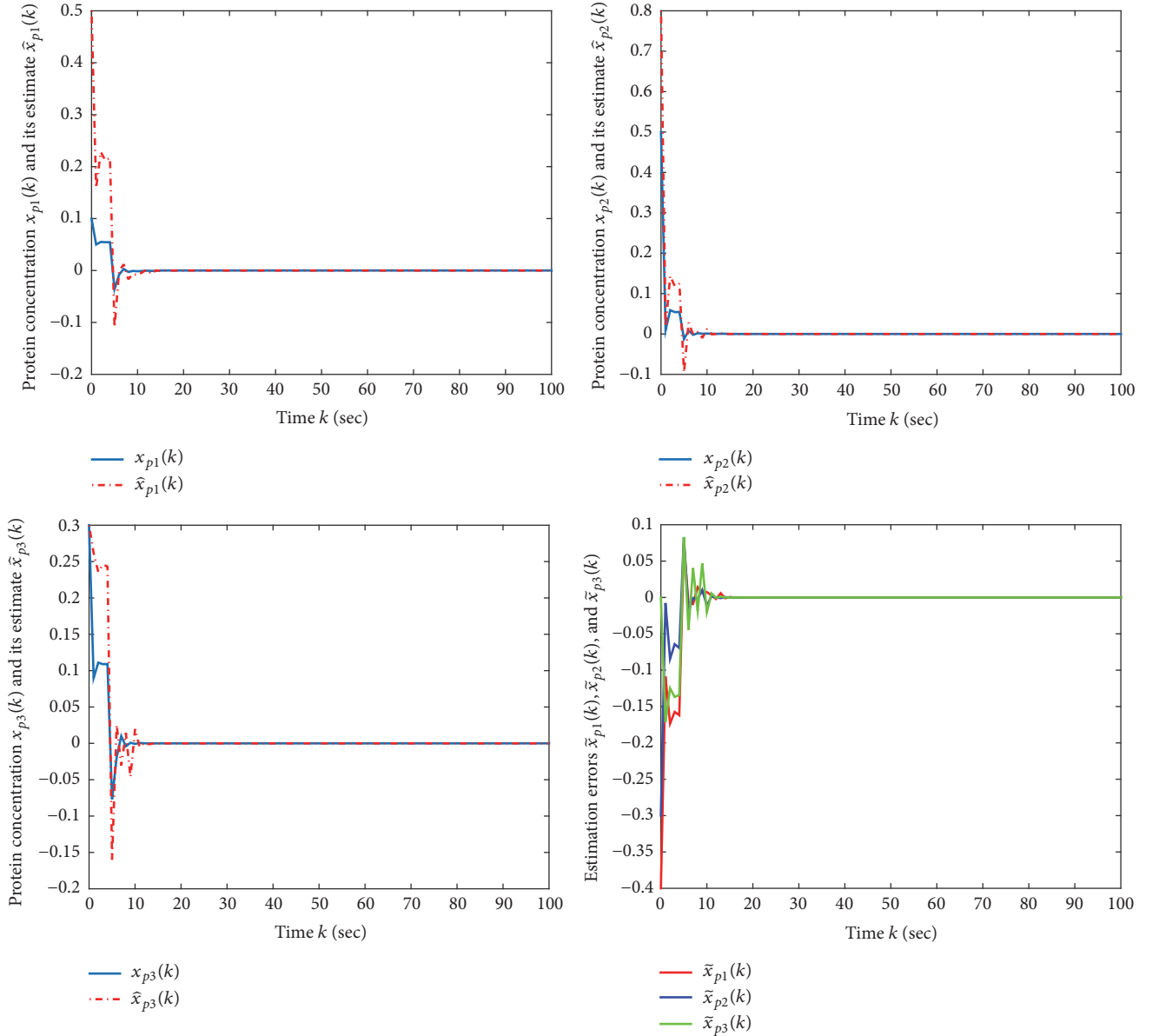


FIGURE 2: Protein concentrations, its estimates, and the estimation error of Example 1.

and the estimator system approaches zero asymptotically. It should be noticed that when we do not consider the nonfragility of estimator gain perturbations, the admissibility of upper bounds of time delays (under different lower bounds of time delays) guaranteeing the globally asymptotical stability of the resulting estimation error system for the addressed GRNs can be obtained and listed in Table 1 by using Theorem 2 in [27] and Theorem 8 in this paper, respectively, which further illustrate the advantages and less conservatism of the newly proposed result.

5. Conclusion

In this paper, we have investigated the nonfragile state estimator design for a class of the discrete-time GRNs

TABLE 1: Admissible upper bound δ_M with different δ_m , when $\tau_m = 2$ and $\tau_M = 5$.

δ_m	2	4	6	8	10
[[27], Theorem 2]	6	8	10	12	14
Theorem 8	7	9	11	13	15

with interval time-varying delays and randomly occurring uncertainties. The norm-bounded uncertainties enter into the GRNs in random ways, and such randomly occurring uncertainties have been characterized by certain mutually uncorrelated Bernoulli-distributed white noise sequences. Under these circumstances, the state estimator has been

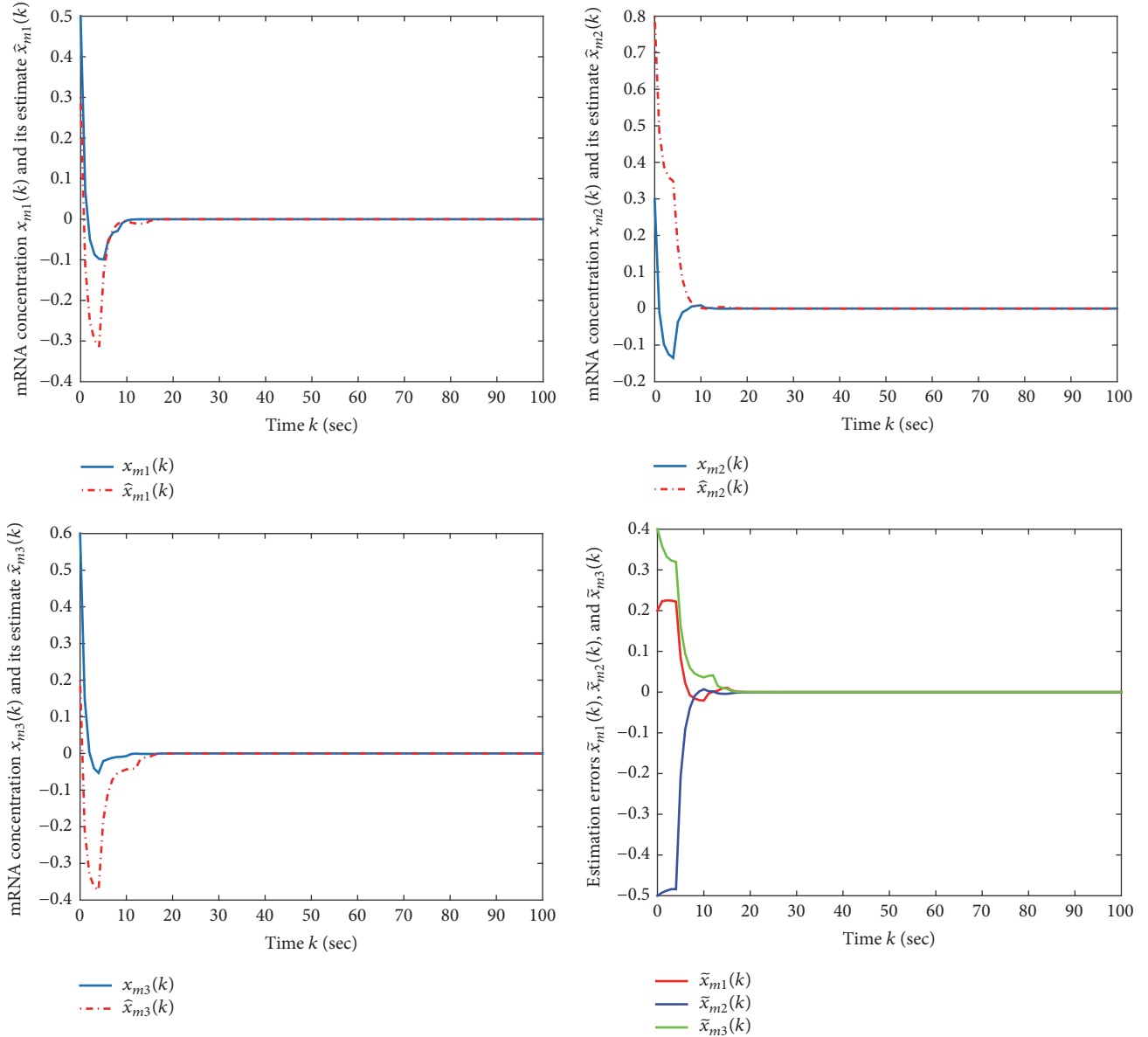


FIGURE 3: mRNA concentrations, its estimates, and the estimation error of Example 2.

designed to estimate the true concentration of the mRNA and the protein from available measurements. By constructing a Lyapunov-Krasovskii functional, new delay-dependent estimation criterion has been obtained in terms of LMIs by using the discrete-time Wirtinger-based inequality and the reciprocally convex approach. A new constraint condition on feedback regulatory function has been used with hope to reduce the conservativeness of the estimation criterion. Then, the desired state estimation algorithm has been provided in view of the solutions to LMIs, which can ensure that the estimation error dynamics are globally asymptotically stable. Finally, numerical simulations have been given to illustrate the applicability and usefulness of the developed

theoretical results. It is worth mentioning that the proposed method in this paper can be extended to address the nonfragile state estimation problems for Markovian jump systems as in [40, 41] with different types of time delays and time-varying systems in [42–44] with unreliable measurements, which constitute interesting topic for future research.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

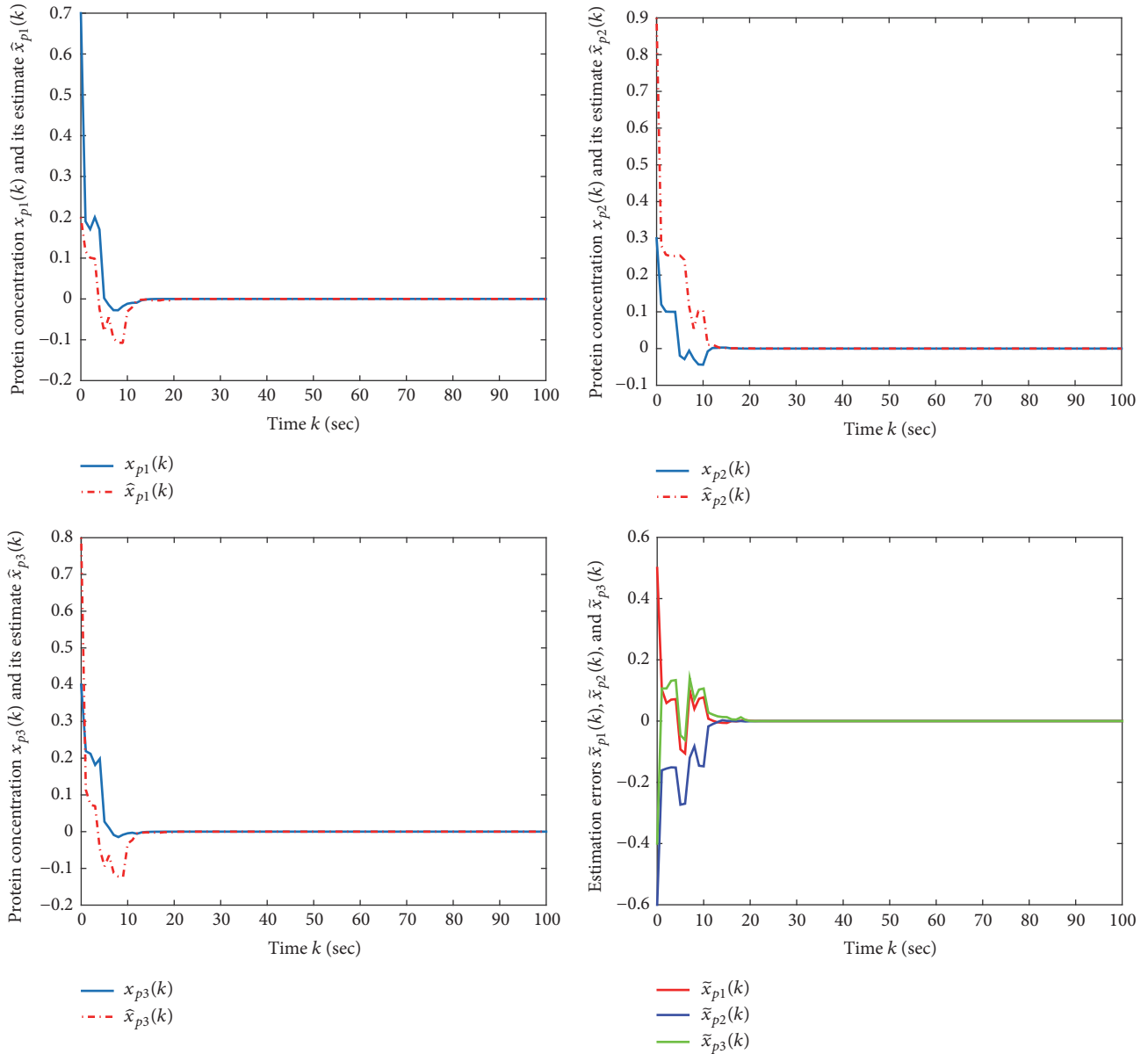


FIGURE 4: Protein concentrations, its estimates, and the estimation error of Example 2.

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