

Research Article

An Integer-Order Memristive System with Two- to Four-Scroll Chaotic Attractors and Its Fractional-Order Version with a Coexisting Chaotic Attractor

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First, based on a linear passive capacitor C , a linear passive inductor L , an active-charge-controlled memristor, and a fourth-degree polynomial function determined by charge, an integer-order memristive system is suggested. The proposed integer-order memristive system can generate two-scroll, three-scroll, and four-scroll chaotic attractors. The complex dynamics behaviors are investigated numerically. The Lyapunov exponent spectrum with respect to linear passive inductor L and the two-scroll, three-scroll, and four-scroll chaotic attractors are yielded by numerical calculation. Second, based on the integer-order memristive system with a four-scroll attractor, a fractional-order version memristive system is suggested. The complex dynamics behaviors of its fractional-order version are studied numerically. The largest Lyapunov exponent spectrum with respect to fractional-order p is yielded. The coexisting two kinds of three-scroll chaotic attractors and the coexisting three-scroll and four-scroll chaotic attractors can be found in its fractional-order version.

1. Introduction

Chaos is an interesting phenomenon in nonlinear systems. High irregularity, unpredictability, and complexity are the typical characteristics of chaotic systems [1, 2]. These typical characteristics have great applications in the following fields: data encryption [3], secure communication [4–7], power grid protection [8, 9], and so on [10–16]. Therefore, more and more attentions have been attracted on the study of chaotic systems in the last few decades [17–20]. In 1971, Chua reported the fourth circuit element named memristor [21], and a solid-state implementation of a memristor has been successfully realized in Hewlett-Packard in 2008 [22]. After then, the applications of a memristor have caught many attentions in nonlinear science [23–28]. Meanwhile, chaotic and hyperchaotic attractors have been found in many memristor-based circuits [21, 23–26]. For example, Muthuswamy and Chua provided a memristor-based circuit with a

single-scroll chaotic attractor [24], Bao et al. reported a memristor-based circuit with a double-scroll chaotic attractor [25], Teng et al. found a memristor-based circuit with double-scroll and four-scroll chaotic attractors [26], and so on [27, 28]. On the other hand, many real physical systems such as electromagnetic wave propagation, dielectric polarization, and heat conduction can be described by fractional-order differential equations [29, 30]. Meanwhile, chaotic phenomenon has been discussed in many fractional-order nonlinear systems such as the fractional-order electronic circuits [31], the fractional-order gyroscopes [32], the fractional-order chaotic brushless DC motor [12], the fractional-order microelectromechanical system [33], and the fractional-order neural networks [34, 35]. So, more attentions have been paid to research the chaotic behaviors of fractional-order nonlinear systems.

Motivated by the above considerations, first, based on a memristor-based chaotic circuit reported by Muthuswamy

and Chua [24], Bao et al. [25], and Teng et al. [26], an integer-order memristive chaotic system with two-scroll, three-scroll, and four-scroll chaotic attractors is provided in this paper. It is noticed that there is only a single-scroll chaotic attractor in [24], only a double-scroll chaotic attractor in [25], and only double-scroll and four-scroll chaotic attractors in [26]. However, there are two-scroll, three-scroll, and four-scroll chaotic attractors in our memristive system. Meanwhile, the Lyapunov exponent spectrum, and phase diagram for our memristive chaotic system are obtained. Second, based on the proposed integer-order memristive chaotic system with a four-scroll chaotic attractor, a fractional-order version chaotic system is suggested. We find that the coexisting three-scroll and four-scroll chaotic attractors and coexisting two kinds of three-scroll chaotic attractors are emerged in the fractional-order version. To the best of our knowledge, this result is rarely reported.

The outline of this paper is organized as follows. In Section 2, the concept of a memristor and some memristor-based system are briefly reviewed. Based on the review, we present an integer-order memristive chaotic system with two-scroll, three-scroll, and four-scroll chaotic attractors and some basic dynamics behaviors are obtained. In Section 3, based on the integer-order memristive chaotic system with a four-scroll chaotic attractor, we present its fractional-order version and we find that there are coexisting chaotic attractors in its fractional-order system. In Section 4, the conclusion is given.

2. An Integer-Order Memristive Chaotic System

The charge-controlled memristor [24, 26] is described by a nonlinear I - V characteristic as $V_M = M(q)I_M$ and $\dot{q} = F(q, I_M)$. Here, V_M , I_M , and q are the voltage, current, and charge associated to the device, respectively. $M(q)$ is the memristance, and $F(q, I_M)$ is the internal state function. In [24, 26], two schematics of the simplest memristor-based chaotic circuit with a linear passive inductor, linear passive capacitor, and a nonlinear active memristor have been reported. The state equations represent the current-voltage relation for the linear passive capacitor, and the inductor is described as

$$\begin{aligned} \frac{CdV_C}{dt} &= I_L, \\ \frac{LdI_L}{dt} &= -(V_C + M(q)I_L), \end{aligned} \quad (1)$$

where V_C denotes the voltage of the linear passive capacitor C and I_L denotes the current of the linear passive inductor L .

In [24], the memristance $M(q)$ is defined as $M(q) = \beta(q^2 - 1)$, and the internal state function $F(q, I_M)$ is defined as $F(q, I_M) = I_M - (\alpha + I_M)q$, where $I_M = -I_L$. The memristor-based circuit in [24] has a single-scroll chaotic attractor (for more details, see [24]), and its dynamics are described by

$$\begin{aligned} \frac{CdV_C}{dt} &= I_L, \\ \frac{LdI_L}{dt} &= -(V_C + \beta(q^2 - 1)I_L), \\ \frac{dq}{dt} &= -I_L - (\alpha - I_L)q. \end{aligned} \quad (2)$$

In [26], the memristance is chosen as $M(q) = \delta q^4 + \gamma q^2 - \beta$ and the internal state function is chosen as $F(q, I_M) = I_M - (\alpha - I_M^2)q$, where $I_M = -I_L$. The memristor-based circuit in [26] has double-scroll and four-scroll chaotic attractors (for more details, see [26]), and its dynamics are shown as

$$\begin{aligned} \frac{CdV_C}{dt} &= I_L, \\ \frac{LdI_L}{dt} &= -(V_C + (\delta q^4 + \gamma q^2 - \beta)I_L), \\ \frac{dq}{dt} &= -I_L - (\alpha - I_L^2)q. \end{aligned} \quad (3)$$

Now, based on [24, 26], an integer-order memristive system is suggested in our paper. The memristance is defined as $M(q) = \delta q^4 - \beta$, and the internal state function is defined as $F(q, I_M) = I_M - (\alpha - I_M^2)q$. So, the integer-order memristive chaotic system in this paper is suggested as

$$\begin{aligned} \frac{CdV_C}{dt} &= I_L, \\ \frac{LdI_L}{dt} &= -(V_C + (\delta q^4 - \beta)I_L), \\ \frac{dq}{dt} &= -I_L - (\alpha - I_L^2)q, \end{aligned} \quad (4)$$

where $C = 1F$, $\delta = 0.5$, $\beta = 2.4$, $\alpha = 0.75$, and $1H \leq L \leq 8H$.

The equilibrium points of system (4) can be calculated by

$$\begin{aligned} I_L &= 0, \\ -(V_C + (\delta q^4 - \beta)I_L) &= 0, \\ -I_L - (\alpha - I_L^2)q &= 0. \end{aligned} \quad (5)$$

Obviously, only $(I_L, V_C, q) = (0, 0, 0)$ is the equilibrium point in system (4). The Jacobian matrix J at this equilibrium point is

$$J = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{1}{L} & \frac{2.4}{L} & 0 \\ 0 & -1 & -0.75 \end{bmatrix}, \quad (6)$$

and its eigenvalues are $\lambda_1 = 0.5(2.4/L) + \sqrt{(2.4/L)^2 - 4/L}$, $\lambda_2 = 0.5(2.4/L) - \sqrt{(2.4/L)^2 - 4/L}$, and $\lambda_3 = -0.75$. If $(2.4/L)^2 - 4/L \geq 0$, then $\lambda_{1,2} > 0$. If $(2.4/L)^2 - 4/L < 0$, then $\text{Re}(\lambda_{1,2}) > 0$. So, the equilibrium point $(I_L, V_C, q) = (0, 0, 0)$ in system (4) is unstable.

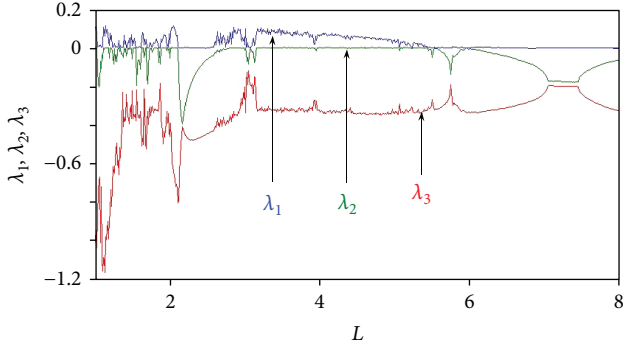


FIGURE 1: The Lyapunov exponent spectrum varies as linear passive inductor L .

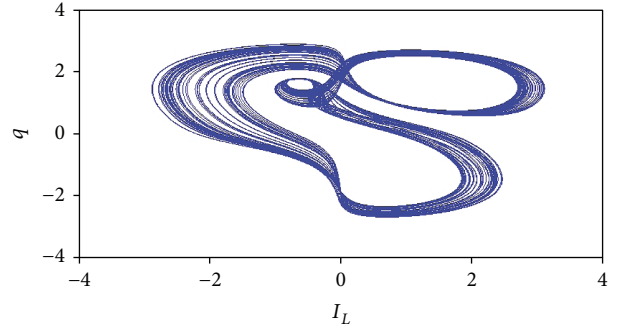


FIGURE 3: Three-scroll chaotic attractor in integer-order memristive system (4).

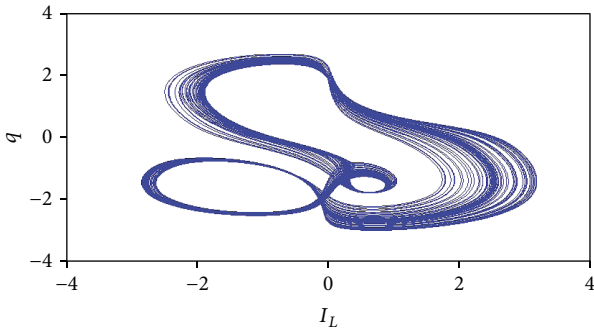


FIGURE 2: Three-scroll chaotic attractor in integer-order memristive system (4).

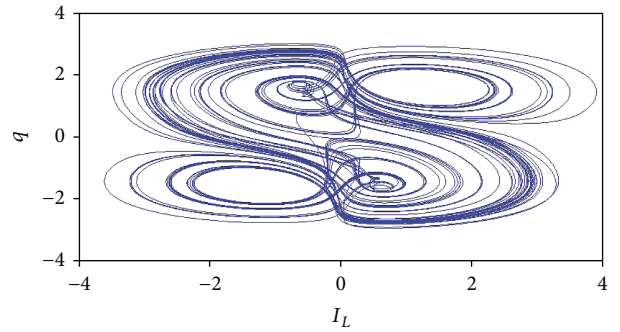


FIGURE 4: Four-scroll chaotic attractor in integer-order memristive system (4).

By numerical calculation, the Lyapunov exponent spectrum of integer-order memristive system (4) with respect to linear passive inductor L can be obtained and is displayed in Figure 1.

According to Figure 1, the maximum Lyapunov exponent λ_1 is positive for the suitable L . The positive Lyapunov exponent λ_1 indicates that the chaotic attractor is emerged in system (4). Next, some results are shown as follows:

2.1. Two Kinds of Three-Scroll Chaotic Attractors Are Emerged in System (4). Letting $L = 1.734$, the Lyapunov exponents are $\lambda_1 = 0.0168$, $\lambda_2 = 0$, and $\lambda_3 = -0.3275$. The Lyapunov dimension is $D_L = 2 + \lambda_1/|\lambda_3| = 2.051$; so, system (4) is fractal. The chaotic attractor is shown in Figure 2. The result in Figure 2 indicates that the three-scroll chaotic attractor is emerged in system (4).

Letting $L = 1.8$, the Lyapunov exponents are $\lambda_1 = 0.0246$, $\lambda_2 = 0$, and $\lambda_3 = -0.3152$. The Lyapunov dimension is $D_L = 2 + \lambda_1/|\lambda_3| = 2.078$; so, system (4) is fractal. The chaotic attractor is shown in Figure 3. The result in Figure 3 indicates that the three-scroll chaotic attractor is emerged in system (4).

According to Figures 2 and 3, we find that two kinds of three-scroll chaotic attractors are emerged in our integer-order memristive chaotic system.

2.2. The Four-Scroll Chaotic Attractor Is Emerged in System (4). Letting $L = 1.4$, the Lyapunov exponents are $\lambda_1 = 0.0663$,

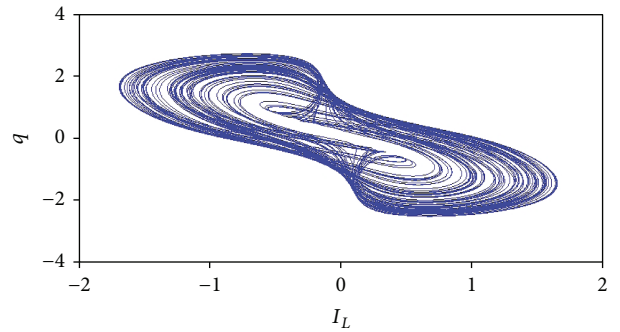


FIGURE 5: Two-scroll chaotic attractor in integer-order memristive system (4).

and $\lambda_3 = -0.3593$. The Lyapunov dimension is $D_L = 2 + \lambda_1/|\lambda_3| = 2.1845$; so, system (4) is fractal. The chaotic attractor is displayed in Figure 4. The result in Figure 4 indicates that the four-scroll chaotic attractor is emerged in system (4).

2.3. The Two-Scroll Chaotic Attractor Is Emerged in System (4). Letting $L = 4$, the Lyapunov exponents are $\lambda_1 = 0.0397$, $\lambda_2 = 0$, and $\lambda_3 = -0.3364$. The Lyapunov dimension is $D_L = 2 + \lambda_1/|\lambda_3| = 2.1180$; so, system (4) is fractal. The chaotic attractor is displayed in Figure 5. The result in Figure 5 indicates that the two-scroll chaotic attractor is emerged in system (4).

According to the above results, the proposed integer-order memristive chaotic system (4) in this paper can generate two- to four-scroll chaotic attractors. This result is different with many previous results [21, 23–28].

3. A Fractional-Order Memristive Chaotic System with Coexisting Chaotic Attractors

In this section, based on integer-order memristive chaotic system (4), a fractional-order version with coexisting chaotic attractors is given.

According to Figure 4 in Section 2, the four-scroll chaotic attractor is emerged in integer-order memristive system (4) with $C = 1F$, $\delta = 0.5$, $\beta = 2.4$, $\alpha = 0.75$, and $L = 1.4$. Now, based on this case, a fractional-order version memristive system is suggested, which is shown as follows:

$$\begin{aligned}\frac{d^p V_C(t)}{dt^p} &= I_L(t), \\ \frac{d^p I_L(t)}{dt^p} &= -\frac{[V_C(t) + (0.5q^4(t) - 2.4)I_L(t)]}{1.4}, \\ \frac{d^p q(t)}{dt^p} &= -I_L(t) - (0.75 - I_L^2(t))q(t).\end{aligned}\quad (7)$$

Here, $0.92 \leq p \leq 1$ is the fractional-order version and $d^p V_C(t)/dt^p = \int_0^t (t-\tau)^{-p} dV_C(\tau)/\Gamma(1-p)$, $d^p I_L(t)/dt^p = \int_0^t$

$(t-\tau)^{-p} dI_L(\tau)/\Gamma(1-p)$, and $d^p q(t)/dt^p = \int_0^t (t-\tau)^{-p} dq(\tau)/\Gamma(1-p)$.

Now, by the improved version of Adams-Bashforth-Moulton numerical algorithm [36], nonlinear fractional-order system (7) with initial condition $(I_L(0), V_C(0), q(0))$ can be discretized as follows:

$$\begin{aligned}V_C(n+1) &= V_C(0) + \frac{\tau^p}{\Gamma(p+2)} \left[I_L^s(n+1) + \sum_{j=0}^n \alpha_{j,n+1} I_L(j) \right], \\ I_L(n+1) &= I_L(0) + \frac{\tau^p}{\Gamma(p+2)} \\ &\quad \cdot \left[\frac{-(V_C^s(n+1) + (0.5(q^s(n+1))^4 - 24)I_L^s(n+1))}{1.4} \right. \\ &\quad \left. + \sum_{j=0}^n \frac{\alpha_{j,n+1} (-(V_C(j) + (0.5 - (q(j))^4 - 24)I_L(j)))}{1.4} \right], \\ q(n+1) &= q(0) + \frac{\tau^p}{\Gamma(p+2)} \\ &\quad \cdot \left[I_L^s(n+1) - (0.75 - (I_L^s(n+1))^2)q^s(n+1) \right. \\ &\quad \left. + \sum_{j=0}^n \alpha_{j,n+1} (-I_L(j) - (0.75 - (I_L(j))^2)q(j)) \right],\end{aligned}\quad (8)$$

where

$$\begin{aligned}V_C^s(n+1) &= V_C(0) + \frac{1}{\Gamma(p)} \sum_{j=0}^n \beta_{j,n+1} I_L(j), \\ I_L^s(n+1) &= I_L(0) + \frac{1}{\Gamma(p)} \sum_{j=0}^n \beta_{j,n+1} \left(\frac{-(V_C(j) + (0.5(q(j))^4 - 2.4)I_L(j))}{1.4} \right), \\ q^s(n+1) &= q(0) + \frac{1}{\Gamma(p)} \sum_{j=0}^n \beta_{j,n+1} (-I_L(j) - (0.75 - (I_L(j))^2)q(j)), \\ \alpha_{j,n+1} &= \begin{cases} n^{p+1} - (n-p)(n+1)^p, & j=0, \\ (n-j+2)^{p+1} + (n-j)^{p+1} - 2(n-j+1)^{p+1}, & 1 \leq j \leq n, \\ 1, & j=n+1, \end{cases} \\ \beta_{j,n+1} &= \frac{\tau^p [(n-j+1)^p - (n-j)^p]}{p}, \quad 0 \leq j \leq n.\end{aligned}\quad (9)$$

The approximation error is as follows:

$$\begin{aligned}|V_C(t_n) - V_C(n)| &= o(\tau^{1+p}), \\ |I_L(t_n) - I_L(n)| &= o(\tau^{1+p}), \\ |q(t_n) - q(n)| &= o(\tau^{1+p}).\end{aligned}\quad (10)$$

In this numerical algorithm, T is the total time length of numerical calculation, N is the iterative calculation

time, and $\tau = T/N$ is the step length. So, $t_n = n\tau$ ($n = 0, 1, 2, \dots, N$).

Next, we study the dynamical behaviors for fractional-order system (7) by the improved version of Adams-Bashforth-Moulton numerical algorithm [36]. First, using numerical calculation, the largest Lyapunov exponents (Largest LE) of fractional-order system (7) with respect to fractional-order p can be obtained, which is shown in Figure 6.

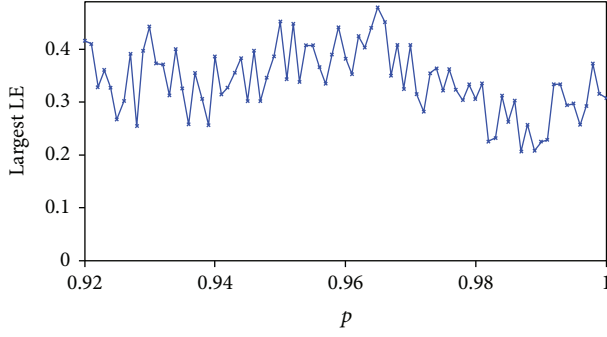


FIGURE 6: The Largest LE varies as fractional-order p .

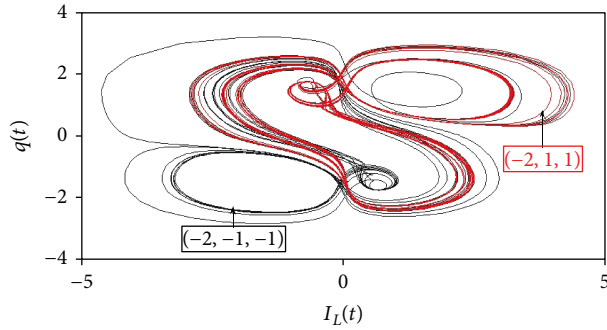


FIGURE 7: The coexisting three-scroll and four-scroll chaotic attractors in system (7).

According to Figure 6, the largest Lyapunov exponent is larger than zero for $0.92 \leq p \leq 1$. The positive largest Lyapunov exponent indicates that the chaotic attractor is emerged in fractional-order system (7). Next, some results are shown as follows:

3.1. Coexisting Three- and Four-Scroll Chaotic Attractors in System (7) for $p = 0.935$. Letting $p = 0.935$, the Largest LE is 0.3251. Therefore, fractional-order system (7) has chaotic behavior. The chaotic attractor can be obtained by numerical calculation. Here, we find that there are coexisting three-scroll and four-scroll chaotic attractors which depend on the initial conditions. For example, let the initial condition be $(-2, -1, -1)$ and $(-2, 1, 1)$. The four-scroll chaotic attractor (black line) and three-scroll chaotic attractor (red line) are shown in Figure 7.

3.2. Coexisting Two Kinds of Three-Scroll Chaotic Attractors in System (7) for $p = 0.94$. Letting $p = 0.94$, the Largest LE is 0.3864. Therefore, fractional-order system (7) has chaotic behavior. The chaotic attractor can be obtained by numerical calculation. Here, we find that there are coexisting two kinds of three-scroll chaotic attractors which depend on the initial conditions. For example, let the initial condition be $(-2, -1, -1)$ and $(-2, 1, 1)$. The two kinds of three-scroll chaotic attractors (black line, red line) are shown in Figure 8.

3.3. Four-Scroll Chaotic Attractor in System (7) for $p=0.99$. Letting $p = 0.99$, the Largest LE is 0.2247. Therefore, fractional-order system (7) has chaotic behavior. By numerical

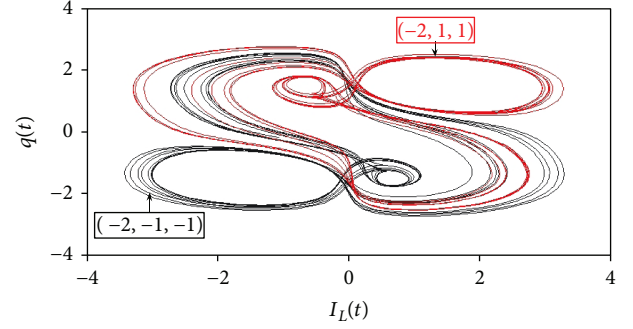


FIGURE 8: The coexisting two kinds of three-scroll chaotic attractors in system (7).

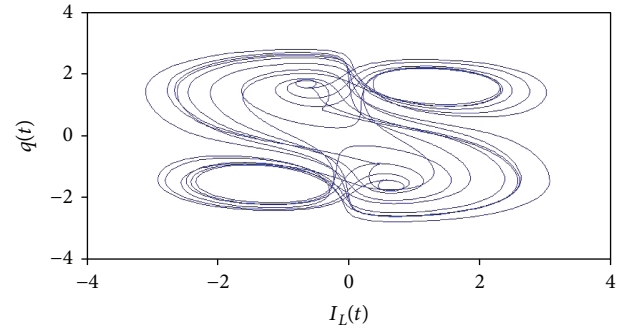


FIGURE 9: Four-scroll chaotic attractor in system (7).

calculation, we find that the four-scroll chaotic attractor is emerged in fractional-order system (7). The four-scroll chaotic attractor is shown in Figure 9.

According to Figure 9, four-scroll chaotic attractor is emerged in fractional-order system (7). This result is just as that of integer-order memristive chaotic system (4) with $L = 1.4$.

According to Figure 8, the coexisting two kinds of three-scroll chaotic attractors are obtained in fractional-order system (7) and the two kinds of three-scroll chaotic attractors do not exist in integer-order memristive chaotic system (4) with $L = 1.4$. So, two kinds of three-scroll chaotic attractor are newly produced.

According to Figure 7, the coexisting three-scroll and four-scroll chaotic attractors are emerged in fractional-order system (7). But, there is only a four-scroll chaotic attractor in integer-order memristive chaotic system (4) with $L = 1.4$. So, the three-scroll chaotic attractor is newly produced.

In summary, for integer-order memristive chaotic system (4) with $L = 1.4$, there is only a four-scroll chaotic attractor. However, for its fractional-order version, it can produce two kinds of new three-scroll chaotic attractors and has coexisting three-scroll and four-scroll chaotic attractors. These results in Section 3 are rarely reported in the previous literature.

4. Conclusions

By a linear passive capacitor C , a linear passive inductor L , and an active-charge-controlled memristor, an integer-order

memristive system is devised in this paper. The memristance $M(q)$ is defined as a fourth-degree polynomial function determined by charge, that is, $M(q) = \delta q^4 - \beta$. By numerical calculation, the Lyapunov exponent spectrum of the proposed memristor-based chaotic circuit with respect to linear passive inductor L is yielded. The proposed integer-order memristive system can generate two-scroll, three-scroll, and four-scroll chaotic attractors for suitable linear passive inductor L .

Furthermore, based on the proposed integer-order memristive system with a four-scroll chaotic attractor for $L = 1.4$, a fractional-order version memristive system is given. By numerical calculation, we obtain the largest Lyapunov exponent with respect to fractional-order p . This fractional-order version memristive system can newly produce two kinds of three-scroll chaotic attractors, and the coexisting three-scroll and four-scroll chaotic attractors are obtained.

Disclosure

The data used in our manuscript is obtained by MATLAB program.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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