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Research Article

Event-Triggered Control for the Stabilization of Probabilistic Boolean Control Networks

Shiyong Zhu,[1] **Jungang Lou**,[2] **Yang Liu**,[1] **Yuanyuan Li**,[3] **and Zhen Wang**[4]

[1]*College of Mathematics, Physics and Information Engineering, Zhejiang Normal University, Jinhua 321004, China*
[2]*School of Information Engineering, Huzhou University, Huzhou 313000, China*
[3]*Department of Applied Mathematics, Nanjing Forestry University, Nanjing 210037, China*
[4]*College of Mathematics and Systems Science, Shandong University of Science and Technology, Qingdao 266590, China*

Correspondence should be addressed to Zhen Wang; wangzhen@sdust.edu.cn

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This paper realizes global stabilization for probabilistic Boolean control networks (PBCNs) with event-triggered state feedback control (ETSFC). Via the semitensor product (STP) of matrices, PBCNs with ETSFC are converted into discrete-time algebraic systems, based on which a necessary and sufficient condition is derived for global stabilization of PBCNs. Furthermore, an algorithm is presented to design a class of feasible event-triggered state feedback controllers for global stabilization. Finally, an illustrative example shows the effectiveness of the obtained result.

1. Introduction

As a central focus of genomic research, the way cellular systems fail in diseases has attracted much attention [1, 2]. Various mathematical and computational models have been constructed to describe the behavior of gene regulatory networks, such as Bayesian networks [3], differential equation [4], and Boolean networks (BNs) [5]. By right of its simple representation, BNs have attracted much attention, where the state of each gene is described by two levels, either inactive (0 or OFF) or active (1 or ON). The evolution of each node is related to the states of some other nodes, including itself sometimes, and determined by a series of logical functions at each discrete-time point. BNs with control inputs are named by Boolean control networks (BCNs), which are essentially switched systems with switching among different BNs [6].

Recently, a new matrix product, named the semitensor product (STP), is presented by Cheng et al. [7]. It converts the logical form of BCNs into the algebraic state space representation (ASSR). Compared with other methods, such as algebra geometry and symbolic dynamics, STP is more convenient and scientific in the study of BCNs. In this area, many interesting researches have been made, such as controllability [8–10], stabilization [11–16], optimal control [17], disturbance decoupling problems [18, 19], and function perturbation [20]. Besides, it has been proved that STP is effective in the study of logical systems [21, 22], game theory [23–25], fault detection [26], nonlinear shift register [27, 28], and so on.

However, the determinacy of a Boolean function limits the application of BCNs, due to the existence of the randomness and measuring noise in the real word. Therefore, it is necessary to extend the BCN to the PBCN, which can be regarded as an undetermined system switching between different constituent BCNs in terms of the probabilistic structure. Similar to BCNs, many extended results have been derived for PBCNs, such as controllability [29–31] and stabilization [31–33].

The stability and stabilization problems are two fundamental issues in the control of PBCNs. Reference [32] presented a necessary and sufficient condition to judge whether a PBCN can achieve global stabilization with probability one in the finite time by state feedback controllers, and the controller design is also realized. As mentioned in [31], a reachability matrix was defined to study the asymptotic stability and stabilization of PBCNs. But it is worth mentioning that the state feedback controllers here charge too much cost and need a relatively long transfer period sometimes. Synchronously, many interesting controllers were favorable for their unique properties [18, 34, 35]; motivated by which, we aim to design the ETSFC to overcome the cost of control and transfer period. The event-triggered control not only has a wide application in BCNs [18, 36] but also has smart grids [37], multiagent systems [38–43], and so on. For example, [18] first presented two kinds of event-trigger controllers to study the disturbance decoupling problems of BCNs, and some necessary and sufficient conditions were also obtained. In [36], authors used a class of event-triggered controls to realize global convergence for finite evolution and minimized the event set triggering the state at a special case. But the stabilization of PBCNs with ETSFC is still open but meaningful in the realistic world.

The main constructions of this paper mainly focus on the following two points.

(i) A series of reachability sets are defined, via which, a necessary and sufficient condition for global stabilization of PBCNs controlled by ETSFC is designed.

(ii) An algorithm is presented to design a class of event-triggered state feedback controllers to realize global stabilization of PBCNs.

The remaining part of this paper is constructed as follows. In Section 2, some notations are given and the STP of matrices is introduced. In Section 3, we devote to investigate the necessary and sufficient condition for stabilization of PBCNs with ETSFC and design controllers to realize global stabilization. Besides, an illustrative example is also given. The conclusion is provided in Section 4.

Notation 1. $\mathscr{D} := \{0, 1\}$ and $\mathscr{D}^n = \underbrace{\mathscr{D} \times \cdots \times \mathscr{D}}_{n}$. Denote the ith column of matrix A by $\mathrm{Col}_i(A)$ and the set of all columns of matrix A by $\mathrm{Col}(A)$. $\Delta_n := \mathrm{Col}(I_n)$, where I_n is an $n \times n$ identity matrix. $\beta = \mathrm{lcm}\,(w, q)$ is the least common multiple of w and q. Denote the set of $m \times n$ logical matrices by $\mathscr{L}_{m\times n}$, where $m \times n$ logical matrix B satisfies $\mathrm{Col}(B) \subseteq \mathrm{Col}(I_m)$. Denote the column vector of length k with all entries equaling 1 and 0 by $\mathbf{1}_k$ and $\mathbf{0}_k$, respectively. $\mathbf{r} = (r_1, \ldots, r_k)^T$ is called a k-dimensional probabilistic vector if $r_i \geq 0$ and $\Sigma_{i=0}^{k} r_i = 1$, and the set of k-dimensional probabilistic vectors is denoted by Υ_k. A $m \times n$ matrix C is called a probabilistic matrix if $\mathrm{Col}_i(C) \subseteq \Upsilon_m$ holds for any $i = 1, 2, \ldots, n$. The set of all $m \times n$ probabilistic matrices is denoted by $\Upsilon_{m\times n}$. Define operator "$\circ$" of two probabilistic column vectors E and F as $E \circ F = (p_1 \wedge q_1, \ldots, p_n \wedge q_n)^T$, where $p_i \wedge q_i = 1$ if and only if $p_i q_i > 0$, else $p_i \wedge q_i = 0$. The Khatri-Rao product of n matrices $\widehat{M}_1 * \cdots * \widehat{M}_n$ is defined as $\mathrm{Col}_j(L) = \ltimes_{i=1}^{n} \mathrm{Col}_j(\widehat{M}_i)$.

2. Preliminaries

2.1. STP of Matrices.

Definition 1 (see [7]). Define the STP of matrix $A \in R_{m\times w}$ and $B \in R_{p\times q}$ by

$$A \ltimes B = \left(A \otimes I_{\beta/w}\right)\left(B \otimes I_{\beta/p}\right), \quad \beta = \mathrm{lcm}\,(w, q), \tag{1}$$

and $\otimes$ is the tensor (or Kronecker) product.

Remark 1. If $n = p$, $A \ltimes B = (A \otimes I_1)(B \otimes I_1) = AB$. Thus, STP is a generalization of the conventional matrix product. If no confusion arises, the symbol "$\ltimes$" can be omitted.

Comparing with the general matrix product, the following pseudocommutative properties of STP are presented.

Proposition 1 (see [7]). *Multiply a column matrix $X \in R_{m\times 1}$ and any matrix* N, *then*

$$X \ltimes N = (I_m \otimes N)X. \tag{2}$$

Proposition 2 (see [7]). *Multiply two column matrices $X \in R_{m\times 1}$ and $Y \in R_{n\times 1}$, then*

$$Y \ltimes X = W_{[m,n]} \ltimes X \ltimes Y, \tag{3}$$

where $W_{[m,n]} = [I_n \otimes \delta_m^1, \ldots, I_n \otimes \delta_m^m]$.

Let $1 \sim \delta_2^1$ and $0 \sim \delta_2^2$, then $\mathscr{D} \sim \Delta_2$. Any logical functions with n variables $f : \mathscr{D}^n \to \mathscr{D}$ can be expressed as the equivalent algebraic form by the following lemma.

Lemma 1. *For $f(x_1, x_2, \ldots, x_n)$: $\mathscr{D}^n \to \mathscr{D}$, there exists a unique matrix $M_f \in \mathscr{L}_{2\times 2^n}$, called the structure matrix of function f, such that*

$$f(x_1, x_2, \ldots, x_n) = M_f \ltimes_{i=1}^{n} x_i, \tag{4}$$

where $x_i \in \Delta_2$ and $\ltimes_{i=1}^{n} x_i = x_1 \ltimes \cdots \ltimes x_n \in \Delta_{2^n}$.

2.2. Algebraic Representation of PBCNs by ETSFC. A probabilistic Boolean model with n nodes is

$$\begin{aligned} x_1(t+1) &= f_1^{k_1}(x_1(t), \ldots, x_n(t)), \\ x_2(t+1) &= f_2^{k_2}(x_1(t), \ldots, x_n(t)), \\ &\vdots \\ x_n(t+1) &= f_n^{k_n}(x_1(t), \ldots, x_n(t)), \\ t &\geq 0, \end{aligned} \tag{5}$$

where $x_i \in \mathscr{D}$ are logic states. At each time step, f_i^j are chosen from a given finite logical function set $\mathscr{F}_i = \{f_i^1, f_i^2, \ldots, f_i^{l_i}\}$ with probability p_i^j, where $\Sigma_{j=1}^{l_i} f_i^j = 1$ holds for any $i = 1, 2, \ldots, n$. Ultimately, there are a total of $\pi = \Pi_{i=1}^{n} l_i$ models. If we use the following matrix T to denote the index set of π models:

$$T=\begin{bmatrix} 1 & 1 & \cdots & 1 & 1 \\ 1 & 1 & \cdots & 1 & 2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & \cdots & 1 & l_n \\ 1 & 1 & \cdots & 2 & 1 \\ 1 & 1 & \cdots & 2 & 2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & \cdots & 2 & l_n \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ l_1 & l_2 & \cdots & l_{n-1} & l_n \end{bmatrix}, \tag{6}$$

then the λth model can be defined by the λth row of matrix T, denoted by $\Sigma_\lambda = \{f_1^{\lambda_1}, f_2^{\lambda_2}, \ldots, f_n^{\lambda_n}\}$, where $\lambda = \Sigma_{i=1}^{n-1}(\lambda_i - 1)\Pi_{j=i}^{n-1} l_{j+1} + \lambda_n$ and $1 \le \lambda_i \le l_i$. Moreover, the probability of the λth model selected is computed to

$$P_\lambda = \prod_{i=1}^{n} p_i^{\lambda_i}. \tag{7}$$

Using Lemma 1, let $x(t) = \ltimes_{i=1}^n x_i(t)$; we convert the logical form (5) into an algebraic form as

$$\begin{aligned} x_1(t+1) &= M_1 x(t),\\ x_2(t+1) &= M_2 x(t),\\ &\vdots\\ x_n(t+1) &= M_n x(t), \end{aligned} \tag{8}$$

where M_i are chosen from $\{M_i^1, \ldots, M_i^{l_i}\}$ with probability $\{p_i^1, \ldots, p_i^{l_i}\}$. M_i^j are the structure matrices of Boolean functions f_i^k, $i = 1, \ldots, n$, $j = 1, \ldots, l_i$.

Denote $\mathrm{E}x(t)$ as the mathematical expectation of state at time t; the evolution of state expectation is

$$\mathrm{E}x_i(t+1) = E(M_i x(t)) = \widehat{M_i} x(t), \tag{9}$$

where $\widehat{M_i} = \Sigma_{j=1}^{l_i} \mathrm{p}_i^j M_i^j$ and $i = 1, 2, \ldots, n$.

Further, we multiply equations in (9), which leads to the following equation:

$$\mathrm{E}x(t+1) = L\mathrm{E}x(t), \tag{10}$$

where $L = \widehat{M}_1 * \cdots * \widehat{M}_n \in \Upsilon_{2^n \times 2^n}$ and * is the Khatri-Rao product.

Similarly, PBCNs with m inputs can be equivalently described as

$$\begin{aligned} x_1(t+1) &= f_1^{k_1}(x_1(t), \ldots, x_n(t), u_1(t), \ldots, u_m(t)),\\ x_2(t+1) &= f_2^{k_2}(x_1(t), \ldots, x_n(t), u_1(t), \ldots, u_m(t)),\\ &\vdots\\ x_n(t+1) &= f_n^{k_n}(x_1(t), \ldots, x_n(t), u_1(t), \ldots, u_m(t)), \end{aligned} \tag{11}$$

where $x_i(t)$ and $u_j(t) \in \mathscr{D}$ are states and inputs, respectively, and functions $f_j^i : \mathscr{D}^{n+m} \to \mathscr{D}^n$ are logical functions. By Lemma 1, let $x(t) = \ltimes_{i=1}^n x_i(t)$ and $u(t) = \ltimes_{i=1}^n u_i(t)$; the dynamic of PBCNs (11) can be converted into the following algebraic discrete-time system:

$$\begin{aligned} x_1(t+1) &= L_1^{k_1} u(t) x(t),\\ x_2(t+1) &= L_2^{k_2} u(t) x(t),\\ &\vdots\\ x_n(t+1) &= L_n^{k_n} u(t) x(t), \end{aligned} \tag{12}$$

where L_i^j are the structure matrix of logical function f_i^j. Taking the expected value on both sides of (12), we have

$$\begin{aligned} \mathrm{E}x_1(t+1) &= L_1 u(t) \mathrm{E}x(t),\\ \mathrm{E}x_2(t+1) &= L_2 u(t) \mathrm{E}x(t),\\ &\vdots\\ \mathrm{E}x_n(t+1) &= L_n u(t) \mathrm{E}x(t), \end{aligned} \tag{13}$$

where $L_i == \Sigma_{j=1}^{l_i} p_i^j L_i^j$, $i = 1, 2, \ldots, n$.

Multiplying (13) together, the algebraic state space representation is as follows:

$$\mathrm{E}x(t+1) = \tilde{L} u(t) \mathrm{E}x(t), \tag{14}$$

where $u(t) \in \Delta_{2^m}$ is the control input and $\tilde{L} = L_1 * \cdots * L_n \in \Upsilon_{2^n \times 2^{n+m}}$.

The principle of ETSFC is that the controller only works if $x(t)$ equals to some designated states, consisting of set Λ, and, in this case, the system switches into (14). Otherwise, the system remains (10) and the controller does not work. The state feedback controller $u(t)$ is

$$\begin{aligned} u_1(t) &= k_1(x_1(t), \ldots, x_n(t)),\\ u_2(t) &= k_2(x_1(t), \ldots, x_n(t)),\\ &\vdots\\ u_m(t) &= k_m(x_1(t), \ldots, x_n(t)),\\ &\quad t \ge 0, \end{aligned} \tag{15}$$

where $k_i : \mathscr{D}^n \to \mathscr{D}$ are logical functions. Assume K_i is the structure matrix of logical function k_i, (15) can be converted into

$$u(t) = Kx(t), \quad t \ge 0, \tag{16}$$

where the state feedback gain matrix $K = K_1 * \cdots * K_m \in \mathscr{L}_{2^m \times 2^n}$.

Define control sign $f_{x(t)} \in \Delta_2$ depending on $x(t)$. If $f_{x(t)} = \delta_2^1$, the system corresponds to dynamic (10), and $f_{x(t)} = \delta_2^2$ makes the system correspond to PBCN (14). Therefore, the dynamic of PBCNs controlled by ETSFC is presented as

$$x(t+1) = \widehat{L} f_{x(t)} x(t), \tag{17}$$

where $\widehat{L} = [L\tilde{L}u(t)]$, $u(t)$ is the ETSFC which only works when $f_{x(t)} = \delta_2^2$, that is,

$$f_{x(t)} = \begin{cases} \delta_2^1, & x(t) \in \Delta_{2^n} \setminus \Lambda, \\ \delta_2^2, & x(t) \in \Lambda, \end{cases} \tag{18}$$

where set Λ consists of the states triggering the state feedback controller (16).

3. Main Results

In this section, the event-triggered stabilization of PBCNs is investigated.

Definition 2. PBCN (14) is said to be globally stabilized to state s^* with probability one, if for any initial state $x_0 \in \Delta_{2^n}$, there is a positive integer U and a control sequence $\mathbf{u}$ such that $P[x(t; x_0, \mathbf{u}) = s^*] = 1$, $\forall t \geq U$.

Then, for a nonempty set $A \subseteq \Delta_{2^n}$, let $R_0(A) = A$, and $R_k(A)$ can be expressed by induction as follows:

$$R_k(A) = \left\{ \delta_{2^n}^j \middle| \mathrm{Col}_j(L) \circ \left(\mathbf{1}_{2^n} - \sum_{a \in \bigcup_{i=0}^{k-1} R_i(A)} a \right) = \mathbf{0}_{2^n} \right\}. \tag{19}$$

Furthermore, define $R(A)$ as

$$R(A) = \bigcup_{i=1}^{2^n} R_i(A). \tag{20}$$

Remark 2. $R_k(A)$ can also be represented as follows:

$$R_k(A) = \left\{ \delta_{2^n}^j \middle| P\left[x(t+1) \in \bigcup_{i=0}^{k-1} R_i(A), x(t) = \delta_{2^n}^j \right] \right\} = 1. \tag{21}$$

Assuming $s^* = \delta_{2^n}^r$ and the basins of s^* by $R(\{\delta_{2^n}^r\})$, for $i \geq 1$, a series of sets are defined as follows:

$$\begin{aligned} S_i' &= \left\{ \delta_{2^n}^j \in \frac{\Delta_{2^n}}{\Omega_{i-1}} \middle| \text{there exists a } \delta_{2^m}^{v_j} \text{ such that } \mathrm{Col}_j\left(Blk_{v_j}\left(\tilde{L}\right)\right) \right. \\ &\qquad \left. \circ \left(\mathbf{1}_{2^n} - \sum_{a \in \Omega_{i-1}} a \right) = \mathbf{0}_{2^n} \right\}, \\ S_i &= \frac{R\left(S_i' \cup \Omega_{i-1}\right)}{S_i'}, \end{aligned} \tag{22}$$

where $S_0 = \{\delta_{2^n}^r\}$, $\tilde{S}_0 = R(\{\delta_{2^n}^r\})$, $\Omega_0 = S_0 \cup S_0'$, $\tilde{S}_i = S_i' \cup S_i$, and $\Omega_i = \bigcup_{k=0}^{i} \tilde{S}_k$.

For the above sets, we have some useful properties below.

Proposition 3 (see [32]). *One has*

(i) $\Omega_1 = \{\delta_{2^n}^r\}$, *then* $\Omega_t = \{\delta_{2^n}^r\}$, $\forall t \geq 1$,

(ii) if $\Omega_{t-1} = \Omega_t$, *for some* $t \geq 1$, *then* $\Omega_j = \Omega_t$, $\forall j \geq t$.

Theorem 1. *System (17) is globally stabilized to a designated state* $s^* = \delta_{2^n}^r$ *with probability one, if and only if, the following conditions are satisfied:*

(i) $\mathrm{Col}_r(L) = \delta_{2^n}^r$ *or there exists an integer* $1 \leq v_r \leq 2^m$ *satisfying* $\mathrm{Col}_r(Blk_{v_r}(\tilde{L})) = \delta_{2^n}^r$.

(ii) There exists an integer $0 \leq N \leq 2^n - 1$ *such that* $\Omega_N = \Delta_{2^n}$.

Proof 1. Sufficiency: condition (i) shows that $P[x(t+1) = \delta_{2^n}^r \mid x(t) = \delta_{2^n}^r] = 1$ holds by the designed controller. According to condition (ii) and the constructing procedure of S_i' and S_i, for any initial state $x_0 = \delta_{2^n}^j \in \Delta_{2^n} \setminus \{\delta_{2^n}^r\}$, it claims that $x_0 \in \tilde{S}_0$ or $x_0 \in \bigcup_{i=1}^N S_i'$ or $x_0 \in \bigcup_{i=1}^N S_i$. Therefore, there are three cases.

Case 1: If $x_0 = \delta_{2^n}^j \in \tilde{S}_0$, there exists a positive integer k_j satisfying $\delta_{2^n}^j \in R_{k_j}(\{\delta_{2^n}^r\})$, which implies that $P[x(k_j) = \delta_{2^n}^r \mid x(0) = x_0] = 1$.

Case 2: If $x_0 = \delta_{2^n}^j \in S_{i_j}'$, we can find a control $u = \delta_{2^m}^{v_j}$ such that $\tilde{L}ux_0 = L \ltimes \delta_{2^m}^{v_j} \ltimes \delta_{2^n}^j = \mathrm{Col}_j(Blk_{v_j}(\tilde{L})) \in \Omega_{i-1}$.

Case 3: If $x_0 = \delta_{2^n}^j \in S_{i_j}$, there exists T_j' such that $P[x(T_j') \in S_{i_j}' \cup \Omega_{i-1} \mid x(0) = x_0] = 1$. The case in S_{i_j}' has been discussed before. Alternatively, the state in Ω_i will enter Ω_{i-1}.

In a word, every state $x_0 \in \Delta_{2^n}$ will arrive at $\delta_{2^n}^j$ with probability one.

Necessity: system (17) is globally stabilizable to $s^* = \delta_{2^n}^r$ with probability one; $\delta_{2^n}^r$ should be fixed.

Assume that condition (ii) does not hold; any state in $\Delta_{2^n} \setminus \Omega_N$ cannot be steered into $\delta_{2^n}^r$ via the intermittent control, which is in contradiction with Definition 2.

In the following, we aim to prove that the smallest integer N^* satisfying condition (ii) is no more than $2^n - 1$. It is enough to show that

$$|\Omega_\tau| \geq \tau + 1, \tag{23}$$

where $0 \leq \tau \leq N$. We will show it by induction. When $\tau = 1$, if $|\tilde{S}_0 \cup \tilde{S}_1| < 2$, that is, $|\tilde{S}_0 \cup \tilde{S}_1| = 1$, thus $\tilde{S}_0 \cup \tilde{S}_1 = \{\delta_{2^n}^r\}$. By Proposition 3, it holds that $\Omega_N = \{\delta_{2^n}^r\}$, which is a contradiction with Definition 2.

Suppose that $|\Omega_{\tau-1}| \geq \tau$, for some $0 \leq \tau \leq N$. It can be immediately obtained that

$$|\Omega_\tau| \geq |\Omega_{\tau-1}| \geq \tau. \tag{24}$$

If $|\Omega_\tau| \leq \tau + 1$, then

$$|\Omega_\tau| = |\Omega_{\tau-1}| = \tau, \tag{25}$$

which means that $\Omega_\tau = \Omega_{\tau-1}$. Proposition 3 implies that $\Omega_{\tau-1} = \Omega_N = \Delta_{2^n}$, which is a contradiction to the minimality of integer N.

Moreover, it also holds that $|\Omega_N| = |\Delta_{2^n}| = 2^n \geq N + 1$, that is to say, $N \leq 2^n - 1$.

Assume that conditions (i) and (ii) in Theorem 1 hold, then the event-triggered state feedback controllers can be designed by the following algorithm.

Proof 2. We only need to prove that system (17) is globally stabilized to $\delta_{2^n}^r$ under the controller designed above. For the desired state $\delta_{2^n}^r$, if $\mathrm{Col}_r(L) = \delta_{2^n}^r$, along the trajectory of (10), $x(t\,; x_0) = \delta_{2^n}^r$, for any $t \geq 0$, only if $x_0 = \delta_{2^n}^r$, that is, $\delta_{2^n}^r \in \Lambda$; else, find a $\delta_{2^n}^{v_r}$ such that $\mathrm{Col}_r(Blk_{v_r}(\tilde{L})) = \delta_{2^n}^r$. Thus, system (17) can stay at the desired state $\delta_{2^n}^r$ with probability one.

For any initial state $\delta_{2^n}^j \in \Delta_{2^n} \setminus \{\delta_{2^n}^r\}$, we split it into three cases: $x_0 \in \tilde{S}_0$ or $x_0 \in \bigcup_{i=1}^N S_i'$ or $x_0 \in \bigcup_{i=1}^N S_i$.

Case 1: For $x_0 \in \tilde{S}_0$, according to the proof of Theorem 1, it will reach $\delta_{2^n}^r$.

Case 2: For $x_0 = \delta_{2^n}^j \in \bigcup_{i=1}^N S_i'$, there exists a z_j such that $\delta_{2^n}^j \in S_{z_j}'$. Because $\delta_{2^n}^j \in \Lambda$, it will trigger the controller (16), under which, $\mathrm{Col}_j(Blk_{v_j}(\tilde{L})) \in \Omega_{z_j - 1}$.

Case 3: For $x_0 = \delta_{2^n}^j \in \bigcup_{i=1}^N S_i$, there exists an integer i_j such that for $\delta_{2^n}^{i_j}$, we claim that it will enter into the set $S_{i_j}' \cup \Omega_{i_j - 1}$ and the state in S_{i_j} will enter into $\Omega_{i_j - 1}$, which has been discussed in Case 2.

By mathematical induction, system (17) will stabilize to $\delta_{2^n}^r$.

Remark 3. If $\mathrm{Col}_r(L) = \delta_{2^n}^r$, the controller (16) will be triggered forever and we can only cut down the cost on transient routes.

Remark 4. When $l_1 = l_2 = \cdots = l_n = 1$, PBCN (14) deduces a conventional BCN. The obtained results also can be applied to the event-triggered stabilization of BCNs. Thus, our result can be regarded as a generalization of that in [36] to some extent.

Example 1. Consider the following probabilistic Boolean network:

$$\begin{aligned} X(t+1) &= f_1(X(t), Y(t), Z(t)), \\ Y(t+1) &= f_2(X(t), Y(t), Z(t)), \\ Z(t+1) &= f_3(X(t), Y(t), Z(t)), \end{aligned} \tag{26}$$

where $f_1 \in \{f_1^1, f_1^2\}$ with probabilistic 1/2, 1/2, respectively, and $f_2 \in \{f_1^2, f_2^2\}$ with probabilistic 2/3, 1/3, respectively.

$$\begin{aligned} & f_1^1(X, Y, Z) = (X \wedge Z) \vee (Y \wedge \neg Z) \vee (\neg Y \wedge Z), \\ & f_1^2(X, Y, Z) = (X \wedge Y) \vee (\neg X \wedge \neg Y \wedge Z), \\ & f_2^1(X, Y, Z) = (X \wedge Y \wedge Z) \vee (X \wedge \neg Y \wedge \neg Z) \vee (\neg X \wedge Y \wedge \neg Z), \\ & f_2^2(X, Y, Z) = X \wedge \neg Y \wedge \neg Z, \\ & f_3(X, Y, Z) = 1. \end{aligned} \tag{27}$$

Then we can obtain

$$L = \begin{pmatrix} \frac{2}{3} & 0 & 0 & 0 & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & \frac{1}{6} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 1 & \frac{1}{6} & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{28}$$

Assume the control system is given as

$$\begin{aligned} X(t+1) &= u_1(t) \wedge f_1(X(t), Y(t), Z(t)), \\ Y(t+1) &= u_2(t) \wedge f_2(X(t), Y(t), Z(t)), \\ Z(t+1) &= f_3(X(t), Y(t), Z(t)), \end{aligned} \tag{29}$$

Similarly, we obtain $\tilde{L}$ as follows:

$$\tilde{L} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & \frac{2}{3} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & 1 & \frac{1}{3} & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{30}$$

In a word, the dynamics of this system can be described as

$$x(t+1) = \left[L\tilde{L}u(t)\right] f_{x(t)} x(t), \tag{31}$$

Next, we will design the ETSFC to stabilize system (31) to δ_8^7. By Algorithm 1, Ω_i, $i = 0, 1, \ldots$, can be calculated as follows.

Step 1: Initialize $\Lambda = \varnothing$, if $\mathrm{Col}_r(L) = \delta_{2^n}^r$, then $\Lambda = \bigcup_{i=1}^N S_i'$; otherwise, $\mathrm{Col}_r(K) = \delta_{2^m}^{v_r}$ satisfying $\mathrm{Col}_r(Blk_{v_r}(\tilde{L})) = \delta_{2^n}^r$ and $\Lambda = \{\delta_{2^n}^r\} \cup (\bigcup_{i=1}^N S_i')$. Then go to Step 2.

Step 2: For every state $\delta_{2^n}^j$ in $\Delta_{2^n} \setminus \{\delta_{2^n}^r\}$, find a unique z_j such that $\delta_{2^n}^j \in S_{z_j}'$. Then, let $\mathrm{Col}_j(K) = \delta_{2^m}^{v_j}$ satisfying $\mathrm{Col}_j(Blk_{v_j}(\tilde{L})) \circ (\mathbf{1}_{2^n} - \sum_{a \in \Omega_{z_j-1}} a) = \mathbf{0}_{2^n}$. Then go to Step 3.

Step 3: For remainder $\delta_{2^n}^j \in \Delta_{2^n} \setminus \Lambda$, let $\mathrm{Col}_j(K) = \delta_{2^n}^{v_j}$ be a random vector in Δ_{2^m}. Return set Λ and matrix K, where Λ is the event-set-triggering control (15) and matrix K is the state feedback gain matrix.

Algorithm 1

Step 1: Since $\mathrm{Col}_7(L) \neq \delta_8^7$, then $\delta_8^7 \in \Lambda$ and there exists $u = \delta_2^1$ such that $\mathrm{Col}_7(Blk_1(\tilde{L})) = \delta_8^7$.

Step 2: By the dynamic $x(t+1) = Lx(t)$, we can calculate $R_1(\{\delta_8^7\}) = \{\delta_8^5, \delta_8^8\}$, $R_2(\{\delta_8^7\}) = \{\delta_8^4\}$, and $R_k(\{\delta_8^7\}) = \{\delta_8^4, \delta_8^5, \delta_8^8\}, k \geq 3$, so $R(\{\delta_8^7\}) = \{\delta_8^4, \delta_8^5, \delta_8^7, \delta_8^8\}$.

Step 3: Let $\Omega_0 = \tilde{S}_0 = R(\{\delta_8^7\})$, then $\Delta_{2^n} \setminus \Omega_0 = \{\delta_8^1, \delta_8^2, \delta_8^3, \delta_8^6\}$. Next, we consider dynamic $x(t+1) = \tilde{L}u(t)x(t)$. For δ_8^1, there exists $u = \delta_2^1$ such that $\mathrm{Col}_1(Blk_1(\tilde{L})) \in \Omega_0$. For δ_8^2, there exists $u = \delta_2^1$ such that $\mathrm{Col}_2(Blk_1(\tilde{L})) \in \Omega_0$. For δ_8^3, there exists $u = \delta_2^1$ such that $\mathrm{Col}_3(Blk_1(\tilde{L})) \in \Omega_0$. For δ_8^6, there exists $u = \delta_2^2$ such that $\mathrm{Col}_6(Blk_2(\tilde{L})) \in \Omega_0$.

So $S_1' = \{\delta_8^1, \delta_8^2, \delta_8^3, \delta_8^6\}$ and $\Omega_1 = \Delta_8$. Thus, the PBCN (31) is globally stabilized to δ_8^7 via ETSFC by Theorem 1. In terms of Algorithm 1, set Λ can be expressed by $\Lambda = \{\delta_8^1, \delta_8^2, \delta_8^3, \delta_8^6, \delta_8^7\}$ and one of the feasible state feedback matrices K can be $\delta_2[1, 1, 1, 2, 2, 2, 1, 1]$.

4. Conclusion

In this technique, the global stabilization of PBCNs has been investigated by ETSFC. Under the framework of STP, the algebraic representation of PBCNs has been obtained from the logical form. A necessary and sufficient condition has been derived based on reachability sets. Finally, we have designed a class of feasible event-triggered state feedback controls to realize global stabilization. A numerical example has shown the effectiveness of our results.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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